Learning About Voter Rationality*

Scott Ashworth† Ethan Bueno de Mesquita‡ Amanda Friedenberg§

July 12, 2016

Abstract

An important empirical literature asks whether voters are rational. It tries to answer by examining the response of electoral outcomes to events outside the control of politicians, e.g., natural disasters or economic shocks. The argument is that rational voters should not base electoral decisions on such events and, so, evidence that these events affect electoral outcomes is evidence of voter irrationality. We show that such events can affect electoral outcomes, even if voters are rational and have instrumental preferences. The reason is that these events change voters’ opportunities to learn new information about incumbents. Thus, identifying voter (ir)rationality requires more than just identifying the impact of exogenous shocks on electoral fortunes. Our analysis highlights systematic ways in which electoral fortunes are expected to change in response to events outside incumbents’ control. Such results can inform empirical work attempting to identify voter (ir)rationality.

*We are particularly indebted to Anthony Fowler. We have also benefited from comments by John Bullock, Alan Gerber, Greg Huber, Claire Lim, Andrew Little, Neil Malhotra, Alessia Russo, Ed Schlee, Mike Ting, Dustin Tingley, Stephane Wolton, seminar audiences at Cornell, Emory, the Juan March Institute, Princeton, Stanford, Vanderbilt, Virginia, Yale, and the participants in the EITM Summer Institutes at Princeton (2012) and Mannheim (2014). Friedenberg thanks Caltech for generous hospitality.
†Harris School of Public Policy Studies, University of Chicago, email: sashwort@uchicago.edu
‡Harris School of Public Policy Studies, University of Chicago, email: bdm@uchicago.edu.
§W.P. Carey School of Business, Arizona State University, email: amanda.friedenberg@asu.edu
The literature on voter behavior has long been interested in evaluating voters’ competence to fulfill their electoral responsibility. The early literature focused on whether voters are competent, in the sense of being “sufficiently informed” (Campbell et al., 1960; Fair, 1978; Kinder and Sears, 1985; Popkin, 1991; Sniderman, Brody and Tetlock, 1993; Lupia, 1994; Delli Carpini and Keeter, 1996). A more recent literature focuses on whether voters are competent, in the sense of being “sufficiently rational” (Achen and Bartels, 2004; Wolfers, 2002; Leigh, 2009; Healy, Malhotra and Mo, 2010).

The question of voter rationality is important for two reasons. First, the answer is central to normative debates about the merits of electoral democracy (Downs, 1957; Campbell et al., 1960; Key, 1966; Fiorina, 1981; Fair, 1978; Kinder and Sears, 1985; Popkin, 1991; Sniderman, Brody and Tetlock, 1993; Lupia, 1994; Delli Carpini and Keeter, 1996). Second, there is an extensive theoretical literature that assumes voter rationality and claims to provide insight into a variety of political phenomena, e.g., the incumbency advantage, term limits, judicial review, delegation, campaign finance, local public good provision, special interest politics, the effects of the media, electoral pandering, federalism, and electoral rules. If actual voter behavior dramatically diverges from the assumptions in those models, then there is reason to be skeptical about that research agenda.

An important empirical literature has developed to assess whether voters are indeed rational. That literature examines the response of electoral outcomes to exogenous shocks outside the control of politicians. The idea is that, if voters are rational, then incumbents’ electoral fortunes should be unaffected by such shocks. While several studies find that incumbent electoral fortunes are unaffected by exogenous shocks to voter welfare (Abney and Hill, 1966; Ebeid and Rodden, 2006; Kayser and Peress, 2012), the dominant view is that incumbent electoral fortunes do suffer following such exogenous shocks (Achen and Bartels, 2004; Wolfers, 2002; Leigh, 2009; Healy, Malhotra and Mo, 2010). The literature has interpreted this finding as evidence for voter irrationality.

This paper takes the identified impact of shocks on electoral fortunes as given, and focuses on what it tells us about about voter rationality or irrationality. We argue that—even if voters are rational—exogenous shocks should be expected to affect incumbents’ electoral fortunes. This is because such exogenous shocks can change the voters’ opportunities to learn new information about an incumbent’s quality (or characteristics). We show that this change in voter information is typically sufficient to change both voter behavior and the incumbent’s electoral fortunes. Importantly, this is the case even if (as is standard in the formal literature) voters only have instrumental

---

1 For papers covering these various topics see, among many others, Banks and Sundaram (1993); Lohmann (1998); Persson and Tabellini (2000); Canes-Wrone, Herron and Shotts (2001); Besley and Burgess (2002); Coate (2004); Maskin and Tirole (2004); Ashworth (2005, 2006); Ashworth and Bueno de Mesquita (2006, 2008); Besley (2006); Besley and Prat (2006); Myerson (2006); Gehlbach (2007); Gordon, Huber and Landa (2007); Fox and Van Weelden (2010); Daley and Snowberg (2011); Fox and Jordan (2011); Fox and Van Weelden (2012); Fox and Stephenson (2011, 2015); Almendares and Le Bihan (2015); Eggers (2015).

2 These shocks typically take the form of either a natural disasters (Abney and Hill, 1966; Achen and Bartels, 2004; Healy and Malhotra, 2010; Healy, Malhotra and Mo, 2010; Bechtel and Haimmueller, 2011; Gasper and Reeves, 2011; Cole, Healy and Werker, 2012; Huber, Hill and Lenz, 2012; Chen, 2013) or economic shocks originating outside of the local economy (Ebeid and Rodden, 2006; Wolfers, 2002; Leigh, 2009; Kayser and Peress, 2012).
preferences—i.e., preferences for governance outcomes—and do not blame the incumbent for events outside of her control. Hence, our model suggests that evidence of electoral fortunes responding to events outside an incumbent’s control does not, on its own, entail the conclusion that voters are irrational.

To get a sense of the argument, consider the following example. The occurrence of a hurricane is a random shock, outside the control of the incumbent. But the damage caused by the hurricane depends on the quality of infrastructure maintenance, emergency preparedness, and so on. These, in turn, depend on the preparedness of the government.

Suppose that there are two types of politicians: high quality and low quality. Voters would like to elect high quality politicians, since they are expected to provide good governance outcomes in the future. At the same time, high quality incumbents are better at preparing for storms than are low quality incumbents. Thus, the presence of a hurricane gives voters the opportunity to learn about the quality of the incumbent which, in turn, affects their expectations of future government performance.

To make this more concrete, suppose that the hurricane provides voters with stark information about the level of preparedness: If a hurricane does not occur, voters remain uninformed about the level of preparedness. But, if a hurricane does occur, voters observe the effects of the storm, perfectly learning the level of preparedness. Absent a hurricane, the voters learn nothing new about the incumbent’s quality. But, with a hurricane, they can infer her quality—be it high or low. Rational voters should use this additional information about the incumbent’s quality in forming their assessments of the incumbent’s expected future performance. As a result, the occurrence of a hurricane may well influence rational voters’ electoral decisions.

This change in voter behavior can affect the incumbent’s expected electoral fortunes. To see this, suppose that, ex ante, the voters believe that the incumbent is more likely to be high quality than a future electoral challenger. Then, if there is no hurricane, they reelect the incumbent. If, however, there is a hurricane, the voters learn about the incumbent’s quality. If the level of preparedness is high, they learn the incumbent is high quality and reelect her. But, if the level of preparedness is low, they learn she is low quality and replace her. By giving the voters new information, the hurricane creates the possibility that the incumbent will lose—something that does not happen in the absence of a hurricane. (Notice, if the voters started with the belief that the incumbent was less likely to be high quality than the challenger, then the effect of the hurricane would be reversed.)

Now imagine that an empiricist collects data generated by this example. Some locations have a hurricane and some do not. (For simplicity, assume that the probabilities of high and low quality incumbents is the same in each location.) The empiricist first calculates the reelection rate in locations that did not have a hurricane and finds it to be one. She then calculates the reelection rate in locations that did have a hurricane. It is equal to the share of incumbents who were of high quality.

Our formal model does not use of such stark differences in information. We do so here to make the point as clearly as possible.
high quality, i.e., a number less than one. Hence—just as in the literature—the empiricist correctly concludes that the occurrence of a hurricane causes a reduction in expected incumbent electoral fortunes. This is in spite the fact that the incumbent bears no responsibility for the hurricane. And this observation is not evidence of voter irrationality. In fact, quite the opposite: it is the result of voters rationally trying to select high quality politicians.

Our subsequent analysis builds on this simple example. Specifically, we assume voter rationality and use that hypothesis to deduce how incumbent electoral fortunes should be affected by exogenous shocks. We show that, in many environments, exogenous shocks to voter welfare should be expected to impact incumbent electoral fortunes; in those environments, evidence that electoral fortunes do not suffer would be evidence of voter irrationality. Likewise, in some (but not all) of those environments, incumbent electoral fortunes should decline with exogenous shocks; in those environments, evidence that they do in fact decline would support the rational voter hypothesis.

Our analysis highlights the fact that there are systematic ways in which electoral fortunes should be impacted by events outside the incumbents’ control. In particular, how electoral fortunes should be impacted depends on both how governance outcomes are produced and on prior beliefs about candidates. Thus, identifying voter (ir)rationality requires more than simply identifying the impact of exogenous shocks on electoral fortunes. The hope is that the results here can serve to guide future empirical work and improve the identification of voter (ir)rationality.

In what follows, we focus on how voters learn about characteristics of incumbent politicians from exogenous shocks. We abstract away from the fact that—after the shock and prior to the election—the incumbent may engage in disaster relief. When the incumbent can engage in disaster relief, the relief might also provide information about the quality of the incumbent. So, even in an extreme case where the disaster itself provides no information about the quality of the incumbent, the disaster relief can provide such information and, thereby, affect the incumbent’s electoral fortunes (positively or negatively) (Ashworth, Bueno de Mesquita and Friedenberg, Forthcoming).

The potential for disaster relief complicates the researcher’s ability to identify voter (ir)rationality by identifying the effect of exogenous shocks on reelection fortunes. This fact is important for the literature: A series of papers finds that controlling for the quality of the disaster relief mitigates the (negative) correlation between shocks to voter welfare and incumbent electoral fortunes (Healy and Malhotra, 2010; Bechtel and Hainmueller, 2011; Gasper and Reeves, 2011; Cole, Healy and Werker, 2012). These results are interpreted as evidence of voter rationality. But, such inferences are also premature. Whether this is or is not evidence of voter rationality, again, depends on how governance outcomes are produced and prior beliefs.

\[4\] In the context of a different model of accountability, Gailmard and Patty (2014) make a related but distinct point: Rational voters may well reward politicians who do not engage in disaster prevention but do engage in disaster relief.
1 Motivating Examples

We will analyze how disasters affect voter behavior and electoral fortunes, using the Bayesian learning framework that is standard in the literature (Achen, 1992; Bartels, 1993; Gerber and Green, 1999). We begin by adapting the specific model in Wolfers (2002), since it speaks directly to the issue of observable shocks and electoral fortunes. This specific model justifies the conventional wisdom: if voters are rational, electoral fortunes do not depend on observable shocks. Then, as a robustness exercise, we consider a minor modification to the example. We show that the conclusion is not robust, i.e., the conventional wisdom does not obtain. The comparison of the two examples will raise a series of questions, to be addressed by our subsequent analysis.

1.1 Example 1: When Shocks Have No Effect

There are two politicians, an incumbent and a challenger. Each is either a good type ($\theta$) or a bad type ($\overline{\theta}$). The probability that politician $P$ is a good type is $\pi_P \in (0, 1)$. (The canonical model in the literature instead has a normally distributed prior on type. The fact that there are two types is the only change we make relative to the canonical model. It will serve to simplify the analysis later.)

In each period, the voter observes a governance outcome, which for concreteness we sometimes refer to as public goods. The governance outcome in period $t = 1, 2$, written $g_t$, depends on the incumbent’s type $\theta_t$, as well as two shocks. Specifically,

$$g_t = \theta_t - \omega_t + \epsilon_t.$$

The first shock is an observable disaster. We refer to a particular realization of this random variable, $\omega_t$, as the disaster intensity in period $t$. Note, the disaster intensity is positive, so that higher disaster intensities lower the governance outcome. There is also an unobservable shock that is distributed according to the standard normal distribution. Its realization in period $t$ is $\epsilon_t$. All the random variables are independent of one another.

Between the two governance periods, there is an election. In the election, the voter reelects the incumbent if the voter’s posterior belief that the incumbent is a good type is higher than his prior belief that the challenger is a good type. Using Bayes’ rule, the voter reelects if and only if

$$\pi_I \varphi(g_1 + \omega_1 - \overline{\theta}) \geq \pi_C, \quad (1)$$

where $\varphi$ is the pdf of the standard normal distribution.

It is worth pausing here to see what the rational voter is doing. The voter prefers the candidate with the highest type. He does not observe the type of each candidate. But, he does observe the disaster intensity $\omega_1$ and the governance outcome $g_1$. Note, $g_1$ is a signal of the incumbent’s type—one that is biased by the disaster intensity $\omega_1$. The voter uses this information to try to learn about the incumbent’s type $\theta_I$. He does so by “filtering out” the bias caused by the disaster—adding $\omega_1$
to \( g_1 \)—and then forming his posterior beliefs based on the “filtered out outcome.”

Rearranging Equation (1), the voter reelects if and only if

\[
\frac{\varphi(g_1 + \omega_1 - \theta)}{\varphi(g_1 + \omega_1 - \bar{\theta})} \geq \frac{\pi_C}{1 - \pi_C} \frac{1 - \pi_I}{\pi_I}.
\]

The left-hand side of this inequality is strictly increasing and continuous in \( g_1 + \omega_1 \). So there is a unique number, \( r \), such that Equation (2) holds with equality if and only if \( g_1 + \omega_1 = r \).

Notice that \( r \) represents a benchmark that the observed variables—governance outcomes and disaster intensity—must meet. If there is no disaster (\( \omega_1 = 0 \)), the voter would adopt \( r \) as a benchmark for governance outcomes, i.e., reelecting the incumbent if and only if the governance outcome meets the benchmark \( r \). But, if there is a disaster (\( \omega_1 > 0 \)), the voter ratchets down that benchmark by exactly the level of disaster intensity (\( \omega_1 \)).

To see how this model gives rise to the standard intuitions, we show analogues of Wolfers’s (2002) claim that “events unrelated to a governor’s competence should have no effect on the voting decisions of rational agents” [p. 4, emphasis in original].

First, the voter adopts a reelection threshold that governance outcomes must meet. He updates this threshold to exactly offset the effect of disaster intensity on governance outcomes. To see this, suppose the disaster intensity is \( \omega_1 \) and set \( \hat{g}(\omega_1) \equiv r - \omega_1 \). Then, \( \hat{g}(\omega_1) \) represents the voter’s reelection threshold in terms of governance outcomes. That is, the voter reelects if and only if the observed level \( g_1 \) meets this threshold, i.e.,

\[
g_1 \geq \hat{g}(\omega_1) = r - \omega_1.
\]

So, if a disaster reduces government performance by 1 unit, the voter reduces the reelection threshold by exactly 1 unit.

Second, the incumbent’s electoral fortunes do not depend on the presence or magnitude of a disaster. The probability the incumbent is reelected (given that \( \omega_1 \) is observed) is the probability that \( g_1 \geq \hat{g}(\omega_1) \). That is,

\[
\Pr(\theta_I - \omega_1 + \epsilon_1 \geq \hat{g}(\omega_1)) = \Pr(\theta_I + \epsilon_1 \geq r).
\]

This probability is \textit{constant} in \( \omega_1 \), so incumbent electoral fortunes are indeed constant in disaster intensity. This is exactly the standard intuition.

1.2 Example 2: When Shocks Have An Effect

In the above example, governance outcomes were an additive function of type and disaster intensity. Such additive separability is assumed not for verisimilitude, but for tractability. And it does make for an elegant model that is easy to work with. As such, there is a temptation to conclude that

---

\(^5\)This follows from the fact that the normal distribution has the monotone-likelihood ratio property.
its implications must be robust. If this is correct then, given extant empirical findings, the model suggests voters are irrational. But if additive separability is driving the results, there may be reason to be concerned about this interpretation.

Below we take a first step toward showing that the impression of robustness is spurious. We do so with a second example, which nests the additively separable example as a special case. We show that the conclusions of the additively separable model need not follow.

We make only one change to Example 1. The level of public goods in period \( t \) is now

\[
g_t = (\theta_t - \omega_t)^k + \epsilon_t,
\]

where \( k > 0 \) and each possible disaster intensity \( \omega_t \) is between 0 and \( \theta_t \). (This ensures that public goods are decreasing in disaster intensity.) The additively separable model is the special case where \( k = 1 \).

Repeating the logic that led to Equation (2), the voter reelects the incumbent if and only if

\[
\varphi(g_1 - (\theta - \omega_1)^k) = \frac{\pi_C}{1 - \pi_I} - \frac{\pi_I}{1 - \pi_C} \geq \pi_C - \pi_I.
\]

Using the formula for the pdf of the standard normal, this means the voter reelects if and only if

\[
e^{-\frac{(g_1 - (\theta - \omega_1)^k)^2}{2}} \geq \frac{\pi_C}{1 - \pi_C} - \frac{\pi_I}{1 - \pi_I}.
\]

Taking logs on both sides, we can calculate the voter’s reelection threshold. It is

\[
\hat{g}(\omega_1) = \frac{\log \frac{\pi_C}{1 - \pi_C} - \frac{\pi_I}{1 - \pi_I}}{2 \frac{(\theta - \omega_1)^k - 2 (\theta - \omega)^k}{2}} + \frac{(\theta - \omega_1)^k + (\theta - \omega)^k}{2}.
\]

The probability the incumbent is reelected (given that \( \omega_1 \) is observed) is the probability that \( g_1 \geq \hat{g}(\omega_1) \), or, equivalently, the probability that \((\theta_I - \omega_1)^k + \epsilon_1 \geq \hat{g}(\omega_1)\). This can be written as

\[
1 - \pi_I \Phi(\hat{g}(\omega_1) - (\theta - \omega_1)^k) - (1 - \pi_I) \Phi(\hat{g}(\omega_1) - (\theta - \omega_1)^k),
\]

where \( \Phi \) is the CDF of the standard normal distribution.

Taken together, Equations (3)-(4) allow us to directly compute the probability of reelection as a function of the level of disaster intensity \( \omega_1 \). Figure 1.1 does so for two values of \((\pi_I, \pi_C)\) and three values of \( k \). The left-hand cell is a case where \( \pi_I = \frac{1}{3} \) and \( \pi_C = \frac{1}{2} \), so that \( ex \ ante \) the Challenger is more likely than the Incumbent to be a high type. The right-hand cell is a case where \( \pi_I = \frac{2}{3} \) and \( \pi_C = \frac{1}{2} \), so that \( ex \ ante \) the Incumbent is more likely than the Challenger to be a high type. Within each cell, there are graphs of the probability of reelection for \( k = \frac{1}{2} \), \( k = 1 \), and \( k = \frac{3}{2} \).

The figures confirm our analysis of Example 1. When \( k = 1 \) the probability of reelection is constant in \( \omega_1 \). But this is not the case when \( k = \frac{1}{2} \) or when \( k = \frac{3}{2} \). In those cases, \((\pi_I, \pi_C)\) influences
the effect of disaster intensity on the probability of reelection. When \( k = \frac{1}{2} \), the probability of reelection is decreasing in the left-hand cell and increasing in the right-hand cell. When \( k = \frac{3}{2} \) the situation is exactly reversed.

### 1.3 Back to the Question

The examples lead to different conclusion about voter rationality. On the one hand, if \( k = 1 \) and electoral fortunes respond to observable shocks, then there is evidence of voter irrationality. On the other hand, if \( k \) is either \( \frac{1}{2} \) or \( \frac{3}{2} \) and voters are rational, then electoral fortunes should respond to observable shocks.

A priori, there is no principled reason to prefer some particular value of \( k \). As such, it is difficult to know what the model implies about the relationship between existing empirical results and inferences about voter rationality. To have a view on the correct choice of \( k \), one would have to have a substantive argument about which value of \( k \) best approximates the world being represented by the model. But in the highly computational approach we’ve taken thus far, the choice of \( k \) appears entirely unmotivated.

To start thinking about this issue, recall the hurricane example from the Introduction. There, disaster intensity altered how much information the voters had about the incumbent’s type: When there was no disaster (\( \omega_1 = 0 \)), the voter had no information about preparedness. But when there was a disaster (\( \omega_1 > 0 \)), the voter had information about preparedness. As a result, we saw that disaster intensity could affect electoral fortunes. In what follows, we will show that this qualitative feature of the political environment—how disaster intensity affects voter information—is the key for understanding more generally how disasters affect voter behavior and incumbent electoral fortunes.

These results will make it clear that the choice of \( k \) in the example of this section is in fact a highly substantive one. In particular, it translates directly into an assumption about qualitative features of the political environment which determine how disasters affect voter information. It turns out that, if \( k < 1 \), then larger disaster intensities imply that voters learn more information about the incumbent’s type. If \( k > 1 \), then larger disaster intensities imply that voters learn less information about the incumbent’s type. The case of \( k = 1 \) is the unique instance in which disaster intensity has no effect on how much information the voter learns. Moreover, as we will show, it is
this knife-edge case—where disasters provide no information about the incumbent’s type—that is responsible for the conventional wisdom, i.e., that electoral fortunes are unaffected by disasters.

Once we establish these qualitative results we will be able to return to the substantive question motivating the literature: What information would an empiricist need to make inferences about voter rationality? Our results will show that, in all but the additively separable case, simply observing a relationship between electoral fortunes and observable shocks is not sufficient. But we will also be able to say something more constructive about what additional information would suffice—e.g., information about \( \pi_I, \pi_C \), or information about how governance outcomes are produced.

2 The General Model

In this section, we generalize the model discussed in Section 1. In particular, we will back away from specific functional forms for the technology. This will allow for a more qualitative approach. Specifically, it will allow us to understand the effect of observable shocks on the voter’s information about the incumbent’s type. This will, in turn, permit us to provide a transparent link between, on the one hand, substantively interpretable features of the technology and, on the other hand, a formal measure of informativeness.

2.1 Set Up

There is an Incumbent (I), a Challenger (C), and a Voter (V). We refer to each Politician (P) as “she” and the Voter as “he.” In each of two governance periods, the Voter receives a governance outcome that depends on the type of the Politician in office, an observable level of natural disaster, and an unobservable, idiosyncratic shock. There is an election between the governance periods.

The set of types is \( \{ \theta, \bar{\theta} \} \), where \( \theta \) is the bad type and \( \bar{\theta} > \theta \) is the good type. Write \( \pi_P \in (0,1) \) for the probability that Politician P is type \( \bar{\theta} \). These probabilities are commonly understood by the players. We will say that the Incumbent enters the first governance period ahead if \( \pi_I > \pi_C \) and enters the first governance period behind if \( \pi_C > \pi_I \).

The intensity of the natural disaster in period \( t \) is \( \omega_t \in \Omega \), where \( \Omega \) is a nonempty, open interval in \( \mathbb{R} \). Each \( \omega_t \) is the realization of a random variable that is independent of the Politician’s ability. (The particular distribution will not be relevant.)

The governance outcome in a period is a function of a production technology and a random shock. The production function \( f : \{ \theta, \bar{\theta} \} \times \Omega \rightarrow \mathbb{R} \) is strictly increasing in type (\( \theta \)) and is strictly decreasing in disaster intensity (\( \omega \)). We will be interested in whether natural disasters amplify or mute the effect of type on governance outcomes.

Definition 2.1.

(i) Disasters amplify the effect of type on governance outcomes if, for all \( \omega' > \omega \),

\[
f(\bar{\theta}, \omega') - f(\theta, \omega') > f(\bar{\theta}, \omega) - f(\theta, \omega).
\]
Disasters mute the effect of type on governance outcomes if, for all \( \omega' > \omega \),
\[
f(\overline{\vartheta}, \omega') - f(\overline{\vartheta}, \omega') < f(\overline{\vartheta}, \omega) - f(\overline{\vartheta}, \omega).
\]
When the production function is additive, \( f(\overline{\vartheta}, \omega') - f(\overline{\vartheta}, \omega') = f(\overline{\vartheta}, \omega) - f(\overline{\vartheta}, \omega) \) for all \( \omega, \omega' \). Thus, in that case, disasters neither amplify nor mute the effect of type.\(^6\)

If, in period \( t \), the type of the Politician in office is of type \( \theta_t \), the disaster intensity is \( \omega_t \), and the random shock is \( \epsilon_t \), the governance outcome in that period is \( f(\theta_t, \omega_t) + \epsilon_t \).

Each \( \epsilon_t \) is the realization of a random variable. These random variables are independent of each other, of the Politicians’ abilities, and of the disaster intensities. They are distributed according to an absolutely continuous CDF, \( \Phi \), with a continuously differentiable PDF, \( \phi \). This distribution satisfies three additional requirements: First, for each \( x > x' \geq 0 \), the associated likelihood ratio defined by
\[
g \mapsto \frac{\phi(g - x)}{\phi(g - x')}
\]
is onto with non-zero derivatives. Second, the distribution satisfies the (strict) monotone likelihood ratio property (MLRP) relative to all possible realizations of production: If \( x > x' \), then the associated likelihood ratio
\[
\frac{\phi(g - x)}{\phi(g - x')}
\]
is strictly increasing in \( g \). Third, the PDF is symmetric: for each \( x \in \mathbb{R} \), \( \phi(x) = \phi(-x) \). (The standard normal pdf, \( \varphi \), satisfies these conditions.)

Prior to the game being played, Nature determines the realizations of each Politician’s type and of the random shocks (in all periods). These realizations are not observed by any of the players. Figure 2.1 depicts the timeline: In the initial governance period, the Voter observes the disaster intensity and the governance outcome. This leads to the electoral stage, in which the Voter chooses to reelect the Incumbent or replace her with a Challenger. The winner of the election is the Politician in office in the second governance period. Again, the Voter observes the disaster intensity and the governance outcome, \( g_2 \).

1st Governance Period | Election | 2nd Governance Period

Figure 2.1. Timeline

The Voter’s payoffs are the sum of governance outcomes in the two periods.

\(^6\)Definition 2.1 is a global requirement, i.e., requiring the function satisfy a condition for all \( \omega' > \omega \). We can relax this requirement to instead consider the following localized definition: Say disasters amplify the effect of type at \( (\omega', \omega'') \) with \( \omega'' > \omega' \) if, for all \( \omega \in (\omega', \omega'') \), \( f(\overline{\vartheta}, \omega'') - f(\overline{\vartheta}, \omega') > f(\overline{\vartheta}, \omega) - f(\overline{\vartheta}, \omega) \). And, analogously, for the mutes case.
2.2 Comments on the Model

Before turning to the analysis, we reflect on several features of the model.

First, this is a model of “pure selection.” In the model, the governance outcome is determined entirely by the politician’s type, the disaster, and the shock. The politician in office cannot influence the governance outcome by taking costly actions (either prior to or in response to the disaster). This assumption is made only for simplicity, and does not affect the message of the paper. To see why, recall a well-known feature of agency models of elections: At the time of the election, rational voters are concerned only with electing the politician who affords the highest expected future performance. So, in a two-period model, if there is any uncertainty about the politician’s type, voters are concerned only with “selecting good types” (Fearon, 1999). Thus, the fact that politicians may be able to undertake costly actions does not change the fact that, at the point of the election, rational voter behavior is determined by voter learning (about incumbent quality). When politicians can take costly actions, the actions can also provide information about the incumbent’s type, i.e., above the information provided by the disaster itself (Ashworth, Bueno de Mesquita and Friedenberg, Forthcoming). This can further complicate the voters’ inference problem. But, because the logic of voter behavior does not change in a fundamental way, the main insights of the paper do not change.

Second, when we move away from the additive model, disasters can amplify or mute the effect of type. As a result, it is important to understand what amplify and mute mean, substantively. If disasters amplify the effect of type, then incumbent competence has a larger effect on governance outcomes when disasters are large (versus when they are small). Suppose, as in the example in the Introduction, that good types are better at maintaining infrastructure. In normal times, governance outcomes are relatively unresponsive to how well infrastructure has been maintained. But severe weather events will be particularly devastating if there are weaknesses in the infrastructure—levees will hold or break, for instance. In this case, disasters amplify the effect of type. By contrast, if disasters mute the effect of type, then incumbent competence has a smaller effect on governance outcomes when disasters are large (versus when they are small). For instance, suppose that good types are better at attracting investment. In normal times, good types will oversee better economic performance. But a natural disaster might stop investment, irrespective of the type of the incumbent. In this case, disasters mute the effect of type.

These examples also provide a link to how governance outcomes are informative about the Incumbent’s type. When disasters amplify (resp. mute) the effect of type, the difference in expected performance between competent and incompetent types is larger (resp. smaller) following larger disasters. Thus, during larger disasters, it is easier (resp. harder) for voters to infer the incumbent’s competence from noisy governance outcomes. Section 3 formalizes this link.

Third, we describe the Incumbent as ahead if $\pi_I > \pi_C$ and behind if $\pi_C > \pi_I$. There are two reasons this language makes sense. First, if the Incumbent is ahead, then ex ante she is more likely to win than to lose (and the opposite for an Incumbent who is behind). Second, if the Incumbent is ahead (resp. behind), then absent any new information, the Incumbent wins (resp. loses) reelection.
2.3 Analysis

As in the additive model, the Voter adopts a cutoff rule in the space of governance outcomes. We can characterize this cutoff with a condition analogous to Equation (2). The Voter reelects the Incumbent if the first-period governance outcome, \( g_1 \), is greater than or equal to the (unique) \( \hat{g}(\omega_1) \) given by:

\[
\frac{\phi(g_1) - f(\theta, \omega_1)}{\phi(g_1) - f(\theta, \omega_1)} = \frac{1 - \pi_I}{1 - \pi_C}.
\]

Notice, in the additively separable model of Section 1, \( \hat{g}(\omega_1) \) is \( r - \omega_1 \).

For the analysis of Voter behavior, it will be convenient to work with the loglikelihood ratio,

\[
\ell(g, \omega_1) \equiv \log \frac{\phi(g - f(\theta, \omega_1))}{\phi(g - f(\theta, \omega_1))}.
\]

We can, equivalently, define \( \hat{g}(\omega_1) \) as the solution to \( \ell(\hat{g}(\omega_1), \omega_1) = \beta(\pi_I, \pi_C) \), where

\[
\beta(\pi_I, \pi_C) \equiv \log \left[ \frac{1 - \pi_I}{1 - \pi_C} \right].
\]

Observe that \( \beta(\pi_I, \pi_C) < 0 \) if the Incumbent is ahead and \( \beta(\pi_I, \pi_C) > 0 \) if the Incumbent is behind.

Finally, we calculate the ex-ante probability the Incumbent wins reelection given a disaster intensity \( \omega \). The CDF of governance outcomes is given by

\[
\Gamma(g; \omega) = \pi_I \Phi(g - f(\theta, \omega)) + (1 - \pi_I) \Phi(g - f(\theta, \omega)).
\]

Since the Voter adopts \( \hat{g}(\omega) \) as the benchmark for reelection, the probability that the Incumbent is reelected (given \( \omega \)) is \( 1 - \Gamma(\hat{g}(\omega); \omega) \). (This is the analogue of Equation (4) from Example 2.)

3 Voter Behavior

In Section 1, we saw that, in the case of additively separable production, the Voter’s reelection threshold was monotonic in disaster intensity: Worse disasters were associated with lower reelection thresholds. This is intuitive. For any given level of noise, an increase in disaster intensity lowers the governance outcome. So, an increase in disaster intensity makes the distribution of outcomes “worse.” As a consequence, the Voter might reelect the Incumbent after observing a particular governance outcome that follows a large disaster, despite the fact that he would have replaced the Incumbent had that same governance outcome followed a minor disaster.

Indeed, in the additive model, a stronger property holds: In a natural sense, the Voter’s reelection threshold changes in a way that exactly offsets the effect of the change in the distribution of outcomes. This fact appears to fit with the intuition motivating much of the empirical literature. As such, it will be useful to be more precise about the idea.

For any given level of disaster intensity, \( \omega \), there is an outcome \( g = v(\omega) \) such that, when the
Voter observes $\nu(\omega)$, his posterior belief about the Incumbent’s type equals his prior belief about the Incumbent’s type. Refer to this outcome, i.e., $\nu(\omega)$, as the **neutral news** outcome. We refer to outcomes greater than the neutral news outcome (i.e., $g > \nu(\omega)$) as **good news** outcomes and outcomes less than the neutral news outcome (i.e., $g < \nu(\omega)$) as **bad news** outcomes.

By definition, at the neutral news outcome, the posterior probability that the Incumbent is a good type equals the prior probability. Applying Bayes’ rule, this implies that

$$\frac{\phi(\nu(\omega) - f(\bar{\theta}, \omega))}{\phi(\nu(\omega) - f(\bar{\theta}, \omega))} = 1. \quad (7)$$

Outcomes that are good news (i.e., $g > \nu(\omega)$) raise the Voter’s posterior belief about the Incumbent’s type and outcomes that are bad news (i.e., $g < \nu(\omega)$) lower the Voter’s posterior belief about the Incumbent’s type. (These facts follow from the MLRP.)

For a given level of disaster intensity $\omega$ and a given type $\theta$, $\phi(\nu(\omega) - f(\theta, \omega))$ is the likelihood that the outcome is $\nu(\omega) = f(\theta, \omega) + \epsilon$. So, Equation (7) says that, for a given disaster intensity $\omega$, the likelihood of the neutral news outcome given a good type equals the likelihood of the neutral news outcome given a bad type. Equation (7) and symmetry of $\phi$ imply that we can write the neutral news outcome as

$$\nu(\omega) = \frac{f(\bar{\theta}, \omega) + f(\bar{\theta}, \omega)}{2}.$$  

An implication of this explicit solution is that the neutral news outcome is decreasing in disaster intensity—the more intense the disaster, the lower the neutral news outcome.

Refer back to the additive production function in Section 1: There, the neutral news outcome is $\nu(\omega) = \frac{\pi + g}{2} - \omega$, and the reelection threshold is $\hat{g}(\omega) = r - \omega$. So, if the disaster intensity increases from $\omega$ to $\omega'$, both the neutral news outcome and the reelection threshold decrease by the same amount (specifically, by $|\omega' - \omega|$). In this sense, the change in the Voter’s reelection threshold exactly offsets the change in disaster intensity.

**Definition 3.1.** Let $\omega' > \omega$. Voter behavior **exactly offsets** the effect of disasters if:

$$\hat{g}(\omega') = \hat{g}(\omega) + \nu(\omega') - \nu(\omega).$$

Voter behavior **more than offsets** the effect of disasters if the left-hand side is less than the right-hand side and **less than offsets** the effect of disasters if the left-hand side is greater than the right-hand side.

In the additive model, Voter behavior exactly offsets the effect of disasters. Quite generally, this fact will not hold beyond the additive model. Indeed, without additive separability, the Voter’s reelection threshold need not even be monotonic with disaster intensity—larger disasters need not lead the Voter to lower his benchmark for reelection. The key is that, for any production function that is not additively separable, there is a second effect at work. A change in disaster intensity both shifts the neutral news outcome and changes how informative the outcome is about the Incumbent’s type (in the sense of Blackwell, 1951).
To illustrate these two effects, fix a level of disaster intensity $\omega$. Each type $\theta$ generates a distribution on outcomes, i.e., the probability density of an outcome $g = f(\theta, \omega) + \epsilon$ is the probability density of $\epsilon = g - f(\theta, \omega)$. The Voter’s challenge is to figure out how likely it is that a particular governance outcome came from the distribution associated with the good type (namely, $\phi(g - f(\bar{\theta}, \omega))$) versus the distribution associated with the bad type (namely, $\phi(g - f(\bar{\theta}, \omega))$).

Refer to Figure 3.1. The top picture depicts the densities of outcomes associated with the good type and the bad type when the level of disaster intensity is $\omega$. The bottom picture depicts the densities of outcomes when the disaster intensity is $\omega' > \omega$. In each picture, the density to the left corresponds to the bad type and the density to the right corresponds to the good type. Note, because the level of disaster intensity is higher under $\omega'$ versus $\omega$, the mean of each density is lower under $\omega'$ versus $\omega$, i.e., for each type $\theta$, $f(\theta, \omega') < f(\theta, \omega)$. As a consequence, the neutral news outcome is lower under $\omega'$ versus $\omega$. (Equation (7) says that the neutral news outcome is found at the intersection of these two densities.)

Figure 3.1. Amplifies: Effect of Increase in Disaster on Conditional Distributions

Now let’s turn to the informational effect. Figure 3.1 was drawn for the case when disasters amplify the effect of type. As a consequence, the distance between the mean of the two densities is larger for $\omega'$ versus $\omega$. This increased distance between the two densities corresponds to an
improvement in Voter information—informally, when the means of the densities are further apart, it is easier for the Voter to determine which density a given outcome came from.

It will be convenient to measure this informativeness effect as:

$$\iota(\omega) = f(\theta, \omega) - f(\theta, \omega).$$

By Theorem 3.1 of Ashworth, Bueno de Mesquita and Friedenberg (Forthcoming), if $\omega' > \omega$ and $\iota(\omega') > \iota(\omega)$, then outcomes are more informative (in the sense of Blackwell, 1951) about the incumbent’s type when the disaster intensity is $\omega'$ versus when the disaster intensity is $\omega$. It’s easy to see that disasters amplify (resp. mute) the effect of type if and only if $\iota(\cdot)$ is increasing (resp. decreasing) in disaster intensity. As a consequence, governance outcomes are more informative (resp. less informative) to the Voter following larger disasters, if disasters amplify (resp. mute) the effect of type.

Observe that:

$$f(\theta, \omega) = \nu(\omega) + \iota(\omega) \quad \text{and} \quad f(\theta, \omega) = \nu(\omega) - \iota(\omega).$$

So, for each $\omega \in \Omega$, we can express the loglikelihood ratio in terms of the variables $(g, \nu(\omega), \iota(\omega))$,

$$\ell(g, \omega) = \log \frac{\phi(g - \nu(\omega) + \iota(\omega))}{\phi(g - \nu(\omega) - \iota(\omega))}.$$

This will allow us to separately analyze how changing the disaster intensity changes the Voter’s behavior via a change in the neutral news outcome (i.e., $\nu(\omega)$) and a change in informativeness (i.e., $\iota(\omega)$).

### 3.1 The Neutral News Effect

Increasing disaster intensity from $\omega$ to $\omega'$ decreases the neutral news outcome from $\nu(\omega)$ to $\nu(\omega')$. To understand how this effect impacts the reelection rule, let’s consider a thought experiment in which there is a change to the neutral news outcome with no corresponding change in information. To do so, define a new function:

$$\tilde{\ell}(g) = \log \frac{\phi(g - \nu(\omega') - \iota(\omega))}{\phi(g - \nu(\omega') + \iota(\omega))}.$$

This function is obtained from $\ell(\cdot, \omega)$ by decreasing the neutral news outcome from $\nu(\omega)$ to $\nu(\omega')$, but leaving $\iota(\omega)$ fixed. That is, this function is obtained by shifting $\ell(\cdot, \omega)$ up. (See Lemma B.1, which shows that, for any given $g$, $\tilde{\ell}(g) > \ell(g, \omega)$.)

This upward shift is depicted in Figure 3.2. The dark solid line is the function $\ell(\cdot, \omega)$; the fact that it is increasing follows from the MLRP. The light solid line is the function $\tilde{\ell}(\cdot)$.\(^7\) Let us point to several additional features: First, the neutral news outcome, $\nu(\omega)$, is the outcome where the

\(^7\)In the figure, the two functions are parallel lines. This is true when $\phi$ is a normal density, but not true more generally. Our argument will not rely on this feature of the graphs.
loglikelihood ratio $\ell(\cdot, \omega)$ equals zero. Second, the reelection threshold, $\hat{g}(\omega)$, is the outcome where the loglikelihood ratio $\ell(\cdot, \omega)$ equals $\beta(\pi_I, \pi_C)$. This figure is drawn for a case where the Incumbent is ahead—i.e., ex-ante the Incumbent is more likely than the Challenger to be a good type. In this case $\beta(\pi_I, \pi_C) < 0$ and, as a consequence, the reelection threshold, $\hat{g}(\omega)$, is below the neutral news outcome, $\nu(\omega)$. That is, the Incumbent can gain reelection even if she produces a governance outcome that is somewhat bad news. Third, because $\tilde{L}(\cdot)$ lies above $\ell(\cdot, \omega)$, $\tilde{L}(\cdot)$ intersects $\beta(\pi_I, \pi_C)$ at a governance outcome, $\tilde{g}$, that is below $\hat{g}(\omega)$.

Notice that the outcome $\tilde{g}$ would leave the Voter indifferent between the Incumbent and Challenger, if he accounted for the change in neutral news outcome but ignored any change in informativeness. The fact that $\tilde{g}$ lies below $\hat{g}(\omega)$ reflects the intuition with which we began. When the level of disaster intensity increases, the Voter wants to “filter out” the mechanical effect of the shock on the distribution of outcomes—i.e., he doesn’t blame the Incumbent for events outside of her control. If he ignores the effect of informativeness, then he is willing to reelect the Incumbent at a lower level of performance—i.e., to use a more lax reelection standard. Moreover, the difference between $\hat{g}(\omega)$ and $\tilde{g}$ is exactly the change in the neutral news outcome, i.e., $\hat{g}(\omega) - \tilde{g} = \nu(\omega) - \nu(\omega')$. (See Lemma B.1.) So, if the Voter adopted the benchmark $\tilde{g}$, then the Voter would exactly offset the effect of the disaster.

### 3.2 The Informativeness Effect

To add the informativeness effect, we begin with the function $\tilde{L}$ and allow the level of informativeness to change from $\iota(\omega)$ to $\iota(\omega')$. This gives the function $\ell(\cdot, \omega')$. Figure 3.3 illustrates this change in $\iota(\cdot)$, both for the case where disasters amplify the effect of type and for the case where disasters mute the effect of type. In each of these cases, the function $\ell(\cdot, \omega')$ intersects the function $\tilde{L}$ at the neutral news outcome $\nu(\omega')$. This is because the functions $\ell(\cdot, \omega')$ and $\tilde{L}$ differ only in the level of informativeness; as such, the functions cannot differ at the neutral news outcome. Moreover, in each of these cases, the change in informativeness serves to rotate the likelihood function around the neutral news outcome $\nu(\omega')$. But, the nature of the rotation depends on whether disasters
amplify versus mute the effect of type.

Figure 3.3. Informativeness Effect

Focus on the case where disasters amplify the effect of type (Figure 3.3a). In that case, the increase in disaster intensity corresponds to an increase in informativeness. Thus, the Voter is better able to determine whether the governance outcome was drawn from the distribution associated with the good type versus the distribution associated with the bad type. First consider a good news outcome $g > \nu(\omega')$. After observing $g$, the Voter’s posterior probability that the Incumbent is a good type increases relative to his prior. By Lemma A.1, when informativeness increases, this good news becomes better news—that is, the Voter’s posterior moves even further from his prior. As the analysis in Section 2.3 highlights, a greater increase in the Voter’s posterior must be the consequence of a larger value of the loglikelihood ratio. Hence, for any $g > \nu(\omega')$, $\ell(g, \omega')$ is greater than $\tilde{L}(g)$. An analogous argument applies to a bad news outcome, $g < \nu(\omega')$. Since a bad news outcome is worse news when informativeness increases, for any $g < \nu(\omega')$, $\ell(g, \omega')$ is less than $\tilde{L}(g)$. Putting these facts together, an increase in informativeness corresponds to a steepening of the graph of the loglikelihood ratio. An analogous argument applies to the case where disasters mute the effect of type (Figure 3.3b). In that case, an increase in disaster intensity serves to decrease informativeness and, thereby, serves to flatten the graph of the loglikelihood ratio.

This rotation of the loglikelihood ratio is important for understanding how the change in informativeness impacts the Voter’s reelection threshold. Recall that, without the informativeness effect, the Voter would have been indifferent between the Incumbent and Challenger at the governance outcome $\tilde{g}$. So the impact of informativeness on the Voter’s reelection threshold is reflect by the relationship between $\tilde{g}$ and the Voter’s new reelection threshold $\hat{g}(\omega')$.

Focus on the case where the Incumbent is ahead. In that case, $\tilde{g}$ is less than the neutral news outcome $\nu(\omega')$. We will argue that, if disasters amplify the effect of type, then $\hat{g}(\omega') > \tilde{g}$ and, if disasters mute the effect of type, then $\hat{g}(\omega') < \tilde{g}$. So, in the former case, informativeness increases the reelection threshold and, in the latter case, it decreases the reelection threshold.

To understand these changes, focus on the case where disasters amplify the effect of type—

---

8 Recall, $\tilde{g}$ is the outcome at which $\tilde{L}$ intersect $\beta(\pi_I, \pi_C)$ and $\nu(\omega')$ is the outcome at which $\tilde{L}$ and $\ell(\cdot, \omega')$ both intersect 0. When the Incumbent is ahead $\beta(\pi_I, \pi_C) < 0$, from which it follows that $\tilde{g} < \nu(\omega')$. 

so the increase in disaster intensity corresponds to an increase in informativeness. Because the Incumbent is ahead, \( \tilde{g} \) is a bad news outcome (relative to the neutral news outcome \( \nu(\omega') \)). The increase in informativeness makes bad news worse. So, increasing informativeness lowers the Voter’s expectation of the Incumbent’s type at \( \tilde{g} \). Hence, given a governance outcome \( \tilde{g} \), the Voter now strictly prefers the Challenger to the Incumbent. This is why, in Figure 3.3a, \( \ell(\tilde{g}, \omega') < \beta(\pi_I, \pi_C) \). This means that \( \tilde{g} \) cannot be the Voter’s equilibrium threshold since, at the equilibrium threshold, the Voter is indifferent between the Incumbent and the Challenger. To restore indifference, the Voter must use a higher benchmark for reelection. Formally, since \( \ell(\hat{g}(\omega'), \omega') = \beta(\pi_I, \pi_C) \), it follows from the MLRP that \( \hat{g}(\omega') > \tilde{g} \).

Intuitively, if the Incumbent is ahead, the Voter is indifferent between the Incumbent and the Challenger when the news about the Incumbent is somewhat bad. As the governance outcome becomes more informative—as it does when disasters amplify the effect of type—this bad news becomes worse news and so the Voter strictly prefers the Challenger. Hence, the Voter shifts to a higher benchmark for reelection. By contrast, if the governance outcome becomes less informative—as it does when disasters mute the effect of type—this bad news is tempered and so the Voter strictly prefers the Incumbent. Hence, the Voter shifts to a lower benchmark for reelection.

### 3.3 The Overall Effect

Let us sum up. We have seen that the neutral news effect serves to lower the Voter’s benchmark for reelection. When the Incumbent is ahead, the informativeness effect can serve to increase or decrease the Voter’s benchmark for reelection: It increases the Voter’s benchmark when disasters amplify the effect of type and decreases the benchmark for reelection when disasters mute the effect of type.

Notice that, when the Incumbent is ahead and the disaster amplifies the effect of type, the neutral news effect and informativeness effect work in opposite directions—the former serves to lower the Voter’s benchmark for reelection and the latter serves to increase it. The overall effect is that \( \hat{g}(\omega') > \hat{g} \) and so Voter behavior less than offsets the effect of a disaster. More surprisingly, as Figure 3.4 illustrates, it is possible for the informativeness effect to swamp the neutral news effect.\(^9\) That is, the Voter can hold the Incumbent to a more stringent reelection threshold following a more intense disaster—i.e., \( \hat{g}(\omega') > \hat{g}(\omega) \).

Taken together, Figures 3.3a and 3.4 provide counterintuitive results relative to standard intuitions about rational voter behavior. Figure 3.3a shows that, when disasters amplify the effect of type and the Incumbent is ahead, rational Voter behavior less than offsets the effect of disasters. Thus, contrary to the message taken from the additive model, it cannot be presumed that rational Voter behavior exactly offsets the effect of disaster. Figure 3.4 takes this point even further, showing that the rational Voter’s reelection threshold need not be decreasing in disaster intensity. An increase in disaster intensity can cause a large informational effect which, in turn, can lead the Voter to increase his benchmark for reelection.

\(^9\) Appendix B.1 provides the numerical example that generates this figure.
Figure 3.4. Informational Effect Dominates Neutral News Effect

If, instead, the Incumbent is ahead and disaster mutes the effect of type, then the situation is reversed: Both the neutral news and the informativeness effects work in the same direction. As a consequence, rational Voter behavior more than offsets the effect of disasters. This is illustrated in Figure 3.3b.

Now, consider the case where the Incumbent is behind, so that $\beta(\pi_I, \pi_C) > 0$. The neutral news effect still serves to decrease the benchmark for reelection—i.e., $\tilde{g} < \hat{g}(\omega)$. But now $\tilde{g} > \nu(\omega')$ and so $\tilde{g}$ is a good news outcome. The implications for the informativeness effect are now reversed: An increase in informativeness serves to make good news even better news. As a consequence, at $\tilde{g}$, an increase in informativeness makes it more likely that the Incumbent is a good type and this further lowers the Voter’s benchmark. So, in that case, the Voter’s benchmark more than offsets the effect of the disaster. By contrast, a decrease in informativeness serves to temper good news. As a consequence, at $\tilde{g}$, a decrease in informativeness makes it less likely that the Incumbent is a good type and, thereby, raises the Voter’s benchmark. So, in that case, the Voter’s benchmark less than offsets the effect of the disaster. (Again, it is possible that, counterintuitively, the Voter’s reelection threshold may increase in response to an increase in disaster intensity.)

<table>
<thead>
<tr>
<th>Amplifies Mutes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ahead</td>
</tr>
<tr>
<td>Less than offsets</td>
</tr>
<tr>
<td>Behind</td>
</tr>
<tr>
<td>More than offsets</td>
</tr>
</tbody>
</table>

Table 3.1. How Voter Behavior Responds to Disasters when $\pi_I \neq \pi_C$.

Table 3.1 summarizes these results, which are formalized as Proposition B.1 in the Appendix. When $\pi_I = \pi_C$, Voter behavior exactly offsets the effect of disaster. However, when $\pi_I \neq \pi_C$ and $f$ is not additively separable, Voter behavior either more than or less than offsets the effect of disaster.
4 Electoral Fortunes

We next investigate how a change in the level of disaster intensity effects the Incumbent’s electoral fortunes. We will see that, when disasters provide information about the Incumbent’s type, the Incumbent’s electoral fortunes are typically altered by a change in the level of disaster intensity.

For a given level of disaster intensity $\omega$, the probability of reelection is determined by two factors. The first is the Voter’s benchmark for reelection $\hat{g}(\omega)$ and the second is the distribution of governance outcomes. This distribution, given in Equation (6), can be rewritten as

$$\Gamma(g; \omega) = \pi_I \Phi(g - \nu(\omega) - \iota(\omega)) + (1 - \pi_I) \Phi(g - \nu(\omega) + \iota(\omega)).$$

So, when the level of disaster intensity is $\omega$, the probability that the Incumbent is reelected is $1 - \Gamma(\hat{g}(\omega); \omega)$.

Changes to disaster intensity affect the probability of reelection via both channels: the Voter’s reelection rule and the distribution of governance outcomes. Section 3 explored how a change in the level of disaster intensity affects the Voter’s reelection rule. We now focus on how a change in the level of disaster intensity impacts the distribution on governance outcomes. After doing so, we will put the two effects together, to understand the overall impact on electoral fortunes.

A change in the level of disaster intensity has two effects on the distribution of governance outcomes: a neutral news effect and an informativeness effect. We begin with the neutral news effect. We will see that this is associated with a “neutral news shift” in the distribution of outcomes. If the change in disaster intensity only shifted the neutral news outcome, there would be no effect on electoral fortunes. We then turn to the informativeness effect. We will see that, even if the reelection rule did not change, the informativeness effect would change the distribution of outcomes in a way that impacts that probability of reelection.

4.1 The Neutral News Effect

Suppose that there is an increase in disaster intensity, from $\omega$ to $\omega'$. Consider a hypothetical situation in which the Voter ignores the change in informativeness—holding it fixed at $\iota(\omega')$—and instead simply filters out the effect of a disaster on the likelihood of observing any given level of public goods. In this case, the relevant CDF of governance outcomes is

$$\tilde{\Gamma}(g) = \pi_I \Phi(g - \nu(\omega') - \iota(\omega)) + (1 - \pi_I) \Phi(g - \nu(\omega') + \iota(\omega)).$$

Notice that, for any $g$, $\Gamma(g; \omega) < \tilde{\Gamma}(g)$, so that this new CDF can be seen as a worsening of the original CDF of governance outcomes $\Gamma(\cdot, \omega)$.$^{10}$ That is, the distribution of outcomes gets worse when disaster intensity increases. (Formally, $\Gamma(\cdot, \omega)$ first-order stochastically dominates $\tilde{\Gamma}(\cdot)$.) If the Voter filters out the effect of a disaster on the likelihood of observing any given level of public goods and ignores the informational effect, he would adopt a benchmark for reelection that exactly

$^{10}$Recall, $\nu(\omega') < \nu(\omega)$.
offsets the effect of the disaster—i.e., \( \tilde{g} = \hat{g}(\omega) - \nu(\omega) + \nu(\omega') \). Overall, these two changes—in the distribution of outcomes and the reelection rule—leave the probability of reelection unchanged. That is, \( 1 - \tilde{\Gamma}(\tilde{g}) = 1 - \Gamma(\hat{g}(\omega); \omega) \).

Notice that the analysis of the neutral news effect corresponds exactly to the additive model. There, changing the level of disaster intensity leaves the level of informativeness unchanged. So the new distribution of governance outcomes, \( \Gamma(g; \omega') \), corresponds exactly to the distribution \( \tilde{\Gamma} \). Moreover, as we have seen, in that case, the Voter’s actual new benchmark \( \tilde{g}(\omega') \) exactly offsets the effect of disaster (i.e., \( \tilde{g}(\omega') = \tilde{g} \)). Thus, \( 1 - \tilde{\Gamma}(\tilde{g}) = 1 - \Gamma(\hat{g}(\omega'); \omega') \). That said, when changing the level of disaster changes the level of informativeness, these two properties no longer hold: The new actual distribution \( \Gamma(\cdot, \omega') \) is typically different from the ‘shifted down’ distribution \( \tilde{\Gamma} \) and the new benchmark for reelection \( \hat{g}(\omega') \) typically does not exactly offset the effect of the disaster.

4.2 The Informativeness Effect

To add the informativeness effect, we begin with the hypothetical “neutral news” distribution \( \tilde{\Gamma} \) and allow the level of informativeness to change from \( \iota(\omega) \) to \( \iota(\omega') \); this leads to the new distribution \( \Gamma(\cdot; \omega') \). Figure 4.1 illustrates this change in the level of informativeness. The new distribution is a rotation of \( \tilde{\Gamma} \) around some point, labeled \( \overline{g} \). (The particular point depends on both \( \omega \) and \( \omega' \).) The nature of the rotation depends on whether disasters amplify vs. mute the effect of type. Figure 4.1a focuses on the case of amplifies. There, when the governance outcome \( g \) is below \( \overline{g} \), the new true distribution lies above the neutral news distribution—that is, if \( g > g^* \) and \( \overline{g} > g^* \) for any \( g^* < \overline{g} \) (resp. \( g^* > \overline{g} \)). To see this, suppose that disasters amplify the effect of type, so that there is an increase in

![Diagram](image-url)

(a) Disasters Amplify the Effect of Type    (b) Disasters Mute the Effect of Type

Figure 4.1. How Informativeness Changes the Distribution of Outcomes

The change in informativeness serves to systematically change the likelihood of “tail events,” i.e., the likelihood that the outcome \( g \) is less than (resp. more than) \( g^* \) for any \( g^* < \overline{g} \) (resp. \( g^* > \overline{g} \)).
informativeness. Refer to Figure 4.1a and observe that the likelihood of these tail events is higher under the new distribution $\Gamma(\cdot, \omega')$ than under the neutral news distribution $\tilde{\Gamma}(\cdot)$. (For $g_* < \tilde{g}$, $\Gamma(g_*, \omega') > \tilde{\Gamma}(g_*)$; for $g_* > \tilde{g}$, $1 - \Gamma(g_*, \omega') > 1 - \tilde{\Gamma}(g_*)$.) Hence, the increase in disaster intensity makes both very bad and very good governance outcomes more likely. Analogously, when disasters mute the effect of type, increasing disaster intensity decreases informativeness and, so, tail events become less likely.\(^{11}\)

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure4.2.png}
\caption{Amplify and $\tilde{g} < \hat{g}(\omega') < \tilde{g}$: Probability of Reelection Falls}
\end{figure}

This change in the distribution of outcomes has a direct effect on the probability of reelection. That is, it would directly change the probability of reelection, even if the Voter were to adopt a threshold for reelection that exactly offsets the effect of disaster. To see this, refer to Figure 4.2, which depicts the case where disasters amplify the effect of type. If $\tilde{g}$ (i.e., threshold that exactly offsets the effect of disaster) is below $\tilde{g}$, the change in the distribution of outcomes results in a lower probability of reelection, i.e., $1 - \Gamma(\tilde{g}; \omega') < 1 - \tilde{\Gamma}(\tilde{g}) = 1 - \Gamma(g; \omega)$. But if $\tilde{g}$ is above $\tilde{g}$, the change in the distribution of outcomes would result in a higher probability of reelection, i.e., $1 - \Gamma(\tilde{g}; \omega') > 1 - \tilde{\Gamma}(\tilde{g}) = 1 - \Gamma(\hat{g}(\omega); \omega)$.

Whether $\tilde{g}$ is below or above $\tilde{g}$ is, in part, determined by the likelihood that the Challenger is a high type (i.e., $\pi_C$). When the Challenger is sufficiently weak (i.e., $\pi_C$ is sufficiently low), $\tilde{g} < \tilde{g}$ and, when the Challenger is sufficiently strong (i.e., $\pi_C$ is sufficiently high), $\tilde{g} > \tilde{g}$.\(^{12}\) When the Challenger is very weak, the Incumbent only loses when there is a very bad realization of $g$. Hence, if tail events are more likely, it is more likely that the Incumbent will lose; this is detrimental for the Incumbent. By contrast, when the Challenger is very strong, the Incumbent only wins when there is a very good realization of $g$. Hence, if tail events are more likely, it is more likely that the Incumbent will win; this is beneficial to the Incumbent.

In sum, when disasters amplify the effect of type, an increase in disaster intensity shifts probability mass towards the tails of the distribution. This is bad for the Incumbent when the Challenger

\(^{11}\)The shift in the probability mass need not be symmetric. As a consequence, $\tilde{g}$ need not equal $1/2$.

\(^{12}\)To understand this fact note that $\tilde{g}$ is independent of $\pi_C$ (Lemma C.1). The key is that $\hat{g}$ can be seen as an unbounded and strictly increasing function of $\pi_C$. For this, use Equation (5), the MLRP, the fact that the likelihood ratio is unbounded, and the fact that $\nu(\omega), \nu(\omega')$ are independent of $\pi_C$. 

21
is very likely to be a low type and good for the Incumbent when the Challenger is very likely to be a high type. By contrast, when disasters mute the effect of type, an increase in disaster intensity serves to move probability mass away from the tails of the distribution. This is good for the Incumbent when the Challenger is very likely to be a low type and bad for the Incumbent when the Challenger is very likely to be a high type.

4.3 The Overall Effect

The analysis above focused on the direct effect of a change in disaster intensity on the reelection probability—that is, how a change in disaster intensity directly impacts the distribution of outcomes and, thereby, changes the probability of reelection. But there is a second, indirect effect: A change in disaster intensity also changes the Voter’s benchmark for reelection. In particular, as we saw in Section 3, the Voter’s threshold does not exactly offset the effect of disaster, i.e., \( \hat{g}(\omega') \neq \tilde{g} \).

Suppose that disasters amplify the effect of type and \( \overline{g} > \hat{g} \). In this case, the direct effect (captured by the change in the distribution and the fact that \( \overline{g} > \hat{g} \)) causes the probability of reelection to fall. If, as in Figure 4.2, Voter behavior less than offsets the effect of disaster (i.e., \( \hat{g}(\omega') > \tilde{g} \)), then the indirect effect (captured by the fact that the benchmark for reelection is greater than \( \tilde{g} \)) also causes the probability of reelection to fall.\(^{13}\) If, instead, Voter behavior more than offsets the effect of disaster (i.e., \( \tilde{g} > \hat{g}(\omega') \)), then the indirect effect works in the opposite direction. Specifically, it causes the Voter to lower his benchmark for reelection and, thereby, increase the probability that the Incumbent is reelected. In Figure 4.3a the direct effect dominates the indirect effect (causing the overall probability of reelection to fall) and in Figure 4.3b the indirect effect dominates the direct effect (causing the overall probability of reelection to rise).

Table 4.1 summarizes when the Incumbent’s fortunes will increase versus decrease, based on whether (a) disasters amplify or mute the effect of type, (b) the outcome that exactly offsets the effect of disaster lies above or below the rotation point, and (c) Voter behavior more than or less

\(^{13}\)Formally, \( \Gamma(\hat{g}(\omega'); \omega') > \Gamma(\tilde{g}; \omega') > \tilde{\Gamma}(\hat{g}) \).

22
Table 4.1. Change in Electoral Fortunes

<table>
<thead>
<tr>
<th>Amplifies Mutes</th>
<th>Amplifies Mutes</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tilde{g} &gt; \bar{g}$</td>
<td>$\tilde{g} &gt; \bar{g}$</td>
</tr>
<tr>
<td>$\tilde{g} &lt; \bar{g}$</td>
<td>Reelec Prob $\downarrow$</td>
</tr>
</tbody>
</table>

Table 4.1 indicated that (c) is determined by both (a) and whether the Incumbent is ahead or behind. In Section 4.2, we pointed out that (b) depends on the likelihood that the Challenger is a high type. In fact, (a)-(b)-(c) are jointly determined by the production technology (i.e., whether disasters amplifies or mutes the effect of type), the likelihood that the Incumbent is a high type (i.e., $\pi_I$) and the likelihood that the Challenger is a high type (i.e., $\pi_C$).

Refer to Figure 4.4, which indicates whether Incumbent fortunes will increase versus decrease as a function of $(\pi_I, \pi_C)$. If disasters amplify the effect of type, then the shaded region corresponds to the pairs $(\pi_I, \pi_C)$ for which the reelection probability rises and the non-shaded region corresponds to the pairs for which the reelection probability falls. If disasters mute the effect of type, then the shaded region corresponds to the pairs $(\pi_I, \pi_C)$ for which the reelection probability falls and the non-shaded region corresponds to the pairs for which the reelection probability rises. The sole exception is $(\pi_I, \pi_C) = (\frac{1}{2}, \frac{1}{2})$: In that case, the neutral news outcome $\tilde{g}$ correspond exactly to the rotation point $\bar{g}$ and the Voter exactly offsets the effect of disaster; so that case acts as the additive model and the Incumbent neither benefits from nor is hurt by the effect of disaster.

Notice that, when disasters amplify the effect of type and $\pi_C \geq \max\{\frac{1}{2}, \pi_I\}$, the Incumbent’s electoral fortunes necessarily benefit from the increased disaster intensity. In that case, both the direct and indirect effects work in the same direction. Because the Challenger is sufficiently strong, the direct effect benefits the Incumbent’s electoral fortunes. Moreover, because the Incumbent is behind, the indirect effect also benefits the Incumbent’s electoral fortunes. Likewise, if disasters amplify the effect of type but $\min\{\frac{1}{2}, \pi_I\} \geq \pi_C$, then the direct and indirect effects serve to hurt the Incumbent’s electoral fortunes. This follows since the Challenger is sufficiently weak and the Incumbent is ahead. (Proposition C.1 in the Appendix shows these claims.) An reverse argument applies when disasters mute the effect of type. (See Proposition C.2 in the Appendix.)

5 Conclusion

In their seminal paper, Achen and Bartels (2004, pp.7–8) write, “To the extent that voters engage in sophisticated attributions of responsibility they should be entirely unresponsive to natural
Figure 4.4. **Amplifies**: Reelection probability rises in shaded and falls in unshaded.  
**Mutes**: Reelection probability falls in shaded and rises in unshaded.

Disasters, at least on average; to the extent that they engage in blind retrospection, they should exhibit ‘systematic attribution errors’.” This claim is emblematic of a large empirical literature. That literature is rooted in a simple argument: Because natural disasters are outside the control of incumbents, their electoral fortunes will suffer after a natural disaster only if voters are irrational. This argument underlies the interpretation of the empirical results as pointing toward voter irrationality.

This paper suggests that this interpretation is not warranted, or is at least premature. We study a canonical model of voter learning, and show that, in all but knife-edged cases, rational voter behavior implies that disasters—indeed, any shocks outside the control of policy makers—will affect incumbent electoral fortunes. As such, identifying the impact of natural disasters on incumbent electoral fortunes is not sufficient to infer voter irrationality.

While our results raise challenges for empirical efforts to assess voter rationality, they also suggest a way forward. With a substantive understanding of how disasters affect informativeness about type (e.g., do they amplify or mute the effect of type), the researcher can identify both the impact on electoral fortunes and the *ex ante* distributions of quality and use this to identify voter (ir)rationality. In the specific case where the exogenous shock is not expected to provide information about the quality of the incumbent, the researcher only needs to identify impact on electoral fortunes. This is the case in Healy, Malhotra and Mo (2010) and Fowler and Montagnes (2015), where the shock is college sports losses.\(^{14}\)

An alternative approach would contrast the implications of voter rationality with a specific model of bounded rationality. Toward that end, a growing literature provides theoretical models of electoral accountability with boundedly rational voters (Callander and Wilson, 2008; Patty, 2006;  

\(^{14}\)Healy, Malhotra and Mo (2010) find evidence that incumbent electoral fortunes do depend on those shocks. However, Fowler and Montagnes (2015) find that the effect is not systematic.
Ashworth and Bueno de Mesquita, 2014; Diermeier and Li, 2013; Ortoleva and Snowberg, 2015; Levy and Razin, 2015a,b; Lockwood, 2015). While this literature is currently focused on normative questions, as it progresses, it may point to tests of specific models of boundedly rational voters.
A Preliminary Results

In this Appendix, we report a technical result that will be useful in the subsequent analysis. The result is Lemma B.2 in Ashworth, Bueno de Mesquita and Friedenberg (Forthcoming) and so the proof is omitted.

Lemma A.1. Consider a function $\mathcal{L} : \mathbb{R} \times \mathbb{R}_+ \rightarrow \mathbb{R}_+$ given by

$$\mathcal{L}(x, \delta) = \frac{\phi(x - \delta)}{\phi(x + \delta)}.$$  

(i) $\mathcal{L}$ is increasing in $\delta$ if $x > 0$.

(ii) $\mathcal{L}$ is constant in $\delta$ if $x = 0$.

(iii) $\mathcal{L}$ is decreasing in $\delta$ if $x < 0$.

B Supporting Material for Section 3

The following Proposition formalizes the results reported in Table 3.1:

Proposition B.1. Fix $\omega' > \omega$.

(i) Suppose disasters amplify the effect of type. Then $\pi_I \geq \pi_C$ if and only if $\nu(\omega) - \nu(\omega') \geq \hat{g}(\omega) - \hat{g}(\omega')$ and $\pi_C \geq \pi_I$ if and only if $\hat{g}(\omega) - \hat{g}(\omega') \geq \nu(\omega) - \nu(\omega')$.

(ii) Suppose disasters neither amplify nor mute the effect of type. Then, for any $\pi_I$ and $\pi_C$,

$$\nu(\omega) - \nu(\omega') = \hat{g}(\omega) - \hat{g}(\omega').$$

(iii) Suppose disasters mute the effect of type. Then $\pi_I \geq \pi_C$ if and only if $\hat{g}(\omega) - \hat{g}(\omega') \geq \nu(\omega) - \nu(\omega')$ and $\pi_C \geq \pi_I$ if and only if $\nu(\omega) - \nu(\omega') \geq \hat{g}(\omega) - \hat{g}(\omega')$.

Observe that the statement of the proposition has implications for strict inequalities: Suppose disasters amplify the effect of type and $\pi_I > \pi_C$. Then we can apply the fact that “$\pi_I \geq \pi_C$” and “not $\pi_C \geq \pi_I$” to obtain that $\nu(\omega) - \nu(\omega') > \hat{g}(\omega) - \hat{g}(\omega')$.

This Proposition follows from two Lemmata. The first formalizes the discussion surrounding Figure 3.2.

Lemma B.1. Fix $\omega' > \omega$.

(i) For each $g$, $\tilde{L}(g) > \ell(g, \omega)$.

(ii) If $\tilde{L}(\hat{g}) = \beta(\pi_I, \pi_C)$, then $\hat{g}(\omega) - \hat{g} = \nu(\omega) - \nu(\omega')$.

Proof. Observe that, for each $g$, there exists some $g' > g$ so that $g = g' + (\nu(\omega') - \nu(\omega))$. Since

$$\tilde{L}(g) = \log \frac{\phi(g - \nu(\omega') - i(\omega))}{\phi(g - \nu(\omega') + i(\omega))} = \log \frac{\phi(g' - \nu(\omega) - i(\omega))}{\phi(g' - \nu(\omega) + i(\omega))} = \ell(g', \omega).$$
and (by the strict MLRP) $\ell(g', \omega) > \ell(g, \omega)$, it follows that $\tilde{L}(g) > \ell(g, \omega)$.

If $\tilde{L}(\tilde{g}) = \beta(\pi_I, \pi_C)$, then

$$
\log \frac{\phi(\tilde{g} - \nu(\omega') - \ell(\omega))}{\phi(\tilde{g} - \nu(\omega) + \ell(\omega))} = \log \frac{\phi(\tilde{g}(\omega) - \nu(\omega) - \ell(\omega))}{\phi(\tilde{g}(\omega) - \nu(\omega) + \ell(\omega))}.
$$

By the strict MLRP, this implies that $\tilde{g} - \nu(\omega') = \hat{g}(\omega) - \nu(\omega)$. ■

The second lemma formalizes the “rotation” effect discussed surrounding Figures 3.3a, 3.4, and 3.3b.

**Lemma B.2.** Fix $\omega' > \omega$.

(i) Suppose disasters amplify the effect of type. Then $g \geq \nu(\omega')$ if and only if $\ell(g, \omega') \geq \tilde{L}(g)$ and $g \leq \nu(\omega')$ if and only if $\ell(g, \omega') \leq \tilde{L}(g)$.

(ii) Suppose disasters mute the effect of type. Then $g \geq \nu(\omega')$ if and only if $\ell(g, \omega') \leq \tilde{L}(g)$ and $g \leq \nu(\omega')$ if and only if $\ell(g, \omega') \geq \tilde{L}(g)$.

**Proof.** Immediate from Lemma A.1. ■

Note, again, that while these statements involve weak inequalities they have implications about strict inequalities. For instance, if disasters amplify the effect of type and $g > \nu(\omega')$ then, using the fact that “not $g \leq \nu(\omega')$” it follows that $\ell(g, \omega') > \tilde{L}(g)$.

**Proof of Proposition B.1.** We show the result for the case where disasters amplify the effect of type and $\pi_I \geq \pi_C$ (or, equivalently, $\beta(\pi_I, \pi_C) \leq 0$). The other cases are analogous.

In this case, we want to show that $\nu(\omega') - \nu(\omega') \geq \hat{g}(\omega') - \hat{g}(\omega')$. By Lemma B.1(ii), this is true provided $\hat{g}(\omega) - \hat{g} \geq \hat{g}(\omega) - \hat{g}(\omega')$ or, equivalently, provided $\hat{g}(\omega') \geq \hat{g}$. To show this, observe that

$$
\ell(\nu(\omega'), \omega') = 0 \geq \beta(\pi_I, \pi_C) = \ell(\hat{g}(\omega'), \omega').
$$

So, by the MLRP, $\nu(\omega') \geq \hat{g}(\omega')$. From this, the fact that disasters amplify the effect of type, and Lemma B.2, $\tilde{L}(\hat{g}(\omega')) \geq \ell(\hat{g}(\omega'), \omega')$. It follows that

$$
\tilde{L}(\hat{g}(\omega')) \geq \ell(\hat{g}(\omega'), \omega') = \beta(\pi_I, \pi_C) = \tilde{L}(\hat{g}).
$$

Since $\tilde{L}$ is increasing, it follows that $\hat{g}(\omega') \geq \hat{g}$, as desired. ■

**B.1 Computational Examples**

Figures 3.2–3.4 were drawn based on numerical examples. We now give details on the numerical examples, to illustrate that the effects depicted can indeed arise.
The examples have several features in common: In each, \( \phi \) is the pdf of a standard normal distribution:
\[
\phi(x) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{x^2}{2}\right).
\]
Also, in each \( \pi_C = \frac{1}{2}, \omega' = \frac{1}{2}, \) and \( \omega'' = 1. \) The examples vary in the production function and the choice of \( \pi_I. \) These choices are described in Table B.1.

<table>
<thead>
<tr>
<th>( \pi_I )</th>
<th>( f(\theta, \omega) )</th>
<th>( f(\theta', \omega) )</th>
<th>( \pi_C )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{1}{1+\exp(-2.5)} )</td>
<td>( 1 - \omega )</td>
<td>( 1 - 4\omega )</td>
<td>( \frac{1}{1+\exp(-2.5)} )</td>
</tr>
<tr>
<td>( \frac{1}{1+\exp(-2)} )</td>
<td>( 4 - 2\omega )</td>
<td>( 1 - \omega )</td>
<td>( \frac{1}{1+\exp(-2)} )</td>
</tr>
</tbody>
</table>

Table B.1. Parameters for the numerical examples.

We can analytically solve for the Voter’s reelection threshold. Using the formula for the normal pdf, \( \ell(g, \omega) = 2\iota(\omega)(g - \nu(\omega)). \) So we can solve explicitly for \( \hat{g}: \)
\[
\hat{g}(\omega) = \frac{\beta(\pi_I, \pi_C) + \nu(\omega)}{2\iota(\omega)}.
\]

Figure 3.4 depicted a situation in which an increase in disaster intensity caused the benchmark for reelection to rise. With this in mind, we will show that \( \hat{g}(\omega'') > \hat{g}(\omega'). \) Using the explicit solution for \( \hat{g}(\omega), \) and the last column of Table B.1, \( \nu(\omega) = (2-3\omega)/2, \iota(\omega) = \omega/2, \) and \( \beta(\pi_I, \pi_C) = -2. \) So,
\[
\hat{g}(\omega') = \frac{-15}{4} < \frac{-10}{4} = \hat{g}(\omega'').
\]

Thus, as the disaster becomes more intense, the reelection threshold becomes more stringent.

### C Proofs for Section 4

Say that the election is even Steven if \( \pi_I = \pi_C = 1/2. \) In what follows, we will restrict attention to elections that are not even Steven. This Appendix will show the following results.

**Proposition C.1.** Suppose disasters amplify the effect of type and the election is not even Steven.

(i) If \( \pi_C \geq \max\{\frac{1}{2}, \pi_I\}, \) then \( 1 - \Gamma(\hat{g}(\omega'); \omega') > 1 - \Gamma(\hat{g}(\omega); \omega). \)

(ii) If \( \min\{\frac{1}{2}, \pi_I\} \geq \pi_C, \) then \( 1 - \Gamma(\hat{g}(\omega); \omega) > 1 - \Gamma(\hat{g}(\omega'); \omega'). \)

**Proposition C.2.** Suppose disasters mute the effect of type and the election is not even Steven.

(i) If \( \min\{\pi_I, \frac{1}{2}\} \geq \pi_C, \) then \( 1 - \Gamma(\hat{g}(\omega'); \omega') > 1 - \Gamma(\hat{g}(\omega); \omega). \)

(ii) If \( \pi_C \geq \max\{\frac{1}{2}, \pi_I\}, \) then \( 1 - \Gamma(\hat{g}(\omega); \omega) > 1 - \Gamma(\hat{g}(\omega'); \omega'). \)

We begin by analyzing how the change in informativeness—going form \( \hat{\Gamma} \) to \( \Gamma(\cdot, \omega') \)—rotates the distribution of public goods. We then use this fact to conclude whether the reelection probability
would increase or decrease, depending on (a) where $\tilde{g}$ lies relative to the rotation point and (b) whether the incumbent is ahead versus behind. Finally, we conclude by providing parameters of the model that pin down where $\tilde{g}$ lies relative to the rotation point.

**The Effect of Informativeness: Rotating the Distribution $\tilde{\Gamma}$** Write

$$G(x; \delta) = \pi I \Phi(x - \delta) + (1 - \pi I) \Phi(x + \delta).$$

Observe that

$$\frac{\partial G}{\partial \delta}(x; \delta) = -\pi I \phi(x - \delta) + (1 - \pi I) \phi(x + \delta).$$

Notice that $\tilde{\Gamma}(g) = G(g - \nu(\omega'), \iota(\omega))$ and $\Gamma(g; \omega') = G(g - \nu(\omega'), \iota(\omega')).$ Thus, we will be interested in how the function $G(x; \delta)$ responds to changes in $\delta$.

**Lemma C.1.** Fix $\delta' \neq \delta$. There is exactly one $\pi$ such that $G(\pi; \delta') = G(\pi; \delta)$. Moreover:

(i) For $\delta' > \delta$: If $\pi > x$ then $G(\pi; \delta') > G(x; \delta)$ and if $\pi > x$ then $G(x; \delta) > G(x; \delta').$

(ii) For $\delta > \delta'$: If $\pi > x$ then $G(\pi; \delta) > G(x; \delta')$ and if $\pi > x$ then $G(x; \delta') > G(x; \delta)$.

The level $\pi$ that solves $G(\pi; \delta') = G(\pi; \delta)$ will typically depend on both $\delta$ and $\delta'$.

To show Lemma C.1, it will be useful to fix $\delta' \neq \delta$ and define auxiliary functions $\Delta(\cdot) : \mathbb{R} \to \mathbb{R}$ and $\Xi(\cdot) : \mathbb{R} \to \mathbb{R}$ so that

$$\Delta(x) = G(x; \delta') - G(x; \delta)$$

$$= \pi I [\Phi(x - \delta') - \Phi(x - \delta)] + (1 - \pi I) [\Phi(x + \delta') - \Phi(x + \delta)]$$

and

$$\Xi(x) = \frac{\Phi(x - \delta') - \Phi(x - \delta)}{\Phi(x + \delta') - \Phi(x + \delta)}.$$}

We begin with the following observation, which follows directly from calculations:

**Lemma C.2.** Suppose $\delta' > \delta$.

(i) $\Delta(x) \geq 0$ if and only if $\frac{1 - \pi I}{\pi I} \geq \Xi(x)$.

(ii) $\Delta(x) \leq 0$ if and only if $\frac{1 - \pi I}{\pi I} \leq \Xi(x)$.

**Lemma C.3.** Suppose $\delta > \delta'$.

(i) $\Delta(x) \geq 0$ if and only if $\frac{1 - \pi I}{\pi I} \leq \Xi(x)$.

(ii) $\Delta(x) \leq 0$ if and only if $\frac{1 - \pi I}{\pi I} \geq \Xi(x)$.

**Lemma C.4.**
(i) If $\delta' > \delta$, the function $\Xi(\cdot)$ is strictly increasing in $x$.

(ii) If $\delta > \delta'$, the function $\Xi(\cdot)$ is strictly decreasing in $x$.

**Proof.** Observe that 
\[
\Xi'(x) = -\frac{(\phi(x - \delta') - \phi(x - \delta))(\Phi(x + \delta') - \Phi(x + \delta)) - (\Phi(x - \delta') - \Phi(x - \delta))(\phi(x + \delta') - \phi(x + \delta))}{(\Phi(x + \delta') - \Phi(x + \delta))^2}.
\]

We will show: $\delta' > \delta$ implies that $\Xi'(x) > 0$ for all $x$. Reversing the inequalities below gives the desired result when $\delta > \delta'$.

Observe that $\Xi'(x) > 0$ if and only if
\[-(\phi(x - \delta') - \phi(x - \delta))(\Phi(x + \delta') - \Phi(x + \delta)) + (\Phi(x - \delta') - \Phi(x - \delta))(\phi(x + \delta') - \phi(x + \delta)) > 0.
\]

Since $\delta' > \delta$, $\Xi'(x) > 0$ if and only if or if and only if
\[
\frac{\phi(x - \delta') - \phi(x - \delta)}{\Phi(x - \delta') - \Phi(x - \delta)} > \frac{\phi(x + \delta) - \phi(x + \delta')}{\Phi(x + \delta) - \Phi(x + \delta')}.
\]

Applying Cauchy’s Mean Value Theorem, there exists $\xi \in (x - \delta', x - \delta)$ and $\bar{\xi} \in (x + \delta, x + \delta')$ such that
\[
\frac{\phi(x - \delta') - \phi(x - \delta)}{\Phi(x - \delta') - \Phi(x - \delta)} = \frac{\phi'(\xi)}{\phi(\xi)}
\]
and
\[
\frac{\phi(x + \delta) - \phi(x + \delta')}{\Phi(x + \delta) - \Phi(x + \delta')} = \frac{\phi'(\bar{\xi})}{\phi(\bar{\xi})}.
\]

Observe that $\xi < \bar{\xi}$ and $\phi$ is log-concave (see Lemma B.1 in Ashworth, Bueno de Mesquita and Friedenberg, Forthcoming). So
\[
\frac{\phi(x - \delta') - \phi(x - \delta)}{\Phi(x - \delta') - \Phi(x - \delta)} > \frac{\phi'(\xi)}{\phi(\xi)} > \frac{\phi'(\bar{\xi})}{\phi(\bar{\xi})} = \frac{\phi(x + \delta) - \phi(x + \delta')}{\Phi(x + \delta) - \Phi(x + \delta')},
\]
implying that $\Xi'(x) > 0$. ■

**Lemma C.5.** There exist some $\bar{x}$ so that $\Delta(\bar{x}) = 0$.

**Proof.** Observe that $\Delta(\bar{x}) = 0$ if and only if $\Xi(\bar{x}) = \frac{(1 - \pi_t)}{\pi_t}$. So it suffices to show that $\Xi(\cdot)$ is onto. To do so, observe that $\Xi(\cdot)$ is continuous. We will show it is not bounded on $\mathbb{R}$, from which it follows that the function is onto. We show this when $\delta' > \delta$; a symmetric argument applies for the reverse inequality.

Observe that, by the Cauchy Mean Value Theorem, for any given $x$, there exists some $\tilde{\delta}(x) \in (\delta, \delta')$ so that 
\[
\Xi(x) = \frac{\phi(x - \tilde{\delta}(x))}{\phi(x + \tilde{\delta}(x))}.
\]

30
It follows from Lemma A.1 that, when \( x \geq 0 \) (resp. \( x \leq 0 \)), \( \Xi(x) \) is greater than or equal to (resp. less than or equal to)

\[
\frac{\phi(x - \delta)}{\phi(x + \delta)}.
\]

Now, using the fact that the likelihood ratio is unbounded, for any given \( K > 0 \), there exists some \( x(K) \) so that

\[
\Xi(x(K)) \geq \frac{\phi(x(K) - \delta)}{\phi(x(K) + \delta)} \geq K.
\]

Likewise, for any given \( K < 0 \). Thus, \( \Xi(\cdot) \) is unbounded.

**Proof of Lemma C.1.** Immediate from Lemmata C.2-C.3-C.4-C.5.

We can apply Lemma C.1 to the functions \( \tilde{\Gamma}(g) = \mathcal{G}(g - \nu(\omega'), \iota(\omega)) \) and \( \Gamma(g; \omega') = \mathcal{G}(g - \nu(\omega'), \iota(\omega')) \).

**Corollary C.1.** Suppose disasters amplify the effect of type. If \( \omega' > \omega \), then there exists some \( \overline{g}(\omega, \omega') \) so that the following hold:

(i) If \( g \geq \overline{g}(\omega, \omega') \), then \( \tilde{\Gamma}(g) \geq \Gamma(g; \omega') \).

(ii) If \( g \leq \overline{g}(\omega, \omega') \), then \( \tilde{\Gamma}(g) \leq \Gamma(g; \omega') \).

**Corollary C.2.** Suppose disasters mute the effect of type. If \( \omega' > \omega \), then there exists some \( \overline{g}(\omega, \omega') \) so that the following hold:

(i) If \( g \geq \overline{g}(\omega, \omega') \), then \( \tilde{\Gamma}(g) \leq \Gamma(g; \omega') \).

(ii) If \( g \leq \overline{g}(\omega, \omega') \), then \( \tilde{\Gamma}(g) \geq \Gamma(g; \omega') \).

**Implications for Reelection Probabilities**  

The following Lemmata follow immediately from Corollaries C.1-C.2, the analysis in the main text, and Proposition B.1.

**Lemma C.6.** Suppose that disasters amplify the effect of type.

(i) If \( \tilde{g} \geq \overline{g}(\omega, \omega') \) and \( \pi_C \geq \pi_I \) with at least one strict inequality, then \( \Gamma(\tilde{g}(\omega), \omega) > \Gamma(\tilde{g}(\omega'), \omega') \).

(ii) If \( \overline{g}(\omega, \omega') \geq \tilde{g} \) and \( \pi_I \geq \pi_C \) with at least one strict inequality, then \( \Gamma(\tilde{g}(\omega'), \omega') > \Gamma(\tilde{g}(\omega), \omega) \).

**Lemma C.7.** Suppose that disasters mute the effect of type.

(i) If \( \tilde{g} \geq \overline{g}(\omega, \omega') \) and \( \pi_C \geq \pi_I \) with at least one strict inequality, then \( \Gamma(\tilde{g}(\omega'), \omega') > \Gamma(\tilde{g}(\omega), \omega) \).

(ii) If \( \overline{g}(\omega, \omega') \geq \tilde{g} \) and \( \pi_I \geq \pi_C \) with at least one strict inequality, then \( \Gamma(\tilde{g}(\omega), \omega) > \Gamma(\tilde{g}(\omega'), \omega') \).
Where Does $\tilde{g}$ Lie Relative to $\overline{g}(\omega, \omega')$ We begin with a preliminary result that will be of use.

**Lemma C.8.** There is an $\tilde{\omega} \in (\omega, \omega')$ such that

$$\frac{\phi(\overline{g}(\omega, \omega') - \nu(\omega') - \iota(\tilde{\omega}))}{\phi(\overline{g}(\omega, \omega') - \nu(\omega') + \iota(\tilde{\omega}))} = \frac{1 - \pi_I}{\pi_I}.$$  

**Proof.** By Lemmata C.2-C.3,

$$-\frac{\Phi(\overline{g}(\omega, \omega') - \nu(\omega') - \iota(\tilde{\omega})) - \Phi(\overline{g}(\omega, \omega') - \nu(\omega') + \iota(\tilde{\omega}))}{\Phi(\overline{g}(\omega, \omega') - \nu(\omega') + \iota(\tilde{\omega})) - \Phi(\overline{g}(\omega, \omega') - \nu(\omega') + \iota(\tilde{\omega}))} = \frac{1 - \pi_I}{\pi_I}.$$

Applying the Cauchy Mean Value Theorem to the functions given by

$$h_-(x) \equiv \Phi(\overline{g}(\omega, \omega') - \nu(\omega') - \iota(x)) \quad \text{and} \quad h_+(x) \equiv \Phi(\overline{g}(\omega, \omega') - \nu(\omega') + \iota(x)),$$

we see that there exists some $\tilde{\omega} \in (\omega, \omega')$ such that

$$-\frac{\Phi(\overline{g}(\omega, \omega') - \nu(\omega') - \iota(\tilde{\omega})) - \Phi(\overline{g}(\omega, \omega') - \nu(\omega') + \iota(\tilde{\omega}))}{\Phi(\overline{g}(\omega, \omega') - \nu(\omega') + \iota(\tilde{\omega})) - \Phi(\overline{g}(\omega, \omega') - \nu(\omega') + \iota(\tilde{\omega}))} = \frac{1 - \pi_I}{\pi_I}.$$

From this the claim follows. \[\square\]

In light of Lemma C.8, in all subsequent results, we fix $\tilde{\omega} \in (\omega, \omega')$ so that

$$\frac{\phi(\overline{g}(\omega, \omega') - \nu(\omega') - \iota(\tilde{\omega}))}{\phi(\overline{g}(\omega, \omega') - \nu(\omega') + \iota(\tilde{\omega}))} = \frac{1 - \pi_I}{\pi_I}.$$  

**Lemma C.9.** Fix $\omega' > \omega$.

(i) If $\pi_I \geq \frac{1}{2}$, then $\overline{g}(\omega, \omega') \leq \nu(\omega')$.

(ii) If $\pi_I \leq \frac{1}{2}$, then $\overline{g}(\omega, \omega') \geq \nu(\omega')$.

**Proof.** We show that $\overline{g}(\omega, \omega') \leq \nu(\omega')$ if and only if $\pi_I \geq \frac{1}{2}$. Reversing the inequalities gives that $\overline{g}(\omega, \omega') \geq \nu(\omega')$ if and only if $\pi_I \leq \frac{1}{2}$.

Observe that $\overline{g}(\omega, \omega') \leq \nu(\omega')$ if and only if

$$1 = \frac{\phi(\nu(\omega') - \nu(\omega') - \iota(\tilde{\omega}))}{\phi(\nu(\omega') - \nu(\omega') + \iota(\tilde{\omega}))} \geq \frac{\phi(\overline{g}(\omega, \omega') - \nu(\omega') - \iota(\tilde{\omega}))}{\phi(\overline{g}(\omega, \omega') - \nu(\omega') + \iota(\tilde{\omega}))} = \frac{1 - \pi_I}{\pi_I},$$

where the first equality follows from symmetry, the inequality follows from the MLRP, and the last equality is by definition of $\tilde{\omega}$. It follows that $\overline{g}(\omega, \omega') \leq \nu(\omega')$ if and only if $\pi_I \geq \frac{1}{2}$. \[\square\]

**Lemma C.10.** Fix $\omega' > \omega$ and let $\frac{1}{2} \geq \pi_I$.

(i) Suppose that disasters amplify the effect of type and $\pi_C > \frac{1}{2}$. Then $\tilde{g} > \overline{g}(\omega, \omega')$.

(ii) Suppose that disasters mute the effect of type and $\frac{1}{2} > \pi_C$. Then $\overline{g}(\omega, \omega') > \tilde{g}$.  

32
Proof. We begin with Part (i). Observe that
\[
\frac{\phi(\overline{g}(\omega, \omega') - \nu(\omega') - \iota(\overline{\omega}))}{\phi(\overline{g}(\omega, \omega') - \nu(\omega') + \iota(\overline{\omega}))} = \frac{1 - \pi_I}{\pi_I} < \frac{1 - \pi_I}{\pi_I} \frac{\pi_C}{1 - \pi_C} = \frac{\phi(\tilde{g}(\omega) - \nu(\omega) - \iota(\omega))}{\phi(\tilde{g}(\omega) - \nu(\omega) + \iota(\omega))} = \frac{\phi(\tilde{g} - \nu(\omega') - \iota(\omega))}{\phi(\tilde{g} - \nu(\omega') + \iota(\omega))},
\]
where the first line follows from Lemma C.8, second line follows from the fact that \(\pi_C > \frac{1}{2}\), the third line follows from the definition of \(\tilde{g}(\omega)\), and the last line follows from the definition of \(\tilde{g}\).

With this, to show that \(\tilde{g} > \overline{g}(\omega, \omega')\), it suffices to show that
\[
\frac{\phi(\overline{g}(\omega, \omega') - \nu(\omega') - \iota(\overline{\omega}))}{\phi(\overline{g}(\omega, \omega') - \nu(\omega') + \iota(\overline{\omega}))} \geq \frac{\phi(\overline{g}(\omega, \omega') - \nu(\omega') - \iota(\omega))}{\phi(\overline{g}(\omega, \omega') - \nu(\omega') + \iota(\omega))}. \tag{8}
\]
In that case, the inequalities above imply that
\[
\frac{\phi(\tilde{g} - \nu(\omega') - \iota(\omega))}{\phi(\tilde{g} - \nu(\omega') + \iota(\omega))} > \frac{\phi(\overline{g}(\omega, \omega') - \nu(\omega') - \iota(\omega))}{\phi(\overline{g}(\omega, \omega') - \nu(\omega') + \iota(\omega))}
\]
From this, the MLRP implies that \(\tilde{g} > \overline{g}(\omega, \omega')\), as desired.

By Lemma C.9 and the fact that \(\frac{1}{2} \geq \pi_I\), \(\overline{g}(\omega, \omega') \geq \nu(\omega')\). With this, the function
\[
\delta \mapsto \frac{\phi(\overline{g}(\omega, \omega') - \nu(\omega') - \delta)}{\phi(\overline{g}(\omega, \omega') - \nu(\omega') + \delta)}
\]
is (weakly) increasing in \(\delta\). (See Lemma A.1.) Now using the fact that disaster amplifies the effect of type, it follows that \(\iota(\overline{\omega}) > \iota(\omega)\). From this, Equation (8) follows.

Part (ii) involves reserving certain inequalities in the above argument. First, since \(\frac{1}{2} > \pi_C\),
\[
\frac{\phi(\overline{g}(\omega, \omega') - \nu(\omega') - \iota(\overline{\omega}))}{\phi(\overline{g}(\omega, \omega') - \nu(\omega') + \iota(\overline{\omega}))} > \frac{1 - \pi_I}{\pi_I} \frac{\pi_C}{1 - \pi_C} = \frac{\phi(\tilde{g} - \nu(\omega') - \iota(\omega))}{\phi(\tilde{g} - \nu(\omega') + \iota(\omega))}.
\]
Thus, it suffices to show that
\[
\frac{\phi(\overline{g}(\omega, \omega') - \nu(\omega') - \iota(\omega))}{\phi(\overline{g}(\omega, \omega') - \nu(\omega') + \iota(\omega))} \geq \frac{\phi(\overline{g}(\omega, \omega') - \nu(\omega') - \iota(\overline{\omega}))}{\phi(\overline{g}(\omega, \omega') - \nu(\omega') + \iota(\overline{\omega}))}. \tag{9}
\]
If so, the MLRP analogously implies that \(\overline{g}(\omega, \omega') > \tilde{g}\).

Now observe that \(\overline{\omega} > \omega\) and disaster mutes the effect of type; so \(\iota(\omega) > \iota(\overline{\omega})\). Recall, since
\( \frac{1}{2} \geq \pi_I, \tilde{g}(\omega, \omega') \geq \nu(\omega'). \) With this, the function
\[
\delta \mapsto \frac{\phi(\tilde{g}(\omega, \omega') - \nu(\omega') - \delta)}{\phi(\tilde{g}(\omega, \omega') - \nu(\omega') + \delta)}
\]
is (weakly) increasing in \( \delta \) and so Equation (9) holds, as desired. ■

Several remarks are in order. First, observe that the proof of Lemma C.10 also gives the following:

**Remark C.1.** Fix \( \omega' > \omega \) and let \( \frac{1}{2} > \pi_I \).

(i) Suppose that disasters amplify the effect of type and \( \pi_C \geq \frac{1}{2} \). Then \( \tilde{g} > \tilde{g}(\omega, \omega') \).

(ii) Suppose that disasters mute the effect of type and \( \frac{1}{2} \geq \pi_C \). Then \( \tilde{g}(\omega, \omega') > \tilde{g} \).

Moreover, reversing the argument in the proof of Lemma C.10, we get the following:

**Lemma C.11.** Fix \( \omega' > \omega \) and let \( \pi_I > \frac{1}{2} \).

(i) Suppose that disasters amplify the effect of type and \( \frac{1}{2} \geq \pi_C \). Then \( \tilde{g}(\omega, \omega') > \tilde{g} \).

(ii) Suppose that disasters mute the effect of type and \( \pi_C \geq \frac{1}{2} \). Then \( \tilde{g} > \tilde{g}(\omega, \omega') \).

We will not need to make use of part (ii), as it will be redundant. We only include it for completeness.

**Remark C.2.** Fix \( \omega' > \omega \) and let \( \frac{1}{2} \geq \pi_I \).

(i) Suppose that disasters amplify the effect of type and \( \frac{1}{2} > \pi_C \). Then \( \tilde{g}(\omega, \omega') > \tilde{g} \).

(ii) Suppose that disasters mute the effect of type and \( \pi_C > \frac{1}{2} \). Then \( \tilde{g} > \tilde{g}(\omega, \omega') \).

We will give a further condition under which \( \tilde{g}(\omega, \omega') > \tilde{g} \) (resp. \( \tilde{g}(\omega, \omega') > \tilde{g} \)).

**Lemma C.12.**

(i) \( \tilde{g} \leq \nu(\omega') \) if and only if \( \pi_I \geq \pi_C \).

(ii) \( \tilde{g} \geq \nu(\omega') \) if and only if \( \pi_C \geq \pi_I \)

**Proof.** Note that \( \tilde{g} \leq \nu(\omega') \) if and only if
\[
1 = \frac{\phi(\nu(\omega') - \nu(\omega') - \nu(\omega))}{\phi(\nu(\omega') - \nu(\omega') + \nu(\omega))} \geq \frac{\phi(\tilde{g}(\omega) - \nu(\omega') - \nu(\omega))}{\phi(\tilde{g}(\omega) - \nu(\omega') + \nu(\omega))},
\]
where the inequality follows from the MLRP and the equality follows from symmetry. Using the fact that
\[
\frac{\phi(\tilde{g}(\omega) - \nu(\omega') - \nu(\omega))}{\phi(\tilde{g}(\omega) - \nu(\omega') + \nu(\omega))} = \frac{\phi(\tilde{g}(\omega) - \nu(\omega) - \nu(\omega))}{\phi(\tilde{g}(\omega) - \nu(\omega) + \nu(\omega))} = \frac{1 - \pi_I}{\pi_I} \frac{\pi_C}{1 - \pi_C}
\]

34
it follows that \( \tilde{g} \leq \nu(\omega') \) if and only if

\[
1 \geq \frac{1 - \pi_I}{\pi_I} \frac{\pi_C}{1 - \pi_C}
\]

or if and only if \( \pi_I \geq \pi_C \). Reversing the inequalities gives part (ii). \( \blacksquare \)

Lemma C.13.

(i) If \( \pi_C \geq \pi_I > \frac{1}{2} \), then \( \tilde{g} > \bar{g}(\omega, \omega') \).

(ii) If \( \frac{1}{2} > \pi_I \geq \pi_C \), then \( \bar{g}(\omega, \omega') > \tilde{g} \).

Proof. We show part (i); part (ii) follows by reversing inequalities: Since \( \pi_C \geq \pi_I \), \( \tilde{g} \geq \nu(\omega') \). (See Lemma C.12(i).) Since, \( \pi_I \geq \frac{1}{2} \), \( \tilde{g} > \nu(\omega') > \bar{g}(\omega, \omega') \). (See Lemma C.9.) \( \blacksquare \)

Conclusion of Proof Proposition C.1 follows from Lemmata C.6, Lemma C.10-Remark C.1, Lemma C.11-Remark C.2, and Lemma C.13. Proposition C.2 follows from Lemmata C.7, Lemma C.10-Remark C.1, Lemma C.13 and the following additional result.

Lemma C.14. Suppose disasters mute the effect of type and \( \pi_I > \frac{1}{2} \geq \pi_C \). Then \( \Gamma(\hat{g}(\omega), \omega) > \Gamma(\hat{g}(\omega'), \omega') \).

Proof. If \( \bar{g}(\omega, \omega') \geq \tilde{g} \), then Lemma C.7 says that \( \Gamma(\hat{g}(\omega), \omega) > \Gamma(\hat{g}(\omega'), \omega') \). Suppose that \( \tilde{g} > \bar{g}(\omega, \omega') \). We will show that \( \bar{g}(\omega, \omega') > \hat{g}(\omega') \). With this,

\[
\Gamma(\hat{g}(\omega), \omega) = \tilde{\Gamma}(\bar{g}) > \bar{\Gamma}(\bar{g}(\omega, \omega')) = \Gamma(\bar{g}(\omega, \omega'), \omega') > \Gamma(\hat{g}(\omega'), \omega'),
\]

where the first inequality follows from the assumption that \( \tilde{g} > \bar{g}(\omega, \omega') \) and the MLRP, the second equality follows by Corollary C.2, and the last inequality follows from the fact that \( \bar{g}(\omega, \omega') > \hat{g}(\omega') \) (that we will show) and the MLRP.

We now turn to show that \( \bar{g}(\omega, \omega') > \hat{g}(\omega') \): By Lemma C.8, there exists some \( \bar{\omega} \in (\omega, \omega') \) so that

\[
\frac{\phi(\bar{g}(\omega, \omega') - \nu(\omega') - i(\bar{\omega}))}{\phi(\bar{g}(\omega, \omega') - \nu(\omega') + i(\bar{\omega}))} = \frac{1 - \pi_I}{\pi_I}.
\]

Since \( \frac{1}{2} \geq \pi_C \), this implies that

\[
\frac{\phi(\bar{g}(\omega, \omega') - \nu(\omega') - i(\bar{\omega}))}{\phi(\bar{g}(\omega, \omega') - \nu(\omega') + i(\bar{\omega}))} > \frac{1 - \pi_I}{\pi_I} \frac{\pi_C}{1 - \pi_C} = \frac{\phi(\bar{g}(\omega') - \nu(\omega') - i(\omega'))}{\phi(\bar{g}(\omega') - \nu(\omega') + i(\omega'))}.
\]

So by the MLRP \( \bar{g}(\omega, \omega') > \hat{g}(\omega') \). \( \blacksquare \)
References


