ABSTRACT. The currency composition of sovereign external debt in emerging economies is tilted towards foreign currency. Yet, the share of debt denominated in local currency has increased in the last decade, more so in booms than in busts. We study these facts through the lens of a quantitative model of optimal currency-composition of sovereign debt when the government lacks commitment regarding debt and monetary policy. High levels of debt in local currency give rise to incentives to dilute debt repayment through nominal currency depreciation, which can be attained through inflation or real exchange rate depreciation. Governments tilt the currency-composition of debt towards foreign currency to avoid inflationary costs and real exchange rate distortions. This is done at the expense of foregoing the hedging properties of debt in local currency. Our model is used shed light on the recent dynamics of the debt currency composition and on its cyclical behavior. Alternative linked-debt instruments are discussed.

Keywords: Sovereign debt, currency composition, monetary policy, time inconsistency.

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1. Introduction

Two decades ago the currency composition of sovereign debt was associated with debt dollarization. The predominant view at that time was that governments have difficulties in placing debt denominated in local currency, a phenomenon termed the ‘original sin’ (Eichengreen et al. (2002)). This raised concerns since high levels of debt denominated in foreign currency were associated with higher vulnerabilities to debt and exchange rate crises. This picture has changed significantly in recent years as the currency composition of sovereign debt has become an active margin of debt management.

In this paper we study the trade-offs associated to the management of the currency composition of sovereign debt. We propose a theoretical framework to jointly analyze the government’s debt issuance choices by currency and the conduct of monetary policy, and argue that even if governments could issue debt in local currency, they would not be willing to do so to a great extent. A key reason for why governments tilt their debt portfolio towards foreign currency is that they want to avoid falling in the temptation of engaging in distortionary nominal exchange rate depreciation. When governments face high levels of debt in local currency they may want to dilute the value of their debt by generating costly inflation or keeping the real exchange rate excessively depreciated. The shift in the currency composition of debt towards foreign currency debt comes at the expense of foregoing consumption insurance associated to borrowing in local currency. A quantitative analysis of the model for an average emerging economy shows that the presence of these trade-offs can rationalize the observed average currency composition of sovereign external debt, its recent dynamics as well as its cyclical properties.

The paper begins by documenting three stylized facts about the currency composition of sovereign debt in emerging economies. Using data for a panel of 18 countries we show that: (i) Debt currency composition tends to be tilted towards foreign currency: on average, around two thirds of sovereign external debt is denominated in foreign currency. (ii) The original sin has gradually dissipated: in the last decade the average share of debt in local currency increased from 9% to 39%. (iii) The currency composition of debt has a strong cyclical component: in economic booms the share of debt denominated in local currency is higher than in recessions. The average correlation between output and the share of debt denominated in local currency is 40%.

See, for example, Calvo (2005).
We then layout a general equilibrium model of sovereign debt and monetary policy to study the trade-offs associated with the currency composition of debt and shed light on the above-mentioned facts. In the model the government lacks commitment and chooses debt issuance in foreign and local currency and the exchange rate is determined endogenously through the interaction of debt choices and the conduct of monetary policy. We study two variants of equilibrium, one in which monetary policy is taken as given and another monetary policy is chosen optimally.

We first focus on the equilibrium given a particular monetary policy that replicates the co-movement of income and the nominal exchange rate. This approach highlights the role of the hedging properties of debt in local and foreign currency in determining optimal portfolio positions. Given that emerging economies tend to experience currency depreciation in recessions and appreciation during booms, debt in local currency constitutes a useful hedge against income risk. If the agent is indebted in local currency, when a negative income shock is realized, the exchange rate will likely depreciate which in turn lowers the value of debt repayments in local currency and attenuates the original income shock. The solution to this portfolio problem calls for issuing large levels of debt in local currency and little debt (if any) in foreign currency. This position is at odds with observed average levels of debt by currency in emerging economies.

This apparent disconnect is due to the fact that the attractive hedging properties of debt in local currency are partially offset by its associated perverse incentive problems. The government, through its choice of debt issuance and monetary policy can dilute the value of existing debt in local currency. One way to dilute debt in local currency is through inflation. Inflation, although costly, induces nominal exchange rate depreciation which in turn dilutes the value of debt in local currency. Another less obvious way to dilute debt in local currency is through inducing real exchange rate depreciation. By reducing the level of overall debt issuance the government post-pones consumption of tradable goods for households which in turn depreciates the real exchange rate and reduces the real value of debt in local currency. Both inflation and excessive real exchange rate depreciation are distortionary from an ex-ante point of view since the effects of debt dilution are anticipated by foreign investors that ask for lower prices for debt in local currency. The government takes this into account when choosing debt in the previous period and tilts the currency composition of debt towards foreign currency denomination to avoid engaging in costly inflation and distorting the real exchange rate in equilibrium. This shift in the currency composition of debt is done at the expense of further consumption insurance given that the model features an endogenous exchange rate that is counter-cyclical.
A calibrated version of the model, which takes into account the welfare costs of inflation estimated from previous literature and targets the total level of external public debt, shows that the equilibrium currency-composition of sovereign debt predicted by the model is in line with that observed in the data. The model also correctly predicts the cyclical pattern of the currency composition of sovereign external debt observed in the data. The government optimally chooses to issue a larger share of debt in local currency in booms. This cyclical behavior is not linked to the hedging properties of debt in local currency over the cycle but rather to the cyclical properties of the benefits associated to debt dilution. In recessions the benefit of diluting debt via nominal depreciation is higher since saving resources for consumption is more valuable given that the marginal utility of consumption is higher in those states. This implies an optimal inflation rate that is counter-cyclical. The government internalizes this when choosing debt by currencies and chooses to reduce its debt issuance in local currency to avoid high levels of costly inflation in recessions.

The model is also used to shed light on the recent observed increase in the share of debt in denominated in local currency. This change in the currency composition of external public debt happened simultaneously with a generalized process of deleveraging in emerging economies through which the level of total external public debt was reduced by 6 percentage points of GDP. We argue that these two phenomena are related to each other, since a common feature of episodes of total debt reduction is an increase in the share of debt denominated in local currency. Faced with a desire to reduce the overall level of debt, the government optimally chooses to do so by rolling-over debt in local currency and issuing less debt in foreign currency. The reason is that while the optimal level of debt in local currency is determined by weighing its benefits and costs, the optimal level of foreign currency is set to satisfy the desired inter-temporal consumption allocations. According to our calibrated model, one third of the observed increase in the share of debt in local currency can be explained by deleveraging shocks (i.e. shocks to the discount factor) that reproduce the observed dynamics of total external public debt.

In the last part of the paper we comment on the trade-offs associated to the use of alternative debt instruments. It could be thought that one way to get around the possibility of diluting the value of debt in local currency is to issue inflation-linked debt. We argue that this is true only to a partial extent, since the issuance of inflation-linked bonds does not fully overcome its incentive problems. While the government can no longer dilute the value of debt through inflation, it can still do so by postponing tradable consumption and inducing real exchange rate depreciation. We also consider the case of debt linked to the level of output.
We also briefly discuss the use of debt linked to the evolution of output and debt denominated in foreign currencies that have strong co-movements with the local currency. GDP-linked debt can get around the time inconsistency problem in the case of endowment economies, but no so in the case of economies with production. In the later the government has incentives to cool down the level of economic activity to repay less debt. Finally, we argue that issuing debt in a foreign currency that is correlated with the local currency completely eliminates the incentive problems but at the expense of exposing the economy to additional sources of risk.

Related Literature. This paper builds upon the literature on sovereign debt and time inconsistency of monetary policy. It is also closely related to a number of recent papers that study the joint determination of sovereign debt issuance and inflation choices.

Following the original framework of sovereign defaultable debt developed in Eaton and Gersovitz (1981), a recent body of literature has studied the quantitative dynamics of sovereign debt. Arellano (2008) and Aguiar and Gopinath (2006) analyze sovereign debt and business cycle properties in emerging economies. Several studies have extended the framework to study different aspects related to debt management such as its maturity composition, its composition by residence of creditors, its liquidity management and post-default debt renegotiations. Arellano and Ramanarayanan (2012) argue that the problem of debt maturity choice involves a trade-off between hedging and disciplining properties. While short-term debt is useful for providing incentives, long-term debt is useful for hedging consumption. Our currency-composition problem bares a resemblance to that of debt maturity choice. Here debt in foreign currency does not give rise to perverse incentive problems and debt in local currency has useful hedging properties. A few papers focus attention on the currency-composition of sovereign debt in real debt models. Bohn (1990) and Korinek (2009) study how debt denominated in different currencies can have different hedging properties for the borrower. The contribution of this paper is the analysis of a quantitative model of currency composition of sovereign debt in a context in which the government can also choose its optimal monetary policy.

The paper is also builds the literature on the time inconsistency of government polices. Following the seminal contributions of Kydland and Prescott (1977) and Barro and Gordon

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2For example, Broner et al. (2013) and Aguiar et al. (2016) study the optimal maturity composition of sovereign debt. Broner and Ventura (2011) and Perez (2015) analyze how the residence composition of government’s creditors can shape the incentives to default. Bianchi et al. (2014) quantify the optimal stock of international reserves as a buffer for exogenous shutdowns of the market for sovereign debt. Yue (2010) and Pitchford and Wright (2012) introduce the dimension of post-default debt renegotiation into defaultable debt models. Aguiar and Amador (2014) survey recent advances in the field.
a large strand of macroeconomic research has analyzed various contexts in which the ideal policies that a government would want to implement differ from the policies he can credibly promise. In those situations the government would like agents to trust his announcements but agents do not do so because they anticipate the government will have incentives to deviate from that announcement once they have taken their actions. In this paper the source of time inconsistency comes from the fact the government would like foreign investors and even himself to trust his desire of not to engage in distortionary inflation in the future but this is not possible as ex-post, once debt has been issued, the incentives to inflate arise to avoid paying to foreign investors.

Finally, the paper is also related to a set of papers that study the effects of partially defaulting on nominal debt by engaging in inflation, an idea that was explored in early work by Fischer (1983) and Calvo (1988). Our paper is mostly related to two contemporaneous papers that study the currency composition of sovereign debt. One is Du et al. (2016), that argue that various degrees of credibility regarding monetary policy can help reconcile cross-sectional returns of nominal bonds. Another is Engel and Park (2016), that study an optimal contract problem with the fear of outright default and debt dilution through inflation. We contribute to this literature along two dimensions. First, we analyze the optimal currency composition of debt in a context in which the exchange rate is endogenous and depends on economic fundamentals. This setup allows us to characterize the choice of currency composition through a trade-off between the hedging benefits and incentive problems of debt in local currency. Additionally, it gives rise to a novel channel of debt dilution through real exchange rate depreciation. Second, we present a quantitative dynamic model of sovereign debt that can reconcile the average currency composition of debt in emerging economies, its recent dynamics as well as its cyclical properties.

Examples of applications of time inconsistency problems include capital taxation (Chari and Kehoe (1990)), monetary policy (Rogoff (1985), Chari et al. (2015)) and government default and bailouts (Chari and Kehoe (1993), Farhi and Tirole (2012)). A more general treatment of time inconsistency problems can be found in Phelan (2006).

Other papers that focus on the problem of debt dilution through inflation include Aguiar et al. (2013) and Da-Rocha et al. (2013), that analyze how the ability to dilute debt via inflation can reduce the likelihood of suffering self-fulfilling debt crises; Araujo et al. (2013), that estimate the welfare implications of issuing debt in domestic currency or in a common currency; Du and Schreger (2015), that show that inflation can have associated negative effects on the balance-sheet of corporates; and Sunder-Plassmann (2014), that argues that inflation can help the government repay real debt in the long-run.
Layout. The remaining of the paper is organized as follows. Section 2 presents the data on currency composition of sovereign debt for a panel of emerging economies. Section 3 describes the model and defines the relevant notion of equilibria. Section 4 characterizes the main trade-offs involved in the choice of the currency composition of debt. Section 5 analyzes the quantitative properties of the model and compares them to their data counterparts. Section 6 comments on the use of alternative debt instruments, including inflation-linked debt. Finally, section 7 concludes.

2. Debt Currency-Composition in Emerging Economies: Stylized Facts

This section documents the main patterns of the currency composition of sovereign debt observed in emerging economies. We are interested in studying how large are debt positions in domestic currency relative to foreign currency, how these relative positions have changed over the past decade and how they fluctuate over the business cycle. To this end, we use data on sovereign external debt decomposed by currency of denomination and data on GDP for a panel of emerging economies. The source of the data on debt are Arslanalp and Tsuda (2014), which includes foreign holdings of both internationally- and domestically-issued government debt for the period 2003-2014 on a quarterly basis, and WDI which covers annual data on total external debt for a longer period of time. Data on quarterly real GDP comes from IFS and national sources. All countries included in the sample are middle income economies integrated to global capital markets, as measured for example by the fact that at some point in time they were part of the set of countries included in J.P. Morgan’s EMBI tracking sovereign debt in emerging economies.

Table 1 reports five key moments for a sample of emerging economies. Two of them are related to the total level of external public debt and three are related to the share of debt denominated in local currency. The first column shows the average level of external public debt for all countries. emerging economies display average levels of external public debt of of 22% of GDP. This average figure hides significant levels of heterogeneity across countries. As shown in the second column, the level of debt has fallen significantly over the last decade. The 2014 levels of external public debt are, on average, 6% of GDP lower than the 2004 levels. The fall was generalized, occurring in 13 of the 18 countries (see Figure A.1 for the dynamics of total debt for all countries).
### Table 1. Facts on Sovereign External Debt by Currency

<table>
<thead>
<tr>
<th>Country</th>
<th>Total External Public Debt</th>
<th>Share of Debt in LC</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Average (% of GDP)</td>
<td>Δ 2014-04 (% of GDP)</td>
<td>Average (% of Debt)</td>
</tr>
<tr>
<td>Argentina</td>
<td>28%</td>
<td>-14%</td>
<td>6%</td>
</tr>
<tr>
<td>Brazil</td>
<td>12%</td>
<td>-7%</td>
<td>52%</td>
</tr>
<tr>
<td>Bulgaria</td>
<td>43%</td>
<td>-18%</td>
<td>0%</td>
</tr>
<tr>
<td>China</td>
<td>7%</td>
<td>-4%</td>
<td>11%</td>
</tr>
<tr>
<td>Egypt</td>
<td>34%</td>
<td>-25%</td>
<td>11%</td>
</tr>
<tr>
<td>Hungary</td>
<td>37%</td>
<td>16%</td>
<td>31%</td>
</tr>
<tr>
<td>India</td>
<td>14%</td>
<td>-1%</td>
<td>11%</td>
</tr>
<tr>
<td>Indonesia</td>
<td>30%</td>
<td>-12%</td>
<td>18%</td>
</tr>
<tr>
<td>Lithuania</td>
<td>19%</td>
<td>20%</td>
<td>3%</td>
</tr>
<tr>
<td>Malaysia</td>
<td>23%</td>
<td>-7%</td>
<td>66%</td>
</tr>
<tr>
<td>Mexico</td>
<td>17%</td>
<td>4%</td>
<td>49%</td>
</tr>
<tr>
<td>Peru</td>
<td>31%</td>
<td>-29%</td>
<td>17%</td>
</tr>
<tr>
<td>Philippines</td>
<td>35%</td>
<td>-26%</td>
<td>13%</td>
</tr>
<tr>
<td>Poland</td>
<td>22%</td>
<td>7%</td>
<td>38%</td>
</tr>
<tr>
<td>Russia</td>
<td>5%</td>
<td>-16%</td>
<td>18%</td>
</tr>
<tr>
<td>South Africa</td>
<td>9%</td>
<td>9%</td>
<td>56%</td>
</tr>
<tr>
<td>Thailand</td>
<td>12%</td>
<td>-3%</td>
<td>68%</td>
</tr>
<tr>
<td>Turkey</td>
<td>20%</td>
<td>-4%</td>
<td>31%</td>
</tr>
<tr>
<td>Average</td>
<td>22%</td>
<td>-6%</td>
<td>28%</td>
</tr>
<tr>
<td>Std. Dev.</td>
<td>11%</td>
<td>14%</td>
<td>22%</td>
</tr>
</tbody>
</table>

**Notes:** The average external public debt is computed over the period 1990-2014. The share of debt in LC refers to the share of external public debt denominated in local currency. The correlation between output and the share of debt in LC refers to the correlation between the cyclical component of real GDP and the cyclical component of the share of external public debt denominated in local currency. Both variables are detrended using the HP filter. The share of debt in local currency and the correlation with GDP is computed for the period 2004-2014, when data by currency becomes available. Δ 2014-04 refers to the difference between the relevant variable in 2014 and in 2004. For countries in which the 2004 data was not available, the difference is taken from the earliest data available.
The last three columns illustrate the behavior of debt by currency denomination. The currency composition of external public debt is tilted towards debt in foreign currency. As shown in the third column, on average, less than one third of the public debt held abroad is denominated in local currency. Only 4 of our 18 countries have more than half of their external public debt denominated in local currency. The share of debt in local currency increased significantly in the last decade (fourth column). The average share of debt in local currency was 9% in 2014, compared to 39% in 2004. This increase was generalized, with 14 of the 18 countries shifting towards more local currency denominated debt (see Figure A.2 for the dynamics of the share of debt in local currency for all countries).

Once this generalized increase is filtered out, the share of public debt in local currency is on average procyclical, with booms characterized by larger positions of debt in local currency than busts. For the average country of our sample, the correlation between output and the detrended share of external public debt denominated in local currency is 40%. Again, only a few countries deviate from this pattern and show a countercyclical share in local currency. It could be thought that the strong procyclicality can be due exclusively to the episode of the global financial crisis. To address this issue we analyzed the episode of the Tequila crisis in Mexico which has historical data available. From the beginning to the end of 1994 the share of sovereign debt denominated in foreign currency increased from less than 5% to more than 50% (Figure A.3). Additionally, the procyclicality of the share in local currency is not due to exchange rate movements. Figure A.4 compares the evolution of the share valued at current nominal exchange rates with the share computed at a constant exchange rate over time. The two variables have similar magnitudes and dynamics.

The main focus of the paper is placed on the analysis of the three stylized facts regarding the currency-composition of external sovereign debt, namely, an average currency composition of sovereign external debt that is tilted towards foreign currency, the recent increase in the share of debt denominated in local currency and its the procyclical behavior. Later we argue that these facts need to be analyzed taking into account the behavior of total external public debt.

3. A Model of Debt Currency Composition

In this section we construct a general equilibrium model to study the currency composition of sovereign debt. The environment is a small open economy in which a representative risk-averse household consumes tradable and nontradable goods and faces a stochastic endowment.

5For a detailed study of debt composition during this episode see Calvo and Mendoza (1996).
The government makes the economy’s savings decisions through the choice of debt issuance, and conducts monetary policy through the choice of inflation. The government is benevolent but lacks commitment. There exits a foreign and a local unit of account, in which prices are denominated. Two securities are available for saving and borrowing purposes: one-period non-state contingent debt denominated in foreign and local currency.

3.1. The Model Economy

Households. Preferences are defined over an infinite stream of tradable and non-tradable consumption and inflation

$$\mathbb{E}_0 \left[ \sum_{t=0}^{\infty} \beta^t \{ u(c_t) - l(\pi_t) \} \right],$$

where $\beta \in (0, 1)$ is the subjective discount factor, $c_t$ denotes consumption in period $t$, $u : \mathbb{R}_+ \to \mathbb{R}$ is a twice continuously differentiable, strictly concave function, $\pi_t \equiv \frac{P_t}{P_{t-1}}$ is the gross inflation rate at period $t$, $P_t$ denotes the aggregate level of prices in period $t$ (to be described in more detail later), and $l : \mathbb{R}_+ \to \mathbb{R}_+$ is a convex function with $l'(1) = 0$. The disutility from inflation captures the distortionary costs associated to inflation.

The consumption good is assumed to be a composite of tradable and nontradable goods

$$c_t = C(c_{T,t}, c_{N,t})$$

where $c_{T,t}$ is consumption of tradable goods in period $t$, $c_{N,t}$ is consumption of non-tradable goods in period $t$, and $C : \mathbb{R}_+^2 \to \mathbb{R}_+$ is a twice continuously differentiable function, increasing in both arguments, concave, and homogeneous of degree one.

Households receive a stream of endowments of tradable goods $y_{T,t}$ and non-tradable goods $y_{N,t}$. Households are assumed to be hand-to-mouth and the government makes savings decisions for the households though an active management of active lump-sum transfer to them. This assumption is made for simplicity as the focus of analysis is government rather than household debt. The budget constraint of household each period is then given by:

$$p_{T,t}c_{T,t} + p_{N,t}c_{N,t} = p_t y_{T,t} + p_{N,t} y_{N,t} + T_t,$$

where $p_{T,t}$ and $p_{N,t}$ are prices of tradables and non-tradables and $T_t$ are lump-sum transfers from the government, all measured in local currency.

The household’s problem is to choose state-contingent plans $\{c_t, c_{T,t}, c_{N,t}\}_{t=0}^{\infty}$ that maximize utility (1), subject to the aggregation technology (2), the sequence of budget constraints (3), the given sequence of prices $\{p_{T,t}, p_{N,t}\}_{t=0}^{\infty}$, and the given sequence of endowments and government
transfers \( \{y_{T,t}, y_{N,t}, T_t\}_{t=0}^{\infty} \). Combining the first-order conditions of the household’s problem the optimal choice between tradable and nontradable goods can be described by:

\[
\frac{C_{c, N,t}}{C_{c, T,t}} = \frac{p_{T,t}}{p_{N,t}}
\]

(4)

where we denote \( f_{x,t} \) the derivative of \( f \) with respect to the variable \( x_t \) to simplify notation.

**Government.** The government chooses inflation and sovereign external debt, making lump-sum transfers from the proceeds of net borrowing to households. Two securities are available for saving and borrowing purposes: debt in foreign and local currency. Debt in foreign currency is a one-period security that pays one unit of foreign currency in the following period. Similarly, debt in local currency is a security that pays one unit of local currency in the following period. Let \( b_t^*, b_t \) be the stock of debt in foreign and local currency with which the agent enters the period. The budget constraint of the government expressed in local currency is given by:

\[
T_t = e_t q_t^* b_{t+1}^* + q_t b_{t+1} - e_t b_t^* - b_t
\]

(5)

where \( q_t^* \) is the price of debt in foreign debt measured in foreign currency, \( q_t \) the price of debt in local currency measured in local currency, and \( e_t \) is the exchange rate (i.e., the price of foreign currency in terms of local currency).

**Foreign Lenders.** We assume securities are priced by risk-neutral foreign investors that have access to a risk-less security denominated in foreign currency that pays the international interest rate \( R \). The price of both securities are then given by

\[
q_t^* = \frac{1}{R}
\]

(6)

\[
q_t = \frac{1}{R} E_t \left[ e_t \right]
\]

(7)

**Equilibrium.** The market for nontradable goods clears at all times:

\[
c_{N,t} = y_{N,t}.
\]

(8)

Assuming the law of one price holds for tradable goods and normalizing the international price of tradables to one, we get that \( p_{T,t} = e_t \). Given the assumed preferences and aggregation technology the aggregate price level of the economy is given by \( P_t = e_t C_{cT} \left( \frac{c_{T,t}}{y_{N,t}}, 1 \right) \).\(^6\) It will

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\(^6\) This expression can be derived from a standard consumption expenditure minimization problem or profit maximization problem (see, for example Uribe and Schmitt-Grohe, 2016).
be convenient to use this expression to denote the inverse of the equilibrium nominal exchange rate as:

$$e_t^{-1} = r\left(\pi_t, \frac{c_{T,t}}{y_{N,t}}\right)$$  \hspace{1cm} (9)

where $r : \mathbb{R}^2_+ \rightarrow \mathbb{R}_+$ is a twice continuously differentiable function, decreasing in its first argument and increasing in its second argument and we omit its dependence on $P_{t-1}$.

Aggregating households and government budget constraints and imposing condition (8), we obtain the resource constraint of the economy, expressed in foreign currency as:

$$c_{T,t} = y_{T,t} + \left(q^*_t b^*_{t+1} - b^*_t\right) + r\left(\pi_t, \frac{c_{T,t}}{y_{N,t}}\right) (q_t b_{t+1} - b_t).$$  \hspace{1cm} (10)

We can now define the competitive equilibrium of this economy as follows:

**Definition 1** (Competitive equilibrium). Given initial debt positions, $b^*_0$ and $b_0$, initial price level, $P_0$, a state-contingent sequence of endowments, $\{y_{T,t}, y_{N,t}\}_{t=0}^\infty$, and government policies $\{\pi_t, b^*_{t+1}, b_{t+1}\}_{t=0}^\infty$ a competitive equilibrium is a state-contingent sequence of allocations $\{c_{T,t}, c_{N,t}, T_t\}_{t=0}^\infty$ and prices $\{q^*_t, q_t, e_t, P_t\}_{t=0}^\infty$, such that:

1. Allocations solve the household’s problems at the equilibrium prices,
2. Transfers satisfy the government budget constraint,
3. Debt prices satisfy (6) and (7),
4. The market for non-tradable goods clears.

**3.2. Optimal Government Policies**

In this section we formulate the problem of optimal policy for a benevolent government that lacks commitment. We focus on the notion of a Markov Perfect Equilibrium in which policies depend on payoff-relevant states. We assume $y \equiv \begin{bmatrix} y_T & y_N \end{bmatrix}$ follows a Markov process with transition probability $g_y(y, y')$. Define $s = \{b^*, b, y, P_{-1}\}$ as the aggregate state. Since the planner lacks commitment, it takes as given the optimal monetary policy of future planners. Let $R(b^*, b, y_T, P)$ be the perceived inverse of the nominal exchange rate policy function of future planners. The planner’s problem written in recursive form is given by:

$$V(s) = \max_{b^*, y, \pi, c_T} \{u(C(c_T, y_N)) - l(\pi)\} + \beta \mathbb{E}[V(s')]$$  \hspace{1cm} (P1)

subject to

$$c_T = y_T - b^* - r\left(\pi, \frac{c_T}{y_N}\right) b + \frac{b^*}{R} + \frac{b'}{R} \mathbb{E}[R(s')]$$

$$P = \pi P_{-1}$$
The first restriction corresponds to the resource constraint of the economy where the prices of both debts have been already replaced. The second restriction defines inflation as the increase in the aggregate level of prices. Given the recursive formulation of the planner’s problem we define equilibrium as follows:

**Definition 2** (Markov Perfect Equilibrium). A Markov Perfect Equilibrium consists of a set of policy functions \( \{c_T(s), b^*(s), b'(s), \pi(s)\} \), value function \( V(s) \), perceived policies \( R(s) \) and prices \( \{q^*(s), q(s)\} \) such that

- Policies \( \{c_T(s), b^*(s), b'(s), \pi(s)\} \) solve the planner’s problem \((P1)\).
- Prices \( \{q^*(s), q(s)\} \) solve \((6), (7)\).
- Perceived policies coincide with actual policies: \( R(s) = r(s) \).

To characterize the optimal policy trade-offs, it is also useful to consider the optimal problem of a government that can only choose debt policies for a given inflation policy. Let \( \Pi(s, \epsilon^\pi) \) be the assumed inflation policy function, where \( \epsilon^\pi \) is stochastic and follows a Markov process potentially correlated with \( y \), with transition probability \( g(y, \epsilon^\pi, y', \epsilon^\pi') \). Define \( \tilde{s} = \{s, \epsilon^\pi\} \) as the aggregate state for this problem. The planner’s problem for a given inflation policy written in recursive form is given by:

\[
\tilde{V}(\tilde{s}) = \max_{b^*, b', c_T} \left\{ u(C(c_T, y_N)) - l(\Pi(\tilde{s})) \right\} + \beta \mathbb{E}\left[ \tilde{V}(\tilde{s}') \right] \\
\text{subject to} \\
c_T = y_T - b^* - r \left( \Pi(\tilde{s}), \frac{c_T}{y_N} \right) b + \frac{b'}{R} b' + \frac{b'}{R} \mathbb{E}[R(\tilde{s}')] \\
\quad P = \Pi(\tilde{s}) P_{-1}
\]

We define equilibrium for a given inflation policy as follows:

**Definition 3** (Markov Perfect Equilibrium Given Inflation Policy). A Markov Perfect Equilibrium consists of a set of policy functions \( \{c_T(\tilde{s}), b^*(\tilde{s}), b'(\tilde{s}), \pi(\tilde{s})\} \), value function \( V(\tilde{s}) \), perceived policies \( R(\tilde{s}) \) and prices \( \{q^*(\tilde{s}), q(\tilde{s})\} \) such that

- Policies \( \{c_T(\tilde{s}), b^*(\tilde{s}), b'(\tilde{s})\} \) solve the planner’s problem \((P2)\).
- Prices \( \{q^*(\tilde{s}), q(\tilde{s})\} \) solve \((6), (7)\).
- Perceived policies coincide with actual policies: \( R(\tilde{s}) = r(s) \).
4. The Trade-Offs of the Debt Currency Composition

This section describes the main trade-offs associated to the choice of the currency composition of sovereign debt. We argue that the optimal currency composition of debt weighs the hedging benefits of debt in local currency against its incentive problems. On the one hand, debt in local currency is useful for hedging if the economy features counter-cyclical exchange rates (i.e. local currencies that depreciate in recessions). On the other hand, debt in local currency is harmful by giving rise to incentives to dilute debt repayment through distortionary nominal currency depreciation.

4.1. The Hedging Benefits of Debt in Local Currency

This section highlights the hedging benefits of debt in local currency. To do so we focus on the Markov equilibrium given a particular inflation policy, \( \pi = \Pi(s, \epsilon^\pi) \), that has a stochastic component and is such that

\[
\begin{align*}
\frac{\pi}{y^N} = \epsilon^\pi,
\end{align*}
\]

where \( \epsilon^\pi \) is stochastic and follows a joint Markov process with endowments. This approach facilitates the characterization of the hedging properties in debt in local currency in terms of the co-movement of the inverse of the nominal exchange rate \( e^{-1} = \epsilon^\pi \) and the tradable endowment.

Under this setting the government problem (P2) is equivalent to a portfolio problem of an agent that faces income risk and can choose saving and borrowing in foreign and local currency. This particular type of portfolio problem with incomplete markets and income risk does not have closed form solutions.\(^7\) The optimal consumption process satisfies the following optimality conditions

\[
\begin{align*}
&u'(c_t)C_{ct,t} = \beta R \mathbb{E} [u'(c_{t+1})C_{ct,t+1}] \\
&\text{COV} [u'(c_t)C_{ct,t}, e_{t}^{-1}] = 0.
\end{align*}
\]

Along the optimal consumption path, the consumption process features the Euler equation for tradable goods (equation (11)). Equation (12) comes from the possibility of issuing debt in local currency which is a security that is priced by risk-neutral investors that care about the expected return in foreign currency. According to this equation, along the optimal consumption path the

\(^7\)Duffie et al. (1997) characterize the solution to this class of problem in continuous time. Viceira (2001) provides an analytical characterization of an approximate solution to a similar problem.
marginal utility of tradable consumption is isolated from any risk that stems from fluctuations in the exchange rate.

This consumption path is attained with certain positions in foreign and local currency that depend on the correlation between tradable income and the inverse of the nominal exchange rate. This is made clear in the following proposition that characterizes the optimal debt position in local currency for a particular joint stochastic process for tradable endowment and the inverse of the nominal exchange rate.

**Proposition 1.** Assume $y_{Nt}$ is deterministic and $y_t$ and $e_t^{-1}$ follow a joint stochastic process which is i.i.d. over time. Define the standard deviation of random variable $X$ as $\sigma_X$ and correlation coefficient between $X$ and $Y$ as $\rho(X,Y)$.

1. If $\rho(y_t, e_t^{-1}) = 1$ then $b_t = \frac{\sigma_y}{\sigma_{e^{-1}}} > 0$ is part of the solution set of the government problem.
2. If $\rho(y_t, e_t^{-1}) = 0$ then $b_t = 0$ is part of the solution set of the government problem.
3. If $\rho(y_t, e_t^{-1}) = -1$ then $b_t = -\frac{\sigma_y}{\sigma_{e^{-1}}} < 0$ is part of the solution set of the government problem.

All proofs can be found in Appendix B.

This proposition illustrates the hedging properties of debt in local currency. When tradable income is perfectly positively correlated with the inverse of the nominal exchange rate, the government can attain perfect tradable consumption smoothing for households. It does so by issuing positive debt in local currency. This way, when there is a negative tradable income shock the currency depreciates and the government needs to repay less in terms of tradable consumption, muting this way the negative effect of the income shock. When income is perfectly negatively correlated with the inverse of the exchange rate the government can also attain tradable consumption smoothing for the households by saving in local currency. This way, when a negative tradable income shock realizes the currency appreciates and the return on its savings measured in tradable consumption is higher. When tradable income is uncorrelated with the inverse of the exchange rate the government can also attain tradable consumption smoothing for the households by saving in local currency. This way, when a negative tradable income shock realizes the currency appreciates and the return on its savings measured in tradable consumption is higher. When tradable income is uncorrelated with the exchange rate debt in local currency provides no hedging against income risk. In this case taking any position on local currency exposes households to an additional source of risk without perceiving any excess expected return for this since debt pricing is done by risk-neutral investors.

A well-known stylized fact in emerging economies, documented for our sample of economies in Appendix C, is that exchange rates are negatively correlated with aggregate tradable income (that is, $\rho(y_t, e_t^{-1}) > 0$). Given the results shown in Proposition 1, this introduces a reason for
why debt in local currency can be a useful hedge against income risk. In next section we use the patterns observed in the data between income and exchange rate to conduct a quantitative assessment of this channel based on the debt problem for given monetary policy.

4.2. The Incentive Problems of Debt in Local Currency

This section highlights the incentive problems associated to debt in local currency. These problems arise due to the fact that the value of repayment of debt in local currency, \( r(\pi_t, c_{Tt}) \), can be diluted ex-post. There are two channels of dilution. One is dilution through inflation, which operates through the effect that inflation choices have on the nominal exchange rate, and the other one is dilution through real exchange rate, which operates through the effect that debt choices have on tradable consumption and on the nominal exchange rate. We analyze each channel separately by focusing on the Markov equilibrium with optimal time-consistent monetary policy in a deterministic setting without endowment shocks.

To analyze the dilution through inflation channel we focus on the case in which \( r(\pi_t, c_{Tt}) = 0 \). The trade-offs associated to the choice of monetary and debt policies emerge from the analysis of the first order conditions of the government problem. We provide a derivation of these conditions in Appendix B. Consider a case in which the outstanding level of debt in local currency is positive \( (b_t > 0) \). The trade-off involved in the choice of inflation can be seen from the following first order condition

\[
-l'(\pi_t) = u'(c_t)C_T(c_{Tt}, y_{Nt})r_\pi(\pi_t, c_{Tt})b_t
\]

On the one hand, higher inflation entails direct losses. On the other hand, higher inflation increases the level of prices, one of these being the nominal exchange rate. A nominal depreciation in turn dilutes the value of repayment of local-currency debt measured in tradable consumption units and allows for higher consumption by saving resources. The optimal inflation choice is one in which the marginal cost of inflation (left hand side) equals the marginal benefit derived from diluting debt in local currency through inflation (right hand side). A marginal increase in inflation reduces debt repayment and increases tradable consumption by \( r_\pi(\pi_t, c_{Tt})b_t \). This marginal benefit is higher, the higher the outstanding stock of debt in local currency. This implies that, in most cases, the optimal level of inflation is increasing in the level of debt in local currency.

Additionally, to the extent that tradable consumption is pro-cyclical in equilibrium, the optimal inflation rate will be counter-cyclical. This is due to the fact the benefits of diluting debt through inflation are counter-cyclical. For a given level of debt in local currency the
incentives to dilute debt via inflation differ with the realization of the endowment. In periods of low realizations of the tradable endowment, the marginal utility of consumption is high and so is the marginal benefit of saving resources for consumption by reducing debt repayments. This increases the attractiveness of diluting debt in local currency through inflation.

Inflation is distortionary from an ex-ante perspective. The reason is that in equilibrium foreign investors anticipate the optimal inflation choices and offer lower debt prices for higher levels of debt in local currency to compensate for the future debt dilution. The lower debt prices offset the ex-post benefits of debt dilution. Therefore, from an ex-ante perspective inflation only entails the direct welfare costs. The government takes this into account when making debt choices. In particular, the government chooses a lower level of debt in local currency than what he would choose if he could commit not to inflate ex-post, precisely to avoid incurring in costly inflation in equilibrium. This can be seen in the modified Euler equation that characterizes the choices of debt in local currency:

\[
u'(c_t)C_{ct,t} = \beta R u'(c_{t+1})C_{ct,t+1} \frac{1}{1 + \frac{b_{t+1}r_{\pi,t+1}\pi_{b,t+1}}{r(\pi_{t},CT_t)}}.\]

The disciplining effect here depends on the sensitivity of optimal inflation to debt in foreign currency. Insofar the optimal level of inflation does not depend on the level of outstanding debt in foreign currency, the optimal choice of debt in foreign currency is undistorted, as characterized by the standard Euler equation.

We can characterize the equilibrium debt position in local currency for the particular case in which there are no inflation costs. This is summarized in the following proposition.

**Proposition 2 (Market Shutdown).** If \( l(\pi) = 0 \) for all \( \pi \), the equilibrium features a market shutdown for debt in local currency: \( q(s)b'(s) = 0 \).

The intuition behind this result is simple and relies on the lack of commitment. If the government does not have any cost of inflating debt, it will optimally choose to completely
default on any positive debt in local currency by choosing an arbitrarily large level of inflation. Foreign investors anticipate this and are unwilling to lend in local currency (i.e. \( q(s) = 0 \)).

Now we analyze the dilution through real exchange rate channel by focusing, for simplicity, in the case of \( r_\pi(\pi_t, c_{T_t}) = 0 \), which would correspond to real debt denominated in non-tradable goods. Another way to dilute the value of repayment of debt in local currency is to depreciate the real exchange rate ex-post. This can be done through postponing tradable consumption by reducing debt issuance for the following period at the expense of distorting inter-temporal consumption decisions. This would call for lower levels of debt ex-post. However, this effect is distortionary ex-ante since the benefits of dilution are offset by lower debt prices offered by foreign investors that anticipate this behavior. The government takes this into account and affects its debt decisions to discipline the future incentives to dilute debt. Interestingly, debt choices are both the instruments used to dilute debt through real exchange rate depreciation and the instruments used to discipline the temptation to do so in the future. This can be seen from modified Euler equation for debt in local currency

\[
\begin{align*}
    u'(c_t)C_{c_{T,t}} = \beta Ru'(c_{t+1})C_{c_{T,t+1}} & \quad \left(1 + r_{c,t}b_t\right) \quad \text{Dilution through RXR} \\
    \frac{1}{1 + r_{c,t}b_t} b_{t+1} & \quad \text{Discipline Effect}
\end{align*}
\]

The Euler equation features a component that captures the dilution through real exchange rate, \((1 + r_{c,t}b_t)\) and acts as a perceived higher interest rate when there is a positive outstanding debt in local currency. The presence of this term is due to the fact that raising one additional unit of good with debt increases tradable consumption by \(1/ (1 + r_{c,t}b_t)\) which is less than one. This is because of a second order effect of consumption on debt repayment: higher tradable consumption increases the nominal exchange rate by \( r_c(\pi_t, c_{T_t}) \) (through an increase in the real exchange rate) which in turn increases the value of debt repayment and decreases tradable consumption. The Euler equation also features a component that captures disciplining role of debt current choices on future incentives to manipulate the payoff of debt in local currency through future debt choices. The disciplining effect is present in the case in which future choices of debt in local currency are non-zero, and depends on the elasticity of the payoff of debt in local currency to the real exchange rate and the extent to which current choices of debt in local currency affect future debt choices.
A similar expression characterizes the optimal choice of debt in foreign currency

\[ u'(c_t)C_{cT,t} = \beta R u'(c_{t+1})C_{cT,t+1} \left. \frac{1}{1 + b_{t+1}r_{c,t}} \frac{1}{1 - \partial (r(c_{T,t+2},\pi_{t+2})b_{t+2} + b_{t+2}) b_{t+1}} \right| \text{Dilution through RXR} \]

The dilution effect is the same as the one for debt in local currency. The discipline effect now depends on the extent to which current choices of debt in foreign currency affect future debt choices. Both effects are present insofar outstanding debt in local currency and current choices of debt in local currency are different from zero.

Note that the disciplining effects in the Euler equations, nor the dilution effect through real exchange rate, would not be present if debt choices were made by private agents. If this was the case the debt choices of households would be characterized by standard Euler equations which would in turn lead to over-borrowing in domestic currency and excessively high inflation since they don’t internalize the effect of their debt choices on the optimal monetary policy decisions of the government (see Aguiar et al. (2015)).

In summary, while debt in local currency has useful hedging properties if the nominal exchange rate correlates negatively with income, it also gives perverse incentives to dilute its repayment through inflation or future debt choices. While the former characteristic points toward a higher debt in local currency, the latter calls for a substitution of debt in local currency for debt in foreign currency. The problem of choosing different debt instruments with different hedging properties and incentive problems bares a resemblance to that of debt maturity choice. As argued in Arellano and Ramanarayanan (2012) short-term debt is more useful for providing incentives (as is debt in foreign currency) whereas long-term debt is useful for hedging against future fluctuations in interest rates (as is debt in local currency for hedging against income fluctuations).

5. Quantitative Analysis

In this section we calibrate the model to match the salient features of the average economy studied in Section 2 and evaluate the quantitative results in the light of the data on the currency composition of sovereign external debt.

5.1. Calibration

To better understand the quantitative properties of the model we calibrate two economies: a “full model”, in which the government chooses debt and inflation, as in Definition 2; and a “debt problem”, in which the government only chooses debt, as in Definition 3, with the additional
assumption made in Section 4.1 that the inflation policy is such that the exchange rate is not affected by debt choices. The later is aimed at quantifying the hedging properties of debt in local currency if one takes as given the joint dynamics of exchange rates and income observed in the data. We maintain the calibration of the two economies as close as possible to be able to make comparisons across the two model specifications.

One period corresponds to one quarter. The period utility function is assumed to be given by:

\[ u(c_{T,t}, c_{N,t}) - l(\pi) = \frac{(c_{T,t}^a c_{N,t}^{1-a})^{1-\sigma}}{1-\sigma} - \frac{\psi}{2} (\pi - 1)^2. \]

The parameter values are summarized in Table 2. The international interest rate is set to \( R = 1.01 \) and the risk aversion coefficient \( \sigma \) is set to 2, which are standard values in quantitative macro studies. For simplicity we set equal shares for tradables and non-tradables in the consumption aggregator. The discount factor is calibrated to target an average stock of external public debt of 22% of annual GDP. The calibrated value is \( \beta = 0.99 \).

Table 2. Model Calibration

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Debt Problem</th>
<th>Full Model</th>
<th>Source/Target</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Preferences</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Risk aversion coefficient</td>
<td>( \sigma )</td>
<td>2.00</td>
<td>2.00</td>
</tr>
<tr>
<td>Tradable share in utility</td>
<td>( a )</td>
<td>0.50</td>
<td>0.50</td>
</tr>
<tr>
<td>Discount factor</td>
<td>( \beta )</td>
<td>0.99</td>
<td>0.99 Avg. external debt (22% of GDP)</td>
</tr>
<tr>
<td>Cost of inflation</td>
<td>( \phi )</td>
<td>-</td>
<td>17.0 Burstein &amp; Hellwig (2008)</td>
</tr>
<tr>
<td><strong>Endowments and Interest Rate</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Risk free interest rate</td>
<td>( R )</td>
<td>1.01</td>
<td>1.01</td>
</tr>
<tr>
<td>Non-tradable endowment</td>
<td>( y_N )</td>
<td>2.62</td>
<td>2.62 Share of tradable output (30%)</td>
</tr>
<tr>
<td>Autocorrelation of ( y_T )</td>
<td>( \rho_{y_T} )</td>
<td>-</td>
<td>0.54 Tradable output</td>
</tr>
<tr>
<td>Std. dev. of ( y_T )</td>
<td>( \sigma_{y_T} )</td>
<td>-</td>
<td>0.03 Tradable output</td>
</tr>
</tbody>
</table>

We calibrate the value of \( \psi \), the parameter associated with the disutility of inflation, to obtain a direct welfare cost of inflation that is in line with the cost estimated in Burstein and Hellwig

---

8The choice of a quadratic cost of inflation is motivated by the fact that this can be mapped into a Rotemberg-type of adjustment costs for prices, whose quantitatively properties have been widely studied. See Nuño and Thomas (2015) for this explicit formulation in the context of a model of sovereign debt.
In the main specification of that paper an increase in annual inflation of 10% (e.g. from 0% to 10%) has associated a welfare loss equivalent to a drop in permanent consumption of 1.3%. The value of $\psi$ that yield such welfare loss is 20.

We assume $y_{T,t}$ is stochastic and and $y_{N,t} = y_N$ is constant over time. For the full model this means that the only source of exogenous fluctuations is the level of tradable endowment, which we assume follow an AR(1) process in logs

$$\log y_{T,t} = \rho y_{T}, \log y_{T,t-1} + \epsilon_{t}^{y}$$

where $\epsilon_{t}^{y} \sim N(0, \sigma_{y_{T}})$. This process is estimated with quarterly data on industrial production for the period 1990Q1-2014Q4 for the panel of countries analyzed in Section 2 that have available data. The data of industrial production was seasonally adjusted and detrended using the HP filter. This process is estimated to be persistent. The estimation results are shown in Table 2. The unconditional average level of $y_{T}$ is 1 and we set the level of the non-tradable endowment to $y_{N} = 2.6$ to obtain a share of tradable output of 30% consistent with observed data for the countries in our sample.

For the debt problem, we assume that the shocks to the inflation policy $\varepsilon^{\pi}$ are such that the nominal exchange rate follows the path observed in the data. In this model the exogenous state is thus given by $s_{t}^{x} = (y_{T,t}, e_{t}^{-1})$ which is assumed to follow a first-order VAR process in logs,

$$\log s_{t}^{x} = \Phi_0 + \Phi_1 \log s_{t-1}^{x} + \varepsilon_{t}^{x}, \quad (13)$$

where $\varepsilon_{t}^{x} \sim N(0, \Omega)$. This process is estimated with quarterly data on industrial production (as in the full model) and nominal exchange rate for the panel of countries analyzed in Section 2 for the 1990Q1-2014Q4 period. The inverse exchange rate is measured as the cyclical component of the inverse of nominal exchange rate vis-a-vis the US dollar. Estimation details are provided in Appendix C. Both processes are estimated to be persistent. Additionally, output and the inverse of the nominal exchange rate are positively correlated as reflected by the positive covariance between their respective innovations. This positive correlation is consistent with the evidence shown in Appendix C.

5.2. Model Predictions and Data

We analyze the model’s quantitative performance by comparing moments from the model’s ergodic set with moments of the data. To compute the model’s moments we simulate the exogenous stochastic processes of each economy (tradable income in the full model and tradable income and exchange rate in debt problem) for 10,000 periods and trace the evolution of the
debt positions. The moments are computed by eliminating the first 1,000 observations. The
moments from the data correspond to data on sovereign external debt for the same set of
countries for the period 2004Q1-2014Q4 which is the sample for which the data is available.

Table 3 compares the moments regarding debt positions in the model with their data coun-
terparts. We first analyze the moments from the debt problem in the light of the data moments.
In the data the average level of external sovereign debt is 22% of GDP, of which 28% is de-
nominated in local currency. In both simulated economies the total level of sovereign debt is
roughly the same as in the data since it is a target of the calibration. However, the debt problem
predicts a share of debt in local currency of 57%, significantly higher than the one observed in
the data. The high levels of debt in local currency are due to the strong negative correlation
between output and exchange rate shown in the data. This co-movement makes debt in local
currency an attractive security for smoothing tradable consumption. These results reveal a
simple puzzle, which we interpret as the presence of certain frictions that prevent governments
from exploiting the hedging properties of debt in foreign and local currency.

Additionally, the debt problem is unable to replicate the procyclicality of share of debt in
domestic currency. While in the data the correlation between output and the difference between
debt in local and foreign currency as a share of GDP is 27%, its model counterpart is -1%. This
suggests that the variation of the hedging properties of debt in local currency over the business
cycle cannot account for the cyclical variation of the currency composition of debt.

Table 3. Model Results

<table>
<thead>
<tr>
<th>Moment</th>
<th>Data</th>
<th>Problem</th>
<th>Full Model</th>
<th>β Shocks</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Average Levels</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total Debt (% of GDP)</td>
<td>22%</td>
<td>20%</td>
<td>25%</td>
<td>25%</td>
</tr>
<tr>
<td>Debt in LC (% of Total Debt)</td>
<td>28%</td>
<td>57%</td>
<td>6%</td>
<td>3%</td>
</tr>
<tr>
<td><strong>Second Moments</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Debt Standard Deviation</td>
<td>11%</td>
<td>11%</td>
<td>1%</td>
<td>15%</td>
</tr>
<tr>
<td>Debt in LC - Output Correlation</td>
<td>27%</td>
<td>-1%</td>
<td>77%</td>
<td>14%</td>
</tr>
</tbody>
</table>

*Notes:* Debt currency composition is measured as the ratio between debt in local currency to
total debt. The currency composition-output correlation refers to the correlation between the
output and the ratio of the difference debt in local and foreign currency to GDP.
We now focus on the case of the full model economy. Before analyzing the model’s moments it is important to understand the behavior of the exchange rate in the model and its co-movement with income shocks. In this model the nominal exchange is endogenously determined. According to the equilibrium expression of the exchange rate, equation (9), the level of the exchange rate depends on the inflation rate and on the relative price of non-tradables. The relative price of non-tradables is countercyclical in the model: given that markets are incomplete, tradable consumption co-varies positively with tradable endowment, which in turn delivers a countercyclical relative price of non-tradables. Additionally, as explained in section 4.2, the optimal choice of inflation is also counter-cyclical. Therefore, in this model the nominal exchange rate is counter-cyclical. This feature is relevant in shaping the hedging properties of debt in local currency since the government will find attractive to issue debt in local currency to the extent that the nominal exchange rate rate is negatively correlated with tradable income.

The third column of Table 3 shows the moments generated from the full model economy. This variant of the model sheds light on some of the puzzles generated by the previous model. In particular, as in the data, the simulated full model predicts that the currency composition of external public debt is tilted towards foreign currency. Quantitatively, the model predicts only 6% of debt denominated in domestic currency, lower than the 28% observed in the data. This result suggests that the government’s lack of commitment regarding debt policy and monetary policy is quantitatively relevant in determining the optimal currency composition of government debt.

The full model also generates, as in the data, a procyclical share of debt in domestic currency. Quantitatively, the model predicts a magnitude of this correlation (77%) higher that the one observed in the data (27%). The cyclical behavior of the share of debt in local currency is related to the cyclical properties of the incentive problems associated to this type of debt. For a given level of debt in local currency the incentives to dilute debt via inflation differ with the realization of the endowment. In periods of low realizations of the endowment the marginal utility of consumption is high and so is the marginal benefit of saving resources for consumption by reducing debt repayments. This increases the attractiveness of diluting debt in local currency through inflation. Given that the temptation to engage in costly inflation is higher in bad times the government optimally chooses its currency composition in such a way that the incentives to inflate in bad times are mitigated. If the tradable endowment process is persistent, the government reduces its issuance in local currency in low realizations of the tradable endowment. This way, the incentives to inflate -which are likely to be high
given the persistence of the tradable endowment- are undermined by low levels of debt in local currency. Therefore, the model proposes a quantitative explanation for the cyclical behavior of the currency composition of sovereign debt that is linked to the cyclical properties of the benefits associated to debt dilution by inflation rather than the cyclicity of the hedging properties of debt in local currency.

5.3. Episodes of Debt Deleveraging and Debt Currency Composition

As mentioned in Section 2, the last decade was characterized by a reduction in the total level of external public debt and by an increase in the share of debt in domestic currency. We are interested in studying these facts thought the lens of the model and, in particular, to study whether the increase in the share of debt in domestic currency observed over the last decade can be related to the reduction of the total level of debt. A limitation of our quantitative model in this task it that, as shown in Table 3 the volatility of external debt in the model is too low relative to the data; low debt volatility implies that our simulated economy cannot generate periods of debt deleveraging of the magnitude observed in emerging economies during the last decade. One potential reason why the model predicts a low volatility of total debt is that fluctuations in the model are only driven by endowment shocks. In this section we extend the model to incorporate shocks to the household’s discount factor, in the spirit of Krusell and Smith Jr (1998) that use these shocks to match the dynamics of wealth distribution. Once we match the volatility of sovereign debt observed in the data, we show that episodes of debt deleveraging as the those observed in emerging economies over the last decade are also episodes of a significant increase in the share of debt in local currency.

In this extended version of the model household’s preferences are assumed to be given by:

$$\mathbb{E}_0 \left[ \sum_{t=0}^{\infty} \tilde{\beta}^t \{ u(c_t) - l(\pi_t) \} \right],$$

where $\tilde{\beta}$ follows a first-order Markov process. To calibrate the discount factor shock, we assume for simplicity that $\tilde{\beta} \in \{0.995\beta, 1.005\beta\}$, where, as in the baseline calibration, $\beta$ is set to match an average stock of external sovereign debt of 31% of annual GDP. The parameter on the transition probability matrix of $\tilde{\beta}$ is set to match the standard deviation of sovereign debt. The rest of the parameters are set as in the baseline calibration (Table 2).

The moments of the simulated economy with discount factor shocks are reported in Table 3. Besides matching the standard deviation of sovereign debt observed in the data (which is the new target of the calibration), the simulated economy with discount factor shocks also displays
a debt currency composition close to the one observed in the data, as in the baseline full model. Equipped with a quantitative model that matches the standard deviation of sovereign debt and the average debt currency composition, we can now study periods of debt deleveraging similar to those observed over the last decade, documented in Section 2. To do this, the calibrated version of the model is simulated for 1 million quarters, identifying each period $t$ in which two conditions are satisfied: First, between $t$ and $t+40$, the change in total external public debt is more than a given parameter value $\Delta_b$; second at period $t+40$, the level of external public debt is below a given parameter value $b_T$. The responses of the variables of interest are then averaged across all episodes of debt deleveraging. We choose the values of the parameters $\{\Delta_b, b_T\}$ to match an average fall in external public debt between $t$ and $t+40$ of 11 percent (which is the change observed during the last decade documented in Table 1) and a level of external sovereign debt in $t+40$ of 15 percent (the level observed in 2014).

Table (4) reports the change in external public debt-to-gdp and the change in the share of debt in foreign currency for the average episode of debt deleveraging, and compares it to the one observed in the data for the average EM over the last decade. It also reports the change in the exogenous driving processes, endowment and discount factors, that occurs during these episodes. This exercise reveals that debt deleveraging episodes of the magnitude observed in the data, are mostly driven in the model by discount factor shocks: The average episode of debt deleveraging is characterized by small changes in the endowment process. More importantly, the model predicts that episode of debt deleveraging entails a significant increase in the share of debt in domestic currency, of around one third of that observed in the data. Faced with a desire to reduce the overall level of debt, the government optimally chooses to do so by rolling-over debt in local currency and issuing less debt in foreign currency. The reason is that while the optimal level of debt in local currency is determined by weighing its hedging benefits and incentive costs, the optimal level of foreign currency is set to satisfy the desired inter-temporal consumption allocations, which are affected in periods of deleveraging driven by discount factor shocks.

6. Alternative Debt Instruments

In this section we discuss the use of alternative debt instruments that can serve as substitutes of debt in local currency. An often proposed way to overcome the incentive problems associated to debt in local currency is to issue inflation-linked debt denominated in local currency. This is in fact a security commonly observed in several countries including the US and emerging
### Table 4. Debt Currency Composition During Episodes of Debt Deleveraging: Data and Model

<table>
<thead>
<tr>
<th></th>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>∆ Debt</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total Debt (% of GDP)</td>
<td>-6%</td>
<td>-7%</td>
</tr>
<tr>
<td>Debt in LC (% of Total Debt)</td>
<td>30%</td>
<td>13%</td>
</tr>
<tr>
<td><strong>Shocks (episode avg/ uncond. avg)</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Discount Factor</td>
<td>–</td>
<td>0.4%</td>
</tr>
<tr>
<td>Output</td>
<td>–</td>
<td>0.6%</td>
</tr>
</tbody>
</table>

*Notes:* ∆Debt in the Data column refers to the difference between the relevant variable in 2014 and in 2004 for the countries detailed in Table 1. For countries in which the 2004 data was not available, the difference is taken from the earliest data available. Debt currency composition is measured as the ratio between debt in local currency to total debt. ∆Debt in the Model column refers to the change of a given variable during the average episode of debt deleveraging, as defined in section 5.3. The average level of the stochastic processes (discount factor and tradable endowment) during the average episode of debt deleveraging are measured in log deviation from their unconditional means.

We show that this security partially overcomes the incentive problems associated to dilution. While diluting debt repayment through inflation is no longer possible, the government can still dilute its repayment value through the manipulation of the real exchange rate.

We also briefly discuss the use of debt linked to the evolution of output and debt denominated in foreign currencies that have strong co-movements with the local currency. We argue that GDP-linked debt cannot fully overcome the incentive problems associated to debt dilution in economies in which monetary policy has real effects. Additionally, issuing debt in a foreign currency that is synchronized with the local currency completely eliminates the incentive problems but at the expense of exposing the economy to additional sources of risk.

**CPI-linked debt.** Consider the case of one-period inflation-linked debt in local currency. This is a security issued in period $t$ that pays $\pi_{t+1}$ units of local currency in $t+1$, where $\pi_t$ is the gross inflation rate. The risk-neutral price expressed in local currency of this debt security is

$$q_t^\pi = \frac{1}{R} e_t E \left[ \frac{\pi_{t+1}}{e_{t+1}} \right].$$
Let $b^\pi_t$ be the stock of inflation-linked debt in local currency with which the government enters the period. The resource constraint, expressed in local currency, of an economy in which the government can issue debt in foreign currency as well as inflation-linked debt in local currency is given by

$$e_t c_{T,t} + e_t b^*_t + b^\pi_t \pi_t = e_t y_{T,t} + e_t q^*_t b^*_{t+1} + q^\pi_t b^\pi_{t+1}$$

where we already imposed the law of one price for tradable goods and market clearing for non-tradable goods. Substituting for the expressions for debt prices and using the fact that $e_t = \pi_t C_{C_{T,t}} P_t - 1$ we can express the resource constraint in terms of tradable goods as

$$c_{T,t} + b^*_t + b^\pi_t \frac{1}{C_{C_{T,t}} P_{t-1}} = y_{T,t} + \frac{1}{R} b^*_{t+1} + \frac{1}{R} \mathbb{E} \left[ \frac{1}{C_{C_{T,t+1}} P_t} \right] b^\pi_{t+1}$$  \hspace{1cm} (15)

From this expression we can already see that the repayment of inflation-linked debt in local currency no longer depends on the inflation rate, but still depends on the real exchange rate. Therefore, its repayment value can be diluted by affecting the real exchange rate ex-post with future debt choices as explained in section 4.2. Given that $C_{C_{T,t}}$ is decreasing in tradable consumption, the presence of positive inflation-linked gives rise to an incentive to reduce future debt issuance, hence postpone tradable consumption and keep the real exchange rate depreciated.

Finally, the hedging properties of inflation-linked debt may different of those of debt in local currency. As seen in (15), whether inflation-linked debt is a useful security for hedging purposes depends on the co-movement between the inverse the real exchange rate and tradable endowment.

**Output-linked debt.** Consider now the case of one-period debt in foreign currency linked to the level of tradable output. This is a security issued in period $t$ that pays $y_{T,t+1}$ units of foreign currency in $t+1$. The risk-neutral price expressed in foreign currency of this debt security is

$$q^y_t = \frac{1}{R} \mathbb{E} [y_{T,t+1}] .$$

Consider the case of an endowment economy that can issue debt in foreign currency and debt in local currency linked to tradable output. The resource constraint of this economy in terms of tradable goods is given by

$$c_{T,t} = y_{T,t}(1 - b^y_{t+1}) - b^*_t + \frac{1}{R} b^*_{t+1} + \frac{1}{R} \mathbb{E} [y_{T,t+1}] b^y_{t+1},$$  \hspace{1cm} (16)

where $b^y_{t+1}$ is the stock of output-linked debt in foreign currency with which the government enters the period. Equation (16) already shows that perfect smoothing of tradable consumption can be attained by setting $b^y_{t+1} = 1$ to isolate consumption from fluctuations in the tradable endowment and choosing $b^*_t$ to attain the optimal inter-temporal path for consumption.
Therefore, in the context of an endowment economy the use of GDP-linked debt has desirable hedging properties without giving rise to incentive problems. This is no longer the case in the context of production economies in which the government can affect the allocations of inputs of production through monetary or fiscal policy.

Consider an economy in with the same debt instruments but in which tradable output is produced using labor $h_t$ according to a technology $y_{T,t} = f(h_t)$. $^9$ Substituting the production function into the resource constraint we see that the presence of a positive stock of output-linked debt such that $0 < b_{t+1}^{yr} < 1$ acts as if the aggregate productivity of the economy were lower. In principle, this effect would not be internalized when households make their labor choices. However, a government that lacks commitment would like to discourage the number of hours worked since it knows that higher output has associated a higher repayment of output-linked debt. Whether or not the government can induce lower economic activity depends on the details of the model and on the policy instruments. Two ways the government can implement this ex-post allocation is through the conduct of monetary in economies with nominal wage rigidities or through the conduct of fiscal policy with distortionary labor taxes.

*Debt in correlated currencies.* Finally, another possibility is to issue debt denominated in the currency of another country that has strong co-movements with the local currency. The advantage of this security is that its payoffs cannot be altered ex-post by the government through any policy. Its disadvantage is that by issuing this security the government exposes the economy to idiosyncratic currency risk from the other country.

7. Conclusion

This paper focuses attention on three stylized facts regarding the currency denomination of sovereign external debt in emerging economies. First, the currency denomination of sovereign debt is tilted towards foreign currency. Second, the share of debt in local currency has increased significantly over the past decade. Third, the share of debt denominated in local currency is procyclical.

These facts cannot be rationalized by the hedging properties of both securities in determining optimal portfolio positions. Given that emerging economies tend to experience currency depreciation in recessions and appreciation during booms, debt in local currency constitutes a

$^9$The argument does not rely on labor being the factor of production, as long as the factor is chosen contemporaneously at the time of production, after debt choices have been made.
useful hedge against income risk so an optimal portfolio problem calls for issuing large levels of debt in local currency and little debt (if any) in foreign currency.

We show that what can help rationalize these positions is the presence of a time inconsistency problem that arises when issuing debt in local currency. Using a model of optimal choice of debt denominated in foreign and local currency in which the government also controls monetary policy, we show that for higher levels of local-currency debt with which the government enters the period, the benefits associated with debt dilution both through higher inflation rates or through excessively depreciated real exchange rates are higher. The government internalizes the presence of perverse incentives to engage in costly inflation and distorted inter-temporal consumption ex-post and attenuates them by issuing more debt denominated in foreign currency and only little in domestic currency. This currency composition comes at the expense of foregoing the useful hedging properties of local-currency debt. We calibrate the model for a representative emerging economy and find that the model can account for the main three stylized facts highlighted at the beginning.

Finally, we also show that the incentives to dilute debt still persist even when the government can issue inflation-linked debt or output-linked debt. In this case the dilution does not come through inflation but rather through real exchange rate depreciation (in the case of inflation-linked debt) or lower economic activity (in the case of output-linked debt).
References

Emerging Capital Markets in Turmoil: Bad Luck or Bad Policy?, MIT Press.


CURRENCY COMPOSITION OF SOVEREIGN DEBT

APPENDIX A. ADDITIONAL FIGURES

**Figure A.1. Evolution of Total External Public Debt**

Notes: Based on WDI data. Stock of external public debt as a percentage of annual GDP for the countries in the sample.

**Figure A.2. Evolution of the Currency Composition of Sovereign External Debt**

Notes: Based on Arslanalp & Tsuda (2014). Share of sovereign external debt in local currency as a fraction of total sovereign external debt for the countries in the sample.
**Figure A.3.** Currency Composition of Sovereign Debt: The Case of Mexico During the Tequila Crisis

Notes: Based on data from Banxico. Domestic currency debt includes Ajustabonos, CETES and Bondes. Foreign-currency-denominated debt includes Tesobonos.

**Figure A.4.** Share of Debt in Local Currency: The Role of Exchange Rate Valuations

Notes: Based on Arslanalp & Tsuda (2014). Share of sovereign external debt in local currency as a fraction of total sovereign external debt for the average country in the sample. The blue line is computed at current nominal exchange rates and the green line is computed at the average exchange rate in the sample.
APPENDIX B. OMITTED PROOFS AND RESULTS

Proof of Proposition 1

Define the aggregate exogenous state as \( s^x_t = (y_T t, e^{-1} t) \). The government problem (P2) can be expressed recursively as

\[
V(b^*, b, s^x) = \max_{b^*, b', c_T} u(C(c_T, y_N)) + \beta \mathbb{E} \left[ V(b^{*'}, b', s^{x'}) \right]
\]

s.t.

\[
c_T = y_T - b^* - e^{-1} b + q^* b'' + e^{-1} q b', \quad (17)
\]

\[
b^* + e^{-1} b \leq \bar{b},
\]

\[
\text{Pr}(s^{x'}|s^x) = g(y_T, e^{-1} t, y'_T, e^{-1'} t).
\]

The second restriction implies a borrowing constraint on total debt which prevents Ponzi games. We assume \( \bar{b} \) is large enough such that this constraint is not binding and thus we can ignore this constraint. The optimality conditions that are necessary and sufficient are given by

\[
u'(c_t) C_{c_T, t} q^* = \beta \mathbb{E} [u'(c_{t+1}) C_{c_T, t+1}]
\]

(18)

\[
u'(c_t) C_{c_T, t} e^{-1}_t q = \beta \mathbb{E} [u'(c_{t+1}) C_{c_T, t+1} e^{-1}_{t+1}].
\]

(19)

Now we derive equation (12) from (18), (19) and the definition of debt prices. From the definition of covariance we have

\[
\mathbb{E} [u'(c_{t+1}) C_{c_T, t+1} e^{-1}_{t+1}] = \text{COV} (u'(c_{t+1}) C_{c_T, t+1}, e^{-1}_{t+1}) + \mathbb{E} [u'(c_{t+1}) C_{c_T, t+1}] \mathbb{E} [e^{-1}_{t+1}].
\]

Using (19) and (7) we get

\[
u'(c_t) C_{c_T, t} \mathbb{E} [e^{-1}_{t+1}] = \beta R \left[ \text{COV} (u'(c_{t+1}) C_{c_T, t+1}, e^{-1}_{t+1}) + \mathbb{E} [u'(c_{t+1}) C_{c_T, t+1}] \mathbb{E} [e^{-1}_{t+1}] \right].
\]

Using (18) this last equation is equivalent to

\[
\text{COV} (u'(c_{t+1}) C_{c_T, t+1}, e^{-1}_{t+1}) = 0. 
\]

(20)

To prove the proposition we assume that the optimal controls \( b'', c_T, c_N \) satisfy (17)-(18) and we find \( b' \) that satisfies (20).

Consider the case in which \( \rho(y_T, e^{-1}) = 1 \). Then there exist \( \gamma_0, \gamma_1 > 0 \) such that \( e^{-1} = \gamma_0 + \gamma_1 y_T \), with \( \gamma_1 = (\text{var}(e^{-1})/\text{var}(y_T))^{1/2} \). Substituting this equation into (17) and using (??) we get

\[
c_T = y_T - b^* - (\gamma_0 + \gamma_1 y_T) b + q^* b'' + q^* \mathbb{E} [\gamma_0 + \gamma_1 y'_T y_T] b'.
\]

(21)
Note that the conditional expectation does not depend on \( y_t \) due to the iid assumption. From (21) we can see that by setting \( b' = \gamma_1^{-1} > 0 \) we make \( c_T \) independent of the realization of \( y_T \). Note now that since \( y_N \) is deterministic the only source of randomness of \( u'(c_{t+1})C_{c_{t+1},t+1} \) comes from \( c_T \), which we just showed is deterministic. Hence, it follows that (20) is satisfied.

The proof for the case of \( \rho(y, e^{-1}) = -1 \) is analogous with the difference that now we can write \( e^{-1} = \gamma_0 - \gamma_1 y \), for some \( \gamma_0 \) and \( \gamma_1 = (\text{var}(e^{-1})/\text{var}(y))^{1/2} \).

Now consider the case of \( \rho(y, e^{-1}) = 0 \). We show that for \( b = 0 \) (20) is satisfied. Using (17) and (7) we have that

\[
c_T = y - b^* - e^{-1} b + q^* b' + q^* \mathbb{E}[e^{-1}'] b'
\]

Since \( e^{-1} \) is iid over time then \( c_T \) is independent of \( e^{-1} \) if \( b = 0 \), which implies that (20) is satisfied.

Proof of Proposition 2
First note that the \( \pi^*(s) = \infty \) for any \( s \) with \( b > 0 \). Since in equilibrium foreign lenders must obtain an expected return of \( R \), then in equilibrium it must be the case that \( q(s) = 0 \) whenever \( b'(s) > 0 \). For states in which \( b'(s) = 0 \) then the result follows trivially.

Derivation of Euler Equations
This subsection derives the Euler equations shown in section 4. We derive one generalized Euler equation for each type of debt, that embeds the Euler equations for the case of dilution through inflation and the case of dilution through real exchange rate.

Define \( C(b_{t+1}, b_{t+1}^*) \), \( P(b_{t+1}, b_{t+1}^*) \), \( B(b_{t+1}, b_{t+1}^*) \) the expected consumption, inflation and debt policies. In an equilibrium, these expectations are consistent with optimal policies. The inverse of the nominal exchange rate as:

\[
e_{t}^{-1} = r(\pi_t, c_{tT}), \quad \text{where } r_\pi < 0, r_c > 0 \text{ and we have set } y_{Nt} = 1 \text{ without loss of generality.}
\]

The recursive government problem \( P1 \) is defined as:

\[
V(b_t, b_t^*) = \max_{b_{t+1}, b_{t+1}^*, c_t, \pi_t} u(C(c_{tT}, 1)) - l(\pi_t) + \beta V(b_{t+1}, b_{t+1}^*)
\]

subject to

\[
y_T - c_{tT} + \frac{1}{R} r(P(b_{t+1}, b_{t+1}^*), C(b_{t+1}, b_{t+1}^*))b_{t+1} - r(\pi_t, c_{tT})b_t + \frac{1}{R} b_{t+1}^* - b_t^* = 0
\]
First order conditions:

\[ [c_t] : \quad u'(c_t) C_{c_t,t} = \lambda_t (1 + r_c(\pi_t, c_{Tt}) b_t) \]

\[ [\pi_t] : \quad -l'(\pi_t) = \lambda_t r_{\pi}(\pi_t, c_{Tt}) b_t \]

\[ [b_{t+1}] : \quad \beta V_b(b_{t+1}, b^*_t) = -\lambda_t \frac{1}{R} \frac{\partial r(\mathcal{P}(b_{t+1}, b^*_t), C(b_{t+1}, b^*_t)) b_{t+1}}{\partial b_{t+1}} \]

\[ [b_t] : \quad V_b(b_t, b^*_t) = -\lambda_t r(\pi_t, c_{Tt}) \]

\[ [b^*_t] : \quad V_b^*(b_t, b^*_t) = -\lambda_t \]

Combining these equations we get two Euler equations:

\[ \frac{u'(c_t) C_{c_{t+1},t}}{1 + r_c(\pi_{t+1}, c_{Tt}) b_t} \frac{\partial r(\mathcal{P}(b_{t+1}, b^*_t), C(b_{t+1}, b^*_t)) b_{t+1}}{\partial b_{t+1}} = \beta R \frac{u'(c_{t+1}) C_{c_{t+1},t+1}}{1 + r_c(\pi_{t+1}, c_{Tt+1}) b_{t+1}} \frac{r(\pi_{t+1}, c_{Tt+1})}{b_{t+1}} (22) \]

\[ \frac{u'(c_t) C_{c_{t+1},t}}{1 + r_c(\pi_{t+1}, c_{Tt}) b_t} \left[ 1 + \frac{\partial r(\mathcal{P}(b_{t+1}, b^*_t), C(b_{t+1}, b^*_t))}{\partial b^*_t} b_{t+1} \right] = \beta R \frac{u'(c_{t+1}) C_{c_{t+1},t+1}}{1 + r_c(\pi_{t+1}, c_{Tt+1}) b_{t+1}} (23) \]

The price sensitivity of debt can be calculated. Differentiating the resource constraint at \( t + 1 \) with respect to \( b_{t+1} \) and \( b^*_t \):

\[ \frac{\partial C(b_{t+1}, b^*_t)}{\partial b_{t+1}} = \frac{1}{R} \frac{\partial r(C(B(b_{t+1}, b^*_t), B^*(b_{t+1}, b^*_t)), \mathcal{P}(B(b_{t+1}, b^*_t), B^*(b_{t+1}, b^*_t)))}{\partial b_{t+1}} B(b_{t+1}, b^*_t) + B^*(b_{t+1}, b^*_t) \]

\[ \frac{1}{R} \frac{\partial r(C(B(b_{t+1}, b^*_t), B^*(b_{t+1}, b^*_t)), \mathcal{P}(B(b_{t+1}, b^*_t), B^*(b_{t+1}, b^*_t)))}{\partial b_{t+1}} B(b_{t+1}, b^*_t) + B^*(b_{t+1}, b^*_t) \]

To simplify notation denote

\[ \frac{\partial (c_{Tt+2}, \pi_{Tt+2}) b_{t+2} + b^*_t}{\partial b_{t+1}} = \frac{\partial r(C(B(b_{t+1}, b^*_t), B^*(b_{t+1}, b^*_t)), \mathcal{P}(B(b_{t+1}, b^*_t), B^*(b_{t+1}, b^*_t)))}{\partial b_{t+1}} B(b_{t+1}, b^*_t) + B^*(b_{t+1}, b^*_t) \]
and an analogous expression for the derivative with respect to $b_{t+1}^*$. Applying the chain rule to the second term of the right hand side of equations (24) and (25) yields

\[
\frac{\partial r(C(b_{t+1}, b_{t+1}^*), P(b_{t+1}, b_{t+1}^*))_{b_{t+1}}}{\partial b_{t+1}} = r(C(b_{t+1}, b_{t+1}^*), P(b_{t+1}, b_{t+1}^*)) + b_{t+1}r_c(C(b_{t+1}, b_{t+1}^*), P(b_{t+1}, b_{t+1}^*))C_b(b_{t+1}, b_{t+1}^*) + b_{t+1}r_{\pi}(C(b_{t+1}, b_{t+1}^*), P(b_{t+1}, b_{t+1}^*))P_b(b_{t+1}, b_{t+1}^*)
\]

(26)

\[
\frac{\partial r(C(b_{t+1}, b_{t+1}^*), P(b_{t+1}, b_{t+1}^*))_{b_{t+1}^*}}{\partial b_{t+1}^*} = b_{t+1}r_c(C(b_{t+1}, b_{t+1}^*), P(b_{t+1}, b_{t+1}^*))C_b(b_{t+1}, b_{t+1}^*) + b_{t+1}r_{\pi}(C(b_{t+1}, b_{t+1}^*), P(b_{t+1}, b_{t+1}^*))P_b(b_{t+1}, b_{t+1}^*)
\]

(27)

With these equations we can derive the Euler equations. First we derive the Euler equation for debt in local currency. Combining (24) and (26) we get

\[
\frac{\partial r(C(b_{t+1}, b_{t+1}^*), P(b_{t+1}, b_{t+1}^*))_{b_{t+1}}}{\partial b_{t+1}} = r(C(b_{t+1}, b_{t+1}^*), P(b_{t+1}, b_{t+1}^*)) + b_{t+1}r_c(C(b_{t+1}, b_{t+1}^*), P(b_{t+1}, b_{t+1}^*)) \frac{1}{R} \frac{\partial r(C_{Tt+2}, \pi_{Tt+2})_{b_{t+2}} + b_{t+2}}{\partial b_{t+1}} - b_{t+1}r_c(C(b_{t+1}, b_{t+1}^*), P(b_{t+1}, b_{t+1}^*)) \frac{\partial r(C(b_{t+1}, b_{t+1}^*), P(b_{t+1}, b_{t+1}^*))_{b_{t+1}}}{\partial b_{t+1}} + b_{t+1}r_{\pi}(C(b_{t+1}, b_{t+1}^*), P(b_{t+1}, b_{t+1}^*))P_b(b_{t+1}, b_{t+1}^*)
\]

(28)

We re-arrange the same equation to get

\[
\frac{\partial r(C(b_{t+1}, b_{t+1}^*), P(b_{t+1}, b_{t+1}^*))_{b_{t+1}}}{\partial b_{t+1}} = \frac{1}{1 + b_{t+1}r_c(C(b_{t+1}, b_{t+1}^*), P(b_{t+1}, b_{t+1}^*))} \left[ r(C(b_{t+1}, b_{t+1}^*), P(b_{t+1}, b_{t+1}^*)) + b_{t+1}r_c(C(b_{t+1}, b_{t+1}^*), P(b_{t+1}, b_{t+1}^*)) \frac{1}{R} \frac{\partial r(C_{Tt+2}, \pi_{Tt+2})_{b_{t+2}} + b_{t+2}}{\partial b_{t+1}} + b_{t+1}r_{\pi}(C(b_{t+1}, b_{t+1}^*), P(b_{t+1}, b_{t+1}^*))P_b(b_{t+1}, b_{t+1}^*) \right]
\]

Substituting this equation in (22) we get the modified Euler equation for debt in local currency

\[
ul'((c_t)_{C_t})_{C_t,t} = \beta Ru'((c_{t+1})_{C_{t+1}})_{C_{t+1},t+1} \left( 1 + r_{ct}b_t \right) \frac{1}{b_{t+1}} \left[ r(C_{t+1}, \pi_{t+1})_b b_{t+2} + b_{t+2} \frac{\partial r(C_{Tt+2}, \pi_{Tt+2})_{b_{t+2}} + b_{t+2}}{\partial b_{t+1}} + r_{\pi_{t+1}} \pi_{t+1} P_b(b_{t+1}, b_{t+1}^*) \right] = \beta Ru'((c_{t+1})_{C_{t+1}})_{C_{t+1},t+1} \left( 1 + r_{ct}b_t \right) \frac{1}{b_{t+1}} \left[ r(C_{t+1}, \pi_{t+1})_b b_{t+2} + b_{t+2} \frac{\partial r(C_{Tt+2}, \pi_{Tt+2})_{b_{t+2}} + b_{t+2}}{\partial b_{t+1}} + r_{\pi_{t+1}} \pi_{t+1} P_b(b_{t+1}, b_{t+1}^*) \right]
\]

(28)
Now we derive the Euler equation for debt in foreign currency. We follow a similar approach as before. Combining (25) and (27) we get

\[
\frac{\partial r(C(b_{t+1}, b_{t+1}^*)\mathcal{P}(b_{t+1}), b_{t+1}^*)}{\partial b_{t+1}^*} = b_{t+1}r(c(C(b_{t+1}, b_{t+1}^*)\mathcal{P}(b_{t+1}), b_{t+1}^*)) \frac{1}{R} \frac{\partial r(c_{Tt+2}, \pi_{Tt+2})b_{t+2} + b_{t+2}^*}{\partial b_{t+1}^*} \
- b_{t+1}r(c(C(b_{t+1}, b_{t+1}^*)\mathcal{P}(b_{t+1}), b_{t+1}^*)) \frac{\partial f(C(b_{t+1}, b_{t+1}^*)\mathcal{P}(b_{t+1}), b_{t+1}^*)}{\partial b_{t+1}^*} \
- b_{t+1}r(C(b_{t+1}, b_{t+1}^*)\mathcal{P}(b_{t+1}, b_{t+1}^*)) \
b_{t+1}r(C(b_{t+1}, b_{t+1}^*)\mathcal{P}(b_{t+1}, b_{t+1}^*)) \mathcal{P}_b(b_{t+1}, b_{t+1}^*)
\]

We re-arrange the same equation to get

\[
\frac{\partial r(C(b_{t+1}, b_{t+1}^*)\mathcal{P}(b_{t+1}), b_{t+1}^*)}{\partial b_{t+1}^*} = \frac{1}{1 + b_{t+1}r(c(C(b_{t+1}, b_{t+1}^*)\mathcal{P}(b_{t+1}, b_{t+1}^*)) \frac{1}{R} \frac{\partial r(c_{Tt+2}, \pi_{Tt+2})b_{t+2} + b_{t+2}^*}{\partial b_{t+1}^*} \
+ b_{t+1}r(C(b_{t+1}, b_{t+1}^*)\mathcal{P}(b_{t+1}, b_{t+1}^*)) \frac{\partial f(C(b_{t+1}, b_{t+1}^*)\mathcal{P}(b_{t+1}, b_{t+1}^*))}{\partial b_{t+1}^*} \
+ b_{t+1}r(C(b_{t+1}, b_{t+1}^*)\mathcal{P}(b_{t+1}, b_{t+1}^*)) \mathcal{P}_b(b_{t+1}, b_{t+1}^*)}
\]

Substituting this equation in (23) we get the modified Euler equation for debt in foreign currency

\[
u'(c_{t})C_{ct,t} = \beta Ru'(c_{t+1})C_{ct,t+1} \frac{1 + r_{ct,b_{t+1}}}{\text{Dilution thr. RXR}} \frac{1}{1 + b_{t+1}r(c_{Tt+2}, \pi_{Tt+2})b_{t+2} + b_{t+2}^*} \frac{\partial r(c_{Tt+2}, \pi_{Tt+2})b_{t+2} + b_{t+2}^*}{\partial b_{t+1}^*} + \beta \pi_{t+1} \mathcal{P}_b(b_{t+1}, b_{t+1}^*)
\]

**APPENDIX C. INCOME AND EXCHANGE RATES IN EMERGING ECONOMIES**

Table B1 documents the correlation between GDP and exchange rates in our sample of emerging economies, defined in Section 2. The first column indicates there is on average a negative correlation of -44% between GDP and the nominal exchange rate. This negative co-movement is generalized for most of the analyzed economies, with 17 of our 18 economies displaying a negative comovement between GDP and the nominal exchange rate. The second column indicates a similarly large and negative correlation of -38% between GDP and the real exchange rate\(^{10}\), which is also generalized across countries.

In Section 5 it is assumed that \((y_t, c_t^{-1})\) follow a first-order VAR process in logs as specified in (13). This process is estimated with quarterly data on the cyclical component of real industrial production and the nominal exchange rate vis-à-vis the US dollar for the panel of countries in Table B1 for the 1980Q1-2014Q4 period. The data source is International Monetary Fund.

\(^{10}\)The real exchange rate is computed vis-à-vis the US dollar, i.e. \(RER_t = \frac{c_t}{P_t^*}\), where \(P_t^*\) is the US CPI and \(P_t\) is the country’s CPI.
Table B1. GDP and Exchange Rates in emerging economies

<table>
<thead>
<tr>
<th>Country</th>
<th>$\rho_{y,e}$</th>
<th>$\rho_{y,rxr}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Argentina</td>
<td>-49%</td>
<td>-57%</td>
</tr>
<tr>
<td>Brazil</td>
<td>-16%</td>
<td>-39%</td>
</tr>
<tr>
<td>Bulgaria</td>
<td>-52%</td>
<td>-64%</td>
</tr>
<tr>
<td>China</td>
<td>18%</td>
<td>-25%</td>
</tr>
<tr>
<td>Egypt</td>
<td>-12%</td>
<td>-13%</td>
</tr>
<tr>
<td>Hungary</td>
<td>-48%</td>
<td>-44%</td>
</tr>
<tr>
<td>India</td>
<td>-26%</td>
<td>-19%</td>
</tr>
<tr>
<td>Indonesia</td>
<td>-61%</td>
<td>-58%</td>
</tr>
<tr>
<td>Lithuania</td>
<td>-58%</td>
<td>-15%</td>
</tr>
<tr>
<td>Malaysia</td>
<td>-69%</td>
<td>-65%</td>
</tr>
<tr>
<td>Mexico</td>
<td>-77%</td>
<td>-60%</td>
</tr>
<tr>
<td>Peru</td>
<td>-73%</td>
<td>3%</td>
</tr>
<tr>
<td>Philippines</td>
<td>-55%</td>
<td>-29%</td>
</tr>
<tr>
<td>Poland</td>
<td>-1%</td>
<td>-22%</td>
</tr>
<tr>
<td>Russia</td>
<td>-50%</td>
<td>-54%</td>
</tr>
<tr>
<td>South Africa</td>
<td>-23%</td>
<td>-11%</td>
</tr>
<tr>
<td>Thailand</td>
<td>-84%</td>
<td>-70%</td>
</tr>
<tr>
<td>Turkey</td>
<td>-60%</td>
<td>-47%</td>
</tr>
<tr>
<td><strong>Average</strong></td>
<td><strong>-44%</strong></td>
<td><strong>-38%</strong></td>
</tr>
<tr>
<td><strong>Std. Dev.</strong></td>
<td><strong>28%</strong></td>
<td><strong>22%</strong></td>
</tr>
</tbody>
</table>

*Notes: $\rho_{y,e}$ refers to the correlation coefficient between the cyclical component of real GDP and the cyclical component of the nominal exchange rate, measured as units of domestic currency per unit of foreign currency. $\rho_{y,rxr}$ refers to the correlation coefficient between the cyclical component of real GDP and the cyclical component of the nominal exchange rate, measured as the ratio of the US CPI index expressed in domestic currency to the domestic CPI index. Correlations are computed for the period 1960-2014. Data source: World Development Indicators.*

Cyclical components were computed using the Hodrick-Prescott filter, with a smoothing parameter of 1,600. The estimated process is approximated with a Markov chain, setting a grid of five equally spaced points for $\ln(y_t)$, $\ln(\epsilon_t^{-1})$, yielding 25 exogenous states. Consistent with the results in Table B1, exhibits an implicit negative relationship between the nominal exchange rate and output.
Appendix D. Numerical Solution

Provided that there is positive inflation in equilibrium, the price level and the nominal level of debt denominated in local currency will exhibit a trend. We deal with this property by reducing the dimensionality of the state space of the recursive problem to three variables that are stationary. Define \( \tilde{b} = \frac{b}{n_{n-1}y_N} \), this ratio corresponds to the level of local-currency debt scaled by the lagged aggregate level of prices and other scalars. Then the problem (P1), can be re-expressed as

\[
V(b^*, \tilde{b}, y_T) = \max_{b^{*'}, \tilde{b}^{*'}, \pi, c_T} U(c_T, y_N, \pi) + \beta \mathbb{E} \left[ V(b^{*'}, \tilde{b}^{*'}, y_T') \right] \tag{P1'}
\]

subject to

\[
c_T = y_T - b^* - \frac{\tilde{b}^{1/2} c_T}{\pi} + \frac{\tilde{b}^{*'} R}{\tilde{R}} \mathbb{E} \left[ \mathcal{X}(b^{*'}, b', y_T', P) \right]
\]

where \( \mathcal{X}(b^{*'}, b', y_T', P) \) is the perceived ratio of the square root of future tradable consumption to future inflation. In equilibrium, this should be a fixed point, i.e., \( \mathcal{X}(b^{*'}, b', y_T', P) = \frac{c_T^{1/2}}{\pi} \). We solve for equilibrium using global numerical method based on value function iteration. We first conjecture a guess for \( \mathcal{X} \), iterate the value function until convergence, then update our initial guess of \( \mathcal{X} \) and follow the same procedure until convergence.