International Reserves and Rollover Risk*

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Abstract

We study how a government should manage international reserves in a quantitative model of sovereign default extended with long-term debt and a risk-free asset. Keeping higher levels of reserves provides a hedge against rollover risk, but this is expensive because using reserves to pay down debt allows the government to reduce spreads by reducing future default incentives. Our benchmark economy, parameterized to mimic salient features of a typical emerging economy, can account for a significant fraction of the holdings of international reserves, as well as the cyclicity of the gross debt and asset positions. We also show that income windfalls, larger contingent liabilities, improved policy frameworks, and an increase in the importance of rollover risk imply increases in the optimal level of reserves that are consistent with the growth of reserves in emerging economies observed over the last fifteen years. It is essential for our results that debt maturity exceeds one period, as is the case in the data.

Keywords: Sovereign default, international reserves, rollover risk, safe assets

JEL Codes: F32, F34, F41

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1 Introduction

The large accumulation of safe assets by emerging markets is a striking feature of the international financial system. In 2015, holdings of international reserves reached $8 trillion in emerging markets, after an increase in the reserves-GDP ratio of 9 percentage points since 2000.\(^1\) Given the opportunity costs of holdings reserves, an extended view is that reserves might have reached excessive levels. On the other hand, the persistent instability in international financial markets suggests that emerging markets should keep building their stock of reserves, since they provide a valuable buffer during episodes of financial turmoil.\(^2\)

What constitutes an adequate level of reserves? Specifically, what is the optimal level of reserves for an indebted government facing default risk and the possibility of surges in borrowing costs? And from a positive perspective: what are the main determinants in the secular increase in reserves? Despite extensive debates, the literature lacks a quantitative theoretical framework that can be used to tackle these questions.

This paper studies the optimal amount of international reserves in a workhorse model of sovereign default (Eaton and Gersovitz, 1981; Aguiar and Gopinath, 2006; Arellano, 2008) extended with long-term debt, reserve accumulation, and risk-averse foreign lenders. In this setup, lack of government commitment implies that sovereign spreads must reflect future incentives to default. A shock to domestic fundamentals or to the risk premium required by external bondholders exposes the government to rollover risk, that is, the risk of having to roll over its debt obligations at times when its borrowing opportunities deteriorate. When those states are realized, the government can mitigate the fall in consumption using the reserves previously accumulated. We analyze how the government optimally manages both assets and liabilities, and how equilibrium movements in bond prices play a key role in accounting for the optimal portfolio.\(^3\)

We find that it is optimal for an indebted government paying a significant sovereign spread to hold large amounts of reserves. We calibrate the model by targeting salient features of a typical emerging economy paying a significant spread. Model simulations generate an optimal level of reserves of 6 percent of annual income on average, and reserves can reach values as high as 40 percent in some simulation periods. We find that in the simulations, the government accumulates both reserves and debt in periods of low spread and high income (and vice versa), which we show is consistent with the behavior of debt and reserves in emerging economies.

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\(^1\)The empirical regularities of international reserves are presented in Section 2. The IMF (2001) defines international reserves as “official public sector foreign assets that are readily available.”

\(^2\)Building a buffer for liquidity needs is the most frequently cited reason for reserve accumulation in the IMF Survey of Reserve Managers (80 percent of respondents; IMF, 2011). There is also extensive empirical evidence that supports the precautionary role of reserves (e.g., Aizenman and Lee (2007), Calvo, Izquierdo, and Loo-Kung (2012), Bussiere, Cheng, Chinn, and Lisack (2013), Domínguez, Hashimoto, and Ito (2012), Frankel and Saravelos (2012), and Gourinchas and Obstfeld (2011)).

\(^3\)Alfaro and Kanczuk (2009) study reserve accumulation in a sovereign default model with one-period debt and found that “the optimal policy is not to hold reserves at all.” As we explain below, the rollover risk motive that we study in this paper does not arise in their study.
Finally, we show that the secular increase in reserves is consistent with four recent developments in emerging economies: the exceptional increases in real income caused by the surge in commodity prices, the increase in the public sector’s contingent liabilities driven by the growth of the banking sector and foreign corporate debt, the improvements in policy frameworks that reduced possible impatience bias in policy makers, and the increase in the frequency and intensity of global disruptions to international financial markets, which became apparent after the East Asian financial crises. These developments also imply lower optimal debt levels and spreads, which are consistent with the data.

**Mechanism.** Why is it optimal for an indebted government facing default risk to accumulate reserves? The key feature is countercyclical default risk. In times of high income, sovereign spreads are low—reflecting low incentives to default in the future—and it is less costly for the government to borrow. Conversely, when the government is hit by negative income shocks, borrowing costs increase, making it costly to roll over debt to smooth consumption. Having reserves in those states has the benefit of reducing borrowing needs. Shocks to lenders’ stochastic discount factor also play an important role. When lenders become more risk averse, they require higher spreads to compensate for the risk of default. Anticipating that adverse income shocks or risk premium shocks will cause sharp drops in consumption, a government that wishes to borrow finds it optimal to accumulate reserves at the same time. Doing so prevents the government from having to roll over debt at high rates in adverse states of nature. In a nutshell, reserves provide an indebted government with insurance against rollover risk.

We show that for reserves to provide such insurance, it is essential that debt maturity exceeds one period. In a model with one-period debt, the cost of issuing one extra bond is that the government has to allocate one extra unit of resources to service that bond in the next period, if it decides to repay. The benefit of accumulating one extra unit of reserves is that there is one extra unit of resources available in the next period, regardless of the repayment decision. Thus, by issuing one-period debt and accumulating reserves, the government can only transfer resources between future repayment and default states. We show this channel is not quantitatively important. When we assume one-period bonds and recalibrate the model to match the same moments as in our benchmark model, we find that the level of reserves is close to zero, in line with Alfaro and Kanczuk (2009).

Accumulating reserves has costs. We show that the government pays a higher spread by choosing portfolios with higher debt and reserves positions. Although a higher spread reflects that the government repays in less states, this benefit is offset by the extra default cost the government incurs. As a result, higher gross debt and asset positions translates into higher costs for the government. We find that it is relatively more costly to accumulate reserves when lenders are more risk averse and when income is lower. When lenders’ risk aversion takes a high value, the government has to pay a higher risk premium for the extra risk lenders face. Moreover, we also show that the spread is more sensitive to increases in gross positions at low income levels.
When solving the optimal portfolio problem, the government trades off these costs against the insurance benefits of reserves. In particular, the government buys more insurance against adverse future shocks by choosing portfolios with higher gross positions in good times. Conversely, in bad times, the government tends to deplete reserves and reduce debt. This is consistent with the cyclical behavior of debt and reserves in the data, which is presented in Section 2.

Related Literature. Our paper is related to several strands of the literature. First, we build on the quantitative sovereign default literature that follows Aguiar and Gopinath (2006) and Arellano (2008). They show that predictions of the sovereign default model are consistent with several features of emerging markets, including countercyclical spreads and procyclical borrowing. These papers, however, do not allow indebted governments to accumulate assets. We allow governments to simultaneously accumulate assets and liabilities and show that the model’s key predictions are still consistent with features of emerging markets. Furthermore, our model can account for the accumulation of reserves by indebted governments and the accumulation of reserves in periods of low spreads and high income.

Alfaro and Kanczuk (2009) was the first paper to study the joint accumulation of international reserves and sovereign defaultable debt. Using a benchmark model with a one-period bond, they found that the sovereign should accumulate no reserves at all and found this result to be robust to various features including interest rate shocks and sudden stops, but did not analyze issues of debt maturity. As we show, for reserves to provide a hedge against rollover risk, it is essential that maturity exceeds one period.

There are also closely related studies on sovereign debt maturity. Arellano and Ramanarayanan (2012) study a quantitative model with default risk in which the government balances the incentive benefits of short-term debt and the hedging benefits of long-term debt. Trade-offs between short-term debt and long-term debt are also analyzed in Niepelt (2008), Hatchondo, Martinez, and Sosa Padilla (2015), Broner, Lorenzoni, and Schmukler (2013), Dovis (2013), and Aguiar, Amador, Hopenhayn, and Werning (2016). None of these studies allow the government to accumulate assets for insurance purposes.

Our paper is also related to the literature on reserves that emphasizes precautionary savings and sudden stops. Durdu, Mendoza, and Terrones (2009) study a dynamic precautionary savings model in which a higher net foreign asset position reduces the frequency and the severity of binding credit constraints. In contrast, we study a setup with endogenous borrowing constraints resulting from default risk and analyze gross portfolio positions. Jeanne and Ranciere (2011) present a simple analytical formula to quantify the optimal amount of reserves. They model reserves as an Arrow-Debreu security that pays off in a sudden stop. In a similar vein, Caballero and Panageas (2008) propose a quantitative setup in which the government issues nondefaultable debt that is indexed to the income-growth shock. They find that there are significant gains from

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4Aguiar and Amador (2013a) and Aguiar, Chatterjee, Cole, and Stangebye (2016) analyze, respectively, theoretical and quantitative issues in this literature.
introducing financial instruments that provide insurance against both the occurrence of sudden stops and changes in the sudden-stop probability. Aizenman and Lee (2007) and Hur and Kondo (2011) study reserve accumulation in open economy versions of Diamond and Dybvig, generating endogenous sudden stops.

Our paper complements this literature by considering endogenous borrowing costs due to default risk, the role of debt maturity, and the focus on safe assets rather than insurance arrangements. Overall, a key contribution of our paper is to present a unified framework for studying the joint dynamics of reserves, debt, and sovereign spreads. The quantitative analysis is consistent not only with the increase of reserves but also with the decline in the levels of sovereign debt and the behavior spreads over the last fifteen years.

Other studies emphasize other benefits of reserves accumulation. Korinek and Servén (2011) and Benigno and Fornaro (2012) investigate the “mercantilist motive.” They present models in which learning-by-doing externalities in the tradable sector lead the government to accumulate reserves to depreciate the real exchange rate. In Aguiar and Amador (2011), the accumulation of net foreign assets allows the government to credibly commit not to expropriate capital. These studies, however, do not present endogenous gross debt positions and hence do not address whether governments should accumulate reserves or lower their debt level.

Our work is also closely related to papers that study the optimal maturity structure of government debt in the presence of distortionary taxation. Angeletos (2002) and Buera and Nicolini (2004) study a closed-economy model in which the government can issue non-state-contingent bonds of different maturities under perfect commitment. They present examples in which the government can replicate complete market allocations by issuing nondefaultable long-term debt and accumulating short-term assets. There are important differences in the mechanism in our paper. In their model, changes in the term structure of interest rates, which contribute to offset shocks to the government budget constraint, arise as a result of fluctuations in the marginal rate of substitution of domestic consumers. Quantitatively, the gross positions that sustain the complete market allocations are on the order of a few hundred times total GDP (Buera and Nicolini, 2004).

In contrast, in our model, fluctuations in the interest rate reflect changes in the default premium that foreign investors demand in order to be compensated for the possibility of government default. This not only provides a different reason for the asset spanning motive—without default risk our model would have an indetermined portfolio position—but also lead to an empirically plausible portfolio composition of government assets and liabilities. In addition, reserves affect incentives for debt repayment, a channel absent in this literature. Overall, our paper provides an alternative theory of debt management based on limited commitment, which shifts the focus from minimizing deadweight losses of taxation to default risk.

\footnote{Faraglia, Marcet, and Scott (2010) argue that the qualitative predictions of the “complete market approach” to debt management are sensitive to the type of shocks considered, and Debortoli, Nunes, and Yared (2016) argue that the policy conclusions are sensitive to the assumption that the government can commit to fiscal policy.}
Telyukova (2013) and Telyukova and Wright (2008) address the “credit card puzzle,” that is, the fact that households pay high interest rates on credit cards while earning low rates on bank accounts. In these models, the demand for liquid assets arises because of a transaction motive, since credit cards cannot be used to buy some goods. Although we also study savings decisions by an indebted agent, we offer a distinct mechanism for the demand of liquid assets based on rollover risk, when the borrower uses long-term debt. This mechanism could be relevant for understanding the financial decisions of households and corporate borrowers.

The rest of the article proceeds as follows. Section 2 documents a set of facts for reserves and debt. Sections 3 presents the model. Section 4 describes the calibration. Section 5 and 6 present the quantitative results. Section 7 concludes.

2 Facts on debt and reserves

Here we provide here a set of basic facts for debt and reserves. Our sample includes data for the set of 23 emerging market economies, as classified by the World Economic Outlook (WEO) in 2014: Argentina, Thailand, China, Brazil, Bulgaria, Chile, Colombia, Croatia, Ecuador, Hungary, Malasia, Mexico, Peru, Philippines, Lebanon, Morocco, Panama, Poland, Russia, South Africa, Turkey, Ukraine, and Venezuela.\footnote{We start in 2000 because this is when sovereign spreads become widely available. The strong upward trend in reserves also start around that year. The level of reserves and the trend in reserves look similar for other emerging economies not included in the WEO 2014 classification.} For each of these countries, we use data for reserves, debt spreads and GDP in the period 2000-2014.\footnote{The time series for the spread are taken from the Emerging Markets Bond Index Plus (EMBI+ blended) index for the period 1993-2014. The data for public debt is taken from the IMF WEO database. Data for GDP is from WEO. Data for reserves is from 2015 update of Lane and Milesi-Ferretti (2007).}

We organize the empirical regularities in the following three facts.

1. **Indebted governments that hold reserves pay significant spreads on their debt.** This fact, also highlighted by Rodrik (2006) and Alfaro and Kanczuk (2009) among others, is illustrated in Figure 1: panel (a) shows the reserve levels, panel (b) shows the spread levels; panel (c) shows the debt levels, panel (c) the spread levels, and panel (d) the spread volatility. All panels are sorted starting with the country that has more reserves as a fraction of GDP to the one that has the least. The mean level of reserves is 21.5 percent of GDP while the mean level of debt is 49.3 percent. In addition, the cross-sectional correlation is \(\rho\), which reflects as panels (a) and (c) indicate that countries that have debt, have substantial amount of reserves. Finally, the average spread in the sample is 310 bps and the average volatility of spreads is 200 bps, reflecting significant variations in access to credit markets.

2. **There has been a secular increase in reserves.** Emerging markets have substantially increased their holdings of international reserves, a fact much noted in discussions on global imbalances.
In our sample, the increase in reserves is about 8 percent of GDP on average and reaches two-digit numbers for several countries. Figure 2 (panel a) presents the trend in reserves for the mean of the sample of countries considered, and panel (b) shows the increase in reserves across countries. In addition, it shows a downward trend for debt (panel c) and spreads (panel b).

3. Reserves and debt tend to increase when income is high and spreads are low. Besides the low frequency movement pointed out in fact 2, there is also significant variation in reserves across the cycle, with reserves and debt increasing more when income is high and spreads are low. To show this, we compute the mean increase in reserves and the mean increase in debt for the upper and lower quartiles of income growth and for the upper and lower quartile of the level of spreads. Panel (a) of Figure 3 presents the difference in the increases in reserves between the upper quartiles and the lower quartile. According to these data, reserves increase by 1 percentage point more when spreads are in the lower quartile of the distribution. Panel (b) of Figure 3 shows similarly that reserves increase by 2 percent more when income growth is in the upper quartile. In terms of variation of debt, panels (c) and (d) of Figure 3 show similar facts. Debt increases by 1 percentage point more when spreads are in the lower quartile (panel c) and by 0.4 percentage points more when income growth is in the higher quartile.

In other words, during good times, emerging economies receive capital inflows and increase their capital outflows, and during bad times, capital inflows retrench and there is a reduction in capital outflows. On the debt side, these results are also in line with Aguiar, Chatterjee, Cole, and Stangebye (2016), who found a pooled correlation of spreads, lagged one period, and the percentage change of debt of $-0.19$ in emerging market economies. On the flip side, governments tend to deplete those reserves during episodes of low income and high spreads, as recently experienced in the global financial crises (e.g. Frankel and Saravelos, 2012).

To summarize, these three sets of facts uncover important regularities about the levels, trend and cycle of debt and reserve positions. First, emerging markets simultaneously hold large gross debt and asset positions while paying significant spreads on their debt. Second, there has been an upward trend in reserves. Third, reserves and debt are procyclical. Next, we present a quantitative model of the optimal level of reserves as insurance against rollover risk that will be consistent with these empirical regularities.

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8Broner, Didier, Erce, and Schmukler (2013) also report a high volatility of international reserves and document that a broad measure of capital inflows and capital outflows is procyclical.
Figure 1: International reserves and debt in emerging economies.

Figure 2: Trend in Reserves, Debt, and Spreads
Note: Panels (a), (b), and (c) are simple averages of the sample of countries. Data period: 2000-2014.
Figure 3: Cyclical variations: Additional reserves and debt accumulation with lower spread and higher income.

Note: The figure presents the difference in the increase in reserves and debt between the lower (higher) and higher (lower) quartile of the spread (income) distribution. For emerging economies, reserves and debt accumulation are expressed as a percentage of 2014 GDP, and years of higher income are years of higher GDP growth. Low (high) income (spread) years correspond to years in which the GDP growth (spread) is in the lowest (highest) quartile of the distribution of GDP growth (spread) across countries. Data period: 2000-2014.
3 Model

This section presents a dynamic small open economy model with a stochastic endowment stream in which the government issues non-state-contingent defaultable debt and buys a reserve asset that pays the risk-free interest rate.\(^9\)

3.1 Environment

**Endowments.** Time is discrete and indexed by \( t \in 0, 1, \ldots \). The economy’s endowment of the single tradable good is denoted by \( y \in Y \subset \mathbb{R}_+^+ \). The endowment process follows:

\[
\log(y_t) = (1 - \rho) \mu + \rho \log(y_{t-1}) + \varepsilon_t,
\]

with \( |\rho| < 1 \), and \( \varepsilon_t \sim N(0, \sigma^2_\varepsilon) \).

**Preferences.** Preferences of the government over private consumption are given by

\[
E \sum_{j=t}^{\infty} \beta^{j-t} \mu(c_j),
\]

where \( E \) denotes the expectation operator, \( \beta \) denotes the discount factor, and \( c \) represents private consumption. The utility function is strictly increasing and strictly concave.

**Asset/Debt Structure.** As in Arellano and Ramanarayanan (2012) and Hatchondo and Martinez (2009), we assume that a bond issued in period \( t \) promises a deterministic infinite stream of coupons that decreases at an exogenous constant rate \( \delta \). In particular, a bond issued in period \( t \) promises to pay \( \delta(1 - \delta)^{j-1} \) units of the tradable good in period \( t + j \), for all \( j \geq 1 \). Hence, debt dynamics can be represented by the following law of motion:

\[
b_{t+1} = (1 - \delta)b_t + i_t,
\]

where \( b_t \) is the number of bonds due at the beginning of period \( t \), and \( i_t \) is the number of new bonds issued in period \( t \). The government issues these bonds at a price \( q_t \), which in equilibrium will depend on the governments’ portfolio decisions and the exogenous shocks.

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\(^9\)We abstract from possible conflicts of interest between different branches of the government and treat it as a consolidated entity. In practice, reserves are often held by the monetary authority, while borrowing is conducted by the fiscal authority. However, independence of the monetary authority is often limited. As anecdotal evidence from Argentina, the *New York Times* reported that “President Cristina Fernandez fired Argentina’s central bank chief Thursday after he refused to step down in a dispute over whether the country’s international reserves should be used to pay debt.” Similar debates are also present in developed economies. For instance, the Swedish National Debt Office and the Riksbank discussed whether the debt office should borrow to strengthen liquidity buffers and whether buffers should be at the fiscal authority or the Riksbank’s disposal (Riksgalden, 2013). Such debates are beyond the scope of this paper. Liquidity buffers are also sometimes under the direct control of fiscal authorities. For example, Uruguay’s Debt Management Unit reports that as of September 2015, the central government’s liquid assets represent 133 percent of debt services for the next 12 months, while contingent credit lines represent an additional 122 percent.
The government has access to a one-period risk-free reserve asset that pays one unit of the consumption good in the next period and is traded at a constant price \( q_a \).\(^{10}\) Let \( a_t \geq 0 \) denote the government’s reserve holdings at the beginning of period \( t \). The government faces the following budget constraint during each period that it has access to debt markets:

\[
c_t = y_t - \delta b_t + a_t + i_t q_t - q_a a_{t+1} - g,
\]

where \( g \) denotes a time-invariant expenditure in a public good, which captures rigidities in the government budget constraint.\(^{11}\)

**Default.** When the government defaults, it does so on all current and future debt obligations. This is consistent with the observed behavior of defaulting governments, and it is a standard assumption in the literature.\(^{12}\) As in most previous studies, we also assume that the recovery rate for debt in default (i.e., the fraction of the loan that lenders recover after a default) is zero.\(^{13}\) In the default period, we assume that the government cannot borrow and that it suffers a one-time utility loss \( U^D(y) \), which is increasing in income.\(^{14}\) We think of this utility loss as a form of capturing various default costs related to reputation, sanctions, or misallocation of resources. As pointed out by Bulow and Rogoff (1989), punishments aside from exclusion from borrowing are needed to sustain debt in equilibrium.\(^{15}\)\(^{16}\)

Upon default, the government, retains control of its reserves and access to savings. Hence, the budget constraint becomes

\[
c_t = y_t + a_t - q_a a_{t+1} - g.
\]

**Foreign lenders and Risk premium shocks.** Foreign lenders value a stochastic future stream of payments \( x_{t+n} = \delta (1 - \delta)^{n-1} \) using a one-period-ahead stochastic discount factor \( m_{t,t+1} \),

\(^{10}\)Because reserves are a perfectly liquid risk-free asset that pays a constant interest rate each period, the assumed duration of the reserves is irrelevant. We assume a duration of one period without loss of generality. The key assumption for the mechanisms in the paper is that reserves can be adjusted freely every period, which is consistent with the fact that a large share of reserves are in US Treasury Bills.

\(^{11}\)Rigidities in the government budget constraint play an important role in standard debt sustainability analysis. The IMF, for example, assumes that the government cannot adjust spending for two years in response to macro-fiscal stress arising from shocks to GDP or contingent liabilities (IMF, 2013). With a similar motivation, Bocola and Dovis (2015) introduce a minimum government expenditure.

\(^{12}\)Sovereign debt contracts often contain an acceleration clause and a cross-default clause. The first clause allows creditors to call the debt they hold in case the government defaults on a debt payment. The cross-default clause states that a default in any government obligation constitutes a default in the contract containing that clause. These clauses imply that after a default event, future debt obligations become current.

\(^{13}\)Bai and Zhang (2012), Benjamin and Wright (2008), D’Erasmo (2011), and Yue (2010) present models with endogenous recovery rates.

\(^{14}\)In the calibration, a period in the model is a year and thus the exclusion from debt markets lasts for a year, which is consistent with the literature and empirical estimates (Gelos, Sahay, and Sandleris, 2011).

\(^{15}\)Even permanent exclusion from both borrowing and saving would sustain very little debt in equilibrium (Aguiar and Gopinath, 2006).

\(^{16}\)Representing default costs with the utility loss \( U^D \), as opposed to a pure output loss, is more flexible for our quantitative analysis because it enables us to later recalibrate the model with one-period debt so that it mimics the same targets that we mimic with our benchmark. This exercise is important for gauging the quantitative role that debt duration plays in the optimal reserve accumulation policy.
to be described below. The value of foreign lenders is given by

\[ E_t \sum_{n=1}^{\infty} \tilde{m}_t^n x_{t+n}, \]  

where \( \tilde{m}_t^n = \prod_{j=1}^{n} m_{t+j-1,t+j}(1-d_{t+j}) \) and \( d_{t+j} \) denotes the default decision in period \( t+j \) (\( d_{t+j} = 1 \) means the government defaults).

To capture dislocations to international credit markets that are exogenous to local conditions, we assume a global shock that increases lenders’ risk aversion. Several studies find that investors’ risk aversion is an important driver of global liquidity (Rey, 2013) and that a significant fraction of the sovereign spread volatility in the data can be accounted for by the volatility of the risk premium (Borri and Verdelhan, 2015; Broner, Lorenzoni, and Schmukler, 2013; Longstaff, Pan, Pedersen, and Singleton, 2011; González-Rozada and Levy Yeyati, 2008). A vast empirical literature shows that extreme capital flow episodes are typically driven by global factors (Calvo, Leiderman, and Reinhart, 1993; Uribe and Yue, 2006; Forbes and Warnock, 2012). Aguiar, Chatterjee, Cole, and Stangebye (2016) show that sovereign defaults are not tightly connected to poor fundamentals and that risk premia are an important component of sovereign spreads.

To introduce risk premium shocks, we assume that foreign investors price bonds’ payoffs using the following stochastic discount factor:

\[ m_{t,t+1} = e^{-r - \nu(\kappa \varepsilon_{t+1} + 0.5 \kappa^2 \sigma^2)}, \quad \text{with} \quad \kappa \geq 0, \]  

This formulation introduces a positive risk premium because bond payoffs are more valuable to lenders in states in which the government defaults (i.e., in states where income shocks \( \varepsilon \) are low). Here, \( r \) is the discount rate, \( \kappa \) is the parameter governing the magnitude of risk premia, and \( \nu \) is the “risk premium shock”, which we assume follows a regime-switching Markov process with values \( \nu = 0 \) and \( \nu = 1 \) and transition probabilities \( \pi_{LH}, \pi_{HL} \). When \( \nu = 1 \), lenders become more risk averse and require a higher expected return to buy government bonds. When \( \nu = 0 \), investors are risk neutral.

The risk premium is increasing in \( \kappa \). Notice that when \( \kappa = 0 \), the pricing kernel corresponds to investors with risk-neutral preferences. A higher value of \( \kappa \) can be seen as capturing how correlated the small open economy is with respect to the investors’ income process, or alternatively, the degree of diversification of foreign investors.\(^{17}\)

This specification of the lenders’ stochastic discount factor (SDF) is a special case of the discrete-time version of the Vasicek one-factor model of the term structure (Vasicek, 1977; Backus, Foresi, and Telmer, 1998), and it has been used in models of sovereign default (e.g., Arellano and Ramanarayanan, 2012). For our purpose, this specification is conveniently tractable and delivers

\(^{17}\)Aguiar, Chatterjee, Cole, and Stangebye (2016) explicitly model the investors’ portfolio problem featuring random finite wealth and limited investment opportunities. In their model, shocks to wealth shift the menu of borrowing opportunities.
a risk premium of bonds relative to reserves. We note that this risk premium will be endogenous to the gross positions chosen by the government, and while we call $\nu = 1$ a “risk premium shock,” the risk premium is endogenous in our environment and depends critically on the probability of default. Notice that without default in our model, the risk premium would disappear, and the government portfolio would become undetermined. While not crucial for the core mechanism of the model, this shock will play an important role in our simulations (we study the importance of this shock in Section 5.5.2). In states in which investors demand a higher premium for government bonds, reserves become more valuable to avoid rolling over debt at high rates.

**Timing.** The timing protocol within each period is as follows. First, income and risk premium shocks are realized. After observing these shocks, the government chooses whether to default and makes its portfolio decision (as explained before, a government in default cannot borrow and reserve accumulation is the only decision).

**Discussion on Asset/Debt Structure** The fact that we take $\delta$ as a primitive of the model deserves some comments. Ideally, one would like the government to manage simultaneously the gross asset and debt positions, in addition to the maturity structure. This would require, however, introducing a third endogenous state variable (e.g. by adding a short-term bond in addition to the long-duration bond.) While this is beyond the scope of this paper, for computational reasons, it is still useful to discuss the possible effects that this might have. For example, one would expect that having reserves available would lead the government to choose relatively shorter maturities, since it is better insulated against rollover risk. Notice also that if the government were to choose a very long maturity, this would mitigate the need of reserves to insure against rollover risk. However, longer debt maturity worsens government incentives by making debt dilution more severe.\footnote{See Arellano and Ramanarayanan (2012), Hatchondo, Martinez, and Sosa Padilla (2015), and Aguiar, Amador, Hopenhayn, and Werning (2016).} To the extent that the government would remain exposed to rollover risk, the key forces in our model would also be present in a framework with both active maturity and asset management.

### 3.2 Recursive Government Problem

We now describe the recursive formulation of the government’s optimization problem. The government cannot commit to future (default, borrowing, and saving) decisions. Thus, one may interpret this environment as a game in which the government making decisions in period $t$ is a player who takes as given the (default, borrowing, and saving) strategies of other players (governments) who will decide after $t$. We focus on Markov perfect equilibrium. That is, we assume that in each period, the government’s equilibrium default, borrowing, and saving strategies depend only on payoff-relevant state variables.

Let $s = \{y, \nu\}$ denote the current exogenous state of the world and $V(a, b, s)$ denote the optimal value for the government. For any bond price function $q$, the function $V$ satisfies the following
functional equation:

\[ V(a, b, s) = \max \left\{ V^R(a, b, s), V^D(a, s) \right\}, \] 

where the government’s value of repaying is given by

\[ V^R(a, b, s) = \max_{a' \geq 0, b' \geq 0} \left\{ u(c) + \beta \mathbb{E}_{s'|s} V(b', a', s') \right\}, \] 

subject to

\[ c = y - \delta b + a + q(b', a', s)[b' - (1 - \delta)b] - q_a a' - g. \]

The value of defaulting is given by

\[ V^D(a, s) = \max_{a' \geq 0} \left\{ u(c) - U^D(y) + \beta \mathbb{E}_{s'|s} V(0, a', s') \right\}, \] 

subject to

\[ c = y + a - q_a a' - g. \]

The solution to the government’s problem yields decision rules for default \( \hat{d}(a, b, s) \), debt \( \hat{b}(a, b, s) \), reserves in default \( \hat{a}^D(a, s) \), reserves when not in default \( \hat{a}^R(a, b, s) \), consumption in default \( \hat{c}^D(a, s) \), and consumption when not in default \( \hat{c}^R(a, b, s) \). The default rule \( \hat{d} \) is equal to 1 if the government defaults and is equal to 0 otherwise. In a rational expectations equilibrium (defined below), investors use these decision rules to price debt contracts.

**Equilibrium Bond Prices.** To be consistent with investors’ portfolio conditions, the bond price schedule needs to satisfy

\[ q(a', b', s) = \mathbb{E}_{s'|s} \left[ m(s', s) \left( 1 - \hat{a}(a', b', s') \right) \left[ \delta + (1 - \delta)q(a'', b'', s') \right] \right], \] 

where

\[ b'' = \hat{b}(a', b', s') \]
\[ a'' = \hat{a}^R(a', b', s'). \]

Equation (7) indicates that, in equilibrium, an investor has to be indifferent between selling a government bond today and keeping the bond and selling it in the next period. If the investor keeps the bond and the government does not default in the next period, he first receives a coupon payment of \( \delta \) units and then sells the remaining value of the bond at the market price, which is equal to \( (1 - \delta) \) times the price of a bond issued in the next period.

The investors’ portfolio condition for the risk-free assets yields \( e^{-r} = q_a \), using (6) and log-normality of the lenders’ SDF.
3.3 Recursive Equilibrium

Definition 1 (Equilibrium). A Markov perfect equilibrium is defined by

1. a set of value functions $V$, $V^R$, and $V^D$,
2. rules for default $\hat{d}$, borrowing $\hat{b}$, reserves $\{\hat{a}^R, \hat{a}^D\}$, and consumption $\{\hat{c}^R, \hat{c}^D\}$,
3. and a bond price function $q$,

such that

i. given a bond price function $q$; the policy functions $\hat{d}$, $\hat{b}$, $\hat{a}^R$, $\hat{a}^D$, $\hat{c}^R$, $\hat{c}^D$, and the value functions $V$, $V^R$, $V^D$ solve the Bellman equations $(V)$, $(VR)$, and $(VD)$.

ii. given government policies, the bond price function $q$ satisfies condition (7).

4 Quantitative Analysis

4.1 Computation

As in Hatchondo, Martinez, and Sapriza (2010), we solve for the equilibrium by computing the limit of the finite-horizon version of our economy. That is, the approximated value and bond price functions correspond to the ones in the first period of a finite-horizon economy with a number of periods large enough that the maximum deviation between the value and bond price functions in the first and second period is no larger than $10^{-6}$. The recursive problem is solved using value function iteration. We solve the optimal portfolio allocation in each state by searching over a grid of debt and reserve levels and then using the best portfolio on that grid as an initial guess in a nonlinear optimization routine. The value functions $V^D$ and $V^R$ and the function that indicates the equilibrium bond price function conditional on repayment $q(\hat{b}(\cdot), \hat{a}^R(\cdot), \cdot, \cdot)$ are approximated using linear interpolation over $y$ and cubic spline interpolation over debt and reserves positions. We use 40 grid points for reserves, 40 grid points for debt, and 30 grid points for income realizations. Expectations are calculated using 50 quadrature points for the income shocks.

4.2 Calibration

Our approach for the calibration is to follow the same discipline that is used to calibrate standard quantitative models of sovereign default that lack reserve accumulation. The calibration has two elements. First, we need to set a series of parameters that can be directly pinned down from the data. Second, a set of moments in the data are crucial to quantifying the rollover risk to which governments are exposed, and we set parameters so that the model is able to match those moments. We proceed by specifying the functional forms, and then we address these two elements in the calibration.
**Functional forms.** The utility function displays a constant coefficient of relative risk aversion, that is,
\[ u(c) = \frac{c^{1-\gamma} - 1}{1 - \gamma}, \text{ with } \gamma \neq 1. \]

The utility cost of defaulting is given by \( U^D(y) = \alpha_0 + \alpha_1 \log(y) \). As in Chatterjee and Eyigungor (2012), having two parameters in the cost of defaulting gives us the flexibility to match the behavior of the spread in the data.

**Parameter Values.** Table 1 presents the benchmark values given to all parameters in the model. A period in the model refers to a year. The values of the risk-free interest rate and the domestic discount factor are standard in quantitative business cycle and sovereign default studies, and we set \( r = 0.04 \) and \( \beta = 0.92 \).

We use Mexico as a reference for choosing the parameters that govern the endowment process, the level and duration of debt, and the mean spread. Mexico is a common reference for studies on emerging economies because its business cycle displays the same properties that are observed in other emerging economies (Aguiar and Gopinath, 2007; Neumeyer and Perri, 2005; and Uribe and Yue, 2006), and it is often used in the quantitative sovereign default literature (Mendoza and Yue, 2009, Aguiar, Chatterjee, Cole, and Stangebye, 2016). For other emerging economies, the value of the calibration targets for debt and spreads would be similar to those for Mexico, as Figure 1 shows. In addition, the other set of untargeted moments of interest related to reserve accumulation is also similar to other countries. Unless specified otherwise, the data period that we use is from Mexico from 1993 to 2014.

The parameter values that govern the endowment process are chosen so as to mimic the behavior of GDP in Mexico during the sample period, which is logged and linearly detrended. The estimation of the AR(1) for the cyclical component yields \( \rho = 0.66 \) and \( \sigma_\varepsilon = 0.034 \). The level of public goods \( g \) is set to 12 percent to match the average level of public consumption to GDP in Mexico. We set \( \delta = 0.2845 \). With this value and the targeted level of sovereign spread, sovereign debt has an average duration of 3 years in the simulations, which is roughly the average duration of public debt in Mexico.\(^{19}\)

We use the global EMBI+ index to parameterize the shock process to lenders’ risk aversion (i.e., the risk premium shock). We assume that a period with high lenders’ risk aversion is one in which the global EMBI+ without countries in default is one standard deviation above the median over the sample period (we use quarterly data from 1993 to 2014). With this procedure, we obtain three episodes of a high risk premium every 20 years with an average duration of each episode equal to 1.25 years, which implies \( \pi_{LH} = 0.15 \) and \( \pi_{HL} = 0.8 \). The high risk-premium episodes are observed in 1994-1995 (Tequila crisis), 1998 (Russian default), and 2008 (global financial crisis).

\(^{19}\)We use data from the central bank of Mexico for debt duration and the Macaulay definition of duration that, with the coupon structure in this paper, is given by \( D = \frac{1+i}{\delta+i} \), where \( i \) denotes the constant per-period yield delivered by the bond.
On average, the global EMBI index was 2 percent higher in those episodes than in normal periods.

**Targeted Moments.** We need to calibrate the value of four other parameters: the two parameters of the utility cost of defaulting \( \alpha_0 \) and \( \alpha_1 \), the parameter \( \kappa \) determining the increase in lenders’ risk aversion in periods of high risk premium, and the domestic risk aversion \( \gamma \). As we mentioned above, the goal is to match a set of moments that guides the model in terms of the severity of the rollover risk that the government faces and the amount of smoothing that the government performs.

We use these four parameters \( \{\alpha_0, \alpha_1, \kappa, \gamma\} \) to match four targets in the data: (i) a mean level of public sovereign debt of 43.5 percent of income, (ii) a mean level of spreads of 240 basis points, (iii) an increase in the spread during high risk-premium periods of 2 percent, which is the average increase in spreads observed in Mexico during the three high risk-premium periods we identify in the data, and (iv) a volatility of consumption relative to output equal to one.

In order to compute the sovereign spread that is implicit in a bond price, we first compute the yield \( i_b \), defined as the return an investor would earn if he holds the bond to maturity (forever) and no default is declared. This yield satisfies

\[
q_t = \sum_{j=1}^{\infty} \delta (1 - \delta) j^{-1} e^{-j r_b}.
\]

The sovereign spread is then computed as the difference between the yield \( i \) and the risk-free rate \( r \), i.e., \( r_s^t \equiv i_b - r \).

Debt levels in the simulations are calculated as the present value of future payment obligations discounted at the risk-free rate, that is, \( \frac{\delta}{1-(1-\delta)e^{-r}} b_t \).

The values for the default cost \( d_0, d_1 \), listed in the bottom panel of Table 1, mainly determine the average debt and average spreads, while \( \kappa \) determines the average increase in spreads. We choose to make the domestic risk aversion part of the calibration because it is a key parameter determining the government’s willingness to tolerate rollover risk. The choice of the value for the risk aversion parameter is determined mainly by the consumption-volatility target. We choose to target a volatility of consumption equal to the volatility of income, in line with the findings of Alvarez, Marques, and Toledo (2013).\(^{20}\) The value of the risk aversion parameter that results from the calibration (\( \gamma = 3.3 \)) is within the range of values used for macro models of precautionary savings.

\(^{20}\) Alvarez, Marques, and Toledo (2013) showed that in emerging economies (including Mexico), the volatility of total consumption is higher than the volatility of aggregate income, but the volatility of the consumption of nondurable goods is lower than the volatility of income. Since our model does not differentiate between total and nondurable consumption, we choose to target a relative volatility of 1.
Table 1: Parameter Values.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r$</td>
<td>Risk free rate</td>
<td>0.04</td>
</tr>
<tr>
<td>$\beta$</td>
<td>Domestic discount factor</td>
<td>0.92</td>
</tr>
<tr>
<td>$\pi_{LH}$</td>
<td>Probability of transiting to high risk-premium</td>
<td>0.15</td>
</tr>
<tr>
<td>$\pi_{HL}$</td>
<td>Probability of transiting to low risk-premium</td>
<td>0.8</td>
</tr>
<tr>
<td>$\sigma_\varepsilon$</td>
<td>std. dev of innovation to $y$</td>
<td>0.034</td>
</tr>
<tr>
<td>$\rho$</td>
<td>Autocorrelation of $y$</td>
<td>0.66</td>
</tr>
<tr>
<td>$g$</td>
<td>Government consumption</td>
<td>0.12</td>
</tr>
<tr>
<td>$\delta$</td>
<td>Coupon decaying rate</td>
<td>0.2845</td>
</tr>
<tr>
<td>$\alpha_0$</td>
<td>Default cost parameter</td>
<td>2.45</td>
</tr>
<tr>
<td>$\alpha_1$</td>
<td>Default cost parameter</td>
<td>19</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>pricing kernel parameter</td>
<td>23</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Coeff. of relative risk aversion</td>
<td>3.3</td>
</tr>
</tbody>
</table>

5 Results on Reserve Accumulation

This section shows that the simulations of our calibrated model are consistent with the facts presented in Section 2, in particular the model generates substantial reserve accumulation, and inspects the mechanisms that generate this result.

5.1 Key Statistics: Model and Data

Table 2 reports long-run moments in the data and in the model simulations. The first panel of this table shows that the simulations match the calibration targets. The second panel shows that the model also does a good job in mimicking nontargeted moments. In particular, the simulations generate a volatile and countercyclical spread, and a high correlation between consumption and income.\textsuperscript{21} This is line with previous studies that have shown that the sovereign default model without reserve accumulation can account for these features of the data (Arellano, 2008; Aguiar and Gopinath, 2006). We show that this is still the case when we extend the baseline model to allow for the empirically relevant case in which indebted governments can hold reserves and choose to do so.

\textsuperscript{21}The spread volatility in the model is higher than in Mexico but close to the median for emerging economies (Figure 1). The spread volatility in Mexico is also higher when computed using the stripped EMBI (throughout the paper we use the blended EMBI instead).
Table 2: Basic Statistics: Model and Data

<table>
<thead>
<tr>
<th></th>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Targeted</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\sigma(y)$</td>
<td>3.8</td>
<td>3.8</td>
</tr>
<tr>
<td>mean duration (years)</td>
<td>3.0</td>
<td>3.0</td>
</tr>
<tr>
<td>$\sigma(c)/\sigma(y)$</td>
<td>1.0</td>
<td>1.0</td>
</tr>
<tr>
<td>mean debt (b/y)</td>
<td>43.0</td>
<td>43.5</td>
</tr>
<tr>
<td>mean $r_s$</td>
<td>2.4</td>
<td>2.4</td>
</tr>
<tr>
<td>$\Delta r_s$ with risk-prem. shock</td>
<td>2.0</td>
<td>2.0</td>
</tr>
<tr>
<td><strong>Nontargeted</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\sigma(r_s)$</td>
<td>0.9</td>
<td>2.0</td>
</tr>
<tr>
<td>$\rho(r_s, y)$</td>
<td>-0.5</td>
<td>-0.7</td>
</tr>
<tr>
<td>$\rho(y, c)$</td>
<td>0.8</td>
<td>0.9</td>
</tr>
<tr>
<td>mean reserves (a/y)</td>
<td>8.5</td>
<td>6.0</td>
</tr>
</tbody>
</table>

Note: Moments are computed by generating 1,000 simulation samples of 300 periods each and taking the last 35 observations of samples in which the last default was observed at least 25 periods before the beginning of the sample. The standard deviation of $x$ is denoted by $\sigma(x)$. The coefficient of correlation between $x$ and $z$ is denoted by $\rho(x, z)$.

Reserves. Model simulations generate an average reserve-to-income ratio of 6 percent, which is close to the average ratio observed in Mexico in the post-Tequila period (1996-2014), 8.5 percent, and can account for about a third of the average reserve ratio in emerging economies between 2000-2014 (in Section 6, we show how recent developments in emerging markets can account for significant increases in reserves). Figure 4 shows that the simulations feature periods with reserve levels much higher than the average, of up to 40 percent of annual income. In terms of the cyclical properties, Figure 5 shows that the model can account for the fact that reserve (and debt) accumulation is higher in periods with high income and low spread.
Figure 4: Distribution of Average Reserve Ratios in the Simulations.

(a) Reserves-Spreads Comovement

(b) Reserves-Income Comovement

(c) Debt-Spreads Comovement

(d) Debt-Income Comovement

Figure 5: Comovement of Debt and Reserves with Output and Dpreads: Model vs. Data

Note: See Figure 3 for details.
5.2 Rollover Risk

We now analyze the two sources of rollover risk in the model, which are crucial to understand the optimal portfolio of the government. Figure 6 presents the spread the government is asked to pay as a function of its debt level for different income shocks (Panel a) and risk premium shocks (Panel b) when the government chooses a value of reserves equal to the mean. That is, we plot \( r^*(b', \bar{a}, s) \) as a function of \( b' \) for different values of \( s \).

Panel (a) shows that for the same level of debt, investors demand higher spreads when income is low. This occurs because it is more attractive to default when income is low and income shocks are serially correlated. As emphasized in Arellano (2008) and Aguiar and Gopinath (2006), this allows the model to generate countercyclical spreads, as observed in the data.

Panel (b) shows that the government also faces higher spreads when lenders are more risk averse (and thus demand a larger compensation for default risk). Note that the effect of the exogenous risk premium shock on the endogenous spread is an increasing function of the debt level. This illustrates how even though the risk-premium shock process is exogenous, the incidence of this shock on the domestic economy is a function of the government’s portfolio choices (in particular, if the government were to commit to a portfolio that eliminates default risk, there would be no premium on government bonds).

![Figure 6: Spread Schedule](image)

Note: Spread asked by lenders as a function of the debt level when the government chooses the average level of reserves in the simulations. Debt is presented as a fraction of mean income \( (b_{t+1}/\bar{y}) \). In Panel (a), the lenders’ risk aversion takes the low value. In Panel (b), income takes the mean value.

5.3 Portfolio Policies

Figure 7 illustrates the government’s optimal debt and reserves policies as functions of the lenders’ risk aversion and the level of income (the two sources of rollover risk in the model). Recall that since average output is normalized to one and the time period is annual, all variables can be read as expressed in terms of average annual income.
Panel (a) shows the increase in reserves given initial values of debt and reserves equal to the mean, for the two values of the lenders’ risk aversion, and for income levels between −10 and 10 percent. (Levels of income below −10 percent would lead the government to default.) When the risk aversion is low (blue straight line), the government increases reserve holdings for high income values, and decreases reserve holdings for low income values. When the risk aversion is high (red broken line), the government depletes the stock reserves for the initial values considered. This may look like a drastic response to the risk premium shock but is consistent with governments sharply reducing reserve holdings during times of stress, in line with the empirical findings presented in Figure 3.

Panel (b) shows that when the lenders’ risk aversion is low, the government accumulates debt when income is high and reduces debt when income is low. (Given the initial states considered, the government reduces debt when income is below the mean value and vice versa).\textsuperscript{22}

Overall, Figure 7 shows that the government increases both reserves and debt when the lenders’ risk aversion is low and income is sufficiently high. Therefore, in the simulations, the government accumulate more reserves and more debt when income is higher and when the spread is lower (recall the spread is lower when the lenders’ risk aversion is lower and when income is higher). This enables the model to successfully account for the cyclical properties of the accumulation of reserves and debt in a majority of emerging economies, as illustrated in Figure 5. Moreover, this is also consistent with the complement of capital inflows (sovereign debt) and capital outflows (reserves) in the data (Broner, Didier, Erce, and Schmukler, 2013).

Figure 7: Optimal accumulation of reserves and debt.
Note: The initial levels of debt and reserves are equal to the mean levels in the simulations. The figure displays income levels for which the government chooses to pay its debt.

\textsuperscript{22}Notice that an increase in income may have non-monotonic effects on debt accumulation. If income is very low, an increase in income leads the government to reduce debt decumulation because consumption-smoothing motives lead the government to tolerate high borrowing costs. At some point, the borrowing cost effect dominates and government debt increases with the level of income.
5.4 Inspecting the Mechanism

We now analyze the key forces that shape the optimal portfolio and the fundamental trade-off faced by the government. We will show that keeping higher levels of reserves provides a hedge against adverse shocks that increase the cost of borrowing. Using reserves to pay down debt, however, allows the government to reduce spreads because it weakens its incentives to default in the future.

**Optimality conditions.** To gain insights into the government’s portfolio problem, it will be useful to analyze the optimality conditions. For expositional purposes, we present the optimality conditions for a period in which the government finds it optimal to repay \( \hat{d}(a, b, s) = 0 \) and to accumulate reserves \( \hat{a}R(a, b, s) > 0 \). We also assume that the price function \( q \) and the value functions are differentiable (our numerical solution method does not rely on this). Applying the envelope theorem on (VR) and (VD), we obtain the following Euler equations for debt and reserves:

\[
\begin{align*}
\text{Increase of this-period consumption} & \quad \Rightarrow \quad b' :: u'(c) \left[ q + \frac{\partial q(b', a', s)}{\partial b'} i \right] = \beta \mathbb{E}_{s'|s} \left[ u'(c') \left[ \delta + q'(1 - \delta)(1 - d') \right] \right] \\
\text{Decline of next-period consumption} & \quad \Rightarrow \quad (8)
\end{align*}
\]

\[
\begin{align*}
\text{Decline of this-period consumption} & \quad \Rightarrow \quad a' :: u'(c) \left[ q_a - \frac{\partial q(b', a', s)}{\partial a'} i \right] = \beta \mathbb{E}_{s'|s} u'(c') \\
\text{Increase of next-period consumption} & \quad \Rightarrow \quad (9)
\end{align*}
\]

Equation (8) equates the benefits from issuing one more unit of bonds in the current period to the expected cost of repaying it in the next period. The government issues bonds in exchange for \( q \) units of consumption, but also lowers the price of new issuances, reducing revenue by \( \frac{\partial q(b', a', s)}{\partial b'} i \). The marginal cost from borrowing is given by the costs from paying the coupon that matures in the next period and retiring the remaining \( 1 - \delta \) units of debt at the market price \( q' = q \left( \hat{a}R(a', b', s'), \hat{b}(a', b', s'), s' \right) \).

Equation (9) equates the costs from cutting current consumption to buy one extra reserve asset to the benefits of consuming the proceeds from selling that extra reserve asset in the next period. The marginal cost of buying an extra reserve asset differs from \( q_a \) because current reserve purchases affect the current price at which the government issues debt.

To shed further light on the trade-offs the government faces when it decides its current gross portfolio positions, we compare next across portfolios that generate a current consumption level
equal to the equilibrium consumption. Formally, we compare across portfolios \((a', b')\) that satisfy

\[ y - \delta b + a + q(a', b', s)(b' - (1 - \delta)b) - qa'a - g = \hat{c}R(a, b, s) \tag{10} \]

Let \(\hat{a}(a, b, s, b')\) denote the reserve choice that is consistent with debt choice \(b'\) and equation \((10)\). Applying the implicit function theorem to equation \((10)\) implies that issuing an extra bond in the current period enables the government to purchase additional reserves without deviating current consumption from its equilibrium level. (We omit the arguments of \(\hat{a}\) in equation \((11)\) to simplify notation.) Because any portfolio in the set \((\hat{a}, b')\) delivers the same current utility level, different portfolio choices affect welfare through different continuation values. Thus, the change in welfare implied by the issuance of an additional bond to finance the accumulation of reserves while keeping current consumption constant is given by

\[
\frac{d\mathbb{E}_{s'|s} V(\hat{a}, b', s')}{db'} = \underbrace{\frac{\partial \hat{a}}{\partial b'} \mathbb{E}_{s'|s}[u'(c')] - \mathbb{E}_{s'|s}[u'(c')\delta + (1 - \delta)q](1 - d')}_{\text{Mg. benefit of buying reserves}} \underbrace{\mathbb{E}_{s'|s}[u'(c')](1 - d')}_{\text{Mg. cost of issuing debt}} \tag{12}
\]

Equation \((12)\) is a key expression in our model. Notice that using \((8)-(9)\), we obtain \(d\mathbb{E}_{s'|s} V(\hat{a}, b', s') = 0\) (i.e., at the optimum a marginal increase in debt to buy reserves does not affect welfare).

The first term in \((12)\) indicates the marginal benefit of starting next period with \(\frac{\partial \hat{a}}{\partial b'}\) additional reserves. The second term indicates the marginal cost of starting next period with an additional unit of debt. We inspect this condition below and analyze the costs and benefits of higher gross positions.

**Insurance benefits.** We now show that issuing debt to finance the accumulation of reserves allows the government to transfer resources to future states with low consumption, thereby providing insurance. For that, it is convenient to rearrange equation \((12)\) as

\[
\frac{d\mathbb{E}_{s'|s} V(\hat{a}, b', s')}{db'} = \underbrace{\frac{\partial \hat{a}}{\partial b'} \mathbb{E}_{s'|s}[u'(c')d'] + \mathbb{E}_{s'|s}[u'(c')\delta - (1 - \delta)q'](1 - d')}_{\text{Mg. benefit of buying reserves}} \underbrace{(1 - d')}_{\text{Mg. cost of issuing debt}} \tag{13}
\]

\(^{23}\)This approach is related to the zero-cost trades introduced in Aguiar and Amador (2013b). In their analysis of maturity management, they perturb short-term debt and long-term debt around the optimal portfolio while keeping continuation values constant. They establish that issuing or repurchasing long-term debt shrinks the budget set and this is always undesirable because of incentive reasons.

\(^{24}\)To derive this, we use the chain rule together with envelope conditions \(V_a(a, b, s) = u'(c)\) and \(V_b(a, b, s) = -u'(c)(\delta + q(b', a', s)(1 - \delta))\) for states in which the government repays and \(V_a(a, b, s) = 0\) for states in which the government defaults. It should be clear that all future variables are evaluated at their equilibrium levels in equation \((12)\). That is, \(d' = d(\hat{a}R(a, b, s), b(a, b, s), s')\), \(c' = d'\hat{c}D(\hat{a}R(a, b, s), b(a, b, s), s') + (1 - d')\hat{c}R(\hat{a}R(a, b, s), b(a, b, s), s')\), and \(q' = q(\hat{a}R(a, b, s), b(a, b, s), s'), b(\hat{a}R(a, b, s), b(a, b, s), s'), s')\).
Equation (13) shows that issuing debt to finance the accumulation of reserves allows the government to transfer \( \frac{\partial \tilde{a}}{\partial b} \) resources to default states (first term) and \( \frac{\partial \tilde{a}}{\partial b} - \delta - (1 - \delta)q' \) resources to repayment states (second term). On the second term, notice that the resources transferred to repayment states are decreasing on \( q' \), as long as \( \delta < 1 \). This implies that issuing long-term debt to accumulate reserves allows the government to transfer more resources to next-period states with a higher borrowing cost. Thus, issuing debt to accumulate reserves is an instrument to hedge against rollover risk.

\[ \frac{\partial \tilde{a}}{\partial b} \bigg|_{b'=\hat{b}} - \delta - (1 - \delta)q' \]

Figure 8: Insurance Benefits of Reserves

Note: The figure assumes that in the current period the level of income and the initial levels of debt and reserves are equal to the mean level in the simulations, the lender’s risk aversion is low (\( \nu = 0 \)), and the government chooses the optimal portfolio. Panel (a) presents the resources transferred to the next period by issuing an additional bond to buy reserves, which are given by \( \frac{\partial \tilde{a}}{\partial b} \bigg|_{b'=\hat{b}} \) for default states and by \( \frac{\partial \tilde{a}}{\partial b} - \delta - (1 - \delta)q' \) for repayment states. Panel (b) presents the next-period consumption level. The panels present a discontinuity at the level of next-period income that triggers a default (this income threshold is higher when the lenders’ risk aversion is higher).

Figure 8 shows that indeed issuing debt to accumulate reserves allows the government to transfer resources to low-consumption states. Panels (a) and (b) show respectively how payoffs and consumption vary with next period income for different values of lenders’ risk aversion. Notice that for the lower range of the value of \( y' \) such that the government repays, the government experiences a positive return from issuing debt to accumulate reserves (i.e., \( \delta + (1 - \delta)q' \geq 0 \)). The range of shocks such that this happens is approximately \( 0.95 \geq y' \geq 0.89 \) for low lenders’ risk aversion and \( 0.98 \geq y' \geq 0.90 \) for high lenders’ risk aversion. In contrast, for higher values of income, the government experiences a negative return since bond price increases reflecting lower incentives of future government default. Since consumption is increasing in next-period income in repayment states, issuing bonds to buy reserves yield higher payoffs in states with high marginal utility.
Higher borrowing cost. We next establish that issuing debt to accumulate reserves is costly. To allow for a sharper characterization, we shut down the role played by government’s risk aversion. Formally, we evaluate now equation (13) but assuming a constant marginal utility normalized to one. In this case, equation (13) simplifies to

$$\frac{\partial E_{s'}}{\partial b'} = \frac{\partial \tilde{a}}{\partial b'} - \frac{q}{q_{a}}.$$  \hfill (14)

where the second line follows from the non-negative risk premium required by foreign lenders is non-negative. Let us define $\tilde{q}(b', a, b, s) = q(\tilde{a}'(a, b, s, b'), \tilde{b}', s)$. We can combine (11), (14), with the derivative of $\tilde{q}$ with respect to $b'$ to obtain

$$\frac{\partial E_{s'|s} V (\tilde{a}, b', s')}{\partial b'} = \left( \frac{i}{q_{a}} \right) \frac{\partial \tilde{q}}{\partial b'}.$$  \hfill (15)

Equation (15) shows that the effect on welfare of increasing bond positions has the same sign as $\frac{\partial \tilde{q}}{\partial b'}$.\hfill 25 If the bond price decreases when the government issues debt to buy reserves ($\frac{\partial \tilde{q}}{\partial b'} < 0$), a risk-neutral government issuing debt prefers strictly lower reserves. Notice that for $\frac{\partial \tilde{q}}{\partial b'}$ to be negative, it has to be that $\frac{\partial q}{\partial b'} < -\frac{\partial \tilde{q}}{\partial \tilde{a}} \frac{\partial \tilde{a}}{\partial b'}$ (i.e., the reduction in the bond price due to the accumulation of debt is larger that the increase in the bond price implied by the increase in reserves financed with the accumulation of debt).

Equation (15) shows that the effect of changing gross financial positions on the borrowing cost is therefore a key component of the government’s decision to issue debt in order to finance the accumulation of reserves. Although a higher spread reflects that the government repays in less states, this benefit is offset by the extra default cost the government incurs.\hfill 26 As a result, a portfolio with higher gross debt and asset positions delivering higher spreads means that the government on average is paying higher costs from defaulting. While this provides some justification for the approach in the empirical literature that measures the costs of reserve accumulation using the spread (e.g. Rodrik, 2006), our analysis uncovers that what matters in the sensitivity of the spread to the change in gross positions.

Figure 9 shows how bond spreads vary with government’s gross positions in the model and that indeed $\frac{\partial \tilde{q}}{\partial b'} < 0$. Panel (a) presents the combination of debt ($b'$) and reserves ($a' = \tilde{a}(a, b, s, b')$) that deliver a current consumption level equal to the equilibrium. This plot corresponds to an initial

\hfill 25To see this, notice that rearranging terms $\frac{\partial \tilde{a}}{\partial b'} - \frac{q}{q_{a}} = \frac{i(\frac{\partial \tilde{a}}{\partial b'} - \frac{\partial q}{\partial b'})}{q_{a}}$ and $\frac{\partial q}{\partial b} = \left( \frac{q_{a}}{q_{a} - \frac{\partial \tilde{a}}{\partial b'} q_{a}} \right)$

\hfill 26As is standard in endogenous default models, the fact that default costs are not accrued to creditors implies that the marginal gain from increasing the set of states in which the government will default is compensated exactly by the marginal cost of increasing the set of states in which the government will pay the default cost. Therefore, only consumption smoothing and the effect of financial positions on the bond prices appear in optimality conditions (as shown in equations (8) and (9)).
state for both bonds and reserves equal to the mean, no risk premium shock and two possible values of income. These “isoquants” have a positive slope for the levels of debt considered: the more debt issued, the more reserves the government can buy.27

Panel (b) shows that issuing debt to finance the accumulation of reserves increases the spread the government has to pay (equivalently, $\frac{\partial \tilde{q}}{\partial b} < 0$).28 When income is at the mean value, increasing reserves from 0 to 5 percent raises spreads from 1.7 percent to about 1.8 percent.29

Reserve accumulation is less costly when income is high. The left panel of Figure 9 shows that the sensitivity of the spread to increases in gross positions decreases with income. This induces the government to buy more insurance by increasing its debt and reserve positions in times with higher income (as shown in Figure 7). Periods with higher income also correspond to periods with relatively lower spread. In addition, in periods when lenders’ risk aversion is higher, the cost of keeping large gross positions becomes more significant because the government has to pay a higher compensation to lenders for future default risk. The optimal response in periods in periods with high lenders’ risk aversion is thus to reduce debt and reserves portfolios.

27Beyond some high value of debt (not plotted), the price of debt would fall so much that the amount of reserves that can be purchased falls.

28This in turn implies that the slope of the isoquants in Panel (a) have a slope lower than $q_b/q_a$

29The somewhat modest response is not specific to the states considered in Figure 9. If the government deviated from the optimal portfolio by increasing its debt stock in 1 percent of aggregate income and allocated the extra proceeds to purchase reserves, the mean increase in the spread in the simulations would be equal to 3.4 basis points.
5.5 Role of Maturity and Risk-premium Shocks

We next study the quantitative importance of assuming long-term and shocks to the lenders’ risk aversion. To do so, we study versions of the model (i) with one-period bonds and (ii) without risk-premium shocks.

5.5.1 One-period bonds

We now show that having bonds with maturity exceeding one period is essential for obtaining high levels of reserves in the simulations. This enables us to quantify the importance of reserves to insure against rollover risk, and contrast our results with those presented by Alfaro and Kanczuk (2009).

To see the importance of maturity, recall equation (13). This equation showed that issuing bonds with maturity exceeding one period ($\delta < 1$) to finance the accumulation of reserves allows the government to transfer $\partial \tilde{a} / \partial b' - \delta - (1 - \delta)q'$ resources to repayment states in the next period. Since these resources are a decreasing function of $q'$, issuing debt to accumulate reserves allows the government to transfer more resources to states in which the cost of borrowing is higher, and thus is a hedge against rollover risk. In contrast, with one-period debt ($\delta = 1$), payoffs in repayment states are $\partial \tilde{a} / \partial b' - 1$, and are thus independent from the cost of borrowing next period. Therefore, issuing debt to accumulate reserves does not help the government to hedge against rollover risk. Reserves do allow the government, however, to transfer resources from repayment to default states. As we show next, this incentive to accumulate reserves is not quantitatively important, in line with the results presented in Alfaro and Kanczuk (2009).

To evaluate the quantitative performance of the one-period-bond version of the model ($\delta = 1$), we change the value of parameters to match the same targets we matched in our baseline calibration. The recalibrated parameters are as follows: the parameters that affect the cost of defaulting are $\alpha_0 = 15.7$ and $\alpha_1 = 175.0$, the high level for lenders’ risk aversion is $\kappa = 10.5$, and the risk aversion of domestic consumers is $\gamma = 4.5$. All other parameter values are the same as those assumed in the benchmark calibration. Table 3 shows these parameter values allow the one-period-bond model to match the calibration targets well.

Table 3 shows that with one-period bonds, the average reserve ratio in the simulations falls to 0.3 percent. This is consistent with the zero reserve accumulation obtained by Alfaro and Kanczuk (2009). The fact that reserves are close to zero with one-period bonds implies that the insurance value of transferring resources from repayment states to default states is not quantitatively important. In a nutshell, the insurance value of transferring resources from repayment states to default...
Table 3: Role of Maturity and Risk-premium Shocks

<table>
<thead>
<tr>
<th></th>
<th>Data</th>
<th>Benchmark</th>
<th>One-period Bonds</th>
<th>No Risk-premium Shock</th>
</tr>
</thead>
<tbody>
<tr>
<td>std(y)</td>
<td>3.8</td>
<td>3.8</td>
<td>3.6</td>
<td>3.7</td>
</tr>
<tr>
<td>Mean duration (years)</td>
<td>3.0</td>
<td>3.0</td>
<td>1.0</td>
<td>3.0</td>
</tr>
<tr>
<td>std(cons) / std(y)</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
</tr>
<tr>
<td>Mean debt/y</td>
<td>43.0</td>
<td>43.5</td>
<td>42.6</td>
<td>44.5</td>
</tr>
<tr>
<td>Mean spread</td>
<td>2.4</td>
<td>2.4</td>
<td>2.3</td>
<td>2.4</td>
</tr>
<tr>
<td>Spread increase during high risk-premium</td>
<td>2.0</td>
<td>2.0</td>
<td>2.0</td>
<td>0</td>
</tr>
<tr>
<td>Mean (reserves/y)</td>
<td>8.5</td>
<td>6.0</td>
<td>0.3</td>
<td>3.0</td>
</tr>
</tbody>
</table>

states is outweighed by the costs of facing larger spreads.\textsuperscript{30},\textsuperscript{31}

These differences in reserve accumulation between one-period and long-duration bonds highlight the importance of debt maturity in quantitatively understanding the role of reserves. Reserves provide insurance against rollover risk only if bonds’ maturity exceed one period, and the model with debt maturity calibrated to the data predicts a significant level of reserve accumulation.

### 5.5.2 Role of Risk-premium Shocks

The goal in this section is to assess the importance of the shocks to the lenders’ risk aversion. To do this, we eliminate this shock by assuming that $\nu_t = 0$. We recalibrate the parameters that affect the cost of defaulting ($\alpha_0 = 2.16$ and $\alpha_1 = 18.10$) to match the levels of sovereign debt and spread in the data. All other parameter values are the same as those assumed in the benchmark calibration. Table 3 shows that the average holding of reserves in the simulations of the recalibrated model without shocks to the risk premium is half that of the benchmark simulations. This implies that the lack of a risk premium shock significantly reduces the need for reserves to insure against rollover risk but reserves continue to play an important role in the model.

\textsuperscript{30}Effectively, the government saves resources from not repaying debt in a default state so transferring resources from repayment states to default states has a modest value. Moreover, the spread is more sensitive to changes in debt with one period-bonds, which as shown in eq. 15 is key for the effects on welfare.

\textsuperscript{31}In an earlier version of this paper we showed that if we do not allow the government to use reserves in default states, the government accumulates even more reserves for incentive reasons. If we impose the same restriction with one-period bonds, reserves would have no role and the portfolio would be undetermined.
6 Secular increases in reserves

In this section, we show that the model is capable of rationalizing the upward trend in reserves in emerging markets, as presented in Figure 2.\textsuperscript{32} In particular, we study how four recent developments in emerging economies over the last fifteen years have contributed to this trend: (i) the exceptional increases in income driven by the surge in commodity prices, (ii) the increase in the public sector’s contingent liabilities exemplified by the increase of the size of the banking sector and the growth of the liabilities of the non-financial-corporate sector, and (iii) improved policy frameworks that might have reduced possible impatience bias in policy makers, (iv) the increase global disruptions to international financial markets, which became apparent after the East Asian financial crises.

To investigate the quantitative importance of these developments, we feed in shocks associated with each development and conduct transitional dynamics. Below, we provide an empirical background of each development, and explain how we map them into our model. Notice that we will still use Mexico for calibrating the basic parameter of the model, but we use a broader set of countries to measure these developments, since they are mostly common to emerging markets.\textsuperscript{33} Transitional dynamics in Figure 12 and summarized in Table 4 show that the four developments we study produce not only an increase in the level of reserves but also a decline in the levels of sovereign debt and spread, all consistent with the evolution of these variables documented in Section 2.

6.1 Income windfalls

The first development we explore is the prolonged increase in income experienced by most emerging markets. The boom in commodity prices has played an important role in this development. Adler and Magud (2015) document that many emerging economies enjoyed a significant income windfall as a result of temporary terms-of-trade booms. They identify terms of trade booms as periods in which the terms of trade increase at least 15 percent from start to peak, and the annual average increase in the terms of trade is at least 3 percent. They measure income windfalls as the cumulative difference between aggregate real income and the counterfactual aggregate real income that would have been observed if the terms of trade had remained constant at the pre-boom level. Figure 10 illustrates the large income windfall generated by post-2000 terms-of-trade booms. The

\textsuperscript{32}Recently, developed economies have started accumulating more reserves. For example, in December 2012, the Riksbank requested that the Swedish National Debt Office borrow an equivalent of 2.5 percent of GDP so as to double the stock of international reserves and strengthen their ability to weather financial market turmoil (see Riksgalden, 2013).

\textsuperscript{33}We do not attempt, however, to decompose the contribution of these developments for each country. What complicates this analysis is that estimating the model would require solving the model numerous times, which is already computationally and time intensive.
median income windfall represented 60 percent of GDP in the last pre-boom year.\textsuperscript{34}

To illustrate the effect of income windfalls in the government’s equilibrium choices, we extract from the simulations the samples in which the income level over the last 15 periods is, on average, between 3 and 5 percent higher than the mean. This implies an average income windfall of 60 percent of mean income. Figure 12 (second row) shows that income windfalls generate a significant increase in reserves while debt and spreads decline, which is consistent with the paths observed in the data. Interestingly, inspecting Figures 2 and 10 shows that some of the economies with higher windfalls (e.g., Lebanon, Peru, and Russia) also experienced larger increases in reserves.

![Figure 10: Terms-of-trade Windfalls (2000-2014). Source: Adler and Magud (2015).](image)

### 6.2 Contingent liabilities

The second development we explore is the increase in the public sector’s contingent liabilities, reflecting for instance implicit guarantees to banks, public and private corporations, and public-private partnerships. The motivation to incorporate contingent liabilities should be clear from an extensive empirical literature. For example, Bova, Ruiz-Arranz, Toscani, and Ture (2016) document that the fiscal costs of contingent liability realizations can be as high as 57 percent of GDP. The IMF (2014) report that the average direct fiscal cost of banking crises between 1970 and 2011 was 10 percent of GDP for emerging economies, with peaks close to 60 percent of GDP in Argentina in 1980 and Indonesia in 1997. Furthermore, the fact that contingent liabilities are standard in the IMF public debt sustainability analysis (IMF, 2013) speaks about the importance of their fiscal consequences.

\textsuperscript{34}It should also be mentioned that some economies for which Adler and Magud (2015) do not identify a terms-of-trade boom also had high income and in particular high government revenues during this period. For instance, in Mexico, roughly one-third of federal government revenues are oil related, and Levi, Mahler-Haug, and O’Neil (2014) estimate that a decline in oil prices of 60 USD per barrel would imply a revenue shortfall of up to 3 percent of GDP per year. Furthermore, although some emerging economies did not benefit directly from term-of-trade booms, they are still likely to have transited through periods of high income between 2000 and 2014. WorldBank (2016) state that GDP growth in emerging markets slowed from 7.6 percent in 2010 to 3.7 percent in 2015, with an unlikely return to the high growth rates observed before the global financial crisis of 2008.
Various factors point to a significant increase in governments’ exposure to contingent liabilities. With financial globalization, domestic corporations have increased their access to foreign capital markets, which for instance has been discussed as a key factor behind the East Asian crisis (e.g., Schneider and Tornell, 2004). Du and Schreger (2015) document that higher external foreign currency corporate financing is associated with a higher sovereign default risk. The importance of implicit guarantees to banks in emerging economies is apparent from the funding cost advantage of systemically important banks relative to other banks, which peaked at 1400 basis points in 2009 (GFSR, 2014).

Figure 11 illustrates how contingent liabilities may have increased significantly in a majority of emerging economies. The left panel shows that in most emerging economies, the size of the banking sector increased significantly (see also GFSR, 2014). The right panel shows that non-financial corporate debt also increased significantly. Note that some of the economies with higher levels of reserves have larger banking sectors and higher levels of non-financial corporate debt (e.g., Lebanon, China and Malaysia).

We model a contingent liability shock by subtracting an amount $L$ from the budget constraint of the government in states in which there is a global shock to the risk premium. This is arguably a crude way to model contingent liabilities, but it captures the problem faced by governments that suddenly need to use resources to bail out private institutions. By synchronizing the contingent
liability with global shocks to the risk premium, we economize a state variable.\footnote{Our modeling of the contingent liability shock is isomorphic to assuming a negative effect of the global risk premium on aggregate income. There is ample evidence of a negative effect of global shocks to the risk premium in the real economy. High global risk premium is often associated with credit crunches and deep recessions (Mendoza, 2010). Jeanne and Ranciere (2011) estimate the average accumulated income costs of sudden stops in capital flows above 14 percent of annual income.}

With the contingent liability shock hits, the budget constraints in problems (VR) and (VD) become

\begin{align*}
c &= y - b + a + q(b', a', y, p)(b' - (1 - \delta)b) - qa' - g - L \\
\end{align*}

when the government pays and

\begin{align*}
c &= y + a - qa' - g - L \\
\end{align*}

when the government defaults.

We consider contingent liability shocks of 15 percent of average aggregate income, which is in the lower range of the fiscal costs discussed above.\footnote{Recall however, that in order to economize a state variable, we assume that contingent liability shocks occur with the relatively high frequency of risk premium shocks.} To measure these effects in the model, we assume that the economy is initially at the ergodic mean of our baseline economy without contingent liabilities. Then, agents learn unexpectedly that there is a contingent liability shock that follows the process described above. Thus, we use the optimal policy functions and equilibrium bond prices in the model with contingent liabilities to compute the paths for endogenous variables for 15 periods.

The model simulations presented in Figure 2 (third row) indicate that the emergence of contingent liabilities leads the government to substantially increase reserves. Furthermore, it is also optimal for the government to reduce indebtedness, which is consistent with the decline in government debt in emerging economies. Notice that spreads rise on impact as a result of the increase in the risk of contingent liabilities—without contingent liabilities, the spread at the ergodic mean is 2.4 percent. Spreads later decline following the increase in reserves and the reduction in debt.

6.3 Improved policy frameworks

The third development we investigate is the improvement of policy frameworks in emerging markets, much discussed after the apparent resilience of emerging markets during and after the global financial crisis. For example, Frankel, Vegh, and Vuletin (2013) argue that improved policy frameworks helped emerging economies to create fiscal space between 2000 and 2007, allowing several of these economies to conduct countercyclical fiscal policy for the first time during the global financial crisis. Furthermore, between 2000 and 2014, 37 countries adopted fiscal rules and 28 countries established independent fiscal councils, often involving fiscal responsibility laws and automatic sanctioning and enforcement (Debrun and Kinda, 2014).

We model an improvement in policy frameworks as an increase in the time discount factor of
the government. This approach intends to capture corrections to political myopia, which are a common way to account for political economy aspects in emerging economies (Aguiar and Amador, 2011). Following the same steps as with the contingent liability shock, we assume that the economy starts at the ergodic mean of our baseline and unexpectedly the government becomes more patient.

Figure 12 (fourth row) presents the transitional dynamics for $\beta = 0.95$ (recall that in the baseline $\beta = 0.92$). A more patient government accumulates more reserves and reduces debt levels, which is reflected in lower spreads. Notice that in general the effect of an increase in a government’s patience on the level of reserves is ambiguous. A rise in $\beta$ implies that the government wants to transfer more resources from the present to the future, but this can be done by raising reserves or lowering debt. Moreover, lower debt levels reduce rollover risk and, therefore, the need for reserves. On the other hand, a higher $\beta$ also implies that the government becomes more concerned about future fluctuations in consumption and, therefore, has stronger incentives to self-insure by accumulating reserves. In our model simulations, the second effect is dominant and, in spite of a large decline in debt levels, more patient governments choose to have more reserves. $^{37}$

### 6.4 Higher Exposure to Global Shocks

The fourth and last development we investigate is the increased severity of global disruptions to financial markets. Following the East Asian crisis, there has been a widespread view that emerging markets needed to self-insure against financial market turmoil by accumulating reserves (Feldstein, 1999) and empirical analysis (e.g., Aizenman and Lee (2007), Calvo, Izquierdo, and Loo-Kung (2012)) tends to support that this explanation indeed played an important role in policy making decisions.

To incorporate this development into the model, we consider a bigger magnitude of the risk premium shock. In particular, we adjust $\kappa$ to target an average increase in spreads of 4 percent when the risk premium shock hits, as opposed to an average 2 percent increase in the benchmark. This increase in spread is close to the increase in the ‘stripped’ EMBI spread around the East Asian crisis. Again, the economy starts at the mean of the ergodic set and compute transitional dynamics.

Figure 12 (fifth row) shows that the increase in the magnitude of the risk premium shock implies significant increases in reserves. Similar to the contingent liability shock, spreads rise on impact, but then they take a downward path as the government increases the level of reserves and

$^{37}$Note that Figure 12 shows a decline in the level of debt of 22 percentage points in 14 years. Such a large decline in debt levels is not unprecedented. The average ratio of public debt to GDP in emerging economies declined 17 percentage points between 2002 and 2008 (Figure 2). This trend resumed after 2008 in part because countercyclical measures implemented during the global financial crisis (Frankel, Vegh, and Vuletin, 2013) had a persistent effect on public debt levels.
reduces the level of debt.\textsuperscript{38}

\section*{6.5 Summary of Trend Analysis}

Table summarizes the effects of these four developments. Two forces that stand out quantitatively are the increase in contingent liabilities and the increase in the severity of global shocks, each implying an increase in reserves of about 8 percentage points of income. Overall, a combination of the four mechanisms presented in this section could easily account for the average increase in reserve holdings in emerging economies over the last 15 years. Of course, the importance of each of these mechanisms is likely to vary across countries, in line with the significant variation in both the level and the growth of reserves in emerging economies (Section 2).

Table 4: Trend analysis: 2000-2014

<table>
<thead>
<tr>
<th>Year</th>
<th>Scenario</th>
<th>Mean Reserves</th>
<th>Mean Debt</th>
<th>Mean Spread</th>
<th>Increase in Reserves</th>
<th>Increase in Debt</th>
<th>Increase in Spreads</th>
</tr>
</thead>
<tbody>
<tr>
<td>2000</td>
<td>Data</td>
<td>13.9</td>
<td>51.2</td>
<td>5.5</td>
<td>7.6</td>
<td>-1.9</td>
<td>-2.4</td>
</tr>
<tr>
<td>2000</td>
<td></td>
<td>21.5</td>
<td>49.3</td>
<td>3.1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2000</td>
<td></td>
<td>7.8</td>
<td>44.6</td>
<td>1.9</td>
<td>3.3</td>
<td>-4.6</td>
<td>-.05</td>
</tr>
<tr>
<td>2000</td>
<td></td>
<td>11.2</td>
<td>40.0</td>
<td>2.4</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2000</td>
<td>Contingent liabilities $L = 15%$</td>
<td>5.9</td>
<td>43.5</td>
<td>2.4</td>
<td>7.8</td>
<td>-3.7</td>
<td>0.2</td>
</tr>
<tr>
<td>2000</td>
<td></td>
<td>13.6</td>
<td>39.7</td>
<td>2.6</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2000</td>
<td>Policy framework improvements</td>
<td>6.0</td>
<td>43.6</td>
<td>2.4</td>
<td>2.3</td>
<td>-22.0</td>
<td>-1.2</td>
</tr>
<tr>
<td>2000</td>
<td></td>
<td>8.3</td>
<td>21.6</td>
<td>1.2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2000</td>
<td>Larger risk premium shocks</td>
<td>6.0</td>
<td>43.6</td>
<td>2.4</td>
<td>7.6</td>
<td>-4.0</td>
<td>1.1</td>
</tr>
<tr>
<td>2000</td>
<td></td>
<td>13.6</td>
<td>39.6</td>
<td>3.5</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note: The levels of reserves, debt, and spread are in percentages. Data corresponds to the mean of the emerging economies discussed in Section 2. The levels of debt, spreads and reserves as of 2000 correspond to the mean of the ergodic distribution for all model experiments, except for the income windfalls. See the text for details of the experiments.

\textsuperscript{38}Hur and Kondo (2011) considers a multicountry model with learning about the volatility of liquidity shocks to shed light on the upward trend in the reserves-to-debt ratio.
Figure 12: Transitional Dynamics.
7 Conclusions

We used a quantitative sovereign default model to study the optimal accumulation of international reserves. The model featured long-term debt, risk-free assets, shocks to foreign creditors, and endogenous fluctuations in borrowing conditions as a result of the lack of government commitment. We show that the optimal policy is to borrow and accumulate reserves in good times to hedge against future increases in borrowing costs. Allowing for the maturity of bonds to exceed one period is essential for understanding the role of reserves as a hedge against rollover risk.

The model is calibrated to mimic salient features of a typical emerging economy, including the average public debt and spread levels. The model simulations display an average optimal reserve ratio equal to 6 percent of income. The reserve ratio reaches values close to 40 percent in some simulation periods. Furthermore, the government accumulates (decumulates) both reserves and debt in periods of low (high) spread and high (low) income, which we show is consistent with the behavior of debt and reserves in emerging economies.

The paper also shows that four recent developments in emerging economies (income windfalls, the increase in public sector’s contingent liabilities, improved policy frameworks, and an increase in the severity of global shocks) imply significant increases in the optimal holding of reserves, which is consistent with the behavior observed in emerging economies over the last 15 years. These developments also imply lower optimal debt levels and spreads, which are consistent with the data.

There are several avenues that are left for future research. One promising avenue would be to incorporate self-fulfilling debt crises. Introducing maturity management as an additional portfolio decision of the government and allowing for private capital flows and domestic debt issuances are other interesting avenues. Finally, we think that the mechanisms studied in this paper are not confined to sovereign debt markets and could be relevant for understanding the financial decisions of households and corporate borrowers facing rollover risk.
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RIKGALDEN (2013): “Borrowing to satisfy the Riksbank’s need for foreign exchange reserves Proposed,” Swedish National Debt Office, Memorandum to the Board, January 22.


