Renegotiation of Dynamically Incomplete Contracts

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Abstract

I provide a micro-foundation for dynamically incomplete contracts that are renegotiated over time. The micro-foundation is based on showing that such contracts implement the optimal complete contract in a general dynamic model provided the players have “preference-for-robustness.” Preference-for-robustness is a way of modeling players perpetually having a fuzzy idea as time passes about events in the distant future. As an application, I introduce asymmetric information and show that under preference-for-robustness, the optimal dynamic contract is debt. The debt contract is not just a static, wedge payoff function, but, rather, includes a state-contingent allocation of control rights and a refinance option.

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1 Introduction

Consider the following scenario: An entrepreneur has a project and seeks financing. He secures a one year contract from a financier. Despite the one year window, the project is long-lived and will likely be productive and require further financing a year from now. Both parties understand this and know that a year from now, the maturing contract could be renegotiated, perhaps rolled over to another one year contract. Despite this understanding, the current contract does not explicitly lay out a state-contingent renegotiation plan for next year (e.g. each party receives half of the surplus generated from renegotiation). Instead, the parties agree that if it turns out that next year there is some surplus to be captured by extending the relationship, then they will hammer out the details at that time. Furthermore, between now and next year, the financier and/or the entrepreneur may gain better insight into the viability of the project. As opinions begin to change, the parties will likely end up rewriting the contract before it even expires next year. Again, the parties do not pre-specify such an interim renegotiation, agreeing, instead, to cross that bridge if and when they get there.\textsuperscript{2}

In the above story, the initial one year contract together with the example renegotiation paths constitutes a dynamically incomplete contract being renegotiated over time. Here, the modifier “dynamically” conveys the idea that not only is the initial contract incomplete along the time dimension, but that, over time, as details get filled in and terms are renegotiated, the evolving contract remains incomplete along the time dimension. My aim is to provide micro-foundations for these contracts. I do this by considering optimal complete contracting in a general dynamic financial contracting model and showing that every Pareto-optimal payoff can also be attained by a dynamically incomplete contract being renegotiated over time \textit{if the players have “preference-for-robustness.”} Preference-for-robustness is a class of weakly time-consistent preferences I introduce in the paper that models a decision maker who has a perpetually fuzzy idea as time passes about events in the distant future and is averse to this fuzziness. A decision maker with preference-for-robustness behaves in a way not unlike an ambiguity-averse decision maker. Indeed, preference-for-robustness is a simple way to bring max-min decision making from the one-shot to the repeated setting, allowing it to be fruitfully applied to optimal dynamic contracting.

In addition to showing how optimal contracts in general can be implemented with finite maturity incomplete contracts plus ex-interim and ex-post renegotiation, I also show, more specifically, when the optimal contract can be implemented with debt: As an application, I introduce asymmetric information into the baseline model by assuming that the entrepreneur privately observes the contract relevant state of the world. I then show that the optimal contract looks like a debt contract that can be refinanced and features a state-contingent allocation of control rights. This debt contract differs in key ways from many debt contracts derived in previous security design models featuring complete contracts. Typically, in such a model, when a contract is described as debt, what is meant is that the contract is static with

\textsuperscript{2}While some interim renegotiations may be triggered by pre-specified covenants, Roberts (2015) finds that the majority of loan contract renegotiations occur without an actual or anticipated covenant violation. While a micro-foundation of incomplete contract covenants is a logical next step, I do not study covenants in this paper.
a wedge payoff function of the form \( D \wedge v \) where \( D \) is a constant and \( v \) is the random value of the underlying asset. In my model with asymmetric information, the optimal complete contract that looks like debt is more than this. The implementation of the complete contract specifies an initial short-term incomplete contract with the familiar wedge payoff. However, this payoff is preliminary and subject to renegotiation. Essentially, the wedge payoff sets the stage for renegotiation by serving as the outside option in the ex-post bargaining game. Just as importantly, the implementation allocates bargaining power based on the state. If the project is in technical default - that is, if the current liquidation value is below \( D \) - then the financier gets to make a take-it-or-leave-it offer. Otherwise, the entrepreneur makes the take-it-or-leave-it offer.

The additional control rights facet of the debt contract is aligned with the incomplete contracting literature’s perspective on debt. See, for example, Aghion and Bolton (1992). My work can be seen as providing support for this perspective by showing how a theory of optimal control rights allocation can emerge without assuming a particular incompleteness of the contracting space.

I emphasize that the choice of a state-contingent allocation of control rights does not have any intrinsic payoff impact ex-ante (for players who have preference-for-robustness): If there was no asymmetric information or if the entrepreneur could be forced to always tell the truth, then the debt contract’s particular allocation of control rights can be changed to any other, and the ex-ante value of the contract would be unaffected. Allocating control rights specifically in the way that is done in a debt contract matters purely for incentive-compatibility. For example, I show that if, instead, the entrepreneur gets to make a take-it-or-leave-it offer when in technical default then the resulting misreporting incentives are strong enough to make the “debt” contract worthless to the financier.

The paper is related to various strands of the contracting literature. Were it not for the preference-for-robustness component, the optimal contracting problem would be a standard Bayesian optimal dynamic contracting problem over complete contracts. By moving from Bayesian to preference-for-robustness I show how the typically complex optimal dynamic contract reduces to a simple one that captures the essence of many real-life dynamic financing arrangements.\(^3\)

My paper is also related to the effort to micro-found incomplete contracts, such as the null contract, in the hold-up model. See Hart and Moore (1999) and Segal (1999). The setting is one with observable but unverifiable signals. Since signals are not contractible, contracts can only be written over contractible messages sent by players. The papers show that when the environment is sufficiently “complex,” the optimal renegotiation-proof contract

\(^3\) My implementation result via the renegotiation of dynamically incomplete contracts is distinct from similar sounding results showing how optimal renegotiation-proof contracts in the Bayesian setting are implementable with short-term contracts. See, for example, Fudenberg, Holmstrom, and Milgrom (1990). In that literature, short-term implementability is way to interpret the fact that the optimal contract is recursive over continuation payoffs and the value function is everywhere downward sloping. At any moment in time, in order to compute the current short-term contract, one must know the forward looking value function which effectively means that one has to also compute all possible short-term contracts that can occur in the future. In contrast, my implementation result implies that the parties can literally construct the optimal dynamic contract one incomplete step at a time. See Section 2.2 for a detailed comparison.
over messages is no better than the null contract. Mukerji (1998), also looking at the hold-up model, points out that the observable but unverifiable assumption is not even needed provided players are sufficiently ambiguity averse. By looking at complete contracts under max-min decision making, Mukerji (1998)’s approach is similar to mine but in a one-shot context. The idea is that the null contract is attractive under max-min preferences because it gives each party a fixed share of the future surplus. This anticipates the key insight of Carroll (2015), which looks at a static principal-agent model with moral hazard and max-min preferences, and shows that the optimal contract is linear. There are also a number other recent static contracting papers using max-min preferences to deliver simple, intuitive contracts in various settings. See, for example, Garrett (2012) and Frankel (2014).

Lastly, my analysis of the optimality of debt under asymmetric information links two strands of the security design literature. First, there is the complete contracting approach that aims to show when the wedge payoff structure is optimal. See, for example, Townsend (1979), Gale-Hellwig (1985), DeMarzo and Duffie (1999), and Yang (2016). Second, there is the incomplete contracts approach that takes a particular contractual incompleteness as the primitive and is interested in the optimality of the state-contingent allocation of control rights induced by debt. See, for example, Aghion and Bolton (1992). I take the complete contracts approach but by using preference-for-robustness, I am also able to address issues involving renegotiation and contingent bargaining power allocation.

2 Model

Players. There are two players, call them E for “entrepreneur” and F for “financier.”

Project. There is a project $v$ that spans dates 0, 1, and 2. At date 0, it consists of some amount of capital with liquidation value $v_0 \geq 0$. For any $c_0 \leq v_0$, the project can be partially liquidated generating date 0 consumption $c_0$ and leftover capital $k_0 = v_0 - c_0$. Conversely, additional capital can be injected into the project so that $k_0$ can take any value $\geq 0$. This $k_0$ then randomly generates an amount of date 1 capital with liquidation value $v_1 \geq 0$. Again a portion of the project can be liquidated for consumption or additional capital can be injected leading to $k_1$. Finally, $k_1$ randomly generates an amount of date 2 capital $v_2$ which is automatically liquidated for consumption.

States of the World. A date 2 state of the world $s_2$ is a realized date 2 liquidation value $v_2$. Define $\{s_2\}$ to be the set of all possible date 2 states of the world. A date 1 state of the world $s_1$ is the following object,

$$ (v_1; \Pi_{2,E}, \Pi_{2,F} : k_1 \rightarrow 2^{\Delta(\{s_2\})}). \quad (1) $$

A date 1 state of the world consists of a realized date 1 liquidation value for the project plus belief functions $\Pi_{2,E}$ and $\Pi_{2,F}$ for $E$ and $F$, respectively. A belief function $\Pi_{2,E}$ specifies for each date 1 leftover capital amount $k_1$ a set of beliefs for $E$ about the date 2 state of the world. I assume for every pair $k_1 < k_1'$, if $\pi_2 \in \Pi_{2,E}(k_1)$ then there exists a $\pi_2' \geq_d \pi_2$ that
$\in \Pi_{2,E}(k_1')$, and if $\pi'_2 \in \Pi_{2,E}(k_1')$ then there exists a $\pi_2 \leq_d \pi'_2$ that $\in \Pi_{2,E}(k_1)$. The partial order $\geq_d$ is by first-order stochastic dominance. I also assume $\Pi_{2,E}(0) \equiv 0$. $\Pi_{2,F}$ is defined similarly. Let $\{s_1\}$ denote the set of all possible date 1 states of the world. Finally, fix a subset $S_1 \subset \{s_1\}$. Given $S_1$, a date 0 state of the world $s_0$ is the following object,

$$(v_0; \Pi_{1,E}, \Pi_{1,F} : k_0 \rightarrow 2^{\Delta(S_1)}).$$

(2)

It is defined in a similar way to the date 1 state of the world $s_1$.

**Model of Beliefs.** A model of beliefs specifies a subset $S_1 \subset \{s_1\}$ and date 0 state of the world $s_0$.

**Example 1 (Standard Bayesian).** $S_1$ is the set of all date 1 states satisfying the property that $\Pi_{2,E}$ and $\Pi_{2,F}$ are both singleton valued and coincide with each other. $s_0$ is a date 0 state of the world satisfying the property that $\Pi_{1,E}$ and $\Pi_{1,F}$ are both singleton valued and coincide with each other.

**Example 2 (E and F have Preference-for-Robustness).** $S_1 = \{s_1\}$. $s_0$ is a date 0 state of the world satisfying the following property: For each $k_0 > 0$, if $\pi_1 \in \Pi_{1,E}(k_0)$, and $\pi'_1 \in \Delta(S_1)$ satisfies $\pi'_1 |_{v_1} =_d \pi_1 |_{v_1}$, then $\pi'_1 \in \Pi_{1,E}(k_0)$. A similar condition holds for $\Pi_{1,F}$.

**Example 2.1 (E and F have Preference-for-Robustness, Lower Bound).** $S_1$ is the set of all date 1 states satisfying the following property: For each $k_1 > 0$, if $\pi_2 \in \Pi_{2,E}(k_1)$, and $\pi'_2 \in \Delta(\{s_2\})$ satisfies $\pi'_2 \geq_d \pi_2$, then $\pi'_2 \in \Pi_{2,E}(k_1)$. A similar condition holds for $\Pi_{1,F}$. $s_0$ is a date 0 state of the world such that for each $k_0 > 0$, if $\pi_1 \in \Pi_{1,E}(k_0)$, and $\pi'_1 \in \Delta(S_1)$ satisfies $\pi'_1 |_{v_1} \geq_d \pi_1 |_{v_1}$, then $\pi'_1 \in \Pi_{1,E}(k_0)$. A similar condition holds for $\Pi_{1,F}$.

**Example 2.1.1 (E and F have Preference-for-Robustness, Lower Bound Expected Value)** $S_1$ is the set of all date 1 states satisfying the following property: For each $k_1 > 0$, there exists a number $v_{2,E}(k_1)$ such that $\Pi_{2,E}(k_1) = \{\pi_2 \in \Delta(\{s_2\}) \mid \mathbb{E}_{\pi_2} v_2 \geq v_{2,E}(k_1)\}$. A similar condition holds for $\Pi_{2,F}$. $s_0$ is a date 0 state of the world such that for each $k_0 > 0$, there exists a number $v_{1,E}(k_0)$ such that $\Pi_{1,E}(k_0) = \{\pi_1 \in \Delta(S_1) \mid \mathbb{E}_{\pi_1} v_1 \geq v_{1,E}(k_0)\}$. A similar condition holds for $\Pi_{1,F}$.

Technically speaking, because $S_1$ changes across the examples, Example 2.1 is not a special case of Example 2, and Example 2.1.1 is not a special case of Example 2.1. If the set $S_1$ were unchanged in the three examples, so that the only difference was with $s_0$, then it would be formally true that each example model of beliefs contains the subsequent one as a special case. However, keeping $S_1$ unchanged while changing $s_0$ would imply some underlying inconsistency in the decision maker’s view about the way in which the future is “fuzzy.” To the extent that the environment is stationary, it is most natural to impose whatever constraints imposed on $s_0$ on the set of date 1 states as well.

Moreover, all of the results I prove when assuming Example 2 continue to hold assuming Example 2.1 or 2.1.1. Similarly, results proved assuming Example 2.1 continue to hold assuming Example 2.1.1.

**Preferences.** Fix a model of beliefs $(s_0, S_1)$. A date 2 history is $h_2 = s_1 s_2$ for some $s_1 \in S_1$ and $s_2 \in \{s_2\}$. A date 1 history is $h_1 = s_1$ for some $s_1 \in S_1$. I suppress the dependence on $s_0$ since it is assumed that $s_0$ is realized before the beginning of the model. A recapitalization
plan is a random sequence \( k := \{k_0, k_1(h_1)\} \) of nonnegative reals. A consumption stream is a random sequence \( c := \{c_0, c_1(h_1), c_2(h_2)\} \) of nonnegative reals.

\( E \) has a within period weakly concave, strictly increasing utility function \( u_E : (0, \infty) \to \mathbb{R} \). Given a recapitalization plan \( k \) and a consumption stream \( c \), \( E \)'s continuation payoff process \( U_E \) from receiving \( c \) is defined as follows.

\[
U_{2,E}(h_2) = u_E(c_2(h_2)), \\
U_{1,E}(h_1) = u_E(c_1(h_1)) + \min_{\pi_2 \in \Pi_{2,E}(k_1(h_1))} \mathbb{E}_{\pi_2} U_{2,E}(h_2), \\
U_{0,E} = u_E(c_0) + \min_{\pi_1 \in \Pi_{1,E}(k_0)} \mathbb{E}_{\pi_1} U_{1,E}(h_1).
\]

\( u_F \) and \( U_F \) are defined similarly for \( F \). Normalize \( u_E(0) = u_F(0) = 0 \).

**Lemma 1.** Preferences are weakly time-consistent.

**Proof.** Without loss of generality, I prove the result for \( E \). Fix a stopping time \( \tau \in \{1, 2\} \), a recapitalization plan \( k \), and two consumption streams \( c' \) and \( c'' \). Suppose that \( c'_t = c''_t \) for all \( t < \tau \) and \( U'_{\tau,E}(h_\tau) \leq U''_{\tau,E}(h_\tau) \). Here \( U'_E \) and \( U''_E \) denote the continuation payoff processes from receiving \( c' \) and \( c'' \), respectively. Fix an \( h_1 \) such that \( \tau(h_1) > 1 \). Then \( U'_{1,E}(h_1) = u_E(c'_1(h_1)) + \min_{\pi_2 \in \Pi_{2,E}(k_1(h_1))} \mathbb{E}_{\pi_2} U''_{2,E}(h_2|h_1) \leq u_E(c''_1(h_1)) + \min_{\pi_2 \in \Pi_{2,E}(k_1(h_1))} \mathbb{E}_{\pi_2} U''_{2,E}(h_2|h_1) \leq U''_{1,E}(h_1) \). In addition, for all \( h_1 \) such that \( \tau(h_1) = 1 \), it is assumed that \( U'_{1,E}(h_1) \leq U''_{1,E}(h_1) \). Thus, for every \( h_1 \), \( U'_{1,E}(h_1) \leq U''_{1,E}(h_1) \).

Now, an argument similar to the one used above shows that \( U'_{0,E} \leq U''_{0,E} \). \( \square \)

**Contracts.** A contract specifies consumption streams \((c_E, c_F)\) for \( E \) and \( F \), satisfying the budget constraint \( c_E + c_F \leq v \). The recapitalization plan is then \( k = v - (c_E + c_F) \).

The budget constraint and the recapitalization equation imply that \( E \) and \( F \) cannot access anything in the asset market except the project: The budget constraint assumes that the players have no external sources for consumption. The recapitalization plan assumes that the players have no external projects which may draw capital from the one being modeled.

The second assumption is a common one, particularly in a micro-theory model. However, typically one assumes that players have exogenous external consumption streams \((w_E, w_F)\). The budget constraint relaxes to \( c_E + c_F \leq v + w_E + w_F \) and the recapitalization plan becomes \( k = v + w_E + w_F - (c_E + c_F) \). Oftentimes, this relaxed setup is rewritten so that the budget constraint and recapitalization plan continue to be the original \( c_E + c_F \leq v \) and \( k = v - (c_E + c_F) \), but \( E \) and \( F \) can consume negative amounts, down to \(-w_E\) and \(-w_F\), respectively. The utility functions \( u_E \) and \( u_F \) are then redefined over the intervals \([-w_E, \infty)\) and \([-w_F, \infty)\).

For example, a standard case is to assume \( w_E \equiv 0 \) and \( w_F \equiv \infty \) and let \( u_F \) be the identity function: \( E \) has no external consumption stream and is often described as being “protected by limited liability” and \( F \) is a risk-neutral, deep-pocketed financier. All the results that follow generalize to the case with exogenous external consumption streams. On the other hand, fully embedding the project into a general equilibrium model with multiple assets is beyond the scope of this paper.
In the current perfection information setting, when \( E \) and \( F \) are Bayesian, all Pareto-optimal contracts are renegotiation-proof. However, the min feature of preferences means that, in general, a Pareto-optimal allocation can also be achieved by many contracts, including some that are not renegotiation-proof.

From now on I focus on those contracts achieving Pareto-optimal allocations that are renegotiation-proof. I do this for two reasons. First is for the sake of consistency. In a future extension of the model, I will introduce asymmetric information. In this case, the renegotiation-proof constraint does affect the Pareto-frontier and I choose to focus on renegotiation-proof contracts. Second, in the current perfect information model, given that the renegotiation-proof constraint does not affect the Pareto-frontier, imposing the constraint seems like a natural benchmark case to consider.

**Definition.** From now on, the term Pareto-optimal contract means renegotiation-proof Pareto-optimal contract.

**Theorem 1.** Suppose \( E \) and \( F \) have preference-for-robustness. Any Pareto-optimal allocation \((U_{0,E},U_{0,F}(U_{0,E}))\) can be achieved by a Pareto-optimal contract \((c^*_E,c^*_F)\) satisfying the following properties: There exists a feasible split of \( v_1, \alpha^*_E(v_1) + \alpha^*_F(v_1) = v_1 \), such that

\[
U_{0,E} = u_E(c^*_0, E) + \min_{\pi_1 \in \Pi_{1,E}(k_0^*)} E_{\pi_1} u_E(\alpha^*_E(v_1))
\]

\[
U^*_0(U_{0,E}) = u_F(c^*_0, F) + \min_{\pi_1 \in \Pi_{1,F}(k_0^*)} E_{\pi_1} u_F(\alpha^*_F(v_1)).
\]

Moreover, for all histories \( h_1 \), the continuation contract \((c^*_E,c^*_F)|h_1\) is Pareto-optimal with

\[
(U_{1,E}(h_1),U_{1,F}(h_1)) \geq (u_E(\alpha^*_E(v_1)),u_F(\alpha^*_F(v_1))).
\]

**Proof.** Fix a Pareto-optimal allocation \((U_{0,E},U^*_0(U_{0,E}))\) and a contract \((c_E,c_F)\) that achieves it. For each \( v_1 \), there is a state \( h'_1 \) with liquidation value \( v_1 \) such that the project is fully liquidated at date 1. This will occur whenever both parties are sufficiently pessimistic about the future given any leftover capital \( k_1 \).

For each \( v_1 \), define \( \alpha^*_E(v_1) := c_{1,E}(h'_1) \) and \( \alpha^*_F(v_1) := v_1 - \alpha^*_F(v_1) = c_{1,F}(h'_1) \). For each \( \pi_1 \in \Pi_{1,E}(k_0) \), define \( \pi'_1 \in \Pi_{1,E}(k_0) \) to be a belief with the same marginal with respect to \( v_1 \) but has support only over those states of the form \( h'_1 \). Such a belief can be found in \( \Pi_{1,E}(k_0) \) due to the preference-for-robustness assumption. Then

\[
U_{0,E} = u_E(c_0, E) + \min_{\pi_1 \in \Pi_{1,E}(k_0)} E_{\pi_1} U_{1,E}(h_1)
\]

\[
\leq u_E(c_0, E) + \min_{\pi'_1 \in \Pi_{1,E}(k_0)} E_{\pi'_1} U_{1,E}(h'_1)
\]

\[
= u_E(c_0, E) + \min_{\pi'_1 \in \Pi_{1,E}(k_0')} E_{\pi'_1} u_E(\alpha^*_E(v_1))
\]

\[
= u_E(c_0, E) + \min_{\pi_1 \in \Pi_{1,E}(k_0)} E_{\pi_1} u_E(\alpha^*_E(v_1))
\]

Similarly, \( U^*_0(U_{0,E}) \leq u_F(c_0, F) + \min_{\pi_1 \in \Pi_{1,F}(k_0)} E_{\pi_1} u_F(\alpha^*_F(v_1)). \) But equality can be achieved for both continuation payoffs simply by liquidating the project at date 1 no matter what and
splitting \( v_1 \) according to \((\alpha_E^*(v_1), \alpha_F^*(v_1))\). This new contract achieves the Pareto-optimal allocation but is not Pareto-optimal because it is not renegotiation-proof. To make it renegotiation-proof, weakly Pareto-improve each continuation contract so that it becomes a Pareto-optimal continuation contract with \((U_{1,E}(h_1), U_{1,F}(h_1)) \geq (u_E(\alpha_E^*(v_1)), u_F(\alpha_F^*(v_1)))\). The resulting contract, call it \((c_E^*, c_F^*)\), weakly increases each player’s continuation payoff after every history \(h_1\). Thus, \((c_E^*, c_F^*)\) achieves the Pareto-optimal allocation \((U_0,E, U_0,F)\).

Moreover, since every continuation contract is Pareto-optimal by design, \((c_E^*, c_F^*)\) is, itself, Pareto-optimal.

2.1 Implementation with Incomplete Contracts.

Fix a date 1 history \(h_1\). Given a date 1 outside option \((O_{1,E}, O_{1,F}) \in \mathbb{R}^2\) for \(E\) and \(F\), a renegotiation protocol selects a Pareto-optimal continuation contract that is individually rational, or, if none exist, the date 1 outside option.

A date 0 incomplete contract specifies a date 0 consumption plan plus, to serve as the date 1 outside option, a \(v_1\)-contingent feasible split of the project under full liquidation at date 1. Then, as long as some renegotiation protocol is used at date 1, each player’s valuation of the date 0 incomplete contract is invariant over which protocol is used. This is a direct consequence of preference-for-robustness. Consequently, players can agree to the date 0 incomplete contract without specifying beforehand how they would go about renegotiating it the next date if a renegotiation is warranted. Given this fact, it is well-defined to say,

**Corollary 1.** Any Pareto-optimal allocation can be implemented by an incomplete contract with renegotiation.

**Proof.** Let the date 0 incomplete contract be \((c_{0,E}^*, c_{0,F}^*, \alpha_E^*(v_1), \alpha_F^*(v_1))\).

**Example (Linear Incomplete Contracts).** Suppose \(E\) and \(F\) have preference-for-robustness, lower bound expected value. Then every Pareto-optimal allocation can be achieved by a contract where \((\alpha_E^*, \alpha_F^*)\) is a linear split of \(v_1\), and each date 1 continuation contract’s date 2 consumption plan is a linear split of \(v_2\).

**Proof.** Weak concavity of the utility functions implies that the split can always be chosen so that \(\alpha_E^*\) and \(\alpha_F^*\) are weakly convex. The only such split is the linear split.

The reduction from arbitrary incomplete contract to a linear one is mathematically similar to the ones appearing in Mukerji (1998) and Carroll (2015). Indeed, the example can be seen as a way of extending their linearity result to a dynamic context. However, in my model, neither party is required to be risk-neutral.

The example presents an incomplete contract that dynamically evolves but always remains linear. A natural interpretation of the contract is as an equity contract, where both parties start with some initial equity position \((\alpha_E^*, \alpha_F^*)\) and can rebalance over time based on changing risk-attitudes and opinions about the future viability of the project.

More generally, if \(E\) and \(F\) have preference-for-robustness, lower bound, then the splits of \(v_1\) and \(v_2\) are increasing.
2.2 Comparisons to Contracting with Bayesian Players

How is the implementation results different from those in the Bayesian contracting literature about renegotiation-proof optimal long-term arrangements being implementable as sequences of short-term contracts?

Consider a contracting model where Pareto-optimal long-term contracts are recursive over continuation payoffs. Moreover, assume that the continuation payoff process always lies on the Pareto-frontier. Then there is a way to describe each Pareto-optimal long-term contract so that it is as if one party is contracting the other only through a series of short-term contracts. For example, Pareto-optimal contracts in the Bayesian version of my model satisfy the above assumptions. As a result, Pareto-optimal contracts can be described as follows: At each date \( t \), given a state of the world \( s_t \) and a continuation payoff \( U_{t,E} \) promised to \( E \), \( F \) gives \( E \) a short-term contract specifying a \( c_{t,E}(s_t, U_{t,E}) \), \( k_t(s_t, U_{t,E}) \), and a state-contingent promised continuation payoff \( U_{t+1,E}(s_{t+1}, s_t, U_{t,E}) \) for the next date. Then, when the next date rolls around and \( s_{t+1} \) and \( U_{t+1,E} \) are realized, \( F \) writes another short-term contract and so on.

This type of reinterpretation of optimal long-term arrangements has appeared many times in the Bayesian renegotiation-proof contracting literature. Oftentimes this is interpreted to mean that these arrangements, while derived in a complete contracts, Bayesian setting, can nevertheless be practically thought of as dynamically incomplete contracts being renegotiated over time.

How does this result contrast with Corollary 1, which interprets optimal contracts under preference-for-robustness as dynamically incomplete contracts being renegotiated over time, but is not applicable to Bayesian optimal contracts?

Imagine you are a lawyer hired by \( E \) and \( F \). You have access to all relevant information and your job is to craft a Pareto-optimal contract for the two parties. Before you begin, \( E \) and \( F \) reassure you that it is not such a hard task by explaining to you that at each date, you only need to write a short-term contract that lasts until the next date.

Is this of any comfort to you?

Suppose \( E \) and \( F \) are Bayesian. In order to figure out the optimal state-contingent promised continuation payoff \( U_{t+1,E}(s_{t+1}, s_t, U_{t,E}) \), you need to know the date \( t + 1 \) value function. In order to compute the date \( t + 1 \) value function, you need to know for each possible state \( s_{t+1} \), what short-term contract you would write, which then requires you to know the date \( t + 2 \) value function and so on. Thus, to be able to compute just the single short-term contract for today, you need to compute all possible short-term contracts that can come after it at all dates in the future.

Now suppose \( E \) and \( F \) have preference-for-robustness. In order to compute today’s incomplete contract, you do not need to compute any future incomplete contract. You do not need to consider \( E \) and \( F \)’s beliefs in the future. To compute today’s incomplete contract, all you need to think about is \( E \) and \( F \)’s beliefs today about the liquidation value tomorrow. This allows you to construct the Pareto-optimal contract one incomplete step at a time. As

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4The renegotiation-proof qualifier is needed in order to guarantee that the continuation payoff always lies on the Pareto-frontier, which, recall, is a necessary condition for the reinterpretation. See, for example, Fudenberg, Holmstrom and Milgrom (1990).
a result, you end up only computing the short-term contracts for states that actually get
realized rather than for all possible states that could have been realized.

How is having preference-for-robustness different from believing the world is going to end?

Another concern is how different are Theorem 1 and Corollary 1 from optimal contracts
arising in settings where players are Bayesian but one or both players think “the world is
going to end after tomorrow” (i.e. at date 0, the belief function only places weight on those
states $s_1$ that have $\Pi_{2,1} \equiv 0$).

For example, suppose both $E$ and $F$ are Bayesian, and only $F$ thinks the world is going
to end after date 1. Then the optimal contract will simply load all of $F$’s date 1 consumption
on those states $s_1$ with $\Pi_{1,F} \equiv 0$. In all other states, $E$ gets the entire project.

This does not look at all like the optimal contract arising under preference-for-robustness.

Next, suppose $E$ and $F$ are Bayesian, and both think the world is going to end after date
1. Then over the set of date 1 states where $\Pi_{1,E}$ and/or $\Pi_{1,F} \equiv 0$, the optimal contract fully
liquidates the project and splits the proceeds between $E$ and $F$. For all other date 1 states,
anything goes.

Here, the full liquidation of the project followed by the split of $v_1$ over those states where
$\Pi_{1,E}$ and/or $\Pi_{1,F} \equiv 0$, is reminiscent of the feasible split $(\alpha^*_E(v_1), \alpha^*_F(v_1))$ in Theorem 1.
However, there is nothing tying this split of $v_1$ to the continuation payoffs for $E$ and $F$ in
those states where neither $\Pi_{1,E}$ nor $\Pi_{1,F}$ are trivial. This is unlike what happens under
preference-for-robustness, where it is crucial that all continuation contracts deliver payoffs
that dominate the feasible split $(\alpha^*_E(v_1), \alpha^*_F(v_1))$. The discipline imposed on continuation
payoffs across a wide range of states under preference-for-robustness is what allows the op-
timal contract to be interpreted as a dynamically incomplete contract that is renegotiated
over time. In the end-of-the-world model, one could take an optimal contract and change it
so that neither party gets any consumption if the world doesn’t end after date 1, and the
contract would remain optimal. However, this contract is worth zero to both $E$ and $F$ under
preference-for-robustness.

In the coming sections, I further develop the notion of preference-for-robustness and apply
it to extensions of the model. Optimal contracts will be implemented with longer horizon
incomplete contracts featuring interim renegotiation and refinance-able debt featuring state-
contingent allocation of control rights. These results will help further distinguish optimal
contracting under preference-for-robustness from Bayesian optimal contracting.

3 Incomplete Contracts with Interim Renegotiation

In the baseline model, I showed how optimal complete contracts can be implemented as
incomplete contracts featuring ex-post renegotiation. Empirical work by Roberts and Sufi
(2009) and Roberts (2015) shows, however, that the typical loan contract is renegotiated early
and frequently, before the initial contract matures. Moreover, much of this renegotiation
is not triggered by the actual or anticipated violation of a pre-specified covenant. I now
extend the notion of preference-for-robustness to show how incomplete contracts with interim
renegotiation can also be optimal.

Example 3 (E and F have Long Horizon Preference-for-Robustness). \( S_1 = \{ s_1 \} \). \( s_0 \) is a date 0 state of the world satisfying the following property: Fix an arbitrary \( \pi \in \Delta(S_1) \). Define \( \Pi_2 \circ \pi_1 := \{ \pi_1 \cdot \pi_2 \mid \pi_2(s_1) \in \Pi_2,E(v_1) \} \) for all \( s_1 \} \) to be the set of all probability distributions of \( v_2 \) generated by \( \pi_1 \) assuming the project is left alone at date 1. For each \( k_0 \), there is a set of probability distributions of \( v_2 \), call it \( \Pi_2,E \circ \Pi_1,E(k_0) \), such that \( \pi_1 \in \Pi_1,E(k_0) \) if and only if \( \Pi_2,E \circ \pi_1 \subset \Pi_2,E \circ \Pi_1,E(k_0) \). A similar condition holds for \( \Pi_1,F \).

**Theorem 2.** Suppose \( E \) and \( F \) have long horizon preference-for-robustness. Any Pareto-optimal allocation \( (U_{0,E},U_{0,F}(U_{0,E})) \) can be achieved by a Pareto-optimal contract \((c^*_E,c^*_F)\) satisfying the following properties: There exists a feasible split of \( v_2 \), \( \alpha^*_E(v_2) + \alpha^*_F(v_2) = v_2 \), such that

\[
U_{0,E} = u_E(c^*_{0,E}) + \min_{\pi_2 \in \Pi_2,E \circ \Pi_1,E(k_0^*)} \mathbb{E}_{\pi_2} u_E(\alpha^*_E(v_2)),
\]

\[
U_{0,F}(U_{0,E}) = u_F(c^*_{0,F}) + \min_{\pi_2 \in \Pi_2,F \circ \Pi_1,F(k_0^*)} \mathbb{E}_{\pi_2} u_F(\alpha^*_F(v_2)).
\]

Moreover, for all histories \( h_1 \), the continuation contract is Pareto-optimal with

\[
(U_{1,E}(h_1),U_{1,F}(h_1)) \geq \left( \min_{\pi_2 \in \Pi_2,E(v_1)} \mathbb{E}_{\pi_2} u_E(\alpha^*_E(v_2)), \min_{\pi_2 \in \Pi_2,F(v_1)} \mathbb{E}_{\pi_2} u_F(\alpha^*_F(v_2)) \right). \tag{3}
\]

**Proof.** If either \( U_{0,E} \) or \( U_{0,F}(U_{0,E}) \) is 0 then the theorem is obviously true. So assume both payoffs are strictly greater than 0.

Fix a contract \((c^*_E,c^*_F)\) that achieves \((U_{0,E},U_{0,F}(U_{0,E}))\) with date 0 leftover capital \( k_0^* \). Fix an \( \varepsilon > 0 \) and define the date 1 state of the world

\[
\hat{s}_1^\varepsilon = (\varepsilon; \hat{\Pi}_{2,E}, \hat{\Pi}_{2,F} : k_1 \to 2^{\Delta(s_2)})
\]

with the following properties: \( \hat{\Pi}_{2,E}(\varepsilon) = \Pi_{2,E} \circ \Pi_1,E(k_0^*) \), and \( \hat{\Pi}_{2,E}(\delta) = 0 \) for all \( \delta < \varepsilon \). Similar properties hold for \( \hat{\Pi}_{2,F} \). The belief \( \hat{\pi}_1^\varepsilon \in \Delta(S_1) \) that puts all weight on \( \hat{s}_1^\varepsilon \) is in both \( \Pi_1,E(k_0^*) \) and \( \Pi_1,F(k_0^*) \).

Consider the continuation contract following \( \hat{s}_1^\varepsilon \). If there is any date 1 consumption by either player, then \((U_{1,E}(\hat{s}_1^\varepsilon),U_{1,F}(\hat{s}_1^\varepsilon)) \leq (u_E(\varepsilon),u_F(\varepsilon)) \). This combined with the fact that \( \hat{\pi}_1^\varepsilon \in \Pi_1,E(k_0^*) \cap \Pi_1,F(k_0^*) \) means that if \( \varepsilon \) is sufficiently small, there will be no consumption at date 1 and \( k_1(\hat{s}_1^\varepsilon) = \varepsilon \). In this case, define \( \alpha^*_E(v_2) := c^*_{2,E}(\hat{s}_1^\varepsilon v_2) \) and \( \alpha^*_F(v_2) := c^*_{2,F}(\hat{s}_1^\varepsilon v_2) \). Thus, the continuation payoffs satisfy

\[
U_{1,E}(\hat{s}_1^\varepsilon) = \min_{\pi_2 \in \Pi_2,E \circ \Pi_1,E(k_0^*)} \mathbb{E}_{\pi_2} u_E(\alpha^*_E(v_2)),
\]

\[
U_{1,F}(\hat{s}_1^\varepsilon) = \min_{\pi_2 \in \Pi_2,F \circ \Pi_1,F(k_0^*)} \mathbb{E}_{\pi_2} u_F(\alpha^*_F(v_2)).
\]
Since \( \hat{s}^\varepsilon_1 \) is just one element of \( \Pi_{1,E}(k^*_0) \) and \( \Pi_{1,F}(k^*_0) \), it must be that

\[
U_{0,E} = u_E(c^*_0,E) + U^*_1(s^\varepsilon_1),
\]

\[
U^*_{0,F}(U_{0,E}) \leq u_F(c^*_0,F) + U^*_1(s^\varepsilon_1).
\]

However, equality can be achieved by having the continuation contract after every \( h_1 \) be the same as the one after \( s^\varepsilon_1 \). To see this, note that for every belief \( \pi_1 \in \Pi_{1,E}(k^*_0) \), we have

\[
U_0 = u_E(c^*_0,E) + \min_{\pi_1 \in \Pi_{1,E}(k^*_0)} E_{\pi_1} U_{1,E}(s_1)
\]

\[
= u_E(c^*_0,E) + \min_{\pi_1 \in \Pi_{1,E}(k^*_0)} \min_{\pi_2 \in \Pi_{2,E}(v_1)} E_{\pi_1} E_{\pi_2} u_E(\alpha^*_E(v_2))
\]

\[
= u_E(c^*_0,E) + \min_{\pi_1 \in \Pi_{1,E}(k^*_0)} \min_{\pi_2 \in \Pi_{2,E}(v_1)} E_{\pi_2} u_E(\alpha^*_E(v_2)) \geq u_E(c^*_0,E) + U^*_1(s^\varepsilon_1).
\]

A similar argument proves \( U^*_{0,F}(U_{0,E}) \geq u_F(c^*_0,F) + U^*_1(s^\varepsilon_1) \).

Now the continuation payoff after each history \( h_1 \) is

\[
\min_{\pi_2 \in \Pi_{2,E}(v_1)} E_{\pi_2} \alpha^*_E(v_2), \quad \min_{\pi_2 \in \Pi_{2,F}(v_1)} E_{\pi_2} \alpha^*_F(v_2).
\]

Finally, to make the contract Pareto-optimal, weakly Pareto improve each continuation contract to a Pareto-optimal one.

Fix a date 1 history \( h_1 \) and a date 1 continuation contract to serve as the outside option. An interim renegotiation protocol selects a date 1 Pareto-optimal continuation contract that is individually rational given the outside option.

A date 0 incomplete contract specifies a date 0 consumption plan, leaves the project alone at date 1, and specifies a feasible split of \( v_2 \). Fix a date 0 incomplete contract and, for each history \( h_1 \), let the corresponding continuation contract be the default at date 1. Then, as long as some interim renegotiation protocol is used at date 1, each player’s valuation of the date 0 incomplete contract is invariant over the protocol choice. Just like before, players can agree to the date 0 incomplete contract without specifying beforehand how they would go about renegotiating it the next date if a renegotiation is warranted. Thus, it is well-defined to say,

**Corollary 2.** Any Pareto-optimal allocation can be implemented by an incomplete contract with interim renegotiation.

### 4 Debt and Control Rights

Standard debt has two salient features. First, the payoff function is a wedge function of the form \( D \wedge v \) where \( D \) is a constant and \( v \) is the underlying asset value. Second, the allocation of control rights is done in a state-contingent way where the financier retains control of the
asset if and only if the entrepreneur is in default. Implied by the allocation of control rights is the notion that the wedge payoff function is preliminary and may eventually be renegotiated.

The security design literature looking at debt largely splits into two groups, depending on which of these two features is the focal point. There is a complete contracts approach that focuses on explaining the wedge payoff function. In addition, there is an incomplete contracts approach, where a primary objective is to highlight how variations in the way securities allocate control rights can also have important payoff implications.

My goal is to unify these two approaches. I do this by introducing an asymmetric information problem into the baseline model. The entrepreneur privately observes the true date 1 state of the world and can strategically reveal a contractible reported date 1 state of the world. I show that the resulting optimal complete contract under preference-for-robustness can be implemented by an incomplete contract featuring the wedge payoff function. This contract is then renegotiated according to a protocol that allocates bargaining power in a state-contingent way based on whether the project is in technical default (i.e. when the liquidation value is less than the debt obligation).

Formally, at date 1, $E$ can choose to credibly reveal only a portion $\hat{v}_1 \leq v_1$ of the project’s true date 1 liquidation value. The budget constraint tightens to $c_{1,E} + c_{1,F} \leq \hat{v}_1$. The revealed leftover capital is $\hat{k} := \hat{v}_1 - (c_{1,E} + c_{1,F})$ whereas the true leftover capital continues to be $k_1 := v_1 - c_{1,E} + c_{1,F}$.

**States of the World.** The date 2 state of the world is the same as before and publicly observed. The date 1 state of the world $\hat{s}_1$ is privately observed by $E$ and is the following object,

$$ (v_1; \Pi_{1,E} : k_1 \to 2^{\Delta(\{s_2\})}, \Pi_{1,F} : (\hat{k}_1, \delta) \to 2^{\Delta(\{s_2\})}). \quad (4) $$

The function $\Pi_{1,F}(\hat{k}_1, \delta)$ takes as arguments both the reported leftover capital and the difference $\delta := k_1 - \hat{k}_1 = v_1 - \hat{v}_1$. This function tells $E$ how his misreporting of the liquidation value will affect $F$’s belief function. I impose two mild restriction on the form of $\Pi_{1,F}$. Holding $\delta$ fixed, $\Pi_{1,F}(\cdot, \delta)$ is weakly increasing in $\hat{k}_1$. This means, fixing a report $\hat{v}_1$, the induced belief function for $F$ is weakly increasing in what he thinks is the leftover capital. Holding $k_1$ fixed, $\Pi_{1,F}(\cdot, k_1 - \cdot)$ is weakly increasing in $\hat{k}_1$. This means the less $E$ underreports $v_1$, the less pessimistic is $F$. Given a $\hat{v}_1 \leq v_1$ and a reported belief function $\hat{\Pi}_{1,E}$ for $E$, $\delta$ is determined and the date 1 reported state of the world $\hat{s}_1$ is the following object,

$$ (\hat{v}_1; \hat{\Pi}_{1,E} : \hat{k}_1 \to 2^{\Delta(\{s_2\})}, \hat{\Pi}_{1,F} : \hat{k}_1 \to 2^{\Delta(\{s_2\})}), \quad (5) $$

where $\hat{\Pi}_{1,F}(\hat{k}_1) := \hat{\Pi}_{1,F}(\hat{k}_1, \delta)$ is the induced belief function of $F$. Let $\{s_1\}$ denote the set of all date 1 states of the world. Given a subset $S_1 \subset \{s_1\}$, the definition of $s_0$ is similar to before and is publicly observed.

A contract specifies a consumption $(c_E, c_F)$ along with a report strategy $(\hat{v}_1, \hat{\Pi}_{1,E})$ for $E$. A contract is truth-telling if $\hat{v}_1 = v_1$ and $\hat{\Pi}_{1,E} = \Pi_{1,E}$. As an abuse of notation, I let $s_1 = (v_1; \Pi_{1,E}, \Pi_{1,F} : k_1 \to 2^{\Delta(\{s_2\})})$ denote the reported state when the privately observed
state is \( s_1 = (v_1; \Pi_{1,E} : k_1 \to 2^\Delta(s_2)), \Pi_{1,F} : (\hat{k}_1, \delta) \to 2^\Delta(s_2)) \) and \( E \) tells the truth. A truth-telling contract is incentive-compatible if for every state \( s_1 \), truth-telling maximizes \( E \)'s continuation payoff. I restrict attention to truth-telling contracts that are not only incentive-compatible, but more specifically, renegotiation-proof.

**Definition.** From now on, the term contract means renegotiation-proof truth-telling contract.

Since there is asymmetric information at date 1, defining renegotiation-proofness requires care.

Fix a truth-telling contract with consumption plan \((c_E, c_F)\) and some state \( s_1 \). Consider a (possibly off-equilibrium) reported state \( \hat{s}_1 \). The continuation contract is some

\[
(c_{1,E}(\hat{s}_1), c_{1,F}(\hat{s}_1), c_{2,E}(\hat{s}_1, \cdot), c_{2,F}(\hat{s}_1, \cdot)).
\]

A renegotiation of this continuation contract is some alternate consumption plan

\[
(c'_{1,E}, c'_{1,F}, c'_{2,E}(\cdot), c'_{2,F}(\cdot))
\]

satisfying the budget constraint \( c'_{1,E} + c'_{1,F} \leq \hat{v}_1 \). Under the renegotiation, the true leftover capital is \( k'_1 = v_1 - (c'_{1,E} + c'_{1,F}) \) and the reported leftover capital is \( \hat{k}'_1 = \hat{v}_1 - (c'_{1,E} + c'_{1,F}) \).

Under the renegotiation, \( E \)'s continuation payoff is

\[
u_E(c'_{1,E}) + \min_{\pi_2 \in \Pi_{2,E}(k'_1)} E_{\pi_2} u_E(c'_{2,E}(\cdot)), \tag{6}\]

and \( F \) thinks his continuation payoff is

\[
u_F(c'_{1,F}) + \min_{\pi_2 \in \Pi_{2,F}(\hat{k}'_1)} E_{\pi_2} u_F(c'_{2,F}(\cdot)). \tag{7}\]

\( E \) is strictly better off under the renegotiation if

\[
(6) \geq u_E(c_{1,E}(s_1)) + \min_{\pi_2 \in \Pi_{2,E}(k_1)} E_{\pi_2} u_E(c_{2,E}(s_1, \cdot)),
\]

and \( F \) thinks he is strictly better off under the renegotiation if

\[
(7) \geq u_F(c_{1,F}(\hat{s}_1)) + \min_{\pi_2 \in \Pi_{2,F}(\hat{k}_1)} E_{\pi_2} u_F(c_{2,F}(\hat{s}_1, \cdot)).
\]

**Definition.** A contract is renegotiation-proof if and only if there does not exist a state \( s_1 \) and a renegotiation of the continuation contract following some reported state \( \hat{s}_1 \) that makes \( E \) strictly better off and makes \( F \) think it is strictly better off.

The definition of preference-for-robustness naturally generalizes to this asymmetric information setting.

**Theorem 3.** Suppose \( E \) and \( F \) have preference-for-robustness, lower bound. Then any Pareto-optimal allocation can be achieved by a Pareto-optimal contract with the following
structure: Since, by assumption, the continuation contract maximizes $U_{1,E}(h_1)$ subject to $U_{1,F}(h_1) = u_F(D_1)$ if $v_1 \geq D_1$, and maximizes $U_{1,F}(h_1)$ if $v_1 < D_1$.

Here, the qualifier “lower bound” that is attached to preference-for-robustness imposes a certain monotonicity on the optimal contract. Formally, in the implementation of the optimal contract using a date 0 incomplete contract with renegotiation, the initial split of $v_1$ is now weakly increasing. See Example 2.1 and the discussion at the end of Section 2.1.

Proof. For each date 1 state of the world $s_1$, define $\overline{U}_{1,F}(s_1)$ to be the continuation contract that maximizes $F$’s continuation payoff. Note, in particular, this continuation contract gives nothing to $E$.

Step 1. Fix a contract and a state $s''_1 = (v''_1, \Pi''_{1,E}, \Pi''_{1,F})$ satisfying $U_{1,F}(s''_1) < \overline{U}_{1,F}(s''_1)$. Then for every $v'_1 \geq v''_1$, there exists a state $s'_1 = (v'_1, \Pi'_{1,E}, \Pi'_{1,F})$ satisfying $U_{1,F}(s'_1) \leq U_{1,F}(s''_1)$.

Proof of Step 1. Fix a $v'_1 \geq v''_1$ and consider the privately observed state $s'_1 = (v'_1; \Pi'_{1,E} : k_1 \rightarrow 2^{\Delta(s_2)}; \Pi'_{1,F} : (\hat{k}_1, \delta) \rightarrow 2^{\Delta(s_2)})$ with the following properties: $\Pi'_{1,F}(k_1) = \Pi''_{1,F}([k_1 - (v'_1 - v''_1)] \wedge 0)$ and $\Pi'_{1,F}(\hat{k}_1, \delta) = \Pi''_{1,F}(k_1, 0)$ for all $\delta \leq v'_1 - v''_1$.

Moreover, assume that $\Pi'_{1,E} : k_1 \rightarrow 2^{\Delta(s_2)}$ is such that the continuation contract that maximizes $E$’s continuation payoff subject to delivering continuation payoff $U_{1,F}(s''_1)$ to $F$ satisfies $c_{1,E} + c_{1,F} < v''_1$. That this is possible comes from two observations. First, it is possible to deliver continuation payoff $U_{1,F}(s''_1)$ to $F$ while satisfying $c_{1,F} < v''_1$. To see why, first compute the continuation contract that delivers $\overline{U}_{1,F}(s''_1)$ to $F$. Obviously, $c_{1,F} \leq v''_1$.

Since, by assumption, $U_{1,F}(s''_1) < \overline{U}_{1,F}(s''_1)$, one can just take the continuation contract that delivers $\overline{U}_{1,F}(s''_1)$ to $F$ and decrease $c_{1,F}$ until the continuation payoff decreases to $U_{1,F}(s''_1)$. If after decreasing $c_{1,F}$ all the way to zero the continuation payoff is still larger than $U_{1,F}(s''_1)$, just simply start decreasing date 2 consumption. Second, given that it is possible to deliver continuation payoff $U_{1,F}(s''_1)$ to $F$ while satisfying $c_{1,F} < v''_1$, then by assuming $E$ is very optimistic about date 2, one can ensure that the marginal opportunity cost of consuming at date 1 is arbitrarily high for $E$. This implies that one can always find a $\Pi'_{1,E} : k_1 \rightarrow 2^{\Delta(s_2)}$ such that the continuation contract that maximizes $E$’s continuation payoff subject to delivering continuation payoff $U_{1,F}(s''_1)$ to $F$ satisfies $c_{1,E} + c_{1,F} < v''_1$.

Let $\{c_{1,E}, c_{1,F}, c_{2,E}(\cdot), c_{2,F}(\cdot)\}$ be the continuation contract that maximizes $E$’s continuation payoff subject to delivering continuation payoff $U_{1,F}(s''_1)$ to $F$ when the state is $s'_1$.

I claim that $U_{1,F}(s'_1) \leq U_{1,F}(s''_1)$.

Suppose not. Suppose $s'_1$ is privately observed by $E$. If $E$ misreports to $v''_1$ and $\Pi''_{1,E}$ then the reported state is $s''_1$ and the continuation payoff promised to $F$ is $U''_{1,F}$ which, by assumption, is strictly smaller than what is promised to $F$ if $E$ tells the truth. Now consider the continuation contract $\{c^*_{1,E}, c^*_{1,F}, c^*_{2,E}(\cdot), c^*_{2,F}(\cdot)\}$. If $E$ had told the truth, then $F$ would value this continuation contract at $U_{1,F}(s''_1)$. However, because I assume that $\Pi'_{1,F}(\hat{k}_1, \delta) = \Pi''_{1,F}(k_1, 0)$ for all $\delta \leq v'_1 - v''_1$, even if $E$ misreports to $v''_1$, $F$’s valuation of the continuation contract is unchanged. Thus, by misreporting to $s''_1$ and then renegotiating the continuation contract to $\{c^*_{1,E}, c^*_{1,F}, c^*_{2,E}(\cdot), c^*_{2,F}(\cdot)\}$, $E$ is made strictly better off and $F$ thinks he is equally well-off. Now just tweak $\{c^*_{1,E}, c^*_{1,F}, c^*_{2,E}(\cdot), c^*_{2,F}(\cdot)\}$ slightly so that $F$
gets slightly more than before, and I have shown that the contract is not renegotiation-proof. Contradiction.

Fix a Pareto-optimal contract and define the following constant:

\[ D_1 := \inf \{ s_1 \mid U_{1,F}(s_1) < U_{1,F}(s) \} \]

**Step 2.** \( E \) and \( F \) weakly prefers the contract described in the theorem with the above \( D_1 \) to the Pareto-optimal contract.

Fix a distribution \( \pi_1 \in \Pi_{1,F}(k_0) \). For every \( v_1 < D_1 \), move all the weight \( \pi_1 \) puts on \( v_1 \)-states to a \( v_1 \)-state where \( F \)'s belief function is trivial (i.e. for all \( k_1 > 0, \Pi_{2,F}(k_1) = \Delta(s_1) \)). Fix an \( \epsilon > 0 \). For every \( v \geq D_1 \), move all the weight \( \pi_1 \) puts on \( v \)-states with \( U_{1,F} > D_1 + \epsilon \) to a \( v' \)-state where \( U_{1,F} \leq D_1 + \epsilon \) where \( v' \geq v \). Step 1 implies this is possible. Call the modified distribution \( \pi_1' \). Because \( F \) has preference-for-robustness, lower bound, \( \pi_1' \in \Pi_{1,F}(k_0) \).

Fix any state \( s \) with liquidation value \( v_1 < D_1 \). By assumption, if \( U_{1,F}(s) < U_{1,F}(s) \) then \( U_{1,F} \geq D_1 > v_1 \). On the other hand, \( U_{1,F}(s) \geq v_1 \). Thus, \( U_{1,F}(s) \geq v_1 \). If, furthermore, \( F \)'s belief function is trivial, then \( U_{1,F}(s) = U_{1,F}(s) = v_1 \). This implies that the value of the portion of the Pareto-optimal contract where \( v_1 < D_1 \) weakly decreases moving from \( \pi_1 \) to \( \pi_1' \). It is clear that the value of the portion of the Pareto-optimal contract where \( v_1 \geq D_1 \) weakly decreases moving from \( \pi_1 \) to \( \pi_1' \). Thus, the value of the Pareto-optimal contract weakly decreases moving from \( \pi_1 \) to \( \pi_1' \).

Similarly, the value of the contract described in the theorem weakly decreases moving from \( \pi_1 \) to \( \pi_1' \). Moreover, the value of the contract described in the theorem plus \( \epsilon \) is weakly larger than the value of the Pareto-optimal contract under \( \pi_1' \). Letting \( \epsilon \) tend to zero implies that \( F \) weakly prefers the contract described in the theorem to the Pareto-optimal contract.

Next, look at \( E \). For every \( v_1 < D_1 \), there is a \( v_1 \)-state \( s_{v_1} \) where \( U_{1,E}(s_{v_1}) = U_{1,F}(s_{v_1}) \) and, consequently, \( E \) gets nothing. For every \( v_1 \geq D_1 \), there is a \( v_1 \)-state \( s_{v_1} \) where \( E \) gets at most \( v_1 - D_1 \). Fix a distribution \( \pi_1 \in \Pi_{1,E}(k_0) \). For every \( v_1 \), move all the weight \( \pi_1 \) puts on \( v_1 \)-states with \( U_{1,E} > (v_1 - D_1) \wedge 0 \) in the Pareto-optimal contract to \( s_{v_1} \). Call the modified distribution \( \pi_1' \). \( \pi_1' \in \Pi_{1,E}(k_0) \). The value of the Pareto-optimal contract weakly decreases moving from \( \pi_1 \) to \( \pi_1' \). Define a similar modified distribution \( \pi_1'' \) for the contract described in the theorem. The value of the contract described in the theorem weakly decreases moving from \( \pi_1 \) to \( \pi_1'' \). Moreover, the value of the contract described in the theorem under \( \pi_1'' \) is weakly larger than the value of the Pareto-optimal contract under \( \pi_1' \). Thus, \( E \) weakly prefers the contract described in the theorem.

**Step 3.** The contract described in the theorem is a renegotiation-proof truth-telling contract.

Fix a state with \( v_1 < D_1 \). Then \( E \) gets nothing and the only states that \( E \) can (mis)report to are those where the contract maximizes \( F \)'s continuation payoff subject to \( E \) getting nothing. Thus, there is no way he can (mis)report to a state and renegotiate the contract to make himself get more than nothing.

Fix a state with \( v_1 \geq D_1 \). Then \( E \) gets to maximize his payoff subject to delivering \( D_1 \) to \( F \). Clearly, \( E \) cannot profitably misreport to a state with \( \hat{v}_1 < D_1 \). If he (mis)reports to
any other state, he still has to deliver the same continuation payoff $D_1$ to $F$. Moreover, he can only make $F$ weakly more pessimistic about the project by misreporting. Again, there is no way he can (mis)report to a state and renegotiate the contract to make himself strictly better off.

**Definition.** A debt contract consists of the following two objects:

1. A date 0 incomplete contract where the feasible split of $v_1$ that serves as the date 1 outside option takes the form, $(v_1 - v_1 \land D_1, v_1 \land D_1)$.

2. The date 1 renegotiation protocol that has $E$ make a take-it-or-leave-it offer when $v_1 \geq D_1$ and has $F$ make a take-it-or-leave-it offer when $v_1 < D_1$.

**Corollary 3.** Any Pareto-optimal allocation can be implemented by a debt contract.

Notice this implementation is more specific than the ones that have appeared before in that it specifies a particular renegotiation protocol. This is no mistake. Unlike before, altering the allocation of bargaining power under asymmetric information can seriously affect incentive-compatibility and the value of the contract. For example, letting $F$ make take-it-or-leave-it offers when $v_1 \geq D_1$ will clearly induce $E$ to pretend to be pessimistic about the future by misreporting a low belief function $\hat{\Pi}_{1,E}$. More seriously,

**Corollary 4.** Fix an arbitrary debt contract and change the renegotiation protocol to any one where $E$ makes a take-it-or-leave-it offer when $v_1 < D_1$. Then the modified debt contract does not always induce truth-telling. Moreover, the payoff of the contract to $F$ is zero.

**Proof.** Under the proposed renegotiation protocol, for any $v_1 > 0$, there are $v_1$-states $s_{v_1}$ such that $U_{1,F}(s_{v_1}) < U_{1,F}(s_{v_1})$ and $U_{1,F}(s_{v_1}) \leq v_1$. Step 1 of the proof of Theorem 3 then implies that for every $v'_1 \geq v_1$, there is a $v'_1$-state $s_{v'_1}$ where $U_{1,F}(s_{v'_1}) \leq U_{1,F}(s_{v_1}) \leq v_1$. Thus, the contract is worth at most $v_1$ to $F$. Since $v_1$ is arbitrary, the contract is worth 0 to $F$.

I emphasize that if there was perfect information or if, equivalently, $E$ were somehow forced to truthfully report the date 1 state of the world, then one can change the debt contract’s renegotiation protocol to any other and the ex-ante value of the contract to both parties would be unaffected. The importance of allocating power specifically in the way that is described above comes purely from the need to preserve incentive-compatibility. Thus, I am able to show how the state-contingent allocation of control rights induced by debt emerges as an optimal response to information asymmetry without resorting to ad-hoc assumptions about contractual incompleteness.

In related work, Townsend (1979) and Gale-Hellwig (1985) study security design under asymmetric information with costly state verification. They show in a standard expected-utility setting how debt might also emerge as the optimal contract. However, there are limitations to the result. Both parties must be risk-neutral and random contracts are disallowed. Moreover, the result does not extend to a dynamic setting and the contract is not renegotiation-proof. As is well-known, these assumptions are not mild. For example, Mookherjee and Png (1989) shows that if random state verifications are allowed then the optimal contract looks drastically different from debt. In contrast, my debt result is derived in a dynamic setting, focusing on renegotiation-proof contracts, where players can be risk-averse and contracts can be random.
Future Directions

In this paper I introduced a baseline notion of preference-for-robustness and then looked at two special cases and one extension. There are obviously many more natural extensions and special cases that one can consider, and some of them may lead to surprising new insights. Another direction is to keep the preference-for-robustness as is and make the model richer. For example, one can introduce moral hazard or multilateral financing contracts. It would be interesting to see how the optimal debt contract in the asymmetric information model changes under these enrichments.

References


