Meeting Technologies in Decentralized Asset Markets

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Abstract

We explore a frictional asset market where traders periodically meet in pairs. Before the start of trading, ex-ante homogeneous traders irreversibly choose their meeting rate at some cost. We prove that traders who choose a higher meeting rate emerge as intermediaries, earning profits by taking asset positions that are misaligned with their preferences. When the cost is a differentiable function of the meeting rate, we prove that the meeting rate distribution has no mass points. When the cost is proportional to the meeting rate, we prove that the meeting rate distribution has a Pareto tail with parameter two and that middlemen emerge endogenously: a zero measure of traders choose to have continuous contact with the market and account for a positive fraction of all meetings. The efficient allocation has the same qualitative features as the equilibrium and can be decentralized through taxes that internalize search externalities.

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1 Introduction

This paper examines over-the-counter markets for assets, where potential traders periodically meet in pairs and trade (Rubinstein and Wolinsky, 1987). We are interested in understanding the observed heterogeneity in these markets, whereby we mean that some individuals trade much more frequently and with many more partners than do others. In particular, real world trading networks appear to have a core-periphery structure. Traders at the core of the network act as financial intermediaries, earning profits by taking either side of a trade, while traders in the periphery trade less frequently and their trades are more geared towards obtaining an asset position aligned with their portfolio needs.

We consider a model economy with a unit measure of traders each of whom seeks to trade a single asset for an outside good (money). Following Duffie, Gárleanu and Pedersen (2005) and a large subsequent literature, we assume traders have an intrinsic reason for trade, differences in the flow utility they receive from trading the asset. Moreover, this idiosyncratic valuation changes over time, creating a motive for continual trading and retrading. We add on to this a second source of heterogeneity, in meeting rates. We posit that some traders expect more frequent trading opportunities than others. In addition, we assume that the likelihood of contacting any particular individual is proportional to her meeting rate.

Under these assumptions, we show that intermediation arises naturally. When two traders who have the same flow valuation for the asset meet, the trader who has a higher meeting rate acts as an intermediary, leaving the meeting with holdings that are further from the desired one. This occurs in equilibrium because traders with a faster meeting rate expect to have more future trading opportunities and so place less weight on their current flow payoff. Intermediation thus moves misaligned asset holdings towards traders with higher meeting rates, which improves future trading opportunities. Thus the equilibrium displays a core-periphery structure where the identity of the market participants at the core—fast traders—remains stable over time.

In the full model, we recognize that traders’ meeting rates are endogenous. We consider an initial, irreversible investment in this meeting technology. For example, traders may invest in having faster communication technologies, better visibility through location choices or advertisement, or relationships with more counterparties. While we assume throughout that traders are ex ante identical, we recognize that they may choose different meeting rates, as in a mixed strategy equilibrium. A higher meeting rate gives more trading opportunities but we assume it also incurs a sunk cost.

We prove that if the cost is a differentiable function of the meeting rate, then equilibrium is characterized by an atomless distribution of meeting rates. The force pushing towards
heterogeneity is the gains from intermediation. If everyone else chooses the same meeting rate, an individual who chooses a slightly faster meeting rate achieves a first order gain in profits because she can act as an intermediary for everyone else; while an individual who chooses a slightly slower meeting rate also achieves a first order gain in profits because everyone else can act as an intermediary for him.

We also consider the natural assumption that the cost is proportional to the meeting rate. We prove that the equilibrium distribution of meeting rates has a positive lower bound and is unbounded above, with many individuals choosing a high meeting rate: first, the right tail of the meeting rate distribution is Pareto with tail parameter 2, so the variance in meeting rates is infinite. Second, a zero measure of individuals choose an infinite meeting rate, giving them continuous contact with the market. These “middlemen” account for a positive fraction of all meetings, earning zero profits in each meeting but making it up on volume. We stress that ex ante there is no difference between middlemen (the core of the network) and the periphery; however, they choose to make different investments and so ultimately play a very different role in the market.

We next consider efficient trading patterns and investments. We prove that the equilibrium trading pattern—passing misalignment to traders with higher meeting rates—is efficient. On the other hand, the equilibrium investments are inefficient, but the qualitative behavior of the optimal investment is similar to the equilibrium. If costs are a differentiable function of the meeting rate, there is optimally no mass points in the distribution of meeting rates. If costs are proportional to meeting rates, the optimal meeting rate distribution has a Pareto tail with parameter two and a zero measure of middlemen account for a positive fraction of all meetings. Pigouvian taxes highlight the inefficiencies associated with equilibrium: individuals only capture half the surplus in each meeting, leading to an underinvestment in contacts; but they do not internalize a business stealing effect, which induces them to overinvest in contacts.

Finally, we emphasize the connection between intermediation and dispersion in meeting rates. We consider an economy in which individuals with the same desired asset holdings never meet, which eliminates the possibility of intermediation in our model economy. Under these conditions, we show that if the cost is a weakly convex function of the meeting rate, all individuals choose the same meeting rate, both in equilibrium and optimally. Thus dispersion in meeting rates and intermediation are intimately connected: if there is dispersion in meeting rates, faster traders act as intermediaries; and if intermediation is permitted, meeting rates are naturally disperse.

The last section offers several numerical exercises which characterize socially optimal and equilibrium distribution of search efficiency, and discusses how they compare to each
other. In both cases, faster traders have asset positions that are misaligned with their preferences more often than slower traders. That is, the willingness of faster traders to serve as intermediaries outweighs the fact that they have more trading opportunities. On average, traders in equilibrium contact the market more often than is optimal, but there are both too few very slow and too few very fast traders in equilibrium. The optimal meeting rate distribution is less disperse than the equilibrium one.

**Literature Review** This paper is closely related to a growing body of work on dynamic trading with search frictions that goes back to Rubinstein and Wolinsky (1987) who analyze the role of middlemen in bilateral markets and the endogenous determination of the extent of their activity. Duffie, Gârleanu and Pedersen (2005) and Duffie, Gârleanu and Pedersen (2007) popularized use of search models in studying asset markets, and the subsequent work extends this basic framework in a variety of dimensions in response to newly available empirical evidence on trade and intermediation in over-the-counter markets (Lagos and Rocheteau, 2009; Weill, 2008; Vayanos and Weill, 2008; Neklyudov, 2014; Hugonnier, Lester and Weill, 2014; Üslü, 2015; Chang and Zhang, 2015; Farboodi, Jarosch and Menzio, 2016; Nosal, Wong and Wright, 2016, among many others). However, in these setups either agents’ meeting rates are homogeneous, or there is an exogenous degree of heterogeneity which leads to endogenous intermediation (Neklyudov, 2014; Hugonnier, Lester and Weill, 2014; Üslü, 2015; Chang and Zhang, 2015). Alternatively, some of these models feature an exogenously given middlemen who facilitate trade (Duffie, Gârleanu and Pedersen, 2005; Weill, 2008). We add to this literature by showing that first, heterogeneity arises naturally to leverage the potential gain to intermediation, and second, middlemen arise endogenously in an environment with heterogeneous meeting technologies.


There is also ample empirical evidence on concentration of trade among very few finan-
cial institutions, i.e. middlemen. The largest sixteen derivatives dealers, known as the G16, intermediate more than 80 percent of the global total notional amount of outstanding derivatives (Mengle, 2010; Heller and Vause, 2012). Bech and Atalay (2010) documents that the distributions of node strength has a heavy right tail, and Peltonen, Scheicher and Vuillemey (2014) finds the degree distribution of the aggregate credit default swap network can be scale-free.

Hugonnier, Lester and Weill (2014) show that intermediation chains emerge when agents can have a wide set of different flow valuation rather than just two. In particular, individuals with moderate current tastes act as intermediators, buying and selling to individuals with currently more extreme taste. In contrast to their setup, ours offers a theory where the identity of the individuals at the center of the intermediation chain remains stable over time, a key empirical feature of many decentralized asset markets. See, for instance (Bech and Atalay, 2010) for the federal funds market.

Farboodi, Jarosch and Menzio (2016) study a decentralized asset market where some market participants are better at bargaining than others. This leads to intermediated trades driven by rent-extraction motives which are, at best, neutral for welfare. In contrast, intermediation in our setup improves upon the allocation since misaligned asset positions are traded toward those who are more efficient at offsetting them.

The decentralized interdealer market in Neklyudov (2014) features dealers with heterogeneous search technology. As in our framework, fast dealers endogenously emerge as intermediaries in the interdealer market. The same force is present in Üslü (2015) who offers a setup that allows for rich heterogeneity in pricing and inventories. Chang and Zhang (2015) provide an alternative model of intermediation where agents differ in terms of the volatility of their taste for an asset and agents with less volatile valuation act as intermediaries. We share with these contributions the notion that those who have relatively moderate taste for an asset account for most of the turnover in an asset market with search frictions. In contrast to these contributions we show that the coexistence of heterogeneous market participants is an equilibrium outcome even when agents are ex-ante homogeneous.

Outline The rest of the paper is organized as follows: Section 2 lays out the model. Section 3 defines and characterizes the equilibrium. Section 4 discusses the socially optimal allocation and how it can be decentralized. Section 5 provides a numerical example. Section 6 considers an economy where intermediation is prohibited and shows that this economy also has no dispersion in meeting rates either in equilibrium or in the optimal allocation. Section

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1 A related literature studies the positive and normative consequences of high-speed trading in centralized financial markets; see, for instance, Pagnotta and Philippon (2015).
2 Model

We study a marketplace where time is continuous and extends forever. A unit measure of traders have preferences defined over their holdings of an indivisible asset and their consumption or production of an outside good. The supply of the asset is fixed at \( \frac{1}{2} \) and individual traders’ holdings are restricted to be \( m \in \{0, 1\} \), so at any point in time half the traders hold the asset and half do not. Traders have time-varying taste \( i \in \{h, l\} \) for the asset and receive flow utility \( \delta_{i,m} \) when they are in state \((i, m)\), where \( i \) indexes their taste and \( m \) indexes their asset holding. We assume that \( \Delta \equiv \frac{1}{2}(\delta_{h,1} + \delta_{l,0} - \delta_{h,0} - \delta_{l,1}) \geq 0 \), which implies that traders in the high state are the natural asset owners. Preferences over net consumption of the outside good are linear, so that good effectively serves as transferable utility when trading the asset.

Traders’ taste switches between \( l \) and \( h \) independently at an identical rate \( \gamma > 0 \). This implies that at any point in time, half the traders are in state \( h \) and half are in state \( l \). Thus in a frictionless environment, the supply of assets is exactly enough to satiate the traders in state \( h \). Search frictions prevent this from happening. Instead a typical trader meets another one according to a Poisson process with arrival rate \( \lambda \geq 0 \). Trade may occur only at those moments. We assume that traders choose their meeting rate at time 0. A high meeting rate is costly: a trader who chooses a meeting rate \( \lambda \) pays a cost \( c(\lambda) \) per meeting (so a flow cost \( \lambda c(\lambda) \)).

We allow for the possibility that different traders choose different meeting rates. Let \( G(\lambda) \) denote the cumulative distribution function of meeting rates in the population and \( \Lambda \) denote the average meeting rate. Importantly, we allow for the presence of a zero measure of traders who are middlemen. Middlemen are in continuous contact with the market and may account for a positive fraction of all meetings. That is, we require that \( \Lambda \geq \int_0^\infty \lambda dG(\lambda) \) and allow the inequality to be strict, in which case there are middlemen.

Search is random, so whom the trader meets is independent of her current taste and asset holding, but is proportional to the other trader’s meeting rate. More precisely, conditional on meeting a counterparty, the counterparty’s meeting rate falls into some interval \( [\lambda_1, \lambda_2] \) with probability \( \int_{\lambda_1}^{\lambda_2} \frac{\lambda}{\Lambda} dG(\lambda) \). In addition, the probability of meeting a middleman is \( 1 - \int_0^\infty \frac{\lambda}{\Lambda} dG(\lambda) \). For any function \( f : [0, \infty] \to \mathbb{R} \), it will be convenient to define the expected value of \( f \) in a meeting:

\[
\mathbb{E}(f(\lambda')) \equiv \int_0^\infty \frac{\lambda'}{\Lambda} f(\lambda') dG(\lambda') + \left(1 - \int_0^\infty \frac{\lambda'}{\Lambda} dG(\lambda')\right) f(\infty)
\]
This explicitly accounts for the possibility both of meeting a regular trader and of meeting a middleman. When $\Lambda = 0$, so (almost) everyone chooses a zero meeting rate, we assume that a trader who chooses a positive meeting rate is equally likely to meet any of the other traders and let $\mathbb{E}(f(\lambda'))$ denote the average value of $f$ in the population.

When two traders meet, their asset holdings and preferences are observed by each. If only one trader holds the asset, as will be the case in half of all meetings, the traders may swap the asset for the outside good. We assume that whether trade occurs and what the terms of trade are is determined according to the (symmetric) Nash bargaining solution.

3 Equilibrium

In equilibrium, we need to find two objects. First, let $1_{\lambda,i,m}^{X,i',m'}$ denote the probability that a trader with meeting rate $\lambda \in [0, \infty]$ in preference state $i \in \{h,l\}$ with asset holdings $m \in \{0,1\}$ trades when she contacts a trader with meeting rate $\lambda' \in [0, \infty]$ in preference state $i' \in \{l,h\}$ with asset holdings $m' \in \{0,1\}$. Second, let $p_{\lambda,i,m}^{X,i',m'}$ denote the transfer of the outside good from $\{\lambda, i, m\}$ to $\{\lambda, i', m'\}$ when such a trade takes place. Feasibility requires that $1_{\lambda,i,m}^{X,i',m'} = 1_{\lambda,i,m}^{\lambda',i',m'}$ and $p_{\lambda,i,m}^{X,i',m'} + p_{\lambda,i,m}^{\lambda',i',m'} \geq 0$, where the latter condition ensures that there are no outside resources available to the trading pair. The trading probability and price are determined by Nash bargaining.

3.1 Value Functions and Flows

Let $v_{\lambda,i,m}$ denote the present value of the profits of a trader $(\lambda, i, m)$.

This is defined recursively by

$$
\rho v_{\lambda,i,m} = \delta_{i,m} + \gamma \left( v_{\lambda,\sim i,m} - v_{\lambda,i,m} \right)
+ \lambda \sum_{i' \in \{h,l\}} \sum_{m' \in \{0,1\}} \mathbb{E} \left( 1_{\lambda,i,m}^{X,i',m'} \mu_{X,i',m'} (v_{\lambda,i,m} - v_{\lambda,i,m} - p_{\lambda,i,m}^{X,i',m'}) \right) - \lambda c(\lambda).
$$

(1)

The left hand side of equation (1) is the flow value of the trader. This comes from four sources, listed sequentially on the right hand side. First, he receives a flow payoff $\delta_{i,m}$ that depends on his preferences and asset holdings. Second, his preferences shift from $i$ to $\sim i$ at rate $\gamma$, in which case the trader has a capital gain $v_{\lambda,\sim i,m} - v_{\lambda,i,m}$. Third, he meets another trader at rate $\lambda$, in which case they may swap asset holdings in return for a payment. Here $\mu_{X,i',m'}$ denotes the endogenous fraction of traders with contact rate $\lambda'$ who are in

2We focus throughout on aggregate steady states.
preference state $i'$ and have asset holding $m'$. If the two agree to trade, with probability $1_{\lambda,i,m}^{X',i',m'}$, the trader has a capital gain from swapping assets and transferring the outside good, $v_{\lambda,i,m'} - v_{\lambda,i,m} - p_{\lambda,i,m}^{X',i',m'}$. Finally, the trader pays a cost $c(\lambda)$ in each meeting.

The fraction of traders in different states also depends on the trading probabilities:

$$
\left( \gamma + \lambda \sum_{i' \in \{h,l\}} \mathbb{E} \left( 1_{\lambda,i,m}^{X',i',1-m} \mu_{\lambda,i,m}^{X',i',1-m} \right) \right) \mu_{\lambda,i,m} = \gamma \mu_{\lambda,i,m} + \lambda \sum_{i' \in \{h,l\}} \mathbb{E} \left( 1_{\lambda,i,m}^{X',i',m} \mu_{\lambda,i,m}^{X',i',m} \right) \mu_{\lambda,i,m}. \tag{2}
$$

This equation reflects a balance of inflows and outflows. A trader exits the state $\{\lambda, i, m\}$ either when he has a preference shock, at rate $\gamma$, or when he meets and succeeds in trading with another trader with the opposite asset holding. A trader enters this state when he is in the opposite preference state and has a preference shock or he is in the opposite asset holding state and trades.

Nash bargaining imposes that trades occur whenever this makes both parties better off, and that trading prices equate the gains from trade without throwing away any resources. That is, if there are prices $p_{\lambda,i,m}^{X',i',m'} + p_{\lambda,i,m}^{X',i',m'} = 0$ such that $v_{\lambda,i,m'} - v_{\lambda,i,m} \geq p_{\lambda,i,m}^{X',i',m'}$ and $v_{\lambda,i,m} - v_{\lambda,i,m'} \geq p_{\lambda,i,m}^{X',i',m'}$, trade occurs at a prices such that $v_{\lambda,i,m'} - v_{\lambda,i,m} - p_{\lambda,i,m}^{X',i',m'} = v_{\lambda,i,m} - v_{\lambda,i,m'} - p_{\lambda,i,m}^{X',i',m'}$. Otherwise there is no trade. It follows immediately that

$$
1_{\lambda,i,m}^{X',i',m'} = \begin{cases} 
1 & \text{if and only if } v_{\lambda,i,m'} + v_{\lambda,i,m} \geq v_{\lambda,i,m} + v_{\lambda,i,m'}; \\
0 & \text{otherwise,}
\end{cases}
$$

and that the trading prices satisfy

$$
p_{\lambda,i,m}^{X',i',m'} = \frac{1}{2} \left( v_{\lambda,i,m'} + v_{\lambda,i,m} - v_{\lambda,i,m} - v_{\lambda,i,m'} \right).
$$

Of course, if $m = m'$, there is no possibility of gains from trade. In the remainder of our analysis, we ignore such meetings.

3.2 Symmetry

Call traders’ asset holding positions misaligned both when they hold the asset and are in preference state $l$ and when they do not hold the asset and are in preference state $h$. We focus on allocations in which the two misaligned states and the two well aligned states are treated symmetrically. That is, we look only at equilibria where $1_{\lambda,i,m}^{X',i',m'} = 1_{\lambda,i,m}^{X',i',1-m'}$. That
such equilibria exist is a consequence of our symmetric market structure, where half the traders are in each preference state and half of the traders hold the asset.

In a symmetric equilibrium, equation (2) implies $\mu_{\lambda,i,m} = \mu_{\lambda,1-i,-m}$ for all $\{\lambda, i, m\}$. That is, the fraction of traders with contact rate $\lambda$ in the high state, $i = h$, who hold the asset, $m = 1$, is equal to the fraction of traders with the same contact rate who are in the low state, $i = l$, and do not hold the asset $m = 0$. We call all such traders well-aligned. The remaining traders are misaligned, and again there are equal shares of the two misaligned state for each $\lambda$.

It is mathematically convenient to refer to traders only by their alignment status, where $a = 0$ indicates misaligned and $a = 1$ indicates well-aligned. Let $1_{\lambda,a}$ indicate the trading probability between traders $(\lambda, a)$ and $(\lambda', a')$ conditional on them having the opposite asset holdings. Let $m_\lambda \equiv \mu_{\lambda,l,1} + \mu_{\lambda,h,0}$ denote the fraction of traders with contact rate $\lambda$ who are misaligned. Equation (2) reduces to

$$
\left(\gamma + \frac{\lambda}{2} \mathbb{E}\left(1_{\lambda,0}^{\lambda'} m_{\lambda'} + 1_{\lambda,0}^{\lambda'} (1 - m_{\lambda'})\right)\right) m_\lambda
= \left(\gamma + \frac{\lambda}{2} \mathbb{E}\left(1_{\lambda,1}^{\lambda'} m_{\lambda'} + 1_{\lambda,1}^{\lambda'} (1 - m_{\lambda'})\right)\right) (1 - m_\lambda).
$$

The left hand side is the outflow rate from the misaligned states. This occurs either following a preference shock or a meeting with a trader who has the opposite asset holdings where trade occurs. The right hand side is the inflow rate, again following the same events.

We can also define the average value of a misaligned and well-aligned trader. Let $v_{\lambda,0} \equiv \frac{1}{2}(v_{\lambda,l,1} + v_{\lambda,h,0})$ and $v_{\lambda,1} \equiv \frac{1}{2}(v_{\lambda,l,0} + v_{\lambda,h,1})$. Also define $s_{\lambda} \equiv v_{\lambda,1} - v_{\lambda,0}$, the surplus from being well-aligned rather than misaligned. Taking advantage of symmetry in the misalignment rates and the Nash bargaining solution, these satisfy

$$
\rho v_{\lambda,0} = \delta_0 + \gamma s_{\lambda} + \frac{\lambda}{4} \mathbb{E}\left((s_{\lambda} + s_{\lambda'})^+ m_{\lambda} + (s_{\lambda} - s_{\lambda'})^+(1 - m_{\lambda'})\right) - \lambda c(\lambda)
$$

and

$$
\rho v_{\lambda,1} = \delta_1 - \gamma s_{\lambda} + \frac{\lambda}{4} \mathbb{E}\left((-s_{\lambda} + s_{\lambda'})^+ m_{\lambda} + (-s_{\lambda} - s_{\lambda'})^+(1 - m_{\lambda'})\right) - \lambda c(\lambda),
$$

where $\delta_0 \equiv \frac{1}{2}(\delta_{l,1} + \delta_{h,0})$ and $\delta_1 \equiv \frac{1}{2}(\delta_{l,0} + \delta_{h,1}) = \Delta + \delta_0$. Again, both equations reflect the sum of four terms. The first is the average flow payoff of a misaligned or well-aligned trader. The second is the gain or loss from a preference shock that switches the alignment status. The third is the gain from meetings. This reflects the fact that only half of all meetings are with traders who hold the opposite asset; and in these events each trader keeps half of the increase in the surplus, if any. The $+$-superscript is shorthand notation for the max{$\cdot$, 0} and reflects that meetings result in trade if and only if doing so raises the sum of the surpluses.
The final term is the search cost.

Finally, we can simplify equation (3) using the Nash bargaining solution as well, since trades occur if and only if doing so raises the sum of surpluses:

\[
\left( \gamma + \frac{\lambda}{2} \mathbb{E} \left( \mathbb{I}_{s_{\lambda} + s_{\lambda'} > 0} m_{\lambda'} + \mathbb{I}_{s_{\lambda} > s_{\lambda'}} (1 - m_{\lambda'}) \right) \right) m_{\lambda}
\]

\[
= \left( \gamma + \frac{\lambda}{2} \mathbb{E} \left( \mathbb{I}_{s_{\lambda} < s_{\lambda'}} m_{\lambda'} + \mathbb{I}_{s_{\lambda} + s_{\lambda'} < 0} (1 - m_{\lambda'}) \right) \right) (1 - m_{\lambda}). \quad (6)
\]

Here the indicator function \( \mathbb{I} \) is equal to 1 if the inequality in the subscript holds and is zero otherwise.

### 3.3 Endogenizing the Distribution of Contact Rates

At an initial date 0, all traders choose their meeting rate \( \lambda \) in order to maximize their value. If traders are impatient, that means that their choice will depend on their alignment status at date 0. This would make it necessary to solve for transitional dynamics from this initial condition. We circumvent this issue by focusing on the no-discounting limit of this economy, \( \rho \to 0 \). The surplus \( s_{\lambda} = v_{\lambda,1} - v_{\lambda,0} \) is finite in this limit, while the present value of the gain from switching alignment status, \( \rho (v_{\lambda,1} - v_{\lambda,0}) \) converges to zero. It follows that the trader’s initial asset holdings does not affect their incentive to invest and we may ignore the transitional dynamics.

The focus on the no-discounting limit reflects our expectation that the short-run desire to trade is not an important determinant of the irreversible investment in meeting technologies. We think of the preference shifts as occurring at a much higher frequency than discounting, while trading opportunities may occur at a higher frequency still. This means that other sources of heterogeneity, for example in the importance of holding the asset at the correct time, \( \Delta \), are likely to be a much more important determinant of this investment.

### 3.4 Definition of Equilibrium

We define a steady state equilibrium in the limiting economy with \( \rho \to 0 \). The definition relies only on objects that are well-behaved in this limit.

**Definition 1** A steady state equilibrium is a distribution of meeting rates \( G(\lambda) \), an average meeting rate \( \Lambda \) an allocation of misalignment \( m(\lambda) \), and undiscounted surplus function \( s(\lambda) \), satisfying the following conditions:

1. Balanced inflows and outflows into misalignment as given by equation (6)
2. Consistency of $s_\lambda$ with the value functions (4) and (5),

$$
\Delta = 2\gamma s_\lambda + \frac{\lambda}{4} \mathbb{E} \left( \left( (s_\lambda + s_{\lambda'})^+ - (s_\lambda + s_{\lambda'})^+ \right) m_{\lambda'} + \left( (s_\lambda - s_{\lambda'})^+ - (s_\lambda - s_{\lambda'})^+ \right) (1 - m_{\lambda'}) \right)
$$

3. Optimality of the ex-ante investment decision:

(a) $dG(\lambda) > 0$ only if it maximizes

$$
\delta_1 - \gamma s_\lambda + \frac{\lambda}{4} \mathbb{E} \left( (s_{\lambda} - s_{\lambda'})^+ m_{\lambda'} + (s_{\lambda} - s_{\lambda'})^+ (1 - m_{\lambda'}) \right) - \lambda c(\lambda)
$$

(b) Middlemen make finite profits: $\Lambda \geq \int_0^\infty \lambda dG(\lambda)$ and

$$
\lim_{\lambda \to \infty} \left( \frac{1}{4} \mathbb{E} \left( (s_{\lambda} + s_{\lambda'})^+ m_{\lambda'} + (s_{\lambda} - s_{\lambda'})^+ (1 - m_{\lambda'}) \right) - c(\lambda) \right) \leq 0,
$$

with complementary slackness.

We have already explained the first two conditions. Condition 3(a) ensures that if traders choose a contact rate $\lambda$, it maximizes their average payoff $\lim_{\rho \to 0} \rho v_{\lambda,1} = \lim_{\rho \to 0} \rho v_{\lambda,0}$. Condition 3(b) ensures that if there are middlemen, they earn zero profits in each meeting. Middlemen earn profit by taking half the surplus from meetings where they change the alignment status of their trading partners. If middlemen earned more profit in an average meeting than the cost of a meeting, being a middleman would be arbitrarily profitable, inconsistent with equilibrium. If they earned less, there would be no middlemen, which implies $\Lambda = \int_0^\infty \lambda dG(\lambda)$.

3.5 Equilibrium Trading Patterns

We now begin our characterization of the equilibrium, starting with trading patterns given any distribution $G(\lambda)$.

Proposition 1 When two traders with opposite asset positions meet they

1. always trade the asset if both are misaligned;
2. never trade the asset if both are well-aligned;
3. trade the asset if one is misaligned and the other is well-aligned and the well-aligned trader has the higher meeting rate.
Figure 1: Direction of trade across traders with meeting rate \( \lambda \in \{\lambda_1, ..., \lambda_N\} \) with \( \lambda_1 < \lambda_2 < \ldots < \lambda_N \) and current taste \( i \in \{l, h\} \).

**Proof.** See appendix A.1. ■

The proof shows that the surplus function \( s_\lambda \) is non-negative and decreasing. The result then follows immediately.

The first two parts of the proposition reflect fundamentals. Trade between two misaligned traders creates two well-aligned traders, which improves static welfare. Trade between two well-aligned traders reduces static welfare. The third part of the proposition reflects option-value considerations and is the key feature of the endogenous trading pattern which arises in this environment, namely intermediation. It states that a trader with meeting rate \( \lambda \) buys the asset from a trader with meeting rate \( \lambda' < \lambda \) if both are in state \( l \); and she sells the asset to the trader with a lower meeting rate if both are in state \( h \). These trades do not immediately increase the number of well-aligned traders, but they move misalignment towards traders who expect more future trading opportunities. These trades occur in equilibrium because traders with low contact rates are able to compensate traders with high contact rates for taking the misaligned positions.

The possibility of intermediation implies that a trader’s buying and selling decisions become increasingly detached from her idiosyncratic preferences as her meeting rate increases. In other words, a high meeting rate moderates the impact of the idiosyncratic taste component on a trader’s valuation of the asset. It follows that those who arise as intermediaries at the center of the valuation chain are the traders with frequent meetings. Figure 1 shows the intermediation chain which follows from proposition 1. Slow traders are at the periphery of the trading chain, not trading once their asset position is aligned with their preferences. In turn, the fast traders constitute the endogenous core of the trading network, buying and selling largely irrespective of their preference state. In doing so, they take on misaligned asset positions from types with lower meeting rates and are compensated through the bid-ask spread. This also implies that faster traders not only meet other traders more frequently but also trade more frequently conditional on a meeting because they take on the misalignment from traders with lower search efficiency.
Under the equilibrium trading pattern, the inflow-outflow equality (6) simplifies to

\[ \frac{\lambda}{2} \mathbb{E}(\mathbb{I}_{\lambda' > \lambda})m_\lambda = \left(\gamma + \frac{\lambda}{2} \mathbb{E}(\mathbb{I}_{\lambda' < \lambda})m_\lambda\right)(1 - 2m_\lambda). \] (8)

### 3.6 Equilibrium Distribution of Meeting Rates

We next show that a non-degenerate distribution \( G(\lambda) \), that is the coexistence of traders with different \( \lambda \), occurs in equilibrium even when market participants are ex-ante homogeneous. To do this, we first solve explicitly for the surplus function, taking advantage of the trading patterns highlighted in Proposition 1. We prove in Appendix A.2 that the surplus function satisfies

\[ s_\lambda = \frac{\Delta}{2\gamma} \left(1 - e^{-\int_{\lambda}^{\infty} \phi_{\lambda'} d\lambda'}\right). \] (9)

where

\[ \phi_{\lambda} \equiv \frac{8\gamma}{\lambda \left(8\gamma + \lambda \mathbb{E}\left(1 - \mathbb{I}_{\lambda' < \lambda}(1 - 2m_{\lambda'})\right)\right)}. \] (10)

In addition, part 3(a) of the definition of equilibrium implies that \( G(\lambda) \) is increasing at \( \lambda \) only if \( \lambda \) maximizes

\[ \delta_1 - \gamma s_\lambda + \frac{\lambda}{4} \mathbb{E}(\mathbb{I}_{\lambda' < \lambda}(s_{\lambda'} - s_{\lambda})m_{\lambda'}) - \lambda c(\lambda). \]

We use this to prove the following result:

**Proposition 2** Assume \( c(\lambda) \) is continuously differentiable. Then the equilibrium distribution of search efficiency \( G(\lambda) \) has no mass points, except possibly at \( \lambda = 0 \).

**Proof.** See Appendix A.3.

This proposition implies that although all traders are ex-ante identical, there is no symmetric equilibrium in which all traders choose identical actions. Even stronger, almost all traders choose different types.\(^3\) The proof shows that the gross flow profits have a convex kink at any masspoint. Given a differentiable cost function we hence conclude that masspoints are inconsistent with optimality of the ex-ante investment decision.

To develop an understanding for the result consider an environment where everyone has meeting technology \( \bar{\lambda} \). This turns out to create a convex kink in the value function at \( \bar{\epsilon} \). To understand why, consider the marginal impact of an increase in the meeting rate for a trader with a contact rate \( \bar{\lambda} \). This allows the trader to act as an intermediary for all the other

---

\(^3\)The one caveat is that a positive fraction of traders may choose to live in autarky, setting \( \lambda = 0 \). If this is optimal, all traders must get the same payoff, and so the value of participating in this market must be zero. This is the case when the cost function \( c \) is too high.
traders. Although the gains from intermediation are small, on the order of the difference between the contact rates, the opportunities to intermediate are frequent, whenever she meets a misaligned trader with the opposite asset holdings. Thus a higher meeting rate creates a first order gain. Conversely, consider the marginal impact of a decrease in the meeting rate for a trader with contact rate \( \bar{\lambda} \). This allows all other traders to intermediate for her, dramatically reducing her misalignment probability. Of course, this doesn’t come for free; she pays for these trades using the outside good. Nevertheless, the benefits from the trades are again linear in the difference in contact rates. Thus a lower meeting rate also creates a first order gain. This logic carries over to any masspoint.

A different way to see this is in terms of the number of trades. Setting the same meeting rate as everyone else, trades only occur with both traders are misaligned and holding the opposite asset position. Choosing a slightly different meeting rate allows for gains from a host of other trades, where one party is misaligned and the other well-aligned.

In other words, the nature and frequency of trades depends starkly on the meeting technology compared with other market participants. One gets intermediated by faster traders and intermediates slower traders. As soon as a positive measure of traders has the same meeting rate, this discretely changes the marginal returns to \( \lambda \) and is hence inconsistent with equilibrium under a differentiable cost function.

The absence of a pure strategy equilibrium is a common feature of search models (Butters, 1977; Burdett and Judd, 1983; Burdett and Mortensen, 1998; Duffie, Dworczak and Zhu, 2015). These papers have in common that if all firms charge the same price (or offer the same wage), firms that offering a slightly lower price (higher wage) earn discontinuously higher profits. Our results concern a different object, the meeting rate, and we find that the profit function is continuous but not differentiable. We therefore believe that our finding is distinct from those in the existing literature.

### 3.7 Linear Cost Function

In this section, we restrict the cost function to be linear, so the cost per meeting is constant, \( c(\lambda) = c \). To lead up to our main result we first prove a weaker result which is helpful in understanding the key equilibrium properties of \( G(\lambda) \).

**Proposition 3** Assume \( c(\lambda) = c \). There exists a \( \bar{c} > 0 \) such that if \( c \geq \bar{c} \), everyone lives in autarky, \( G(0) = 1 \); while if \( c < \bar{c} \), the equilibrium distribution of meeting rates \( G(\lambda) \) has a strictly positive lower bound \( \underline{\lambda} \), has no gaps (so \( G \) is strictly increasing), and is unbounded above (so \( G(\lambda) < 1 \) for all finite \( \lambda \)).

**Proof.** See Appendix A.4. ■
To understand the open tail, consider a case where the highest meeting rate in the population is some finite, strictly positive $\bar{\lambda}$ and $\int_{0}^{\infty} \lambda dG(\lambda') = \Lambda$, that is there is no middlemen. The proof shows that the marginal profit of any trader choosing $\lambda > \bar{\lambda}$ is strictly positive and linear in $\lambda$. This means that flow profits are unbounded, inconsistent with optimal ex-ante investment decisions. The reason is that any trader with $\lambda > \bar{\lambda}$ engages in exactly the same pure intermediation activities as the one with $\bar{\lambda}$. She buys and sells in any meeting with a misaligned trader with opposite asset holdings, irrespective of her own alignment status. She never trades in a meeting with a well-aligned meeting partner. It follows that the misalignment rate is constant for all meeting rates $\lambda \geq \bar{\lambda}$. Next, observe that a faster trader has a smaller surplus and hence buys at lower prices but also sells at (symmetrically) lower prices. Because traders with $\lambda > \bar{\lambda}$ buy and sell irrespective of their asset position, the price differential cancels and profits scale linearly with $\lambda$.

Finally, note that the strictly positive lower bound $\bar{\lambda}$ reflects that the value of a trader smoothly converges towards its autarky level as $\lambda \rightarrow 0$. The restriction that costs are not too high ensures that traders fare strictly better than in autarky, which allows us to rule out $\lambda$ close to zero.

The next proposition contains our main result in characterizing $G(\lambda)$:

**Proposition 4** Assume $c(\lambda) = c < \bar{c}$. The equilibrium distribution of search efficiency $G(\lambda)$ has a Pareto tail with tail parameter two. A strictly positive fraction of meetings accrues to a zero measure of middlemen who are in continuous contact with the market, $\Lambda > \int_{0}^{\infty} \lambda dG(\lambda')$.

**Proof.** See Appendix A.5.

The first part of the proposition establishes that, in the tail, the meeting rate is distributed Pareto with tail parameter two. To develop some understanding, note that we have already established that the distribution has no mass points and an open right tail. It thus follows that gross flow values must be linear above the lowest meeting rate $\lambda$. A Pareto with tail parameter two implies that increasing one trader’s meeting rate leaves both the frequency at which one meets a faster trading partner and their relative meeting rates unaltered. On the other hand, it linearly increases the frequency at which one meets a slower trader while the partner’s expected meeting rate converges to a constant (once $\lambda$ is in the tail). Thus, as $\lambda$ increases in the tail, it just linearly increases the meeting frequency with (the same) slower traders hence delivering linear gross flow profits in the tail.

The proof for the emergence of middlemen shows that, for any $G(\lambda)$ and given the intermediated trading pattern, the speed of convergence of the net surplus function is such that $\lambda^2 s'_{\lambda}$ converges to a finite, negative constant. We show that for net profits to be equated in the absence of middlemen, the model requires a speed of convergence sufficiently slow such
that $\lambda^2 s'_\lambda$ explodes. In turn, once middlemen arise, the speed of convergence required for profits to be equated in the tail of the distribution of meeting rates drops. With the right measure of middlemen, profits are equated everywhere. Why do middlemen decelerate the decline in the net surplus function as $\lambda \to \infty$? Note that in the absence of middlemen, the fastest traders in the marketplace trade irrespective of their asset position. For large $\lambda$ their surplus function $s_\lambda$ converges to zero. This is no longer the case when they instead repeatedly meet middlemen that take on their misaligned asset position. Thus, middlemen allow for profits to be equated with an open tail of the meeting rate distribution $G(\lambda)$.

4 Normative Analysis

In this section we study the efficient distribution of meeting rate among traders, as well as the efficient trading pattern. We start with an illustrative example before turning to the full planner’s problem.

4.1 Example

Assume the meeting rate is identical across all traders in the economy, $\lambda = \Lambda$. The fraction of misaligned traders is

$$m_\lambda = \frac{\sqrt{4 + 2\lambda/\gamma} - 2}{\lambda/\gamma}$$

As $\lambda$ increases, the misalignment rate converges to zero at a rate proportional to the square root of the meeting rate, $\lim_{\lambda \to \infty} \sqrt{\lambda}m_\lambda = \sqrt{2}/\gamma$.

To show the scope for intermediation, we hold the average meeting rate constant at $\Lambda$ but distribute the meetings unevenly across the population. In particular, we give a fraction $\alpha$ of the total flow of meetings $\Lambda$ to a fraction $1 - \varepsilon$ of the population (“traders”) and the remaining $1 - \alpha$ to a fraction $\varepsilon$ of the population (“intermediaries”). Let $\lambda_1 \equiv \frac{\alpha}{(1-\varepsilon)}\Lambda$ and $\lambda_2 \equiv \frac{1-\alpha}{\varepsilon}\Lambda$ denote the meeting rate of traders and intermediaries, respectively. In keeping with their names, we assume that the trading pattern is as in equilibrium, that is traders with a higher meeting rate intermediate by taking on misaligned asset positions.

Equating inflows and outflows into misalignment, we can solve explicitly for the fraction of misaligned traders of either type. The resulting solution is cumbersome and we focus on the limiting behavior as the number of intermediaries $\varepsilon$ converges to zero for fixed $\alpha$. Note that in this limit, almost everyone is a trader, but the meeting rate of traders is $\alpha\Lambda$. 

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Optimizing over $\alpha$, we find that $m_{\lambda_1}$ may be as low as

$$m(\lambda_1) = \frac{1}{32} \left( -\sqrt{(\lambda/\gamma)^2 + 48\lambda/\gamma + 64 + \lambda/\gamma + 24} \right).$$

As $\lambda$ increases, the misalignment rate converges to zero at a rate proportional to the meeting rate, $\lim_{\lambda \to \infty} \lambda m_{\lambda} = 8\gamma$. This implies that for sufficiently fast $\gamma$, this binary distribution of meeting rates always reduces misalignment conditional on the total meeting rate. Figure 2 illustrates this numerically.

Why does intermediation improve the allocation for a given aggregate search technology? A key observation is that intermediaries are less likely to hold their desired asset position than are regular traders. The fact that they have many more meetings is outweighed by the fact that they frequently trade away from their desired asset position. Intermediation is useful because it makes it easier for regular traders to obtain their desired asset holdings, not because intermediaries themselves have preferences well-aligned with their asset holdings.

### 4.2 Planner Problem

We now return to the original problem. For simplicity, we work directly with the undiscounted problem where the planner wishes to maximize steady state average utility.\(^4\) Since all traders are identical at time 0, this gives us the Pareto optimal allocation in the undiscounted problem. We also impose that the planner must use a symmetric trading pattern, as in the equilibrium. Thus we simply need to keep track of the number of misaligned traders with each contact rate.

\(^4\)We can also write down the discounted planner’s problem, take the limit as the discount rate converges to zero, and focus on steady states. The results are the same.
The planner’s objective is to maximize

\[ \delta_0 + \Delta \int_0^{\infty} (1 - m_\lambda) dG(\lambda) - \Lambda E(c(\lambda)) \]  

(11)

Each misaligned trader gets a flow payoff of \( \delta_0 \), while each well-aligned trader gets a flow payoff of \( \delta_1 = \Delta + \delta_0 \), expressed in the first two terms. In addition, the planner must pay the search costs, equal to the product of the average search intensity \( \Lambda \) and the cost per meeting in the average meeting.

The planner has two instruments. The first is that she chooses the set of admissible trades. When a trader with meeting rate \( \lambda \) and alignment status \( a \in \{0, 1\} \) meets a trader with meeting rate \( \lambda' \) and alignment status \( a' \in \{0, 1\} \) and the opposite asset position, they trade with probability \( 1_{\lambda,a} \lambda' = 1_{\lambda',a'} \lambda \). This implies that the steady state misalignment rate satisfies

\[
\left( \gamma + \frac{\lambda}{2} \mathbb{E} \left( 1_{\lambda,0} \lambda' + 1_{\lambda,1} (1 - m_{\lambda'}) \right) \right) m_{\lambda} = \left( \gamma + \frac{\lambda}{2} \mathbb{E} \left( 1_{\lambda',0} \lambda + 1_{\lambda',1} (1 - m_{\lambda'}) \right) \right) (1 - m_{\lambda}).
\]  

(12)

Second, the planner chooses the distribution of meeting rates \( G \) and the average meeting rate \( \Lambda \). Note that we allow the planner to set \( \Lambda > \int_0^{\infty} \lambda' dG(\lambda') \), in which case we say that there are middlemen.

We begin by characterizing the optimal trading pattern:

**Proposition 5** When two traders with opposite asset positions meet, it is optimal for them to

1. always trade the asset if both are misaligned;
2. never trade the asset if both are well-aligned;
3. trade the asset if one is misaligned and the other is well-aligned and the well-aligned trader has the higher meeting rate.

**Proof.** See Appendix A.6. ■

The proof proceeds by solving the planner’s constrained optimization problem. The first order conditions yield an expression for the social value of asset ownership, \( S_\lambda \), which is the Lagrange multiplier on the misalignment constraint (12). We then show that the planner requires trade whenever doing so raises the sum of social surpluses. We then show that
the social surplus function looks very similar to the private surplus, and in particular is decreasing and nonnegative.

This Proposition implies that the equilibrium trading pattern is efficient. The intuition for the result is straightforward. The planner’s objective function boils down to minimizing the average rate of misalignment. The planner therefore demands trade if it reduces static misalignment and rejects it if it raises static misalignment. In the case where only one trader is misaligned, the planner moves the misalignment to the trade with more trading opportunities. That is, the planner uses faster traders as intermediaries. We note that the social value in intermediated transactions lies in the future. A trade where misalignment gets swapped is zero-sum in contemporaneous terms. It replaces once misaligned trader with another. The social value lies in the fact that the trader with a higher meeting rate is more likely to subsequently meet a trader with the opposite preference and hence improve upon the allocation; she is also more likely to subsequently meet an intermediator with identical asset position which is another channel through which an intermediated transaction facilitates the flow of the assets to those who value them most.

It thus follows that, given a non-degenerate distribution of meeting rates $G(\lambda)$, the intermediated trading pattern governing equilibrium is efficient. This implies that intermediation has a first order effect on welfare: The allocation would be strictly inferior in an environment where traders would solely engage in fundamental trades.

We turn next to the allocation of meeting rates. We prove in Appendix A.7 that the planner’s surplus function satisfies

$$S_\lambda = \frac{\Delta}{2\gamma} \left( 1 - e^{-\int_0^\infty \Phi_{\lambda'} d\lambda'} \right),$$

(13)

where

$$\Phi_{\lambda} \equiv \frac{4\gamma}{\lambda \left( 4\gamma + \lambda \mathbb{E}(1 - \mathbb{1}_{\lambda' \leq \lambda}(1 - 2m_{\lambda'})) \right)}.$$  

(14)

This is scarcely changed from equations (9) and (10) in the decentralized equilibrium. Moreover, the planner sets $dG(\lambda) > 0$ only if $\lambda$ maximizes

$$-\gamma S_\lambda + \frac{\lambda}{2} \mathbb{E}(\mathbb{1}_{\lambda' < \lambda}(S_{\lambda'} - S_\lambda)m_{\lambda'}) - \lambda \left( c(\lambda) + \gamma \int_{0}^{\infty} S_{\lambda'}(1 - 2m_{\lambda'}) dG(\lambda') \right),$$

(15)

and that $\Lambda \geq \int_{0}^{\infty} \lambda dG(\lambda)$ and

$$\lim_{\lambda \to \infty} \left( \frac{1}{2} \mathbb{E}((S_{\lambda'} - S_\lambda)m_{\lambda'}) - c(\lambda) \right) \leq \frac{\gamma}{\Lambda} \int_{0}^{\infty} S_{\lambda'}(1 - 2m_{\lambda'}) dG(\lambda'),$$

(16)
with complementary slackness. This is the planner’s analog of part 3 of the definition of equilibrium. Given the close mathematical link between the equations describing the equilibrium and optimal allocations, we can easily prove analogs of Propositions 2, 3 and 4 for the social planner:

**Proposition 6** Assume $c(\lambda)$ is continuously differentiable. Then the optimal distribution of search efficiency $G(\lambda)$ has no mass points, except possibly at $\lambda = 0$.

**Proposition 7** Assume $c(\lambda) = c$. There exists a $\hat{c} > 0$ such that if $c \geq \hat{c}$, everyone optimally lives in autarky, $G(0) = 1$; while if $c < \hat{c}$, the optimal distribution of meeting rates $G(\lambda)$ has a strictly positive lower bound $\Lambda$, has no gaps (so $G$ is strictly increasing), and is unbounded above (so $G(\lambda) < 1$ for all finite $\lambda$).

**Proposition 8** Assume $c(\lambda) = c < \hat{c}$. The optimal distribution of search efficiency $G(\lambda)$ has a Pareto tail with tail parameter two. A strictly positive fraction of meetings accrues to a zero measure of middlemen who are in continuous contact with the market, $\Lambda > \int_0^\infty \lambda' dG(\lambda')$.

We omit these proofs, which replicate the corresponding ones for the decentralized equilibrium.

### 4.3 Pigouvian Taxes

Although the equilibrium and efficient distribution of meeting rates are qualitatively similar, the equilibrium distribution is still inefficient. There are two sources of inefficiency. The first comes from bargaining. In the decentralized equilibrium, each trader keeps only half of the surplus from every meeting, while the social planner recognizes the value of the entire surplus. This induces traders to underinvest in meetings in equilibrium. The second is a business stealing effect. When one trader invests more in meetings, he diverts meetings towards himself. The failure to internalize this effect causes traders to overinvest in meetings in equilibrium.

We can directly correct for each of these externalities. First, assume that whenever two traders meet, an outside agent (“the government”) doubles the surplus from the meeting. Second, assume that whenever a trader meets anyone, the government charges him a tax which is independent of his meeting rate, $\bar{\tau} \equiv \frac{\gamma}{2} \int_0^\infty S_{\lambda'} (1 - 2m_{\lambda'}) dG(\lambda')$. Then the equilibrium Bellman equations for misaligned and well-aligned traders become

$$
\rho v_{\lambda,0} = \delta_0 + \gamma s_{\lambda} + \frac{\lambda}{2} E((s_{\lambda} + s_{\lambda'})^+ m_{\lambda'} + (s_{\lambda} - s_{\lambda'})^+ (1 - m_{\lambda'})) - \lambda (c(\lambda) + \bar{\tau})
$$

and

$$
\rho v_{\lambda,1} = \delta_1 - \gamma s_{\lambda} + \frac{\lambda}{2} E((-s_{\lambda} + s_{\lambda'})^+ m_{\lambda'} + (-s_{\lambda} - s_{\lambda'})^+ (1 - m_{\lambda'})) - \lambda (c(\lambda) + \bar{\tau}).
$$
Taking the difference between these equations, we confirm that this reduces to equation (35), and hence the equilibrium and efficient surplus functions are equal, $s_\lambda = S_\lambda$. Moreover, the equilibrium choice of $\lambda$, which must maximize the right hand side of the second Bellman equation, coincides with the optimal choice from conditions (15) and (16).

An equivalent scheme simply offers the taxes and subsidies at a constant rate, independent of whom the trader meets. A misaligned trader pays a per-meeting tax

$$\tau_{\lambda,0} = \frac{\gamma}{\Lambda} \int_0^\infty S_{\lambda'}(1 - 2m_{\lambda'})dG(\lambda') - \frac{1}{2}\mathbb{E}((S_\lambda + S_{\lambda'})m_{\lambda'} + \mathbb{1}_{\lambda' > \lambda}(S_\lambda - S_{\lambda'})(1 - m_{\lambda'})),$$

while a well aligned trader pays a tax

$$\tau_{\lambda,1} = \frac{\gamma}{\Lambda} \int_0^\infty S_{\lambda'}(1 - 2m_{\lambda'})dG(\lambda') - \frac{1}{2}\mathbb{E}(\mathbb{1}_{\lambda' < \lambda}(S_{\lambda'} - S_\lambda)m_{\lambda'}).$$

We can combine these to compute the average per-meeting tax, $T_\lambda = m_\lambda)\tau_{\lambda,0} + (1 - m_\lambda)\tau_{\lambda,1}$. Our numerical results below suggest that this average tax is always positive, so the equilibrium has a higher average meeting rate $\Lambda$ than the optimum.

5 Numerical Illustration

To illustrate the our results, we compute the equilibrium and efficient distribution of search intensities. We set $\delta_1 = 1$ and $\delta_0 = -1$, so in autarky the expected payoff is 0 and the first best payoff is 1. We normalize $\gamma = 1$ and focus on a linear cost function with $c(\lambda) = 0.004$ for all $\lambda$.

As we are particularly interested in behavior of very fast traders, we will use a logarithmic scale for $\lambda$ on the horizontal axis. Start with the equilibrium and efficient distribution of meeting rates. First, we have that in equilibrium, $\int_0^\infty \lambda dG(\lambda) = 55.7 < 69.4 = \Lambda$. In other words, 19.7 percent of all meetings is with a middleman. The corresponding numbers for the optimal solution are $\int_0^\infty \lambda dG(\lambda) = 30.4 < 41.6 = \Lambda$, so 26.9 percent of meetings are with middlemen. Although meeting a middleman is less common in equilibrium, aggregate search intensity is much higher. Figure 3 compares the (tail of the) survival rate, $1 - G(\lambda)$, between the equilibrium and optimal distribution.

Figure 4 plots misalignment rate against $\lambda$ in both equilibrium and the efficient outcome. Importantly traders with a higher meeting rate have higher misalignment rate in both solutions. This is a consequence of intermediation. Intermediaries trade against their desired position, and in so doing improve the allocation of those with lower search efficiency. In equilibrium intermediaries are compensated through bid-ask spread. The reason the two
Figure 3: Meeting Rate. Equilibrium versus Optimal Distribution

curves cross is that at the lower end of distribution traders are better allocated in equilibrium exactly since they more frequently meet faster traders. On the other hand, faster traders more often serve as intermediaries, pushing up their misalignment rate.

Figure 5 puts these two pieces together to see how frequently traders with different trading rates actually trade. The trading rate for an investor with meeting rate $\lambda$ is

$$\frac{\lambda}{2} \left( (1 - \mathbb{E}(\mathbb{1}_{\lambda<\lambda}(1 - m(\lambda)))) m_{\lambda} + \mathbb{E}(\mathbb{1}_{\lambda<\lambda}) m_{\lambda'}(1 - m_{\lambda}) \right).$$

This inherits the Pareto tail of the meeting rate distribution, since the trading probability conditional on a meeting converges to a constant in the tail. It is worth noting that $\gamma = 1$, so each trader only needs a single trade per unit of time to offset the preference shocks. Nevertheless, 93 percent of traders in equilibrium and 77 percent optimally trade more frequently than this.

Figure 6 breaks the total amount of trade by traders into the fraction in which they act as an intermediary (switching from misaligned to aligned), are the counterparty to an intermediary (the counterparty switches from misaligned to aligned), or there is a double coincidence of wants (so both parties switch from aligned to misaligned). Faster traders are more likely to act as an intermediary, less likely to have their trades intermediated, and more likely to enjoy a double coincidence since they are more often misaligned.

Figure 7 plots the average per meeting tax $T_{\lambda} = m_{\lambda} \tau_{\lambda,0} + (1 - m_{\lambda}) \tau_{\lambda,1}$ which internalizes
Figure 4: Misalignment Rate. Equilibrium versus Optimal Distribution

Figure 5: Trading rate. Equilibrium versus Optimal Distribution
the externalities. The tax is positive for all meeting rates, consistent with overinvestment in meetings.

6 Constrained Economy: The Role of Intermediation

To understand the importance of intermediation, consider an economy in which intermediation is not allowed. To be concrete, suppose meetings between two traders with the same preference state simply do not occur. It follows that whenever a misaligned trader meets a well-aligned trader, they have opposite preference states and hence the same asset holdings, and so there is no scope for trade. We show in this section that without intermediation, the equilibrium and optimal distribution of meeting rates are degenerate as long as the cost function $\lambda c(\lambda)$ is weakly concave.
6.1 Equilibrium

We start with equilibrium. Note that there are two relevant types of meetings in this con-
strained economy, those between two misaligned traders with the opposite asset holdings,
and those between two well-aligned traders with the opposite asset holdings. We can extend
our earlier results to prove that the first type of meeting results in trade while the second does
not. Therefore the equilibrium misalignment rate satisfies equation (3), with the restrictions
that $1_{\lambda,0} = 1$ and $1_{\lambda,0} = 1_{\lambda,1} = 1_{\lambda,1} = 0$:

$$\left(\gamma + \frac{\lambda}{2} \mathbb{E}(m_{\lambda'})\right) m_{\lambda} = \gamma (1 - m_{\lambda}). \quad (17)$$

The surplus function similarly satisfies a simpler version of equation (7), dropping the irrel-
levant meetings:

$$\Delta = 2\gamma s_{\lambda} + \frac{\lambda}{4} \mathbb{E}((s_{\lambda} + s'_{\lambda})m'_{\lambda}). \quad (18)$$

And the optimal choice of $\lambda$ satisfies simplified versions of part 3 of the definition of equilib-
rium:

$$dG(\lambda) > 0 \Rightarrow \lambda \in \arg\max \left(\delta_1 - \gamma s_{\lambda} - \lambda c(\lambda)\right); \quad (19)$$

and middlemen make finite profits:

$$\Lambda \geq \int_0^\infty \lambda dG(\lambda) \quad \text{and} \quad \lim_{\lambda \to \infty} \left(- c(\lambda)\right) \leq 0 \quad \text{with complementary slackness.} \quad (20)$$

We look for a distribution function $G$, an average meeting rate $\Lambda$, a surplus function $s$, and
a misalignment rate $m$ that satisfy equations (17)–(20).

Condition (20) immediately implies that there are no middlemen whenever their per-
meeting cost is strictly positive. Moreover, solve equation (18) to get

$$s_{\lambda} = \frac{4\Delta - \lambda \mathbb{E}(s'_{\lambda}m'_{\lambda})}{8\gamma + \lambda \mathbb{E}(m'_{\lambda})}, \quad (21)$$

a decreasing and convex function. Condition (19) then implies that if $\lambda c(\lambda)$ is weakly concave,
all traders choose the same value of $\lambda = \Lambda$. That is, $\mathbb{E}(s'_{\lambda}m'_{\lambda}) = s_{\Lambda}m_{\Lambda}$ and $\mathbb{E}(m'_{\lambda}) = m_{\Lambda}$.

We summarize this result as follows:

**Proposition 9** Consider an economy with no intermediation and a weakly convex cost
$\lambda c(\lambda)$. In equilibrium all traders choose a common value $\lambda = \Lambda$.

To find the equilibrium meeting rate, we solve explicitly for $s_{\lambda}$. First, simplify (21) when
\( \lambda = \Lambda: \)

\[
S_\lambda = \frac{2\Delta}{4\gamma + \Lambda m_\Lambda}.
\]

Then rewrite the expression for a general value of \( \lambda \):

\[
S_\lambda = \frac{\Delta(16\gamma + 4\Lambda m_\Lambda - 2\lambda m_\Lambda)}{(4\gamma + \Lambda m_\Lambda)(8\gamma + \lambda m_\Lambda)}.
\] (22)

Finally, using condition (19), it follows that if \( \lambda c(\lambda) \) is weakly convex, the equilibrium choice of \( \Lambda \) solves

\[
\frac{4\gamma \Delta m_\Lambda}{(4\gamma + \Lambda m_\Lambda)(8\gamma + \lambda m_\Lambda)} = c(\Lambda) + \Lambda c'(\Lambda).
\] (23)

This equation pins down \( \Lambda \).

### 6.2 Optimum

Turn next to the planner’s problem. The objective of the planner is unchanged, given by equation (11). As in equilibrium, we can prove that only meetings between two misaligned traders result in trade. Therefore the planner faces the constraint (17). Replicating the proof of Proposition 5, we get that the optimal surplus function satisfies

\[
\Delta = 2\gamma S_\lambda + \frac{\lambda}{2} \mathbb{E}((S_\lambda + S_{\lambda'})m_{\lambda'}) \Rightarrow S_\lambda = \frac{2\Delta - \lambda \mathbb{E}(S_{\lambda'}m_{\lambda'})}{4\gamma + \lambda \mathbb{E}(m_{\lambda'})},
\]

decreasing and convex. Replicating Appendix A.7, we also obtain that the planner has \( dG(\lambda) > 0 \) only if \( \lambda \) maximizes

\[
-\gamma S_\lambda - \lambda \left( c(\lambda) + \frac{\gamma}{\Lambda} \int_0^\infty S_{\lambda'}(1 - 2m_{\lambda'})dG(\lambda') \right),
\]

analogous to condition (15). Convexity of \( S \) implies that if the cost function is convex, the planner places all weight on a single value of \( \lambda \):

**Proposition 10** Consider an economy with no intermediation and a weakly convex cost \( \lambda c(\lambda) \). All traders optimally choose a common value \( \lambda = \Lambda \).

Replicating the arguments for the equilibrium, we then use this to prove that

\[
S_\lambda = \frac{\Delta(4\gamma + 2\Lambda m_\Lambda - \lambda m_\Lambda)}{(2\gamma + \Lambda m_\Lambda)(4\gamma + \lambda m_\Lambda)}.
\] (24)
It follows that the optimal choice of $\Lambda$ satisfies the first order condition

$$\frac{\gamma \Delta m_\Lambda}{(2\gamma + \Lambda m_\Lambda)(4\gamma + \Lambda m_\Lambda)} = c(\Lambda) + \Lambda c'(\Lambda),$$

(25)

where we simplify the expression slightly using the steady state relationship (17).

We can prove algebraically that for fixed $\Lambda$, the left hand side of equation (23) exceeds the left hand side of (25). It follows immediately that the equilibrium meeting rate weakly exceeds the optimal one and strictly so if the cost function is continuously differentiable and the solution is interior.

7 Conclusions

We study a model of over-the-counter trading in asset markets in which ex-ante identical traders invest in a meeting technology and participate in bilateral trade. We show that when traders have heterogeneous search efficiencies, fast traders intermediate for slow traders: they trade against their desired position and take on misallocation from slower traders. Moreover, we characterize how starting with ex ante homogeneous traders, the distribution of meeting rates is determined endogenously in equilibrium, and how it compares with the corresponding socially optimal distribution. We argue that an economy with homogeneous meeting rates is neither an equilibrium nor socially desirable when the cost of meetings is differentiable. We also characterize the transfer scheme which decentralizes the efficient distribution. Finally, we argue that when intermediation is prohibited, dispersion in meeting rates disappears both in equilibrium and in the optimal allocation.
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Appendix

A.1 Proof of Proposition 1

Proof. We show that \( s_\lambda \) is non-negative and strictly decreasing for all \( \lambda > 0 \). The proposition immediately follows.

If \( s_\lambda < 0 \), \((-s_\lambda + s_{\lambda'})^+ \geq (s_\lambda + s_{\lambda'})^+ \) and \((-s_\lambda - s_{\lambda'})^+ \geq (s_\lambda - s_{\lambda'})^+ \) for all \( \lambda' \). Equation (7) then implies \( s_\lambda \geq 2/\gamma \), a contradiction. This proves \( s_\lambda \) is non-negative. Use that to rewrite equation (7) as

\[
2\gamma s_\lambda = \Delta + \frac{\lambda}{4} \mathbb{E} \left( \left( (s_\lambda + s_{\lambda'})^+ - (s_\lambda + s_{\lambda'}) \right) m_{\lambda'} - (s_\lambda - s_{\lambda'})^+ (1 - m_{\lambda'}) \right).
\]

Use \((-s_\lambda + s_{\lambda'})^+ = s_{\lambda'} - \min\{s_\lambda, s_{\lambda'}\}\) and \((s_\lambda - s_{\lambda'})^+ = s_\lambda - \min\{s(\lambda), s(\lambda')\}\) to rewrite this as

\[
2\gamma s_\lambda = \Delta + \frac{\lambda}{4} \mathbb{E} \left( (s_{\lambda'} - \min\{s_\lambda, s_{\lambda'}\}) - s_\lambda \right) m_{\lambda'} - (s_\lambda - \min\{s(\lambda), s(\lambda')\}) (1 - m_{\lambda'})\).
\]

Grouping terms, this gives

\[
s_\lambda = \frac{4\Delta + \lambda \mathbb{E} \left( \min\{s(\lambda), s(\lambda')\} (1 - 2m_{\lambda'}) \right)}{8\gamma + \lambda}.
\]

View this as a mapping \( s = T(s) \). We claim that for any cumulative distribution function \( G \) and misalignment function \( m \) with range \([0, 1/2]\), \( T \) is a contraction, mapping continuous functions on \([\Delta/2\gamma, \Delta/2\gamma]\) into the same set of functions. Continuity is immediate. Similarly, if \( s \) is nonnegative, \( T(s) \) is nonnegative. If \( s \leq \Delta/2\gamma \),

\[
T(s)_\lambda \leq \left( \frac{8\gamma + \lambda \mathbb{E}(1 - 2m_{\lambda'})}{8\gamma + \lambda} \right) \left( \frac{\Delta}{2\gamma} \right).
\]

Since the misalignment rate is nonnegative, the result follows.

Finally, we prove \( T \) is a contraction. If \( |s_1 - s_2| \leq \varepsilon \) for all \( \lambda \),

\[
|T(s_1)_\lambda - T(s_2)_\lambda| \leq \frac{\lambda \varepsilon \mathbb{E}(1 - 2m_{\lambda'})}{8\gamma + \lambda} \leq \varepsilon \mathbb{E}(1 - 2m_{\lambda'}).
\]

Note that the second inequality uses the fact that the fraction is increasing in \( \lambda \) and hence evaluates it at the limit as \( \lambda \) converges to infinity. Since \( \mathbb{E}(1 - 2m_{\lambda'}) < 1 \), this proves that \( T \) is a contraction in the sup-norm, with modulus \( \int_0^\infty \frac{\lambda'}{\lambda} (1 - 2m_{\lambda'}) dG_{\lambda'} \).

Next we prove that the mapping \( T \) takes nonincreasing functions \( s \) and maps them into
decreasing functions. Take \( \lambda_1 < \lambda_2 \) and let \( E(\lambda) \equiv \mathbb{E}\left( \min\{s(\lambda), s(\lambda')\}(1-2m_{\lambda'}) \right) \). Note that \( m \) nonnegative and \( s(\lambda) \leq \Delta/2\gamma \) implies \( E(\lambda) \leq \Delta/2\gamma \) as well. Similarly, \( s \) nonincreasing implies \( E \) is nonincreasing as well. Then

\[
T(s)(\lambda_1) - T(s)(\lambda_2) = \frac{4\Delta + \lambda_1 E(\lambda_1)}{8\gamma + \lambda_1} - \frac{4\Delta + \lambda_2 E(\lambda_2)}{8\gamma + \lambda_2}
\geq \frac{4\Delta + \lambda_1 E(\lambda_1)}{8\gamma + \lambda_1} - \frac{4\Delta + \lambda_2 E(\lambda_1)}{8\gamma + \lambda_2}
= \frac{4(\lambda_2 - \lambda_1)(\Delta - 2\gamma E(\lambda_1))}{(8\gamma + \lambda_1)(8\gamma + \lambda_2)} > 0,
\]

The first equality is the definition of \( T \). The first inequality uses \( E(\lambda_2) \leq E(\lambda_1) \). The second equality groups the two fractions over a common denominator. And the second equality uses \( E(\lambda) < \Delta/2\gamma \). This proves the result. It follows that the equilibrium surplus function is decreasing.

**A.2 Deriving Equation (9)**

Since the surplus function is nonnegative and nonincreasing, we can rewrite equation (7) as

\[
\Delta = 2\gamma s_\lambda + \frac{\lambda}{4} \mathbb{E}\left( \left( (s_\lambda + s_{\lambda'}) - \mathbb{I}_{\lambda' < \lambda}(s_{\lambda'} - s_\lambda) \right) m_{\lambda'} + \mathbb{I}_{\lambda' > \lambda}(s_\lambda - s_{\lambda'})(1 - m_{\lambda'}) \right)
\]

Placing all the terms involving \( s(\lambda) \) on the left hand side gives

\[
\left( 8\gamma + \lambda \mathbb{E}(1 - \mathbb{I}_{\lambda' \leq \lambda}(1 - 2m_{\lambda'})) \right) s_\lambda = 4\Delta + \lambda \mathbb{E}(\mathbb{I}_{\lambda' > \lambda} s_{\lambda'}(1 - 2m_{\lambda'})).
\]

(26)

Differentiate with respect to \( \lambda \) to get

\[
\left( 8\gamma + \lambda \mathbb{E}(1 - \mathbb{I}_{\lambda' \leq \lambda}(1 - 2m_{\lambda'})) \right) s'_\lambda + \mathbb{E}(1 - \mathbb{I}_{\lambda' \leq \lambda}(1 - 2m_{\lambda'})) s_\lambda = \mathbb{E}(\mathbb{I}_{\lambda' > \lambda} s_{\lambda'}(1 - 2m_{\lambda'}))
\]

Replace the right hand side using equation (26) and simplify to get

\[
s'_\lambda = \frac{8\gamma s_\lambda - 4\Delta}{\lambda \left( 8\gamma + \lambda \mathbb{E}(1 - \mathbb{I}_{\lambda' \leq \lambda}(1 - 2m_{\lambda'})) \right)} = \phi_\lambda \left( s_\lambda - \frac{\Delta}{2\gamma} \right),
\]

where \( \phi_\lambda \) is defined in equation (10). The general solution to this differential equation is

\[
s_\lambda = \frac{\Delta}{2\gamma} - k e^{\int_{\lambda}^{\lambda'} \phi_\lambda d\lambda'}
\]
for fixed $\Delta$ and some constant of integration $k$. Use the fact that $\lim_{\lambda \to \infty} s_\lambda = 0$ to pin down the constant of integration, yielding equation (9).

### A.3 Proof of Proposition 2

**Proof.** We proceed by contradiction. Assume $G(\lambda)$ has a mass point at some $\bar{\lambda} > 0$. From equation (10), we have that $\phi_\lambda$ jumps up at $\bar{\lambda}$ since $2m_\lambda < 1$ for all $\lambda$. Differentiating equation (9), we have that

$$s'_\lambda = -\frac{\Delta}{2\gamma} \phi_\lambda e^{-\int_{\lambda}^{\infty} \phi_{\lambda'} d\lambda'},$$

and so this jumps down at $\bar{\lambda}$. It follows that $s_\lambda$ has a concave kink at $\bar{\lambda}$.

Now consider part 3(a) of the definition of equilibrium. Using the monotonicity and nonnegativeness of $s(\lambda)$, the choice of $\lambda$ must maximize

$$\delta_1 - \gamma s_\lambda + \frac{\lambda}{4} \mathbb{E}(\mathbb{I}_{\lambda' < \lambda}(s_{\lambda'} - s_\lambda)m_{\lambda'}) - \lambda c(\lambda).$$

The first derivative with respect to $\lambda$ is

$$-\left(\gamma + \frac{\lambda}{4} \mathbb{E}(\mathbb{I}_{\lambda' < \lambda}m_{\lambda'})\right) s'_\lambda + \frac{1}{4} \mathbb{E}(\mathbb{I}_{\lambda' < \lambda}(s_{\lambda'} - s_\lambda)m_{\lambda'}) - c(\lambda) - \lambda c'(\lambda).$$

Note that $s'_\lambda$ jumps down at $\bar{\lambda}$ and the other expressions are continuous in $\lambda$. Therefore the slope of the objective function jumps up at $\bar{\lambda}$. That is, $\bar{\lambda}$ represents a local minimum in traders’ objective function, contradicting the assumption that $G$ has a mass point at $\bar{\lambda}$. ■

### A.4 Proof of Proposition 3 [Incomplete]

**Proof.** If $c$ is too large, everyone prefers to live in autarky, well-aligned half their lives and misaligned the other half. Otherwise, below this bound, they are strictly better off not living in autarky, so the lower bound on search intensity $\Delta$ is strictly positive.

To rule out an upper bound on the support we again proceed by contradiction. Assume there is a finite $\bar{\lambda}$ such that $G(\bar{\lambda}) = 1$ and $\Lambda = \int_0^{\bar{\lambda}} \lambda' dG(\lambda')$. Then for all $\lambda \geq \bar{\lambda}$, equation (10) implies

$$\phi_\lambda \equiv \frac{4\gamma}{\lambda \left(4\gamma + \lambda \mathbb{E}(m_{\lambda'})\right)}.$$

Then equation (9) implies

$$s_\lambda = \frac{2\Delta}{4\gamma + \lambda \mathbb{E}(m_{\lambda'})}, \quad \forall \lambda \geq \bar{\lambda}.$$
Since no one chooses these high values of $\lambda$, part 3(a) of the definition implies that for all $\lambda > \bar{\lambda}$, the profits must be lower than choosing $\bar{\lambda}$:

$$-\gamma s_\lambda + \frac{\lambda}{4} \mathbb{E} \left( (s_{\lambda'} - s_\lambda) m_{\lambda'} \right) - \lambda c \leq -\gamma s_\lambda + \frac{\bar{\lambda}}{4} \mathbb{E} \left( (s_{\lambda'} - s_\lambda) m_{\lambda'} \right) - \bar{\lambda} c.$$  

Rearranging terms, replacing $s_\lambda$ using the previous expression, and using $\lambda > \bar{\lambda}$, this reduces to

$$\frac{1}{4} \mathbb{E} (s_{\lambda'} m_{\lambda'}) \leq c. \quad (28)$$

Now compare the value of choosing $\bar{\lambda}$ to the value of autarky, $\lambda = 0$. A necessary condition for $\bar{\lambda}$ to be optimal is that it gives a higher value than autarky:

$$-\gamma s_{\bar{\lambda}} + \frac{\bar{\lambda}}{4} \mathbb{E} \left( (s_{\lambda'} - s_{\bar{\lambda}}) m_{\lambda'} (1 - m_{\lambda'}) \right) - \bar{\lambda} c > -\gamma s_0.$$  

Use equation (7) to eliminate $s_0$ and equation (27) to eliminate $s_{\bar{\lambda}}$:

$$\bar{\lambda} \left( \frac{1}{4} \mathbb{E} (s_{\lambda'} m_{\lambda'}) - c \right) > 0.$$  

This contradicts inequality (28), establishing the contradiction.

**A.5 Proof of Proposition 4**

**Proof.** According to part 3(a) of the definition of equilibrium and Proposition 3, the profit from choosing any value of $\lambda$ in excess of $\underline{\lambda}$ must be the same,

$$\bar{v} = \delta_1 - \gamma s_\lambda + \frac{\lambda}{4} \mathbb{E} \left( \mathbb{I}_{\lambda' < \lambda} (s_{\lambda'} - s_\lambda) m_{\lambda'} \right) - \lambda c, \quad (29)$$

33
where we simplify the expression slightly using the results from Proposition 1. This implies the first order condition

\[ 0 = -\left(\gamma + \frac{\lambda}{4} \mathbb{E}(\mathbb{I}_{\lambda < \lambda} m'_{\lambda})\right) s'_{\lambda} + \frac{1}{4} \mathbb{E}(\mathbb{I}_{\lambda < \lambda} (s_{\lambda} - s_{\lambda}) m'_{\lambda}) - c \]

\[ = -\left(\gamma + \frac{\lambda}{4} \mathbb{E}(\mathbb{I}_{\lambda < \lambda} m'_{\lambda})\right) s'_{\lambda} + \frac{\bar{v} - \delta_1 + \gamma s_{\lambda}}{\lambda} \]

\[ = \frac{\Delta}{2} \left(4\gamma + \lambda \mathbb{E}(\mathbb{I}_{\lambda < \lambda} m'_{\lambda})\right) e^{-\int_{\lambda'}^{\infty} \phi_{\lambda'} d\lambda'} \cdot \frac{\bar{v} - \delta_1}{\lambda} + \frac{\Delta}{2} \left(1 - e^{-\int_{\lambda'}^{\infty} \phi_{\lambda'} d\lambda'}\right) \]

\[ \times \frac{\lambda}{8} \mathbb{E}(\mathbb{I}_{\lambda > \lambda}) s'_{\lambda} + \frac{\bar{v} - \delta_1 + \delta_0}{2} \frac{\Delta}{\lambda}, \]

where the second line simplifies the using first equation (29), the third line replaces \( s_{\lambda} \) and its derivative using equation (9) and replaces \( \phi_{\lambda} \) using equation (10), the fourth line groups terms and recognizes that \( \Delta = \delta_1 - \delta_0 \), and the fifth line simplifies again using equations (10) and (9).

Rewrite this result as

\[ \lambda^2 s'_{\lambda} = \frac{-4(2\bar{v} - \delta_1 - \delta_0)}{\mathbb{E}(\mathbb{I}_{\lambda > \lambda})} \]

(30)

On the other hand, computing \( s'_{\lambda} \) directly from equations (9) and (10) gives

\[ \lambda^2 s'_{\lambda} = \frac{-4\Delta e^{-\int_{\lambda'}^{\infty} \phi_{\lambda'} d\lambda'}}{\frac{8\gamma}{\lambda} + \mathbb{E}(1 - \mathbb{I}_{\lambda \leq \lambda}(1 - 2m'_{\lambda}))} \]

(31)

Equate these two expressions and then take the limit as \( \lambda \) converges to \( \infty \). Note that \( e^{-\int_{\lambda'}^{\infty} \phi_{\lambda'} d\lambda'} \to 1 \) and \( 8\gamma/\lambda \to 0 \). Expanding the expectations operator using its definition and simplifying, we get

\[ 1 - \int_{0}^{\infty} \frac{\lambda'}{\Lambda} dG(\lambda') = \frac{2\bar{v} - \delta_1 - \delta_0}{\delta_1 - \bar{v}} \int_{0}^{\infty} \frac{\lambda'}{\Lambda} m(\lambda') dG(\lambda'). \]

(32)

The left hand side is the fraction of meetings with middlemen, and it is strictly positive whenever the value of traders \( \bar{v} \) exceeds what they can achieve in autarky, \( \frac{1}{2}(\delta_1 + \delta_0) \). It is finite whenever the value of traders is lower than the first best, \( \delta_1 \). This proves that a strictly positive fraction of meetings accrues to a zero measure of market makers who are in continuous contact with the market.
To prove the Pareto tail, again equate (30) and (31) and manipulate to get
\[
\frac{2\bar{v} - \delta_1 - \delta_0}{\lambda E(I_{\lambda' \geq \lambda})} = \frac{\Delta e^{-\int_{\lambda}^{\infty} \phi_{\lambda'} d\lambda'}}{8\gamma + \lambda E\left(1 - I_{\lambda' \leq \lambda}(1 - 2m_{\lambda'})\right)}.
\]
Differentiate with respect to $\lambda$ and evaluate at $\lambda \to \infty$ to get
\[
\lim_{\lambda \to \infty} \lambda^3 dG(\lambda) = \frac{24(2\bar{v} - \delta_1 - \delta_0)\gamma \Lambda}{\Delta - (2\bar{v} - \delta_1 - \delta_0)(1 - 2\lim_{\lambda \to \infty} m_{\lambda})}.
\]
Using the steady state expressions, we find that
\[
\lim_{\lambda \to \infty} m_{\lambda} = \frac{\int_{\lambda}^{\infty} \frac{\lambda'}{\Lambda} m(\lambda') dG(\lambda')}{\int_{0}^{\infty} \frac{\lambda'}{\Lambda} dG(\lambda')} + 2 \int_{0}^{\infty} \frac{\lambda'}{\Lambda} m(\lambda') dG(\lambda') = \frac{\delta_1 - \bar{v}}{\delta_1 - \delta_0}
\]
where the second equation simplifies the first using equation (32). Substituting this into the tail parameter expression, we get
\[
\lim_{\lambda \to \infty} \lambda^3 dG(\lambda) = \frac{24\gamma \Lambda}{\frac{\delta_1 - \delta_0}{2\bar{v} - \delta_1 - \delta_0} - \frac{2\bar{v} - \delta_1 - \delta_0}{\delta_1 - \delta_0}}.
\]
Note that this is positive since traders’ value is less than the first best, $\bar{v} < \delta_1$. This implies that in the right tail, the $G$ distribution is well-approximated by the Pareto distribution
\[
G(\lambda) = 1 - \frac{12\gamma \Lambda}{\frac{\delta_1 - \delta_0}{2\bar{v} - \delta_1 - \delta_0} - \frac{2\bar{v} - \delta_1 - \delta_0}{\delta_1 - \delta_0}} \lambda^{-2}.
\]

A.6 Proof of Proposition 5

**Proof.** We start by solving the problem of maximizing (11) subject to the constraint (12) for all $\lambda$. We do this by writing the Lagrangian, placing a Lagrange multiplier $S_\lambda dG(\lambda)$ on constraint (12). For later, when we endogenize the distribution of meeting rates, we place a multiplier $\theta_0$ on the constraint that $\int_0^\infty dG(\lambda) = 1$ and a multiplier $\theta_1$ on the constraint
that $E(1) = 1$. Then the Lagrangian is

$$\mathcal{L} = \Delta \int_{0}^{\infty} (1 - m_{\lambda})dG(\lambda) - \Lambda E(c(\lambda)) + \theta_{0} \left( 1 - \int_{0}^{\infty} dG(\lambda) \right) + \theta_{1} (1 - E(1))$$

$$+ \int_{0}^{\infty} S_{\lambda} \left( \gamma + \frac{\lambda}{2} E \left( \mathbf{1}_{\lambda,0}^{\lambda,0} m_{\lambda'} + \mathbf{1}_{\lambda,0}^{\lambda,1} (1 - m_{\lambda'}) \right) \right) m_{\lambda}$$

$$- \left( \gamma + \frac{\lambda}{2} E \left( \mathbf{1}_{\lambda,0}^{\lambda,0} m_{\lambda'} + \mathbf{1}_{\lambda,0}^{\lambda,1} (1 - m_{\lambda'}) \right) \right) (1 - m_{\lambda}) \right) dG(\lambda). \quad (33)$$

We start with the first order condition with respect to $m_{\lambda}$. Suppressing the multiplicative constant $dG(\lambda)$, this is

$$\Delta = 2\gamma S_{\lambda} + \frac{\lambda}{2} \mathbb{E} \left( \left( \mathbf{1}_{\lambda,0}^{\lambda,0} (S_{\lambda} + S_{\lambda'}) + \mathbf{1}_{\lambda,1}^{\lambda,0} (S_{\lambda} - S_{\lambda'}) \right) m_{\lambda'} \right)$$

$$+ \left( \mathbf{1}_{\lambda,0}^{\lambda,1} (S_{\lambda} - S_{\lambda'}) + \mathbf{1}_{\lambda,1}^{\lambda,1} (S_{\lambda} + S_{\lambda'}) \right) (1 - m_{\lambda'}) \right) \quad (34)$$

Next take the first order conditions for the indicator functions $\mathbf{1}_{\lambda,a}^{\lambda',a'}$. In doing this, we implicitly use the unstated constraints $0 \leq \mathbf{1}_{\lambda,a}^{\lambda',a'} = \mathbf{1}_{\lambda,a}^{\lambda',a'} \leq 1$. We get

$$S_{\lambda} + S_{\lambda'} \geq 0 \Rightarrow \mathbf{1}_{\lambda,0}^{\lambda,0} = \begin{cases} 1 \\ 0 \end{cases}, \quad S_{\lambda} \geq S_{\lambda'} \Rightarrow \mathbf{1}_{\lambda,0}^{\lambda,1} = \begin{cases} 1 \\ 0 \end{cases}$$

$$S_{\lambda} + S_{\lambda'} \leq 0 \Rightarrow \mathbf{1}_{\lambda,1}^{\lambda,1} = \begin{cases} 1 \\ 0 \end{cases}, \quad S_{\lambda'} \geq S_{\lambda} \Rightarrow \mathbf{1}_{\lambda,1}^{\lambda,0} = \begin{cases} 1 \\ 0 \end{cases}$$

Using this, rewrite the first order condition (34) as

$$\Delta = 2\gamma S_{\lambda} + \frac{\lambda}{2} \mathbb{E} \left( (S_{\lambda} + S_{\lambda'})^+ - (S_{\lambda'} - S_{\lambda})^+ \right) m_{\lambda'}$$

$$+ \left( (S_{\lambda} - S_{\lambda'})^+ - (-S_{\lambda} - S_{\lambda'})^+ \right) (1 - m_{\lambda'}) \right) \quad (35)$$

This is identical to equation (7) for the surplus in the decentralized economy, except that the terms multiplying $\lambda$ are twice as large for the planner. The proof of Proposition 1 implies the surplus function is uniquely defined by this equation and moreover is decreasing and nonnegative. Equilibrium trading patterns follow immediately. ■
A.7 Characterization of Planner’s Problem

We continue manipulating the first order conditions of the Lagrangian developed in Appendix A.6. We start by solving explicitly for the social surplus function, replicating Appendix A.2. Using monotonicity of $S$, rewrite equation (35) as

$$
(4\gamma + \lambda \mathbb{E}(1 - \mathbb{1}_{\lambda \leq \lambda}(1 - 2m_{\lambda}))) S_{\lambda} = 2\Delta + \lambda \mathbb{E}(\mathbb{1}_{\lambda > \lambda}S_{\lambda'}(1 - 2m_{\lambda}))
$$

(36)

Differentiate with respect to $\lambda$ to get

$$
\left(4\gamma + \lambda \mathbb{E}(1 - \mathbb{1}_{\lambda \leq \lambda}(1 - 2m_{\lambda}))\right) S'_{\lambda} + \mathbb{E}(1 - \mathbb{1}_{\lambda \leq \lambda}(1 - 2m_{\lambda})) S_{\lambda} = \mathbb{E}(\mathbb{1}_{\lambda > \lambda}S_{\lambda'}(1 - 2m_{\lambda}))
$$

Replace the right hand side using equation (36) and simplify to get

$$
S'_{\lambda} = \frac{4\gamma S_{\lambda} - 2\Delta}{\lambda \left(4\gamma + \lambda \mathbb{E}(1 - \mathbb{1}_{\lambda \leq \lambda}(1 - 2m_{\lambda}))\right)} = \Phi_{\lambda} \left(S_{\lambda} - \frac{\Delta}{2\gamma}\right),
$$

where $\Phi_{\lambda}$ is given in equation (14). The general solution to this differential equation is

$$
S_{\lambda} = \frac{\Delta}{2\gamma} - Ke^{\int_{\lambda}^{\lambda} \Phi_{\lambda'} d\lambda}
$$

for fixed $\lambda$ and some constant of integration $K$. Use the fact that $\lim_{\lambda \to \infty} S_{\lambda} = 0$ to pin down the constant of integration, equation (13).

Now return to the Lagrangian (33). The first order condition with respect to $dG(\lambda)$ implies $dG(\lambda) > 0$ only if $\lambda$ maximizes

$$
\Delta(1 - m_{\lambda}) - \lambda c(\lambda) - \theta_0 - \theta_1 \frac{\lambda}{\Lambda}
+ \frac{\lambda}{2} \mathbb{E} \left(1_{\lambda,0}^X S_{\lambda'} m_{\lambda} + 1_{\lambda,1}^X S_{\lambda'}(1 - m_{\lambda}) - 1_{\lambda,0}^X S_{\lambda'}(1 - m_{\lambda}) m_{\lambda} - 1_{\lambda,1}^X S_{\lambda'}(1 - m_{\lambda})(1 - m_{\lambda}) \right)
$$

Using Proposition 5, this reduces to

$$
\Delta(1 - m_{\lambda}) - \lambda c(\lambda) - \theta_0 - \theta_1 \frac{\lambda}{\Lambda}
+ \frac{\lambda}{2} m_{\lambda} \mathbb{E} (m_{\lambda} S_{\lambda'}) + \frac{\lambda}{2} (1 - m_{\lambda}) \mathbb{E} (\mathbb{1}_{\lambda < \lambda} m_{\lambda} S_{\lambda'}) - \frac{\lambda}{2} m_{\lambda} \mathbb{E} (\mathbb{1}_{\lambda > \lambda} S_{\lambda'}(1 - m_{\lambda}))
$$

Multiply equation (36) by $\frac{1}{2} m_{\lambda}$ and add to the previous expression. $dG(\lambda) > 0$ only if $\lambda$
maximizes
\[
\Delta + \frac{\lambda}{2} \mathbb{E} (\mathbb{I}_{\lambda'<\lambda} m_{\lambda'} S_{\lambda'}) - \left( 2 \gamma + \frac{\lambda}{2} \mathbb{E} (1 - \mathbb{I}_{\lambda'<\lambda} (1 - 2m_{\lambda'})) \right) S_{\lambda} m_{\lambda} - \lambda c(\lambda) - \theta_0 - \theta_1 \frac{\lambda}{\Lambda}
\]

Using equation (12) under the efficient trading pattern and dropping irrelevant constants, this simplifies to

\[
dG(\lambda) > 0 \Rightarrow \lambda \in \arg \max \left( -\gamma S_{\lambda} + \frac{\lambda}{2} \mathbb{E} (\mathbb{I}_{\lambda'<\lambda} m_{\lambda'} (S_{\lambda'} - S_{\lambda})) - \lambda c(\lambda) - \theta_1 \frac{\lambda}{\Lambda} \right)
\]

(37)

Next, look at the first order condition with respect to \( \Lambda \), which appears implicitly inside each of the expectations operators. Using the efficient trading patterns, this gives

\[
\theta_1 = \int_0^\infty S_{\lambda} \left( \frac{\lambda}{2} \mathbb{E} (m_{\lambda'} + \mathbb{I}_{\lambda'<\lambda} (1 - m_{\lambda'})) \right) m_{\lambda} - \left( \frac{\lambda}{2} \mathbb{E} (\mathbb{I}_{\lambda'<\lambda} m_{\lambda'}) \right) (1 - m_{\lambda}) \ dG(\lambda)
\]

= \gamma \int_0^\infty S_{\lambda} (1 - 2m_{\lambda}) dG(\lambda)

where the second line again uses equation (12) under the efficient trading pattern. Substitute this into equation (37) to get that \( dG(\lambda) > 0 \) only if \( \lambda \) maximizes the expression in (15).

Finally, consider the behavior of (15) for large \( \lambda \). If the inequality (16) is violated, the planner would set \( \lambda \) unboundedly large for everyone. This incurs infinite costs and so cannot be optimal. If the inequality (16) is slack, large values of \( \lambda \) would be inconsistent with condition (15), which implies \( \Lambda = \int_0^\infty \lambda dG(\lambda) \). To have middlemen, the inequality (16) must be binding.