The Employment Cost of Sovereign Default*

Job Market Paper
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Abstract

This paper analyzes the interaction between government default decisions and labor market outcomes in an environment with persistent unemployment and financial frictions. Sovereign risk impairs bank intermediation through balance sheet effects, worsening the conditions for firms to pre-finance wages and vacancies. This generates a new type of endogenous domestic default cost – the employment cost of default. The persistence of unemployment produces serial defaults and rationalizes high debt-to-GDP ratios. In the dynamic strategic game between the government and the private sector, anticipation effects allow the study of both debt crises and outright default episodes. Introducing employment subsidies and bank regulations affect the government’s ability to commit to debt repayment.

Keywords: Sovereign Default, Search and Matching, Financial Frictions
JEL: E24, E44, E62, F34, F41

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1 Introduction

The recent European debt crisis was characterized by high government indebtedness and rising bond spreads in Portugal and Spain and a sovereign default in Greece. In all three countries, the crisis was accompanied by high levels of unemployment. Firm-level evidence documents that sovereign risk depresses job vacancies via bank lending, suggesting a link between a government’s debt policies and labor market outcomes.¹

Such employment effects make debt and default crises costly for an economy.² This cost channel can address the difficulty faced by the literature in rationalizing plausible debt levels and default frequencies in sovereign debt models with limited commitment. These models usually rely on exogenous costs that cannot affect both measures separately: a more severe default punishment increases debt ratios but decreases default frequencies. The persistence of unemployment can disentangle the two and offer a theory of endogenous cost to evaluate debt policies during and in anticipation of default. Yet frictional unemployment as a source of default cost has not received attention in the sovereign default literature.

In this paper, I introduce labor market frictions à la Diamond, Mortensen and Pissarides and frictional financial markets into a default model in the tradition of Eaton and Gersovitz (1981). In the model, firms hire workers subject to matching frictions and borrow from intermediaries to pre-finance wages and vacancies. Government debt acts as liquidity in banks’ balance sheets and facilitates lending to firms. The government understands that its tax, debt and default decision affects labor market outcomes during and in anticipation of default due to the forward-looking behavior of the private sector. The model therefore features a dynamic strategic game between the government and private agents.

The main theoretical contribution is that the model endogenously creates an employment cost of sovereign default. When sovereign bond prices fall due to rising default risk the bankers’ supply of loans to firms falls. Firms post fewer vacancies and fire workers, increasing unemployment. Fixed output sharing prevents the wage from fully adjusting to changes in credit conditions which amplifies the increase in unemployment. Workers face the uninsurable idiosyncratic income risk of becoming unemployed. The rise in unemployment creates an endogenous cost of default and, importantly, cost of default risk. Higher unemployment reduces production and contributes to the recession. This is consistent with evidence showing that a declining labor input significantly contributed to GDP drops during the European debt crisis (Wright (2014)).

The combination of endogenous default costs and anticipation effects allows me to conduct three distinct exercises. First, I use a calibrated version of the model to look at actual default

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¹Domestic banks with a higher exposure to sovereign risk on their balance sheets cut loans to the private sector more aggressively during the crisis and firm-level evidence suggests that this loan contraction depressed job creation, see Acharya, Eisert, Eufinger, and Hirsch (2015) and Bentolila, Jansen, Jiménez, and Ruano (2015).

episodes and contrast them with debt crises – periods of elevated bond spreads that do not lead to default. Second, I address the counterfactually low debt-to-GDP ratios generated by standard models of sovereign default where the cost of default is exogenous. Third, I consider how different labor market policies and bank regulations change the government’s ability to credibly borrow and repay debt.

The forward-looking vacancy-posting behavior of firms is crucial for the model to be able to match the empirically observed decline in employment that accompanies increases in bond spreads before an actual default event (Yeyati and Panizza (2011)). Firms cut vacancy postings when they anticipate default not only because job vacancies become more expensive but also because the expected job separation rate rises. With a static labor market the latter effect would be absent.

The persistence of unemployment is central to the model’s quantitative ability to explain two empirical regularities standard default models have difficulty in replicating. First, persistent unemployment helps the government to sustain high debt-to-GDP ratios because it creates an additional disincentive to default. Defaulting entails prolonged costs because new matches take time to form. Lowering the labor market frictions decreases equilibrium indebtedness to the point where the government is not able to borrow at all. Second, the combination of a higher default probability in high unemployment states and persistently high unemployment after default is responsible for the model’s ability to generate clustered default events – situations where one default follows shortly after another.

Finally, I consider counterfactual labor market policies in the form of wage and employment subsidies that alleviate the employment cost of default if they circumvent the financing constraints of the firms. Furthermore, financial market regulations in the form of higher capital requirements for bankers limit layoffs and the rise in unemployment during default. Since the liquidity function of debt is reduced by tighter bank regulations, which enforce a lower debt exposure of domestic banks, the government’s ability to commit to debt repayment and thus indebtedness increases.

Relation to literature. This paper is related to the literature on sovereign debt as well as the vast literature on financial frictions, labor frictions and the interaction between the two. Following the original default framework of Eaton and Gersovitz (1981), recent papers study the quantitative dynamics of sovereign defaults. Arellano (2008) and Aguiar and Gopinath (2006) analyze sovereign default and business cycle properties in emerging economies. Several studies have extended the framework finding that the presence of reputational costs in the form of exclusion from financial markets cannot explain the high debt levels seen in the data. Extensions focus on e.g. longer debt maturities (Chatterjee and Eyigungor (2012), Hatchondo and Martinez (2009), Arellano and Ramanarayanan (2012)), debt renegotiations (Yue (2010), Benjamin and Wright (2013) and D’Erasmo (2008)), political uncertainty (Cuadra and Sapriza (2008)), inflation and devaluations (Du and Schreger (2015), Na, Schmitt-Grohe, Uribe, and Yue (2015). The handbook chapter by Aguiar, Chatterjee, Cole, and Stangebye (2016) provides an overview.

Bulow and Rogoff (1989) show that without direct sanctions, no debt can be sustained in equilibrium by reputational mechanisms alone if countries are allowed to save after default. This result motivated subsequent
In particular, they conclude that the presence of domestic costs of default is necessary to reconcile high indebtedness with low default frequencies. I propose a new type of domestic default cost by studying the effects of sovereign default on the labor market through the banking system.

Mendoza and Yue (2012) and Perez (2015) also endogenize default costs in an optimal sovereign default framework. Mendoza and Yue (2012) assume that default impairs the import of intermediate goods, which have to be substituted by domestic goods. Since domestic goods are imperfect substitutes, productivity declines. In contrast, I do not impose constraints that are absent in repayment. This is important for generating expectations effects on unemployment before default. As the role of imports does not feature in my model, I regard the proposed employment cost of default as complementary to their trade exclusion costs. Perez (2015) assumes that default hurts the balance sheets of heterogeneous bank-firm entities which results in a misallocation of investment, with production taking place in less productive firms. My work incorporates a similar negative effect that arises from less liquidity in default which hurts banks’ balance sheets. However, it includes a labor margin that contributes to output declines.

The balance sheet mechanism of default has been explored theoretically by Gennaioli, Martin, and Rossi (2014), Basu (2009), Mengus (2014) and Sosa-Padilla (2012). Bocola (2016) contrasts it with a risk effect when bankers perceive the private sector as riskier. My work differs significantly from his work because I endogenize the government’s default decision. In recent work, Niemann and Pichler (2016) include the role of debt as collateral in a quantitative default setting.

The modeling of the banking sector in this paper builds on the literature on financial intermediation dating back to Bernanke and Gertler (1989), Kiyotaki and Moore (1997), and Bernanke, Gertler, and Gilchrist (1999) and is closest to Gertler and Karadi (2011). More recent work includes Mendoza (2010), Gertler and Kiyotaki (2010), and Gilchrist and Zakrjašek (2012). The presence of this friction coupled with banks’ exposure to public debt gives rise to a liquidity function of government debt. When debt prices fall the government’s ability to provide collateral or liquidity to the banking sector is harmed.

The matching frictions in the labor market follow Pissarides (1985) and a large subsequent literature, see e.g. Rogerson, Shimer, and Wright (2005) for a survey. To solve the Shimer (2005) puzzle – that the standard framework is unable to generate enough volatility in labor market tightness relative to productivity – wage rigidity and the interaction of financial and labor market frictions have gained attention. Kehoe, Midrigan, and Pastorino (2015) and Garín (2015) offer explanations for more wage rigidity relative to the standard DMP model, despite determining the wage by period-by-period Nash bargaining. Fixed output sharing captures those amplification mechanisms, while leaving them unmodeled, in this work.

2 Stylized facts about debt and default crises

This section summarizes the empirical regularities during debt and default crises that my model seeks to replicate. I provide evidence that default risk is associated with higher unemployment and worse credit conditions.

Labor markets. Sovereign bond spreads negatively co-move with labor market tightness and are positively correlated with unemployment. Figure 1 plots the sovereign bond spreads of Portugal, Spain, Greece and Iceland over German 10 year bonds together with labor market tightness (the ratio of job vacancies to the unemployment rate) during the recent crisis. Figure 2 shows the time series for the quarterly, non-seasonally adjusted unemployment rate alongside sovereign bond spreads for the same set of countries. These countries were confronted with serious fiscal problems that led to sharp increases in their bond spreads. Iceland was categorized as “near default” between 2007 and 2010 by Reinhart (2010) and Greece underwent a haircut on government bonds in 2012. Sovereign spreads are negatively correlated with labor market tightness and positively correlated with the unemployment rate, as documented in Table 1. The correlations are statistically significant.

Labor is an important determinant of the decline in output that occurs during debt and default crises.\textsuperscript{6} Wright (2014) points out that the two main determinants of lower GDP

\textsuperscript{6}Empirical work on the output loss of default includes for example Borensztein and Panizza (2009),
growth during the recent debt crisis in Europe were labor on the one hand and productivity on the other. I provide a similar decomposition of GDP growth, based on the Conference Board Total Economy Database, in Table 4 in the Appendix.

Unemployment is high during debt crises, independently of the realization of a default ex post. Whether one looks at an actual default event in Greece, a near-default episode in Iceland or the debt crises in Portugal and Spain, there are strong correlations between spreads and labor market variables. Yeyati and Panizza (2011) look at a wider range of historical default events and conclude that defaults usually occur after quarters with increasing or stable unemployment.

Credit markets. As pointed out by Bocola (2016) and Perez (2015), domestic banks’ balance sheets include a large amount of sovereign debt and deteriorate in default, causing the financial sector distress. Borensztein and Panizza (2009) provide evidence that the probability of having a banking crisis conditional on default is 14 percent, an 11 percentage point increase compared to its unconditional probability.

Acharya, Eisert, Eufinger, and Hirsch (2015) document that European banks with higher sovereign risk exposure on their balance sheets cut loans by more than banks with lower sovereign risk exposure, which led to a reduction in the lending volume by 66 and 45 percent

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Table 1: Sovereign spreads: Correlation with labor market tightness and unemployment rate.

<table>
<thead>
<tr>
<th>Country</th>
<th>$\rho(spr, \theta)$</th>
<th>p-value</th>
<th>$\rho(spr, u)$</th>
<th>p-value</th>
<th>Time period</th>
</tr>
</thead>
<tbody>
<tr>
<td>Portugal</td>
<td>−0.5957</td>
<td>0.0000</td>
<td>0.7880</td>
<td>0.0000</td>
<td>1998Q1-2014Q3</td>
</tr>
<tr>
<td>Spain</td>
<td>−0.7325</td>
<td>0.0000</td>
<td>0.8151</td>
<td>0.0000</td>
<td>2005Q2-2015Q2</td>
</tr>
<tr>
<td>Greece</td>
<td>−0.5946</td>
<td>0.0014</td>
<td>0.7360</td>
<td>0.0000</td>
<td>2009Q1-2015Q2</td>
</tr>
<tr>
<td>Iceland</td>
<td>−0.3342</td>
<td>0.0091</td>
<td>0.3383</td>
<td>0.0082</td>
<td>2000Q1-2014Q3</td>
</tr>
</tbody>
</table>

in Spain and Portugal, respectively, over the period from 2008 to 2013. They also present firm-level evidence that this loan contraction depressed job creation.

Finally, when sovereign spreads rise private borrowing becomes more expensive and falls in quantity (Gilchrist and Mojon (2014) and Adelino and Ferreira (2016)). A number of papers have further documented the co-movement of sovereign and private spreads.\footnote{See for example Arellano, Atkeson, and Wright (2015), Klein and Stellner (2014), Bai and Wei (2012), Avino and Cotter (2014) and Bedendo and Colla (2015).}

To sum up, there is ample evidence that in times of higher sovereign risk the borrowing conditions in financial markets deteriorate and unemployment tends to be higher. Given this evidence, I present a model designed to incorporate the co-movement of sovereign spreads with labor market tightness, unemployment and the amount and price of banks’ lending to firms during debt crises and defaults.

### 3 A default model with persistent unemployment

This section outlines a small open economy model populated by five different agents. Workers are either employed and consume their after-tax wage or unemployed and consume unemployment benefits. Heterogeneous firms post vacancies to attract workers and produce output. Domestic banker families consist of depositors who buy government debt and give funds to other banker families and bankers who provide loans to existing and entering firms. Lenders with deep pockets buy government debt and own firms. A government taxes workers and may default on the sovereign debt it issues to depositors and lenders.

Time is discrete and infinite $t = 0, 1, 2, \ldots$. First, aggregate productivity $z_t$ is realized. It is assumed to follow an AR(1) process in logs:

$$\log(z_{t+1}) = \rho \log(z_t) + \sigma \epsilon_{t+1}$$  \hspace{1cm} (1)

Second, government debt is issued to international investors and depositors in the bond market. Third, firms receive within-period lending from bankers in the loan market. Fourth, unemployed workers and firms meet in the labor market.
3.1 Preferences and technology

Workers. There is a continuum of mass 1 of infinitely-lived and risk-averse workers. In each period \( t \), a share \((1-u_t)\) of workers is employed, earns a wage \( w_t \), pays lump-sum taxes \( \tau_t \) and consumes their after-tax income \( c_t = w_t - \tau_t \). The remaining share \( u_t \) is unemployed and consumes unemployment transfers \( c_t = T \). Each worker maximizes the discounted stream of lifetime utility from consumption

\[
\mathbb{E}_t \sum_{s=t}^{\infty} \beta^{s-t} u(c_s)
\]

where preferences exhibit constant relative risk aversion, \( u(c) = \frac{c^{1-\sigma}}{1-\sigma} \).

Workers live by themselves and cannot save. The employed face the risk of becoming unemployed because their job can be destroyed with probability \( \xi \) or because they are fired. The unemployed workers can find a new job with time-varying probability that is determined by the labor market. The government is able to borrow and save and wants to insure the workers against unemployment and wage risk.

Firms. There is an endogenous measure \( H \) of firms. A firm is a job. Firms have a constant returns to scale production technology and the output of a firm is equal to aggregate productivity \( z_t \). In each period, there is a distribution of heterogeneous firms due to idiosyncratic operational cost shocks \( k_{i,t} \in [\underline{k}, \bar{k}] \) that are iid over time. Firms can create vacancies \( v_t \) at a unit cost \( a \). If vacancies are matched with an unemployed worker, production starts in the following period. The labor market is characterized by matching frictions in the DMP tradition. The matching technology is assumed to be Cobb-Douglas,

\[
M(u_t, v_t) = \min(\mu u_t^\psi v_t^{1-\psi}, v_t, u_t)
\]

where \( \mu \) is the match efficiency. The number of matches cannot exceed the number of unemployed workers or posted vacancies. Labor market tightness \( \theta(u_t, v_t) = v_t/u_t \), defined as the ratio of vacancies to unemployment, determines the probability of a match. Let \( \lambda_f(\theta(u_t, v_t)) \) be the probability of filling a vacancy and \( \lambda_w(\theta(u_t, v_t)) \) the probability that an unemployed worker finds a job. Matches are separated exogenously at destruction rate \( \xi \) or endogenously because firms fire a measure \( s_t \) of workers.

Firms face a pre-financing constraint and so borrow to pay for vacancies and wages before production takes place. Borrowing takes the form of within-period loans from bankers at rate \( R_t \). Firms cannot default on loans. Since vacancies are sold to foreign investors before matching takes place, loans are riskless and always paid back at the end of the period.

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8In this work, transfers \( T \) are assumed to be fixed. Balke and Ravn (2016) study how the government trades off intratemporal wedges when it chooses the level of unemployment transfers in each period.

9This assumption is similar to the working capital requirements in Christiano and Eichenbaum (1992) and Neumeyer and Perri (2005).
Firms are unable to issue equity. The flow profit of a matched firm of type \( i \) in period \( t \) therefore needs to be nonnegative:

\[
z_t - R_t w_t - k_{i,t} \geq 0 \quad (4)
\]

If condition (4) is violated, firm \( k \) fires the worker, receives zero payoff and exits.

**Wages.** Wages are determined by a wage function that implements fixed output sharing

\[
w_t = z_t - \omega \quad (5)
\]

The wage function links the wage to aggregate productivity such that firms make a constant expected profit at a zero loan interest rate. The wage is volatile due to its co-movement with productivity and so is not fixed.\(^{10}\) However, it does not fully adjust to financial conditions. This has important implications for situations in which firms face bad credit conditions but need to pre-finance wages and vacancies. In times of high loan rates, it prevents firms and workers from agreeing to a lower wage to avoid separation. As wages cannot fully downward adjust, firms fire workers despite the fact that there would be a positive surplus for the firms and workers of continued employment. This amplifies match destruction.

**Banker families.** There is a continuum of identical banker families. Banker families consist of two types of family members: depositors and bankers. Depositors decide on consumption or saving and bankers intermediate funds between depositors and firms. However, bankers cannot manage the deposits of their own family members. At each point in time, there is a measure 1 of depositors and a measure 1 of bankers. With probability \((1 - \phi)\) depositors become bankers and vice versa, keeping the relative shares constant. The set-up follows Gertler and Karadi (2011) and is described in detail in Appendix 7.2.

Depositors consume goods \( g_t \), buy government bonds \( b_{t+1} \) at price \( q_t \) and make within-period deposits \( X_t \) to bankers from other families to maximize their discounted utility stream:

\[
G_t = \max_{\{g_s, b_{s+1}, X_s\}_{s=t}^{\infty}} \mathbb{E}_t \sum_{s=t}^{\infty} \zeta^{s-t} v(g_s)
\]

s.t. \( g_s = \pi_s + d_s (b_s - q_s b_{s+1}) + R x_s X_s - X_s \quad (7)\)

They discount future utility \( v(\cdot) \) with \( \zeta \). Holding government bonds is risky because the government may not repay \((d_t = 0)\) and cannot exceed aggregate bond issuance \( b_{t+1} \leq B_{t+1} \).

\(^{10}\)Shimer (2005) pointed out that, with fully flexible wages, the employment responses to productivity shocks tend to be small in the standard DMP model. A large literature has since tried to solve the Shimer puzzle, namely the inability of the standard model to generate enough volatility in labor market tightness relative to labor productivity. In recent work Kehoe, Midrigan, and Pastorino (2015) show that one way of rationalizing much bigger responses of employment in models with household-owned firms is to include on-the-job human capital accumulation. Garín (2015) includes credit shocks to the collateral requirement of firms which creates a smaller volatility of wages.
Deposits earn the non-contingent gross return $R_{x,t}$. Depositors receive net payments $\pi_t$ from the bankers who are part of their family. In return for receiving dividends, they endow new bankers in their family with start-up transfers, comprising a constant component $\kappa$ and an amount equal to their holdings of government bonds $q_t b_{t+1}$.

Bankers intermediate between depositors and firms. A banker $j$ starts a period with wealth $W_{j,t}$, obtains deposits $X_{j,t}$ and makes loans $L_{j,t}$ to firms. The growth in equity depends on the difference between the return on loans $R_t$ and the interest rate on deposits $R_{x,t}$ as well as the amount of assets $L_{j,t}$:

$$W_{j,t+1} = R_t L_{j,t} - R_{x,t} X_{j,t} = (R_t - R_{x,t}) L_{j,t} + R_{x,t} W_{j,t}$$

An arbitrage opportunity due to market imperfections makes it optimal for bankers to build up net worth and not to pay dividends until they become depositors. Let $\Lambda_{t,t+i} = \frac{v_{g,t+i}}{v_{g,t}}$ be the stochastic discount factor of the banker family between period $t$ and $t+i$. A banker $j$’s objective is to maximize expected terminal wealth $P_{j,t}$:

$$P_{j,t} = \max_{\{L_{j,s}\}_{s=t}^{\infty}} \mathbb{E}_t \sum_{s=t}^{\infty} (1 - \phi)(s-t)\zeta^{s-t} \Lambda_{t,s} [(R_s - R_{x,s}) L_{j,s} + R_{x,s} W_{j,s}]$$

subject to $P_{j,t} \geq \lambda L_{j,t}$

The incentive constraint (10) limits the banker’s ability to borrow. It is motivated by a moral hazard problem between bankers and depositors. In each period, bankers can choose to divert their assets and depositors can only recover a share $(1 - \lambda)$ but it is too costly for them to enforce the repayment of the remaining share $\lambda$, which is kept by the banker family.

**Investors.** There is a continuum of investors who are risk-neutral. Some investors buy government bonds to break even in expectation. The marginal investor is foreign and determines the price of government debt to be

$$q_t = \mathbb{E}_t \left( \frac{d_{t+1}}{1 + r} \right)$$

where $d_{t+1} \in \{0, 1\}$ is the government’s default decision in the following period and $r$ is the risk-free interest rate. Other investors own firms. They receive the profits, bear the risk of destruction and discount future profits at rate $1/(1 + r)$. However, they cannot provide the within-period loans which has to go through intermediate banks.

**Government.** In each period, the government chooses non-contingent one-period bonds $B_{t+1}$, lump-sum taxes $\tau_t$ paid by employed workers, and default or repayment of inherited
debt \( d_t \in \{0, 1\} \). The government is utilitarian and maximizes social welfare,

\[
    \mathbb{E}_t \sum_{s=t}^{\infty} \beta^{s-t} \left[ (1 - u_s)u(c_s) + u_s u(T) \right]
\]

(12)

where it weights each worker’s utility equally, regardless of their employment status or consumption level. When the government defaults \( (d_t = 0) \) it cannot issue new debt in the same period \( (B_{t+1} = 0) \). It is further constrained by the government budget constraint

\[
    u_t T = (1 - u_t) \tau_t + d_t[q_t B_{t+1} - B_t]
\]

(13)

The government understands how its policy affects the private sector and is constrained by private sector implementability conditions. These involve the response of the private sector to current policy changes that pin down the unemployment rate \( u_t \) and the laws of motion for aggregate employment which determines future unemployment rates and for bankers’ net worth. Section 3.2 describes these constraints in detail. The government further understands the pricing schedule that is dictated by foreign investors who need to break even in expectation according to equation (11).

Importantly, the government cannot commit to any future default, debt or tax policy. I restrict attention to Markovian policies that depend on aggregate productivity, outstanding debt, initial employment and equity of continuing bankers \( W_{e,t} \) summarized by the aggregate state \( \Omega_t = (z_t, B_t, N_t, W_{e,t}) \). Beginning-of-period employment level \( N_t \) and unemployment rate \( u_t \) relate to each other by accounting for firing in the current period \( s_t \) such that \( u_t = 1 - N_t + s_t \). Markov equilibria allow a recursive representation of the workers’, firms’ and government’s problems (see Appendix 7.3). Bankers’ total supply of loans is a linear function of aggregate wealth (20). A government policy \( D \) maps the state into the policy instruments \( D : \Omega_t \rightarrow (B_{t+1}, \tau_t, d_t) \). Henceforth, I will drop time subscripts to denote current period’s states and choices and use “‘” to indicate next period’s states and choices.

**Timing** The timing of the model, illustrated in Figure 3, is as follows. In each period, the aggregate productivity state \( z \) is realized first and the current state is given by \( \Omega = (z, B, N, W_e) \). Next, the government chooses its default \( d_t \), tax \( \tau_t \) and debt \( B_t \) policy. This decision sets the debt level \( B_t \) in next period’s state. Depositors make deposits \( X \) and endow new bankers with funds, leading to the aggregate net worth of the bankers \( W_e \). Firms face idiosyncratic cost shocks \( k \). Then the loan market clears and determines loans \( L \), firing \( s \) and vacancies \( v \). Wages \( w \) are paid before production of output \( z(N - s) \) and consumption \( c \) take place. Loans are repaid and bankers become depositors and vice versa. At the end of the period, a share \( \xi \) of active jobs is destroyed and new matches \( M \) are formed. Together, \( (s, \xi, M) \) determine the next period’s initial employment state \( N' \) and loan market clearing pins down existing banker’s net worth \( W_e' \).
3.2 Equilibrium

I concentrate on Markov equilibria that can be characterized as follows. In each period, the government moves first by choosing current period policies. Its choices are constrained to depend only on the value of the current period’s state – productivity, government debt and the aggregate initial employment and existing wealth levels. After the government has moved, the private agents choose their current period actions. Since the private sector consists of small agents, private agents take future private and government policies and aggregate laws of motion as given. The government, however, correctly anticipates how future policy will depend on current policy through its effect on the state of the economy.

Following Krusell, Quadrini, and Ríos-Rull (1996), Krusell and Ríos-Rull (1999) and Klein, Krusell, and Ríos-Rull (2008) I represent the equilibrium in recursive form in three parts. First, I postulate a government policy as a mapping from the aggregate state variables to borrowing, tax rates and default $D : \Omega \rightarrow (B', \tau, d)$, and I compute the equilibria in the private sector associated with these government policies. I check for multiple equilibria and focus on the one with the highest employment level. Second, I characterize the economic behavior implied by a one-period deviation from this policy mapping. Third, I use these deviations to construct optimal equilibrium policies on the part of the government (fixed-point problem). I will now discuss these steps in detail.

First, consider a government policy $D : \Omega \rightarrow (B', \tau, d)$ such that the state in the following period is given by $\Omega' = (z', B', N', W_e')$.

**Definition 1** Given a government policy $D$, a **private sector equilibrium** $P$ is defined as value functions of firms $J(\Omega; D)$ and $V(\Omega; D)$, workers $E(\Omega; D)$, $U(\Omega; D)$, bankers $P(\Omega; D)$ and depositors $G(\Omega; D)$, policies $s = S(\Omega; D)$, $v = O(\Omega; D)$, $X = X(\Omega; D)$, $b' = b(\Omega; D)$ and $L^b = L(\Omega; D)$, prices $R = R(\Omega; D)$, $R_x = R_x(\Omega; D)$, $q = Q(\Omega; D)$ and $w = w(\Omega; D)$, and laws of motion of aggregate employment $N' = H(\Omega; D)$ and existing wealth $W_e' = W_e(\Omega; D)$ such that:

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11 Alternative equilibrium concepts include for example sustainable equilibria that can depend on the entire previous history of shocks (Chari and Kehoe (1990)).

12 In other work, I focus on the possibility of multiple equilibria in this step and study its implications for bond pricing and sunspot-driven debt crises.
1. $\mathcal{J}(\Omega; D)$ maximizes profits subject to pre-financing constraint (27), $\mathcal{V}(\Omega; D)$ is the value of an open vacancy (28), $\mathcal{E}(\Omega; D)$ and $\mathcal{U}(\Omega; D)$ solve the workers’ Bellman equations (58)-(59), and $\mathcal{P}(\Omega; D)$ and $\mathcal{G}(\Omega; D)$ solve the problem of the banker families (62)-(67);

2. Separation $s$ and vacancies $v$ satisfy the non-negative profit condition and free entry (30)-(31), and deposits $X$, bond holdings $b'$ and loans $L^b$ are optimal, satisfying (22), (23) and (20);

3. The loan rate $R$ clears the loan market (32), $R_x$ clears the deposit market, the debt price $q$ lets lenders break even in expectation (11) and the wage $w$ is set as a function of output (5);

4. Wealth evolves according to (21) and the law of motion of aggregate employment $N'$ is consistent with matching technology and separation: $N' = (1 - \xi)(N - s) + M$.

Second, consider the private sector response to a one-time policy deviation. In the above equilibrium the government policy is set by $D$ at every point in time. In order to define a policy equilibrium I also need to consider the case in which the government sets different policies. Thus, consider now the problem for the private sector that, given the state $\Omega$, faces an arbitrary policy $(\tilde{B}', \tilde{\tau}, \tilde{d})$ in the current period, after which government policy reverts to the equilibrium policy $\bar{D}$. Denote this one-time deviation of the government $\tilde{D} = (\tilde{B}', \tilde{\tau}, \tilde{d}; D)$. The one-period policy deviation affects the next period’s state such that now $\Omega' = (z', \tilde{B}', \tilde{N}')$.

**Definition 2** A private sector response $\tilde{\mathcal{P}}$ to a one-time policy deviation $\tilde{D}$ is a collection of value functions $\tilde{\mathcal{J}}(\Omega; \tilde{D}), \tilde{\mathcal{V}}(\Omega; \tilde{D}), \tilde{\mathcal{E}}(\Omega; \tilde{D}), \tilde{\mathcal{U}}(\Omega; \tilde{D}), \tilde{\mathcal{P}}(\Omega; \tilde{D})$ and $\tilde{\mathcal{G}}(\Omega; \tilde{D})$, policies $\tilde{s} = \tilde{S}(\Omega; \tilde{D}), \tilde{v} = \tilde{O}(\Omega; \tilde{D}), \tilde{X} = \tilde{X}(\Omega; \tilde{D}), \tilde{b}' = \tilde{b}(\Omega; \tilde{D})$ and $\tilde{L}^b = \tilde{L}(\Omega; \tilde{D})$, prices $\tilde{R} = \tilde{R}(\Omega; \tilde{D}), \tilde{R}_x = \tilde{R}_x(\Omega; \tilde{D}), \tilde{q} = \tilde{Q}(\Omega; \tilde{D})$ and $\tilde{w} = \tilde{w}(\Omega; \tilde{D})$, and laws of motion of aggregate employment $\tilde{N}' = \tilde{H}(\Omega; \tilde{D})$ and existing wealth $\tilde{W}'_e = \tilde{W}_e(\Omega; \tilde{D})$ such that:

1. policies are optimal and value functions solve the respective Bellman equations (68)-(71);
2. prices clear bond, deposit and loan markets and satisfy output sharing and free entry;
3. laws of motion are consistent with matching/separation and wealth accumulation.

Note that a private sector response is very similar to a private sector equilibrium $\mathcal{P}$, however it features different current policies. Therefore, all current prices are also different. From the next period onwards the economy reverts back to the original government policy with the original value functions. However, the economy has still undergone a change. Although the functions are the same, the economy will be in a different state in the next period because bond holdings $\tilde{B}'$, the employment state $\tilde{N}'$ and aggregate wealth $\tilde{W}'_e$ may have changed.
Third, I can determine a time-consistent policy equilibrium. Turning to the determination of $D$, suppose the government considers a different policy in the current state. By construction the indirect utilities $\tilde{E}$ and $\tilde{U}$ can be used for this purpose. The most preferred one-time deviation $\delta = (B'_d, \tau_d, d; D)$ solves

\[
W(\Omega) = \max_{d \in \{0,1\}} W_d(\Omega) \tag{14}
\]

\[
W_1(\Omega) = \max_{\tau_1, B'_1} N \tilde{E}(\Omega; B'_1, \tau_1, 1; D) + (1 - N) \tilde{U}(\Omega; B'_1, \tau_1, 1; D) \tag{15}
\]

\[
W_0(\Omega) = \max_{\tau_0} N \tilde{E}(\Omega; 0, \tau_0, 0; D) + (1 - N) \tilde{U}(\Omega; 0, \tau_0, 0; D) \tag{16}
\]

subject to

\[
(1 - N + \tilde{s}) T = (N - \tilde{s}) \tau_d + d[qB'_1 - B] \tag{17}
\]

\[(\tilde{E}, \tilde{U}, \tilde{s}) \text{ is a response } \tilde{P} \text{ to } \delta \tag{18}\]

The function $\delta$ is the most preferred policy of the incumbent government because it gives the highest value to the workers given that its successors will revert to policy $D$.\textsuperscript{13}

**Definition 3** A **time-consistent policy equilibrium** is a government policy $D$ and a private sector equilibrium $P$ such that:

1. $P$ is the private sector equilibrium associated with $D$

2. $D$ is the most preferred one-time deviation: $\delta = D$

This definition of equilibrium amounts to subgame-perfection or time consistency of equilibrium. By varying the policy currently under consideration, the government takes into account the equilibrium response of all future variables.

### 4 Model analysis and solution

**4.1 Loan market clearing**

This section describes the loan market equilibrium and the joint determination of job vacancies $v$, loan price $R$ and firing $s$. The loan market clears when the price of loans $R$ equalizes aggregate loan demand $L^f$ and supply $L^b$.

**Loan supply.** A bank $j$’s optimal supply of loans is a linear function of its net worth if the incentive constraint binds (see Appendix 7.2):

\[
L_j = \chi W_j \tag{19}
\]

\textsuperscript{13}In general, there may be more than one solution to $\delta$. In the numerical computation I verify that $W_1$ is single-peaked.
The leverage ratio $\chi$ – the ratio between assets to equity – is increasing in $R$ and time-varying. However, it does not depend on firm specifics and so the aggregate supply of loans and the evolution of total net worth in the economy are:

$$L^b = \chi W \quad (20)$$

$$W = \phi W_e + (1 - \phi) W_n = \phi [(R^- - R_x^-) \chi^- + R_x^-] W^- + \kappa + \gamma q B' \quad (21)$$

Aggregate loan supply $L^b$ is increasing in the loan rate. Net worth $(21)$ reflects the existing wealth $W_e$ of the share $\phi$ of continuing bankers and new wealth $W_n$ of the share $(1 - \phi)$ of new bankers. New bankers receive a start-up transfer, comprising a constant amount $\kappa$ and the current domestic share $\gamma \in (0,1)$ of aggregate government debt, priced at $q$.\footnote{As long as a share $\gamma < 1$ of all government debt is held domestically, this does not play a role for the pricing of debt because the marginal investor is foreign and debt price equation (23) binds.} The first order conditions of the depositors impose on the within-period deposit return and the debt price given positive domestic bond holdings to satisfy:

$$R_x = 1 \quad (22)$$

$$q = \mathbb{E}\{\zeta \Lambda' d'\} \quad (23)$$

Bankers only operate if $R \geq R_x = 1$.

**Loan demand.** The total loan demand by new and incumbent firms is given by the sum of funds they require to pre-finance wages and vacancies:

$$L^f = w(N - s) + av \quad (24)$$

To determine the number of firm exits $s$, consider the value of a firm with cost $k$, given the aggregate state $\Omega$ and a government policy $\mathcal{D}$:

$$\mathcal{J}(k, \Omega; \mathcal{D}) = \begin{cases} 
  z - Rw - k + \frac{1}{1+\tau} \mathbb{E}_{z,k}\{(1 - \xi)\mathcal{J}(k', \Omega'; \mathcal{D})\} & \text{if } z - Rw - k \geq 0 \\
  0 & \text{if } z - Rw - k < 0 
\end{cases} \quad (25)$$

Once the loan rate exceeds $\hat{R} = \frac{z-w}{w}$ not all firms can operate. Let $\hat{k}$ be the cut-off value of the operational cost above which flow profits are negative. The number of exiting firms $s$ is determined by the share of firms with operational costs higher than $\hat{k}$:

$$s = \frac{\int k \, dk}{\int k \, dk} \quad (26)$$
Figure 4: Loan market equilibrium: Supply $L^b$ (blue solid line) and demand $L^f$ (red dashed line) with loan rate $R$. Above $\hat{R}$ vacancies fall to zero. Above $\check{R}$ some firms would make negative profits and exit.

Expected profits before the realization of types can be expressed recursively as

$$J(\Omega; D) = \left(1 - \frac{s}{N}\right)\left(z - Rw - \mathbb{E}_k\{k|k \leq \hat{k}\} + \frac{1}{1 + r}\mathbb{E}_z\{(1 - \xi)J(\Omega'; D)\}\right)$$

(27)

where $\left(1 - \frac{s}{N}\right)$ is the probability of meeting the profit condition. Given the value $J(\Omega; D)$ the value of a vacancy is:

$$V(\Omega; D) = -Ra + \lambda_f(\theta(1 - N + s, v))\frac{1}{1 + r}\mathbb{E}_z\{J(\Omega'; D)\}$$

(28)

Free entry determines that fewer vacancies are posted when they are expensive and so vacancies are decreasing in the within-period interest rate. There is no vacancy posting if $V < 0$. Let $\check{R}$ be the cut-off loan rate above which vacancies fall to zero, even if the vacancy was filled with certainty, $\lambda_f = 1$.

$$\check{R} = \frac{1}{a(1 + r)}\mathbb{E}_z\{J(\Omega'; D)\}$$

(29)

**Discussion.** In equilibrium $R$, $s$ and $v$ must jointly solve free entry, non-negative firm profits, and loan market clearing:

$$Ra \geq \lambda_f(\theta(1 - N + s, v))\frac{1}{1 + r}\mathbb{E}_z\{J(\Omega'; D)\} \quad (v = 0 \text{ if } >)$$

(30)
\[ s = \frac{\int k \, dk}{\int \frac{k}{\bar{k}} \, dk} \]

\[ \chi W = w(N - s) + av \]  

Figure 4 illustrates the loan market equilibrium. Loan supply (blue solid line) is increasing in the loan rate \( R \). Loan demand (red dashed line) comprises demand for wages and entry costs and is decreasing in the loan rate. The equilibrium return \( R \) is located where aggregate loan demand \( L^d \) crosses the loan supply \( L^b \), point A. Free entry binds \( (V = 0) \) and no firing takes place \( (s = 0) \) because the loan rate lies below \( \hat{R} \) and \( \check{R} \). However, lower loan supply \( L^b' \) increases the equilibrium loan rate and decreases the equilibrium loan amount holding all else equal, point B. Since the associated loan rate is higher than the two threshold levels, the value of a vacancy is negative resulting in zero vacancies \( (v = 0) \) and some firms make negative profits leading to their exit \( (s > 0) \). Some pre-existing jobs are destroyed (blue northeast hatched area).

Wage setting is important for this outcome. If wages could be renegotiated and adjusted downwards in response to high loan rates, firing might be prevented. In this case most labor market dynamics would stem from vacancy posting instead of firing and the unemployment response would be weaker. Here, once the firm makes negative profits it exits although the bilateral bargaining set between the worker and the firms may be non-empty. The wage setting therefore creates inefficiencies.

Firing and firm destruction determine the total probability of separation in equilibrium. While exogenous destruction takes place at the end of each period, firing becomes necessary after productivity is realized and the government sets its current policy at the beginning of a period. This timing makes the government’s current debt and default policy have an immediate effect on current firing.

4.2 Liquidity function of debt and default

This section describes the main trade-off the government faces when making its debt-default decision. The government is concerned with the fact that government bonds are a source of liquidity and aid loan creation, which is needed for the functioning of the labor market and disrupted by either less government debt issuance or lower debt prices.

The complementarity between the public and private sector stems from the fact that the bank’s optimal loan supply depends positively on its assets, which are partially formed by government debt (20). Loan supply therefore depends positively on the value and amount of sovereign debt. Figure 4 illustrates how lower debt prices or less debt issuance suppress loan supply and lead to higher within-period interest rates. The loan supply curve \( L^b \) shifts left to \( L^b' \) (solid blue lines), moving the equilibrium from point A to point B for the same
loan demand $L^f$ (dashed red line). Higher borrowing rates mean higher gross wage costs for a firm and lower incentives to post vacancies. More drastically, if the sovereign debt policy pushes the interest rate above $\hat{R}$ and $\hat{\hat{R}}$, it depresses job vacancies and destroys jobs.

The co-movement of sovereign spreads and loan rates means that the prospect of default alone dries up liquidity and hurts the labor market, independently of whether default takes place or not ex post. This anticipation effect is important for the timing of labor market adjustments around default events but also enables the model to capture debt crises.

When the government is confronted with a low productivity shock, it trades off the cost of increasing taxes to raise enough revenue for debt repayment against the cost of compromising the liquidity function of its debt. In the first case, the government sacrifices utility of the employed who are left with lower after-tax income. In the latter case, the government sacrifices some employed workers’ jobs through firing and reduces the unemployed workers’ probability of finding a job. In equilibrium, defaulting becomes optimal when the utility costs from the tax burden on the employed outweigh the utility costs from higher unemployment.

5 Quantitative evaluation

5.1 Calibration strategy

One period corresponds to a quarter. To give insights into the model predictions for the recent European debt crisis, the model is calibrated to match Portugal’s labor market. I use estimates from other studies whenever possible and calibrate the remaining parameters using indirect inference targeting a set of moments discussed further below. Table 2 summarizes the calibrated parameters and targets. The exact implementation of the model and how it currently differs from the previous section is described in the following. Appendix 7.4 describes the computational strategy.

Workers. The CRRA parameter $\sigma > 0$ determines the intertemporal elasticity of substitution of consumption. I assume $\sigma = 2$ which is in line with a large amount of empirical evidence using either household data or aggregate data, see e.g. Attanasio and Weber (1995) or Eichenbaum, Hansen, and Singleton (1988).

The discount factor $\beta$ is estimated by indirect inference. The main target for $\beta$ is the default probability which I set equal to 3 percent annually. This frequency is consistent with historical default rates in Greece and a standard value in the default literature, see e.g. Arellano (2008). In conjunction with other parameters, I calibrate $\beta$ to be 0.96, showing that domestic workers are impatient relative to the risk-free interest rate, which is calibrated to 4 percent annually, $r = 0.01$. Assuming that the value of unemployment is low, I adopt

15Because of the non-linearity of the moments in the parameters, there is no exact one-to-one mapping from targets to parameters individually. However, some targets are more important for some parameters than others and this is the logic of the discussion.
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Target</th>
</tr>
</thead>
<tbody>
<tr>
<td>Calibrated</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$r = 0.01$</td>
<td>Risk-free interest rate</td>
<td></td>
</tr>
<tr>
<td>$\sigma = 2.00$</td>
<td>Relative risk aversion</td>
<td></td>
</tr>
<tr>
<td>$\psi = 0.40$</td>
<td>Matching parameter</td>
<td></td>
</tr>
<tr>
<td>$\rho = 0.88$</td>
<td>Persistence parameter</td>
<td></td>
</tr>
<tr>
<td>$\sigma_\epsilon = 0.03$</td>
<td>Standard deviation</td>
<td></td>
</tr>
<tr>
<td>$\zeta = 0.98$</td>
<td>Risk-neutral discounting</td>
<td></td>
</tr>
<tr>
<td>$\phi = 0.00$</td>
<td>Survival of bankers</td>
<td></td>
</tr>
<tr>
<td>$\gamma = 0.90$</td>
<td>Domestic debt share</td>
<td></td>
</tr>
<tr>
<td>$k = 0.00$</td>
<td>Operational cost</td>
<td></td>
</tr>
<tr>
<td>Estimated</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\beta = 0.96$</td>
<td>Discount factor</td>
<td>Default probability 3%</td>
</tr>
<tr>
<td>$T = 0.30$</td>
<td>Benefit</td>
<td>Replacement ratio 50%</td>
</tr>
<tr>
<td>$\kappa = 0.60$</td>
<td>Capital requirement</td>
<td>Firing 6%</td>
</tr>
<tr>
<td>$\lambda = 0.93$</td>
<td>Enforcement cost</td>
<td>Maximum loan rate 12%</td>
</tr>
<tr>
<td>$\mu = 0.04$</td>
<td>Match efficiency</td>
<td>Unemployment 7%</td>
</tr>
<tr>
<td>$\xi = 0.03$</td>
<td>Exogenous separation</td>
<td>Inflow rate 0.4 – 1.1%</td>
</tr>
<tr>
<td>$\omega = 0.25$</td>
<td>Wage parameter</td>
<td>Non-negative surplus</td>
</tr>
<tr>
<td>$a = 0.04$</td>
<td>Vacancy costs</td>
<td>Cost-wage ratio 4.5%</td>
</tr>
</tbody>
</table>

Table 2: Calibration strategy: Quarterly. Calibrated parameters are set using outside estimates. Estimated parameter values are found by matching a set of targets using indirect inference.

a transfer level $T = 0.3$ to target an average net welfare benefit replacement rate of 50 percent.\(^{16}\)

**Depositors and bankers.** I assume risk-neutral depositor preferences and discounting, implying $\Lambda_{t,t+i} = 1$ and $\zeta = 0.98$. Bankers stay in business for only one period, $\phi = 0$. Since the domestic share of debt holdings is indeterminate, $\gamma$ is set to 90 percent, reflecting the high exposure of European banks to government debt.\(^{17}\)

The constant start-up wealth of bankers $\kappa$ is estimated by targeting an average increase of the unemployment rate in default of six percentage points. This is in line with the steep increases observed in Southern Europe during the last crisis. The estimate of $\kappa$ is 0.60. Finally, I choose the enforcement cost parameter $\lambda = 0.93$ by targeting a maximum quarterly loan rate of 12 percent.

\(^{16}\)Alternatively, one might consider adopting a Hagedorn and Manovskii (2008) calibration by assuming a much higher replacement rate.

\(^{17}\)Sections 5.2 and 5.4.2 consider alternative values of $\gamma$. 
Firms. I follow Silva and Toledo (2009) and assume that vacancy posting costs correspond to 4.5 percent of quarterly wages. Given this target, I find an estimate of the vacancy posting cost of $a = 0.04$. Wages are set as in (5). The share parameter $\omega$ is set such that workers and firms, facing the risk-free loan rate but zero operational costs, make a surplus at the lowest possible productivity level $z$, i.e. the wage lies within the bargaining set. This yields $\omega = 0.25$.

In the current version of the computational part, I abstract from idiosyncratic operational costs, $k = 0$. Instead I assume that the participation constraint always puts an upper limit on the deposits and that there is a maximum leverage constraint. This yields an upper loan rate $\bar{R}$ of 1.12, which also serves as firing and vacancy threshold, $\bar{R} = \hat{R} = \bar{R}$. Homogeneous firms enter a lottery at $\bar{R}$ over rationed loans.

Technology. I assume that aggregate productivity $z$ follows an AR(1) process as shown in (1). To match the productivity process in Southern Europe I use persistence $\rho = 0.88$ per quarter and a standard deviation $\sigma_{\epsilon} = 0.03$ of a standard normally distributed shock.

The matching function elasticity $\psi$ is calibrated following Merz (1995), $\psi = 0.4$. The match efficiency $\mu = 0.04$ results from targeting an unemployment rate of around 7 percent in the long run without default. I estimate the exogenous job destruction rate $\xi$ to match the rather low monthly inflow rate into unemployment of $0.4 \text{--} 1.1$ percent observed in Portugal and Spain (Elsby, Hobijn, and Sahin (2013)). This yields an estimate of $\xi = 0.03$.

5.2 Simulation results: defaults and debt crises

I use simulation techniques to understand whether the model can replicate the dynamics of labor market variables around debt crises and default events. I draw random productivity shocks for 1,000,000 periods and discard the first 1000 periods, starting the simulations from the mean productivity and zero asset level. I compute the median paths of the economy in four year windows around two different types of episodes: defaults and debt crises. I also report the 25 and 75 percent confidence intervals.

Figure 5 plots the dynamics of the economy around default episodes, where 0 is the period of default. Defaults occur when periods of high or medium productivity are interrupted by a large drop in productivity that sends the economy into a recession. Prior to default, it is very unlikely that productivity suddenly falls due to the high persistence of productivity and spreads stay low. The abrupt big recession implies that default is optimal in these instances because insuring the unemployed becomes too costly. The rapid nature of the productivity loss induces the government to default without firms adjusting to the upcoming lower value of a job early in the run-up of the default. However, when spreads suddenly rise in the two quarters preceding default, firms cut job vacancies by more than three and six percent compared to the initial vacancy level. Although output starts to fall together with productivity, the biggest default cost materializes post-default when many jobs are destroyed such that the economy enters an extended recession with high unemployment levels. I call
Figure 5: Baseline: Default at time 0. Simulation results for preceding as well as succeeding 8 quarters of default. Median (solid black line) as well as 25rd and 75th quantile (dashed blue line).

this the “employment cost of default”. The persistent nature of the output collapse derives from the many workers who lost their job at the time of default and need time to find a new job after a typical default, so that GDP only recovers once jobs are rebuilt in the economy.

Figure 6 instead illustrates instances where sovereign spreads rise substantially but do not lead to default. I define these “debt crises” as episodes where four consecutive quarters exhibit a mean annualized spread of at least five percent but are neither preceded nor succeeded by default in the four year window. I plot the last quarter of these elevated spreads at time 0. In the baseline, debt crises typically occur as a result of a long and moderate fall in productivity. The slow decline in productivity leads the economy into a region where the value of providing insurance still outweighs the cost of default. The government pays therefore an increasing risk premium. In response, firms adjust to higher loan rates and higher default probabilities with lower vacancy postings and labor market tightness decreases slowly. However, lay-offs are not forced on firms. The typical debt crisis ends when the productivity drop reverses and produces a much stronger recovery in the economy than default episodes.

Despite the fact that vacancies and tightness fall by roughly five and three percent, respectively, the employment effects are small because the government fulfills its liquidity function and firing can be avoided over the entire period. This changes substantially if increases in spreads affect the loan market more dramatically, as observed during the recent European debt crisis. By lowering the domestic debt share to $\gamma = 0.1$, the bankers have less collateral for loan creation and employment responds more strongly to rising bond spreads.
I compare the debt crises simulation of the model to Portuguese data between 2010 and 2014 (Figure 7). A striking difference to the baseline is that debt crises can occur both in high as well as low productivity states. The confidence bounds of output cover a wide range of possible paths, including the Portuguese output variation during this period. The unemployment rate peaks one quarter after the end of the crisis period, earlier than in the data where unemployment continues to rise. However, the definition of a crisis used allows the spread to fall back to low levels much faster than was the case in Portugal. The confidence bounds of vacancy postings contain the Portuguese vacancy dynamics but the simulation shows that the model does not exhibit enough volatility in the tightness during this episode. This exercise illustrates that the sensitivity of vacancy postings and tightness are closely linked to the liquidity function of debt because the government’s debt decision affects not only the default probability but also the price of vacancies.

To sum up, there is a striking difference between default episodes and debt crises both in terms of their underlying productivity sequence and in terms of the labor market response. Firms cut vacancies when they anticipate default because of the combination of more expensive job vacancies and a higher expected job separation rate. This strategy avoids firing when a country faces a moderate sequence of adverse productivity shocks but not in the

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18 One potential way of addressing this is to increase the rigidity of the wage, which in the current calibration is an affine function of productivity with slope 1 and thus moves almost as much as productivity.
case of a more abrupt productivity drop. In the latter case the government defaults which disrupts the loan supply to the private sector and triggers more severe employment effects which materialize post-default.

5.3 Mechanism: the importance of frictions

This section serves to highlight the frictions that underlie the mechanism by which default impacts on unemployment via bank lending. It also shows how this model can resolve two of the key quantitative challenges in the sovereign default literature: generating empirically realistic debt ratios and producing clustered default events.

Debt-to-GDP ratios tend to be high and default happens frequently and in clusters, which is hard to rationalize with standard quantitative default models. When debt-to-GDP ratios of around 50 percent for emerging market economies are successfully targeted, it is common for the models to miss the average default frequency by an order of magnitude.\footnote{Aguiar, Chatterjee, Cole, and Stangebye (2016) use the annualized GDP and discuss in detail the difficulty of sovereign default models in matching high debt ratios and simultaneously generate plausible default frequencies.} The quantitative shortcomings of standard default models appear even more severe when one considers that developed countries are on average more indebted than emerging market
Another striking fact is that defaults tend to occur in temporal clusters – the “serial defaults” (Reinhart and Rogoff (2004)).

Although the debt-to-GDP ratio was not targeted in the calibration, the model naturally generates much higher debt ratios, which are consistent with the level of indebtedness in European countries, despite keeping default frequencies realistic. The borrowing ratio to quarterly GDP in the baseline is 2.8. The model also features serial defaults.

To make comparison easier in the following analysis I model the link between sovereign bond prices and the interest rate in the private sector in reduced form by setting the interest rate \( R \) to the inverse of the debt price within \([1, 1.12]\) bounds. While simplifying the model, it captures the counter-cyclicality of the finance premium in the baseline model.

### 5.3.1 Matching frictions

Comparing the implications of different degrees of frictions in the labor market sheds light on the importance of matching frictions and the persistence of unemployment for generating (1) realistic sustainable debt ratios, (2) anticipation effects and (3) serial defaults. In the following exercise I compare two cases that only differ in their match efficiency. In one case, the match efficiency is moderate and employment takes about 8 quarters to recover after default with an average job finding rate of 0.4. In the other case, I increase the match efficiency such that the job finding rate almost doubles and employment recovers much faster.

First, the extent to which frictions are present in the labor market affects the size of default costs and debt ratios. When frictions are low, default triggers a rise in the unem-

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20Moreover, Dias, Richmond, and Wright (2014) find higher debt levels for emerging economies by using more sophisticated measures of indebtedness than face values.

21See also Sturzenegger and Zettelmeyer (2006) and Reinhart (2010). Tomz and Wright (2013) provide a survey of the empirical research on sovereign debt and default.

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Figure 8: Labor frictions and default regions: Default thresholds in the debt-productivity state space given \( N = 0.93 \) (left) as well as for the employment-productivity state space for \( B/Y = 2.8 \) (right). Lowering labor market frictions enlarges shaded default regions.
Figure 9: Expected firm value and default probability: Expected firm value decreases in anticipation of default during debt crisis at time 0. Median (solid black line) as well as 25rd and 75th quantile (dashed blue line). Default probabilities increase shortly after default at time 0, giving rise to clustered default events.

employment rate but the labor market can recover almost fully in the following period. In contrast, in frictional labor markets a one-time disturbance drives up unemployment, generating higher employment cost of default because new matches take longer to form. The persistence of unemployment creates an additional disincentive to default that helps create higher debt ratios in equilibrium. Doubling the match efficiency lowers the debt-to-GDP ratio that is sustained over the ergodic set by about 10 percent and shifts default thresholds. The solid blue line in Figure 8 plots the default threshold for moderate match efficiency in the debt-productivity (left) and employment-productivity (right) space at mean employment and mean indebtedness, respectively. Above the solid blue line it is optimal for the government to repay its debt but to default below, in the blue hatched area. Evidently, default is more likely for higher debt, lower employment and lower productivity. Increasing the match efficiency shifts the thresholds up to the black dashed lines, adding the red shaded intermediate area to the default region. At the same productivity and employment level, less debt can be sustained. Setting the match efficiency so high as to reach a job finding rate close to 1 makes the default region extend to almost the entire state space. The frictionless limit, as in an RBC type of labor market, therefore appears unable to sustain any positive debt level.

Second, in a dynamic labor market firms are forward-looking and discount the future firm value in the run up to a crisis due to higher financing costs and a higher probability of separation in default (Figure 9, left). This can generate fewer vacancy postings and higher unemployment leading up to default, which is consistent with the empirically relevant decline in employment that often accompanies elevated spreads before an actual default. In contrast, in the high match efficiency case where unemployed workers always find a job, default happens at a high employment level by construction. Without the forward-looking

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behavior of the private sector, employment may at most react to changes in productivity or prices but not to default risk because the model lacks anticipation effects.

Third, the combination of a higher default probability in high unemployment states and persistently high unemployment after default is responsible for a higher probability of another default which follows shortly after the first – a “serial default”. Figure 9 (right) shows the new increase in the default probability right after a default event took place. The increase in unemployment in default, which is amplified by the wage function, and the persistence of unemployment due to matching frictions is the main reason that the model is capable of generating clustered default events.

5.3.2 Financial frictions

Frictions in the private lending market are crucial for the effect of default on unemployment for two reasons. First, if the interest rate of firm loans was independent of the debt price or repayment state and there was no firing, then the default decision would affect neither vacancies nor unemployment. Employment would be the same in repayment and default, but the government could reduce taxes in default. This means that the government would always prefer default to repayment. In equilibrium, debt would be unsustainable and there would be no borrowing at all. Intuitively, this is because there would be no punishment for default. It is necessary that government debt fulfills a liquidity function, otherwise there would be no link between the sovereign and the private sector. The inability to issue new debt in the period of default cannot guarantee the sustainability of debt, as shown by Bulow and Rogoff (1989).

Second, without the possibility of firing, default may still increase the interest rate and lower vacancy postings but the government may prefer to default in times of high (not low) employment, which contradicts empirical findings. The reason is that depressed vacancy postings are less costly when only a few workers are unemployed. To match the empirically relevant default region it is therefore crucial to allow for the possibility that firms must fire workers. This is also the reason I impose the cash-in-advance constraint on wages in addition to vacancies, don’t allow firms to issue equity and use a rigid wage function. Without these features there would be less firing. In fact, since more loans are needed when employment is high, the possibility of firing acts as an asymmetric cost of default that is higher in good times. This is crucial for quantitative models (see for example Arellano (2008)).

Unraveling creates another problem when the government defaults in all employment and debt states. If default is more likely in high employment states and if higher borrowing today means higher employment tomorrow, this lowers the current debt price and the government would be less willing to borrow. This results in more unemployment, which decreases the government’s ability to raise revenue. The government might therefore eventually default in both high and low employment states and debt becomes unsustainable.

\(^{22}\)Unraveling creates another problem when the government defaults in all employment and debt states. If default is more likely in high employment states and if higher borrowing today means higher employment tomorrow, this lowers the current debt price and the government would be less willing to borrow. This results in more unemployment, which decreases the government’s ability to raise revenue. The government might therefore eventually default in both high and low employment states and debt becomes unsustainable.
5.4 Policy experiments

Can the government counteract the negative effects of default on the labor market by simultaneously implementing other policies? What implications would these policies have for the government’s ability to commit? In this section, I consider wage and employment subsidies as well as bank regulations, and investigate their effects on labor market outcomes, default frequencies and debt ratios. The results are summarized by Table 3, which lists the average firing rate $s_0$, and unemployment rates in default and repayment $u_0$ and $u_1$, together with the average debt-to-GDP ratio and the default frequency relative to the baseline model.

5.4.1 Labor market policies

The effects of labor market policies depend crucially on their exact implementation, in particular whether subsidy payments are paid upfront or as a reimbursement to firms. A reimbursement subsidy in default fails to extenuate the decrease in loan supply and is ineffective in alleviating the bottleneck in the transmission of sovereign policy to the private sector. For this reason this section is concerned with upfront payments of the government.\(^{23}\)

Wage subsidy. I first consider a wage subsidy – a direct transfer from the government to firms matched with a worker in order to decrease the firm’s wage burden in default. In the example, the government finances five percent of the aggregate wage bill by taxes during defaults. Given the same amount of loan supply but a smaller aggregate wage bill, the number of exiting firms decreases. A wage subsidy therefore moderates the consequences of

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Table 3: Policy experiments: Comparison of average firing in default $s_0$, unemployment rates in repayment and default states $u_1$ and $u_0$, respectively, debt-to-GDP ratio and default frequency relative to baseline.

<table>
<thead>
<tr>
<th></th>
<th>$s_0$</th>
<th>$u_1$</th>
<th>$u_0$</th>
<th>Debt ratio</th>
<th>Default rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Baseline</td>
<td>5.6%</td>
<td>7.7%</td>
<td>12.7%</td>
<td>2.8</td>
<td>1.00</td>
</tr>
<tr>
<td><strong>Labor market policies</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Wage subsidy</td>
<td>3.1%</td>
<td>7.4%</td>
<td>10.1%</td>
<td>2.8</td>
<td>1.08</td>
</tr>
<tr>
<td>Employment subsidy</td>
<td>0.0%</td>
<td>7.1%</td>
<td>7.0%</td>
<td>2.7</td>
<td>1.30</td>
</tr>
<tr>
<td><strong>Bank regulation</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Higher capital requirement</td>
<td>1.3%</td>
<td>7.2%</td>
<td>8.3%</td>
<td>2.7</td>
<td>1.15</td>
</tr>
<tr>
<td>Lower debt exposure</td>
<td>9.5%</td>
<td>11.3%</td>
<td>24.2%</td>
<td>4.6</td>
<td>0.47</td>
</tr>
</tbody>
</table>

---

\(^{23}\)Imagine a subsidy is implemented in a way that firms are reimbursed for parts of their wages or vacancies at the end of the period. This increases the surplus of a firm in default and may increase the number of firm entrants that renders the free entry condition binding. However, as shown in the simulation in the previous section, default episodes are usually accompanied by a slack free entry condition. Matched firms face the same constraints and only the ones able to stay enjoy the benefits of a wage subsidy. There is no effect on entry or firing.
default on the economy. Defaults occur at a higher frequency than in the baseline. Note that the policy also affects the average unemployment rate in repayment, which falls by 0.3 percentage points. This is because a lower separation rate in default increases the value of a job and thus the value of a vacancy. In normal times, when the free entry condition predominantly binds, more firms enter and the unemployment rate decreases.

The policy weakens the link between labor cost and productivity and reduces the cost of labor, relative to the fall in productivity during a default. More importantly, the government redistributes from employed workers to firm owners by changing the after-tax wage. Nevertheless, employed workers may still be better off, despite earning a lower after-tax income, if they would otherwise have been fired and left with a lower consumption level as well as the need to find a new job. Note that, since the government has access to lump-sum taxation, the policy amounts to a simple wage reduction without any further distortionary effect on the economy. A wage subsidy not only softens the cost of default but can also be beneficial even for the workers who have to pay higher taxes, and so be strictly welfare-improving.

**Employment subsidy.** To take this further, I consider the case where the government steps in to make sure all matched firms keep operating during default. I call this policy an *employment subsidy*. The employment subsidy covers the exact amount needed to avoid any firing that would otherwise occur in default. Table 3 shows that by construction firing is now zero in default. The remaining effects are qualitatively the same as the case of a wage subsidy, except that they are more pronounced. Since this intervention is expected by firms it changes their valuation of a job both in default as well as in repayment.

### 5.4.2 Bank regulation

The political discussion in Europe during the recent debt crisis concentrated to a much greater extent on bank regulation rather than labor market reforms. Although the intended goal was to stabilize the banking system as a whole, it is likely that such regulations would also affect the real economy. In this section I look at what alternative banking regulations would imply for the labor market. I consider the implications of two policy counterfactuals by changing two key parameters of the model: bankers’ minimum equity and the sovereign debt share in their balance sheets.

**Higher capital requirement.** I compare the baseline to a model variation with a 10 percent higher minimum wealth holding of bankers, \(\kappa\) (Table 3). We can think of this as a policy that regulates the minimum capital requirement of banks.

In default states, firing falls by 75 percent compared to the baseline in default, implying a smaller default cost. A higher capital requirement therefore decreases the government’s ability to commit to repayment. The average debt ratio falls and the default frequency rises accordingly. In repayment states, the regulation affects the real economy to the extent that it changes loan supply. As long as the government can still fulfill its liquidity function, i.e.
when loan supply is high enough to prevent firing, the effect is small. However, if lower debt ratios prevent the government from providing liquidity and overall loan supply falls, the higher capital requirement is helpful in default but hurts the economy in normal times.\footnote{An important caveat is that this counterfactual disregards what happens if the bankers were unable to have a higher capitalization.}

Debt exposure of banks. This section compares the baseline calibration that features a high share of debt held domestically to a lower debt exposure of banks by reducing the parameter $\gamma$ from 0.9 to 0.1. The results change dramatically as can be seen in Table 3. Most striking is that firing and unemployment rates in default almost double while the government chooses to default only half the time. In normal times, the debt-to-GDP ratio reaches 4.6 and unemployment increases to 11.3 percent on average.

To understand these differences I plot the simulation of default events in Figure \ref{fig:default_events}. Defaults are preceded by good times of rising (rather than falling) productivity but they are still triggered by an abrupt and large productivity drop. Output does not start to fall in the run-up to the default. The government borrows at high risk premia predominantly in good times because a lower domestic debt share hinders the government from providing liquidity. More liquidity is required when high productivity and low unemployment mean that a high aggregate wage bill needs to be pre-financed. The labor market response to default is stronger
because default occurs at higher wage levels. The government’s incentive to issue debt is dominated by liquidity concerns rather than its impatience (as in the baseline).

Since the extent to which the banking sector is exposed to government risk in normal times does not affect the loan supply in default, the unemployment level in the period of default in a given state is unchanged. However, two counteracting effects govern the default frequency. On the one hand, higher debt levels make default more appealing as the savings from refusing repayment are higher. On the other hand, the economy relies more heavily on higher debt issuance and is more sensitive to increases in bond spreads in repayment states because liquidity is more scarce. Lower domestically held debt shares therefore help the government to credibly commit to repay higher debt levels. This is in contrast with the mechanism outlined by Chari, Dovis, and Kehoe (2016) who show that financial repression can increase the government’s commitment ability. The reason for the difference is that, in this paper, financial repression facilitates the provision of liquidity and so weakens one of the reasons for the government to refrain from defaulting.

6 Conclusion

This paper presents a model in the tradition of Eaton and Gersovitz (1981), in which the government not only faces a trade-off between taxation and the financial disruptions caused by defaulting on domestic bankers, but is also concerned with preventing unemployment from rising well in advance of the actual default. I endogenize the cost of default via bank lending to firms and offer a new explanation for the domestic default cost – the employment cost of default. The recent European debt crisis provides ample evidence for this channel.

The model is calibrated to Southern Europe and captures the empirical employment pattern in defaults and debt crises. The forward-looking vacancy-posting behavior of firms, which is induced by matching frictions in the labor market, is crucial for the model’s ability to match the empirically observed decline in labor market tightness that accompanies increases in bond spreads before an actual default event. Persistent unemployment is key for the government to sustain high debt-to-GDP ratios in equilibrium because it creates an additional disincentive to default. It also underlies the ability of the model to generate clustered default events. I consider counterfactual wage and employment subsidies that alleviate the employment costs of default only if they circumvent the financing constraints of the firms. Higher capital requirements of banks limit layoffs and the rise in unemployment during default. A lower debt exposure of domestic banks increases the government’s ability to commit to debt repayment.
7 Appendix

7.1 Decomposition of GDP growth in Europe

<table>
<thead>
<tr>
<th>Contribution</th>
<th>Portugal</th>
<th>Spain</th>
<th>Greece</th>
</tr>
</thead>
<tbody>
<tr>
<td>GDP</td>
<td>−8.52</td>
<td>−6.48</td>
<td>−30.59</td>
</tr>
<tr>
<td>1 Labor services</td>
<td>−5.75</td>
<td>−8.42</td>
<td>−10.95</td>
</tr>
<tr>
<td>1.1 Labor quality</td>
<td>2.83</td>
<td>1.73</td>
<td>1.88</td>
</tr>
<tr>
<td>1.2 Labor quantity</td>
<td>−8.58</td>
<td>−10.15</td>
<td>−12.83</td>
</tr>
<tr>
<td>2 Capital services</td>
<td>5.91</td>
<td>6.16</td>
<td>7.76</td>
</tr>
<tr>
<td>2.1 ICT capital</td>
<td>5.11</td>
<td>1.73</td>
<td>4.76</td>
</tr>
<tr>
<td>2.2 Non-ICT capital</td>
<td>0.80</td>
<td>4.43</td>
<td>3.00</td>
</tr>
<tr>
<td>3 TFP growth</td>
<td>−8.68</td>
<td>−4.24</td>
<td>−27.40</td>
</tr>
</tbody>
</table>

Table 4: Decomposition of growth 2008-2013. Source: The Conference Board Total Economy Database.

The table is calculated using annual data from 2008-2013. GDP growth rates are computed as log differences and stated in percent. The data is taken from The Conference Board Total Economy Database.\(^{25}\)

7.2 Banker families

There is a continuum of identical banker families. Banker families consist of two types of family members: depositors and bankers. Depositors decide on consumption or saving and bankers intermediate funds between depositors and firms. However, bankers cannot manage the deposits of their own family members. At each point in time, there is a measure 1 of depositors and a measure 1 of bankers. Depositors become bankers with probability \((1 - \phi)\) and bankers become depositors with the same probability, keeping the relative shares constant. The set-up follows Gertler and Karadi (2011).

**Depositors.** Depositors consume goods \(g_t\), buy government bonds \(b_{t+1}\) at price \(q_t\) and make within-period deposits \(X_t\) to bankers from other families to maximize their discounted utility stream:

\[
\max_{\{g_s,b_{s+1},x_{s}\}_{s=t}^{\infty}} \mathbb{E}_t^\infty \sum_{s=t}^\infty \zeta^{s-t}v(g_s)
\]

\[
g_s = \pi_s + d_s(b_s - q_s b_{s+1}) + R_{x,s} X_s - X_s
\]

They discount future utility \( v(\cdot) \) with \( \zeta \). Holding government bonds is risky because the government may not repay \( (d_t = 0) \) and cannot exceed aggregate bond issuance \( (b_{t+1} \leq B_{t+1}) \). Deposits earn the non-contingent gross return \( R_{x,t} \). Depositors receive net payments \( \pi_t \) from the bankers that are part of their family. In return, they endow new bankers in their family with start-up transfers, comprising a constant component \( \kappa \) and an amount equal to their holdings of government bonds \( q_t b_{t+1} \).

Let \( \Lambda_{t,t+i} = \frac{v_{g,t+i}}{v_{g,t}} \) be the stochastic discount factor of the banker family between period \( t \) and \( t + i \). The first order conditions are:

\[
q_t = \mathbb{E}_t \{ \zeta \Lambda_{t+1} d_{t+1} \} \quad (35)
\]

\[
R_{x,t} = 1 \quad (36)
\]

**Bankers.** Banker intermediaries lend funds obtained by depositors to firms. A banker \( j \) starts a period with wealth \( W_{j,t} \), obtains deposits \( X_{j,t} \) and makes loans \( L_{j,t} \) to firms. The balance sheet is given by:

\[
L_{j,t} = W_{j,t} + X_{j,t} \quad (37)
\]

Depositors earn a non-contingent gross return \( R_{x,t} \) by giving funds to bankers. The assets \( L_{j,t} \) earn the return \( R_t \). The growth in equity depends on the difference between the return on loans \( R_t \) and the interest rate on deposits \( R_{x,t} \) as well as the amount of assets \( L_{j,t} \):

\[
W_{j,t+1} = R_t L_{j,t} - R_{x,t} X_{j,t}
\]

\[
= R_t L_{j,t} - R_{x,t} (L_{j,t} - W_{j,t})
\]

\[
= (R_t - R_{x,t}) L_{j,t} + R_{x,t} W_{j,t} \quad (39)
\]

A bank supplies loans in period \( i \geq 0 \) if:

\[
\mathbb{E}_t \{ \zeta^i \Lambda_{t+i} (R_{t+i} - R_{x,t+i}) \} \geq 0 \quad (41)
\]

With market imperfection this equation may be slack. An arbitrage opportunity due to market imperfections makes it optimal for bankers to build up net worth until they become depositors and not to pay dividends in the meantime. A banker \( j \)'s objective is therefore to maximize expected terminal wealth \( P_{j,t} \):

\[
P_{j,t} = \max \mathbb{E}_t \sum_{s=t}^{\infty} (1 - \phi) \phi^{s-t} \zeta^{s-t} \Lambda_{t,s} W_{j,s+1}
\]

\[
= \max \mathbb{E}_t \sum_{s=t}^{\infty} (1 - \phi) \phi^{s-t} \zeta^{s-t} \Lambda_{t,s} [(R_s - R_{x,s}) L_{j,s} + R_{x,s} W_{j,s}] \quad (43)
\]
The problem can be reformulated as

\begin{align}
P_{j,t} &= \nu_t L_{j,t} + \eta_t W_{j,t} \quad (44) \\
\nu_t &= \mathbb{E}_t \{(1 - \phi)(R_t - R_{x,t}) + \phi \zeta L_{t,t+1} x_{t,t+1} \nu_{t+1}\} \quad (45) \\
\eta_t &= \mathbb{E}_t \{(1 - \phi) R_{x,t} + \phi \zeta L_{t,t+1} z_{t,t+1} \eta_{t+1}\} \quad (46)
\end{align}

where \(x_{t,t+1} = \frac{L_{j,t+1}}{L_{j,t}}\) is the gross growth rate in assets and \(z_{t,t+1} = \frac{W_{j,t+1}}{W_{j,t}}\) is the gross growth rate in net worth.

An incentive constraint limits the bankers’ ability to borrow from households:

\[ P_{j,t} \geq \lambda L_{j,t} \quad (47) \]

The incentive constraint (47) is motivated by a moral hazard problem between bankers and depositors. In each period, bankers can choose to divert their assets and depositors can only recover a share \((1 - \lambda)\), but it is too costly for them to enforce the repayment of the remaining share \(\lambda\), which is kept by the banker family. The incentive constraint can be expressed as a linear function of net worth:

\[ L_{j,t} \leq \frac{\eta_t}{\lambda - \nu_t} W_{j,t} = \chi_t W_{j,t} \quad (48) \]

Notably, the leverage ratio \(\chi_t\) – the ratio between assets to equity – depends on \(R_t\) and is time-varying. If the incentive constraint binds, the assets a banker can acquire depend positively on the banker’s net worth:

\begin{align}
L_{j,t} &= \chi_t W_{j,t} \quad (49) \\
W_{j,t+1} &= [(R_t - R_{x,t}) \chi_t + R_{x,t}] W_{j,t} \quad (50) \\
z_{t,t+1} &= (R_t - R_{x,t}) \chi_t + R_{x,t} \quad (51) \\
x_{t,t+1} &= \frac{\chi_{t+1} z_{t,t+1}}{\chi_t} \quad (52)
\end{align}

Since \(\chi_t\) does not depend in firm specifics \(j\), aggregate loans and total net worth in the economy are:

\begin{align}
L^b_t &= \chi_t W_t \quad (53) \\
W_t &= \phi [(R_{t-1} - R_{x,t-1}) \chi_{t-1} + R_{x,t-1}] W_{t-1} + \kappa + q_t \gamma_t B_{t+1} \quad (54)
\end{align}

The first term of (54) reflects net worth accumulation of the share of bankers surviving \(\phi\), the right term reflects the share of newly endowed bankers \((1 - \phi)\). Let \(\gamma_t \in [0, 1]\) be the domestic share of total government bonds \(B_{t+1}\). Then new bankers’ wealth comprises the fixed amount \(\kappa\) and all domestically held sovereign priced at \(q_t\).
**Static limit.** In the calibration, I investigate the static limit arising from a zero survival probability of bankers ($\phi = 0$). I further assume risk-neutral family preferences and discounting ($\Lambda_{t,t+1} = 1$ and $\zeta = \frac{1}{1+r}$) and use the first order condition of the depositors for within-period lending, yielding a zero return ($R_{x,t} = 1$). A banker $j$ solves the maximization problem:

$$P_{j,t} = \max_{L_{j,t}} (R_t - 1)L_{j,t} + W_{j,t}$$

s.t. $L_{j,t} \leq \frac{1}{\lambda - R_t - \frac{1}{1+r}} W_{j,t}$

Bankers only supply loans if $R_t \geq 1$ and the incentive constraint is binding for $R_t < \lambda(1+r) + 1$. Aggregate net worth is $W_t = \kappa + q_t \gamma_t B_{t+1}$ and constrained bankers’ total supply of loans is:

$$L^b_t = \frac{\kappa + q_t \gamma_t B_{t+1}}{\lambda - R_t - \frac{1}{1+r}}$$

### 7.3 Recursive formulation

The private sector’s state in its most general form is given by the aggregate state $\Omega = (z, B, N, W_e)$ consisting of productivity, sovereign assets, aggregate employment and total pre-existing net worth, the current government’s policy $(B', \tau, d)$ and a sequence of future policies $(B^{\prime \prime}, \tau', \infty, d', \infty)$ where the superscripts indicate the first and last period of the sequence of future policies. However, in this paper I restrict attention to Markov equilibria. Therefore, it is w.l.o.g. to only consider government policies that are given by a constant function of the state $D : \Omega \rightarrow (B', \tau, d)$ in equilibrium and index the private sector value functions with this government policy. Given $\Omega$ and $D$ the private sector agents know $(B', \tau, d)$. Further, let $\Omega' = (z', B', N', W'_e)$ be next period’s aggregate state.

Let $E(\Omega; D)$ be the value of an employed worker and $U(\Omega; D)$ be the value of an unemployed worker. The workers’ problem can be recursively defined as:

$$E(\Omega; D) = \left(1 - \frac{s}{N}\right) \left( u(w - \tau) + \beta \mathbb{E}_z \{(1 - \xi)E(\Omega'; D) + \xi U(\Omega'; D)\} \right)$$

$$U(\Omega; D) = u(T) + \beta \mathbb{E}_z \{\lambda_w(\theta(1 - N + s, v))E(\Omega'; D)$$

$$+ (1 - \lambda_w(\theta(1 - N + s, v)))U(\Omega'; D)\}$$

Taking government policy $D$ as given, firms choose firing $s$ and vacancies $v$. Expected profit can be expressed recursively as:

$$J(\Omega; D) = \left(1 - \frac{s}{N}\right) \left( z - Rw - \mathbb{E}_k(k|k \leq \hat{k}) + \frac{1}{1+r} \mathbb{E}_z \{(1 - \xi)J(\Omega'; D)\} \right)$$
Given the value $\mathcal{J}(\Omega; \mathcal{D})$ the value of a vacancy is

$$V(\Omega; \mathcal{D}) = - Ra + \lambda_f(\theta(1 - N + s, v)) \frac{1}{1 + r} \mathbb{E}_z \{ \mathcal{J}(\Omega'; \mathcal{D}) \}$$  \hfill (61)$$

The firms’ pre-financing condition shows up in the interest payments $R$ on wages and vacancies and in the probability of separation $(1 - \frac{s}{N})$.

Workers and firms act competitively and take prices as given. They also take the law of motion of the aggregate employment $N' = \mathcal{H}(\Omega; \mathcal{D})$ and wealth $W'_e = W_e(\Omega; \mathcal{D})$ and thus the evolution of the next period’s state $\Omega' = (z', B', N', W'_e)$ as given.

Let $P$ be a banker’s value function and let $G$ be the value function of a depositor. The problems of the banker families can be written recursively:

$$P(\Omega; \mathcal{D}) = \max_{W'_e, L_b} (1 - \phi)W'_e + \phi \zeta \mathbb{E}_z \{ P(\Omega'; \mathcal{D}) \}$$  \hfill (62)$$

s.t. $W'_e = (R - R_x)L_b + R_x W_e$  \hfill (63)$$

$\lambda L_b \leq P(\Omega; \mathcal{D})$  \hfill (64)$$

$$G(\Omega; \mathcal{D}) = \max_{g, b', X} v(g) + \zeta \mathbb{E}_z \{ G(\Omega'; \mathcal{D}) \}$$  \hfill (65)$$

s.t. $g = \pi + d(b - q b' + R_x - X)$  \hfill (66)$$

$b = \gamma B$  \hfill (67)$$

However, the government cannot simply choose the optimal constant government policy $\mathcal{D}$ because it lacks commitment. Therefore, the definition of equilibrium includes the indirect value functions for the private sector that consider one-time deviations, situations in which the government chooses an arbitrary policy $(\tilde{B}', \tilde{\tau}, \tilde{d})$ in this period that deviates from the “constant” policy $\mathcal{D}$ to which it returns in the following period. These value functions of the private sector facing a one-time policy deviation $\tilde{\mathcal{D}} = (\tilde{B}', \tilde{\tau}, \tilde{d}; \mathcal{D})$ are given by:

$$\tilde{J}(\Omega; \tilde{\mathcal{D}}) = \left(1 - \frac{s}{N}\right) \left( z - \tilde{R} \tilde{w} - \mathbb{E}_k(k|k \leq \tilde{k}) + \frac{1}{1 + r} \mathbb{E}_z \{ (1 - \xi)J(\Omega'; \mathcal{D}) \} \right)$$  \hfill (68)$$

$$\tilde{\nu}(\Omega; \tilde{\mathcal{D}}) = - \tilde{R} a + \lambda_f(\tilde{\theta}(1 - N + \tilde{s}, \tilde{v})) \frac{1}{1 + r} \mathbb{E}_z \{ J(\Omega'; \mathcal{D}) \}$$  \hfill (69)$$

$$\tilde{\xi}(\Omega; \tilde{\mathcal{D}}) = \left(1 - \frac{s}{N}\right) \left( u(\tilde{w} - \tilde{\tau}) + \beta \mathbb{E}_z \{ (1 - \xi)E(\Omega'; \mathcal{D}) \} \right)$$

$$+ \left( \frac{s}{N} \right) \tilde{U}(\Omega; \tilde{\mathcal{D}}) + \left( \frac{s}{N} \right) \tilde{U}(\Omega; \tilde{\mathcal{D}})$$  \hfill (70)$$

$$\tilde{U}(\Omega; \tilde{\mathcal{D}}) = u(T) + \beta \mathbb{E}_z \{ \lambda_w(\tilde{\theta}(1 - N + \tilde{s}, \tilde{v}))E(\Omega'; \mathcal{D}) \}$$

$$+ \{ 1 - \lambda_w(\tilde{\theta}(1 - N + \tilde{s}, \tilde{v})) \} \tilde{U}(\Omega; \tilde{\mathcal{D}})$$  \hfill (71)$$

Note that due to a different current government policy, the next period’s aggregate state $\Omega' = (z', \tilde{B}', \tilde{N}', \tilde{W}'_e)$ is likely to differ from before.
7.4 Computation

I use collocation methods to solve for the value functions on a grid for Ω = (z, B, N). I am not only approximating the value functions but also the expected value functions, the law of motions for employment and the price function. The solution algorithm involves the following steps:

1. Grid
   I set up a grid over Ω = (z, B, N) of equi-spaced collocation nodes with grid size $N_z = 21$, $N_B = 25$ and $N_N = 22$. Note that chosen debt levels and next period’s productivity and employment states are not restricted to lie on this grid. The limits for productivity are set such that the lower bound on the probability of $z$ is 0.00001. The space for debt $B$ lies between 0 and 3 in the baseline, making sure this limit is not hit. Employment is chosen to lie between 10 and 99 percent. Note that the range of grids is parameter-dependent.

2. Initialization
   I fix a set of parameter values. Wages as a function of productivity are calculated. Initial guesses for the value functions are determined. Initial debt prices are set to the maximum, default probabilities are set to zero.

3. Pre-computation
   I approximate the response of the private sector to a one-time deviation over states, net worth, continuation values and prices. This includes solving for the loan and labor market outcome such that it delivers an approximation of the law of motion of employment $H$.

4. Solving of value and policy functions by collocation methods
   I approximate each value function in the private sector by solving for the $N_s = N_z \times N_B \times N_N$ coefficients using linear splines. Given a guess for the coefficients, I iterate until the coefficients solve the $N_s$ equations for each value function given by the Bellman equations using standard methods.

   In the computation I limit the innovations to lie within the 0.00001 and 0.99999 interior of the normal cdf that I split into 200 equi-spaced grid points and recover the shock values using the inverse cdf on this grid. Each shock value is associated with a certain productivity level given by the AR(1) process of $z$. I compute expectations using a linear spline to evaluate the expected value functions at these 200 productivity levels and weigh them by the probability mass around the shocks.

5. Approximate implementability constraints
   I use the value functions obtained in the previous step together with the law of motion of employment $H$ from the pre-computation to approximate implementability constraints
that are dependent on the one-time deviation. These include future employment, current firing and current job finding probabilities.

6. Best possible one-time deviation $\delta$
   Using the expected value functions, I compute the best possible one-time deviation of the government using golden search. In this step, for each possible $B'$ the price function delivers $q$, the implementability constraints deliver $N'$, $s$ and $p$ and the government budget constraint delivers $\tau$ to maximize the value of repayment. Note that the search is entirely continuous in debt issuance. I check for multiplicity in employment and a single optimum.

7. Update bond price
   Given the new value of repayment, I maximize over repayment and default states to update the price function.

8. Iterate until convergence
   I compute the indirect value function of private sector. I compare all value and price functions together with the law of motion for employment to the ones of the previous iteration. If the norm is smaller than a fixed convergence criterion I stop, otherwise I go back to step 4.

   In the procedure, I update of the government value functions and debt price schedule slowly to ensure convergence. The underlying difficulty is that since the value functions of the government also enter the constraint (the bond pricing) it is not a contraction.
References

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