Waiting for Affordable Housing in New York City*

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Abstract

The purpose of this paper is to develop and estimate a new dynamic equilibrium model that captures housing markets for low- and moderate-income households and is consistent with the key supply restrictions and search frictions that arise in the markets for public and affordable housing. We provide conditions that guarantee that a unique rationing stationary equilibrium exists and characterize the properties of the wait list. We estimate the model using data from Manhattan collected by the Housing Vacancy Survey in 2002 and 2011. We find that having access to public housing increases household welfare for up to 16 percent. The estimated average time on the wait list for Manhattan is approximately 18 years. The search frictions in the market for rent stabilized imply an average search time of 4 years. Finally, we study the welfare effects of policies aimed at reducing the mismatch in public housing by strictly enforcing eligibility criteria. We show that these policies can lead to a 25 percent decline in the average time on the wait list.

Keywords: Affordable Housing, Excess Demand, Rationing, Search Frictions, Queuing, Welfare Analysis, Stationary Equilibrium.

JEL classification: C33, C83, D45, D58, H72, R31.
1 Introduction

Providing adequate housing and shelter for low- and moderate-income households has been a stated policy goal of most federal, state, and city administrations in the United States since the passage of the Public Housing Act of 1937. There are clear and documented benefits associated with living in high-quality housing communities. Benefits for low-income families, that are typically related to place, include improved housing quality, increased safety, improved property management, better access to neighborhood amenities, and improved mental health from a reduction in stress.\(^1\) Of course, public housing communities have a complicated and sometimes notorious history in the United States, and the potential gains have not always materialized for a variety of reasons such as poor design of communities, neglect, corruption, and mismanagement. Despite its spotty history public housing remains a highly demanded commodity, especially in the most expensive metropolitan areas.

Low-income households are eligible for public housing assistance if their income is below a threshold that depends on household composition and region. The rent charged for public housing is a fixed percentage of household income. Hence, there is no price mechanism to ensure that public housing markets clear. Since demand for public housing vastly exceeds the available supply in many U.S. cities, there are long wait lists to get into public housing communities. Despite the importance of public housing for low-income households, there are few compelling dynamic models that allow us to study housing choices of low- and moderate-income households. The main purpose of this paper is to fill this gap. We develop and estimate a new dynamic equilibrium model that is consistent with the observed market search frictions, the existence of long queues for public housing, and the need to search for a long time to obtain access to affordable housing.\(^2\)

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\(^1\)Whether or not low-income families have benefited economically or educationally is contested. Similarly, there is mixed evidence of the benefits of moving low-income households to more mixed-income neighborhoods. We discuss some of the evidence in detail below.

\(^2\)Geyer and Sieg (2013) consider a static model of public housing with myopic households and provide a
A compelling model needs to capture the dynamic incentives faced by low- and moderate-income households. If households are forward looking, they understand that they will only receive an offer to move into public housing when they reach the top of the wait list. Moreover, it is current policy of most housing authorities to not evict households who have lost their eligibility for housing aid. Public housing thus provides partial insurance against negative income shocks. We show that these incentives of current housing policies give rise to a large degree of mismatch of low- and moderate-income households in housing markets.

To quantify the importance of mismatch in housing markets, we consider the rental and public housing markets in New York City (NYC). Our application thus focuses on one of the most expensive housing markets, that also has the largest stock of public housing among all U.S. cities. In addition, NYC is the only large city in the United States that has strong rent control laws. More than one million households currently live in rent regulated housing units. To understand the effects of public housing policies, our model, therefore, needs to captures the existence of three different housing markets: publicly provided housing, rent-stabilized private housing, and unregulated private housing.

We assume that the unregulated private housing rental market is frictionless. Households can purchase any given level of quantity (or quality) of housing given the prevailing market price. The rental price for stabilized housing is significantly lower than the equivalent price in the unregulated market. Since the demand for rent-stabilized units also exceeds the supply, there are significant frictions in the rent-stabilized market. In contrast to public housing, where rationing is achieved by placing households on wait lists, the key friction here is that finding a rent-stabilized apartment involves significant search efforts as well as luck.\(^3\) We capture these market frictions by endogenizing the probability that a household who is actively detailed discussion of the related literature.

\(^3\)Our paper is also related to search and matching models that have been applied to study housing markets. See, for example, Wheaton (1990), Krainer (2001), or Albrecht, Anderson, Smith, and Vroman (2007). Most of the papers in this literature focus on the markets for owner occupied housing which are distinctly different from affordable rental markets that are the focus of this paper.
searching for rent stabilized housing, will receive an offer to move into a stabilized unit. The length of the wait list for public housing, the probability of finding a rent-stabilized unit, and the price for private housing are, therefore, all endogenously determined in equilibrium of our model.\footnote{Our modeling approach is thus consistent with Glaeser and Luttmer (2003) who show that rent controls lead to misallocations in housing markets. Potential misallocations arise in our model due to search frictions and the existence of public housing.}

We define and characterize a stationary equilibrium of our dynamic model. The equilibrium has the property that everybody in the highest priority group for public housing receives an offer to move into public housing. In addition, a fraction of the households with the second highest priority also get an offer. The discreteness of the priority score effectively partitions the demand for public housing into a finite number of cohorts. To smooth out the flow of households into public housing and equate the inflow with the voluntary flow of households out of public housing, the housing authority needs to randomize among households with the second highest priority score. Of course, all households in the second highest priority group that lose the lottery will obtain an offer in the next period.

In equilibrium low-income household prefer to live in public housing due to the large rent subsidy, which implies a large increase in numeraire consumption, and the relatively high quality of these housing units. Rent stabilized housing appeals to a large range of low- and moderate-income households due the significant rental price discount relative to the unregulated market. Higher income households prefer to rent in the unregulated market since sufficiently large housing units are only available in that market. Due to the existence of rationing in public housing and search frictions in rent stabilized housing a fraction of low- and moderate-income households must also rent in the unregulated market in equilibrium. Due to the no-eviction policy of the housing authority, a significant fraction of higher income households live in public housing. Our model can, therefore, not only generate long wait lists, but is also consistent with observed mismatch in public and rent-stabilized housing markets.
Households not only differ by income, but also by a variety of other characteristics such as family size, race, and gender of the household head. We, therefore, extend our model and capture these differences using discrete household types. Discrete types also allow us to capture differences in preferences over public housing and differences in access to rent stabilized units.\(^5\)

We show that the parameters of the model can be identified based on the observed outcomes in the data. Our proofs of identification are constructive and can be used to define a method of moments estimator. This estimator matches the sorting of households by income and family type among housing options as well as the average time spent in different housing markets. The estimator also matches the average rental payment for each housing type.

Our empirical analysis is based on the 2002 and 2011 samples of the New York City Housing Vacancy Survey (NYCHVS). This survey provides comprehensive data about household characteristics and housing characteristics. In particular, we observe household income and family status, the time that the household spend in the housing unit, as well as a large number of structural characteristics of the housing unit that the household occupies. Most importantly, it allows us to classify households as living in public housing, rent-stabilized housing, or private housing. We implement our estimator focusing on Manhattan since wait lists in NYC are determined at the borough level.\(^6\)

The data show that approximately 10 percent of our sample of low- and moderate-income households live in public housing communities in 2011. 58 percent of households live in rent stabilized units, while 33 percent rent in the unregulated housing market market in Manhattan. At the time of the survey, households have spent, on average, 16 years in public housing, 9.5 years in regulated housing and only 4 years in private housing. Not surprisingly households in public housing are much poorer than households in rent stabilized

\(^5\)This approach could also be used to capture the fact that some households may not consider public housing as a desirable housing choice due to stigma as suggested by Moffitt (1983).

\(^6\)As a robustness check we also estimate our model using all five boroughs in NYC.
and unregulated housing. There are significant frictions in the rent-stabilized market.

We then estimate the structural parameters of the model. We find that our model fits the sorting of households by income among the three housing options. Our model is consistent with the well-known fact that public housing is particularly popular with black, female-headed households. Our model captures differences in rental prices as well as time spent in the housing units. We find that rental prices for stabilized housing are approximately 50 percent of the price in the unregulated market in Manhattan. Thus there is a significant discount which explains the popularity of rent-stabilized unit. The probabilities of finding a rent-stabilized unit is approximately 27 percent per year. Finally, the average quality of rent stabilized units is approximately 30 percent higher than units in public housing in Manhattan.

Overall, public housing communities are attractive options for a large income range of households. We find that welfare of households in public housing exceeds welfare of similar households that must rent in the private market by up to 16 percent since both housing and numeraire consumption are significantly higher. Not surprisingly, there is an high excess demand for public housing is captured in our estimates for the length of the wait list for public housing. We find that average wait time for public housing is 18 years for Manhattan.

One surprising feature of existing public housing policies is that housing authorities rarely ask households to leave public housing once their income exceeds the eligibility threshold. Housing authorities do not evict households from public housing. 17 percent of households living in public housing in Manhattan have income that is larger than 80 percent of the median income. For these types of households housing aid is de facto an open-ended entitlement program.

Our counterfactual experiment therefore considers a policy that restricts access to housing aid to households making less than the median income. We find that such a policy has some interesting distributional effects. The main losers of this policy are households that are currently living in public housing. Their welfare is significantly reduced for two reasons.
First, households that lose eligibility need to move out of public housing and rent in the private or rent stabilized markets. Second, currently eligible households may lose eligibility in the future if they experience a sequence of positive income shocks.

The main beneficiaries of this policy are low-income household that are currently placed on the wait list. Enforcing eligibility criteria creates more vacancies and thus significantly reduces the expected time spent on the wait list. Hence needy households are more likely to obtain access to public housing. Welfare also increases for ineligible households outside of public housing, since public housing becomes more readily available and provides limited insurance against a sequence of negative income shocks.

Our worked is related to a substantial literature that has analyzed policies aimed at providing affordable housing for low-income households. A perceived lack of supply of adequate housing and shelter for low-income households has received much attention among policy makers. For example, the Blasio administration in NYC has announced an aim to significantly increase the supply of affordable housing in NYC. As pointed out by Olsen (2003) one justification for housing subsidies to low-income households is that most taxpayers want to help households in poverty, but feel that, at least, some low-income households undervalue housing. In recent years, proponents of housing subsidies have frequently argued that the primary housing problem of low-income households is an excessive rent-income ratio rather that inadequate housing. Rent stabilization programs directly try to address this concern.

Low-income housing programs can be justified due the potential negative externalities to public health and safety that result from low-cost, high-density housing neighborhoods for poor families. Moreover, children tend to suffer even more than adults from poor neighborhoods with inadequate housing and shelter. These children grow up with less education and lower earning power. They are more likely to have drug addiction, psychological trauma and disease, or wind up in prison.\footnote{For detailed discussion of the literature see, among others, Cutler, Glaeser, and Vigdor (1999) and Oreopoulos (2003).}
There is, however, mixed evidence suggesting negative spill-over effects (such as higher crime rates and lower educational achievement) associated with living in publicly provided housing communities as discussed in Currie and Yelowitz (2000). In contrast Jacob (2004) finds that there are very few positive effects associated with moving out of the projects in Chicago using a variety of different outcomes. Kling, Liebman, and Katz (2007) find that moving to lower poverty neighborhoods improved physical and mental health but produced mixed outcomes for children’s behavior and had little impact on employment outcomes. If public housing is inferior to privately supplied housing an argument can be made for subsidizing the supply of privately provided affordable housing. As detailed in Erickson and Rosenthal (2011), the Low Income Housing Tax Credit program was created in 1986 as part of the Tax Reform Act of 1986 as an alternative to public housing. They find, however, that this program has failed to result in new construction that serves the population served by public housing, largely due to crowd-out effects. Moreover, it is well understood that rent control and stabilization programs are rather crude policy tools creating misallocation in housing markets. Glaeser and Luttmer (2003) find that 21 percent of New York apartment renters live in units with more or fewer rooms than they would if they rented in the private market in 1990.

The rest of the paper is organized as follows. Section 2 provides an overview affordable housing policies in NYC and discusses our data. Section 3 provides our new dynamic model of affordable housing markets. Section 4 discusses identification and estimation of the parameters of our model. Section 5 presents our empirical findings. Section 6 reports the findings from our welfare analysis and considers alternative housing policies. Section 7 offers our conclusions.

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8Earlier work on rent control include Olsen (1972) and Suen (1989). Gyourko (1989) provide an empirical analysis on NYC markets.
2 Affordable Housing in NYC

2.1 Institutional Background

The New York City Housing Authority (NYCHA) provides public housing and administers Section 8 housing vouchers for low- and moderate-income residents throughout the five boroughs of New York City. Households whose income does not exceed 80% (50%) of median income are eligible for the public housing program (voucher program). In addition, income limits are function of family size. For example, in 2011 the income limit for a single person household was $45,850 ($28,500) while it was $65,450 ($40,900) for a family of four.

Applications for public housing are assigned a priority code based upon information such as employment status, income, family size, and quality of previous residence provided. Households are then placed on the housing authority’s preliminary waiting list for an eligibility interview. Households are required to update or renew their applications every two years if they have not been scheduled for an interview. Upon passing the interview and background checks, applicants are then placed on a (borough wide) waiting list.

More than 403,000 New Yorkers reside in NYCHA’s 177,666 public housing apartments across the city’s five boroughs. Another 235,000 residents receive subsidized rental assistance in private homes through the NYCHA-administered Section 8 program. The NYCHA reported that 270,201 families were on the waiting list for conventional public housing and 121,356 families on the waiting list for Section 8. Little is know about the annual flows. The NYT reported on July 23, 2013 that “the queue moves slowly. The apartments are so coveted that few leave them. Only 5,400 to 5,800 open up annually.” As of December 10, 2009 NYCHA stopped processing any new Section 8 applications due to the long waiting list. As consequence, there is almost no mobility in and out of Section 8 housing markets. We, therefore, treat Section 8 housing as a completely separate market and focus on public housing in this paper.
Private housing markets have also been heavily regulated in NYC since the 1930’s. The stock of rent-regulated units includes a relatively small number of rent controlled units - approximately 38,000 - but a much larger number of rent-stabilized units. Rent control primarily affects old units. As of 2011 over one million units were rent-stabilized representing roughly 47 percent of the rental housing stock in NYC.

Rent stabilization generally applies to buildings of six or more units built between February 1, 1947 and December 31, 1973, and to those units that have exited from the rent-control program. Approximately 8 percent of the city’s stabilized units and nearly all stabilized units in buildings constructed after 1974 were voluntarily subjected to rent stabilization by their owners in exchange for tax incentives from the city. Under the 421-a program developers currently have to set aside 20 percent of new apartments for poor and working-class tenants to receive tax abatements lasting 35 years.\(^9\)

Involuntarily stabilized units, representing 92 percent of the stabilized stock, are regulated based on a “housing emergency” declared by the city in 1974 and renewed every three years since. Under New York States Rent Stabilization Law, the city may declare a housing emergency whenever the city’s rental vacancy rate drops below five percent. This law was most recently renewed in June 2015 and affects units with a maximum rent of $2700. Rent stabilization sets maximum rates for annual rent increases. It also entitles tenants to have their leases renewed. The rent guidelines board meets every year to determine how much the landlord can set future rents on the lease.

2.2 Data

The empirical analysis is based on the New York City Housing Vacancy Survey (NYCHVS) in 2002 and 2011. The main advantage of this data set is that it matches household with units, i.e. it contains detailed information about household characteristics and housing char-

\(^9\)The de Blasio administration has been pushing to increase that fraction to 35 percent.
acteristics.

We focus on affordable housing for low- and moderate-income households. As a consequence, we have adopted three sample restrictions. First, we drop households whose average incomes exceed 200% of median income level. This sample restrictions can be motivated by the fact that high-income households are likely to own a condominium or a house and, therefore, face a different choice set as the vast majority of low- and moderate-income households that live in NYC.\(^{10}\) Second, we drop all low-income households that receive vouchers since that market has been closed for at least 6 years. Finally, we drop all households not living in Manhattan since wait lists are operated at the borough level in NYC. These restrictions reduce our sample size to 1557.

Table 1: Descriptive Statistics

<table>
<thead>
<tr>
<th></th>
<th>housing type</th>
<th>rent share</th>
<th>number of years</th>
<th>income head</th>
<th>female</th>
<th>kids</th>
<th>working family</th>
</tr>
</thead>
<tbody>
<tr>
<td>2002</td>
<td>Public</td>
<td>0.10</td>
<td>13.84</td>
<td>26976</td>
<td>0.65</td>
<td>0.95</td>
<td>0.63</td>
</tr>
<tr>
<td></td>
<td>Regulated</td>
<td>0.76</td>
<td>1141</td>
<td>53017</td>
<td>0.54</td>
<td>0.32</td>
<td>0.87</td>
</tr>
<tr>
<td></td>
<td>Unregulated</td>
<td>0.14</td>
<td>2512</td>
<td>62685</td>
<td>0.51</td>
<td>0.19</td>
<td>0.86</td>
</tr>
<tr>
<td>2011</td>
<td>Public</td>
<td>0.10</td>
<td>16.18</td>
<td>32930</td>
<td>0.73</td>
<td>0.92</td>
<td>0.70</td>
</tr>
<tr>
<td></td>
<td>Regulated</td>
<td>0.58</td>
<td>1317</td>
<td>54739</td>
<td>0.53</td>
<td>0.38</td>
<td>0.83</td>
</tr>
<tr>
<td></td>
<td>Unregulated</td>
<td>0.33</td>
<td>2640</td>
<td>71045</td>
<td>0.54</td>
<td>0.17</td>
<td>0.87</td>
</tr>
</tbody>
</table>

Source: New York City Housing Vacancy Survey 2002 and 2011

Rents and income in 2002 are converted to 2011 dollar using CPI.

A household is defined as working if the labor income share is higher than 50 percent of total income.

Regulated units include rent-stabilized units, HUD-regulated units, and Michell-Lama rental units.

\(^{10}\)None of the key findings of this paper qualitatively or quantitatively depend on these choices.
Tables 1 provides some descriptive statistics of the Manhattan housing markets for 2002 and 2011 respectively. Table 1 shows that a large fraction of the rental units in Manhattan are under rent stabilization. The fraction was 76 percent in 2002 and declined to 58 in 2011. At the same time, the average rent increased from $2512 to $2640 in the unregulated market and from $1141 to $1317 in the regulated market.

We also find that households tend to stay for long periods in their apartments. The average number of years that a household has lived in the current apartment is 16.18 for public housing and 9.49 for rent stabilized housing in 2011. The turnover is much higher in private housing markets. Not surprisingly households in public housing are much poorer than household in rent stabilized and unregulated private housing. Families in public housing tend to single parent households, the majority headed by a female. Public housing families also tend to have more children than households in rent-stabilized or private housing.

We have also have detailed data on income categories. We define a household as working if the labor income share is higher than 50 percent of total income. Not surprisingly we find that the fraction of working households is much higher in private and regulated housing than in public housing. Nevertheless, more than 63 (70) percent of all households in public housing receive a significant fraction of their income from labor income in 2002 (2011).

3 A Dynamic Model of Housing Markets

3.1 Supply

We consider a local housing market with three housing options: public housing \((p)\), rent-regulated housing \((r)\), and housing provided by the unregulated market \((m)\). The exogenous housing supply in public and rent regulated housing are given by \(k_p\) and \(k_r\). The assumption of fixed supply of public and rent stabilized housing is appropriate for NYC. There has been few recent construction of new housing communities in NYC. If anything, the supply of rent
stabilized housing has declined in the past decades.\textsuperscript{11} We can, therefore, treat supply as price inelastic and fixed in the short run.

3.2 The Unregulated Private Housing Market

Time is discrete, $t = 0, ..., \infty$. Households are infinitely lived and forward looking. Households have a common discount factor $\beta$ and maximize expected lifetime utility. In the baseline model, households only differ by income, denoted by $y$, which evolves according to a stochastic law of motion that can be described by a stationary Markov process with transition density $f(y'|y)$. Below we extend our model to allow for additional sources of household heterogeneity.

Household flow utility is defined over housing quality, $h$, and a numeraire good, $b$. Consider a household that rents in the private market. Housing services can be purchased at price $p_m$.\textsuperscript{12} Flow utility is, therefore, given by:

$$ u_m(y) = \max_{h,b} U(b, h) $$

s.t. $p_m h + b = y$

Note that we are imposing the realistic assumption that low and moderate–income households do not save and cannot borrow against uncertain future income. They are liquidity constrained and spend their income on housing and consumption goods in each period.

\textsuperscript{11}The New York Times (NYT) reported on July 23, 2015 that the de Blasio administration “had lined up financing for more than 20,000 affordable apartments - about 8,500 to be newly built and 11,800 preserved - through deals with landlords to lock in low rents for decades. That is an aggressive pace. Not since 1989, when a decade-long program begun under Mayor Edward Koch was transforming rubbly mountains of blight into miles of solid apartment blocks, has the city achieved so much in a single year.”

\textsuperscript{12}We implicitly assume that private housing supply is perfectly elastic at price $p_m$. This assumption can be easily relaxed to endogenize the price of housing in the private market by allowing for an upward sloping supply function.
3.3 The Rent Regulated Private Housing Market Market

The flow utility associated with a rent regulated unit of quality $h_r$ and price $p_r < p_m$ is given by:

$$u_r(y) = U(y - p_r h_r, h_r)$$  \hspace{1cm} (2)

The next assumption captures the search frictions in that market.

**Assumption 1**

*Each period, there is a positive probability $q_r$ that a household receives an offer to move into a rent regulated unit of quality $h_r$.***

The probability of receiving an offer to move into a stabilized housing unit is endogenous and depends on the voluntary outflow from regulated housing as discussed below in detail.

3.4 Public Housing

Public housing provides a constant level of housing consumption, $h_p$, and taxes individual at constant rate $\tau$. Per period utility in public housing is, therefore, given by:

$$u_p(y) = U((1 - \tau)y, h_p)$$  \hspace{1cm} (3)

The local housing authority that administers the public housing program manages a wait list. The priority score of a household is a monotonic function of the time spent on the wait list. More formally, let $w$ denote the time that a household has been on the wait list. Let $p(w)$ denote the probability that a household that has been on the wait list for $w$ periods receives an offer to move into public housing. The next assumption captures the behavior of the housing authority.

**Assumption 2**

*a) The housing authority makes take it or leave it offers, i.e. if a household rejects an offer, it...*
will go the end of the wait list \((w = 0)\).

b) The outflow of public housing is voluntary, i.e. the housing authority does not evict households from public housing.

c) Eligibility is determined by an income cut-off, denoted by \(\bar{y}\) and is checked every time period. Loss of eligibility means that the household is removed from the wait list \((w = 0)\).

This assumption is uncontroversial and reflects common practice of housing authorities in NYC and other U.S. metropolitan areas. Note that the distribution of priority scores is endogenous and determined in equilibrium as we discuss below.

### 3.5 Timing of Decisions

The timing of decisions is as follows:

1. Each household gets a realization of income which determines the income distributions at the beginning of the period.

2. Some households get an offer to move into public housing generated with probability \(p(w)\).

3. Some households get an offer to move into rent-regulated housing generated with probability \(q_r\).

4. Households decide to move and obtain the flow utility that depends on their decisions.

5. Wait times are updated.

Note that utility is realized after households have relocated.
3.6 Value Functions and Decision Rules

The two state variables in this model are wait time, \( w \), and income, \( y \). Define the conditional value functions associated with the three choices:

\[
\begin{align*}
\quad v_p(y) &= u_p(y) + \beta \int V_p(y') f(y'|y) \, dy' \\
\quad v_m(y, w) &= u_m(y) + \beta \int V_m(y', w) f(y'|y) \, dy' \\
\quad v_r(y, w) &= u_r(y) + \beta \int V_r(y', w) f(y'|y) \, dy'
\end{align*}
\]

We can derive recursive expressions for the unconditional value functions. The value function of a household with characteristics \( (w, y) \) that rents in the regulated market is given by:

\[
\begin{align*}
V_r(y, w) &= p(w) \, 1 \{y \leq \bar{y}\} \, \max \{v_p(y), v_m(y, 0), v_r(y, 0)\} \\
&\quad + (1 - p(w)) \, 1 \{y \leq \bar{y}\} \, \max \{v_m(y, w + 1), v_r(y, w + 1)\} \\
&\quad + 1 \{y > \bar{y}\} \, \max \{v_m(y, 0), v_r(y, 0)\}
\end{align*}
\]

The value function of a household with characteristics \( (w, y) \) that rents in the private market is then given by:

\[
\begin{align*}
V_m(y, w) &= q_r \, V_r(y, w) \\
&\quad + (1 - q_r) \, p(w) \, 1 \{y \leq \bar{y}\} \, \max \{v_m(y, 0), v_p(y)\} \\
&\quad + (1 - q_r) \, (1 - p(w)) \, 1 \{y \leq \bar{y}\} \, v_m(y, w + 1) \\
&\quad + (1 - q_r) \, 1 \{y > \bar{y}\} \, v_m(y, 0)
\end{align*}
\]

Finally, the value function of a household living in public housing satisfies:

\[
\begin{align*}
V_p(y) &= (1 - q_r) \, \max \{v_p(y), v_m(y, 0)\} \\
&\quad + q_r \, \max \{v_p(y), v_m(y, 0), v_r(y, 0)\}
\end{align*}
\]

These value functions determine the optimal decision rules for each household.
Figure 1 plots the policy function for household in the public housing that do not face eligibility cut-off. The blue vertical line indicates the income eligibility threshold for public housing. Optimal decision rules can be characterized by thresholds. Low-income households prefer to live in public housing, moderate-income households prefer rent-regulated housing while higher income households prefer renting in the private market.

Figure 2 shows the decision rule for a household that has been on the wait list for 5 periods and does not receive an offer to move into public housing. Here we find that low- and high-income households prefer the private markets while moderate-income households prefer units in the rent-regulated market. This results is due to the fact the we have set the quality of housing in the regulated market exceeds the quality in public housing.

### 3.7 A Stationary Equilibrium

Let $g_m(w)$ ($g_r(w)$) denote the marginal distribution of wait times for households in private (rent regulated) housing in stationary equilibrium. Let $g_p(y)$ denote the density of income
of households that are inside public housing at the beginning of each period (before households have moved). Similarly let $g_m(y|w)$ ($g_r(y|w)$) denote the stationary density of income conditional on wait time for households in the private (regulated) market.

The voluntary flow of households out of public housing is given by:

$$
OF_p = k_p (1 - q_r) \int 1\{v_m(y,0) > v_p(y)\} \ g_p(y) \ dy \\
+ k_p \ q_r \ \int 1\{v_m(y,0) \geq \max[v_p(y), v_r(y,0)]\} \ g_p(y) \ dy \\
+ k_p \ q_r \ \int 1\{v_r(y,0) \geq \max[v_p(y), v_m(y,0)]\} \ g_p(y) \ dy
$$

Note that the first two terms is the outflow to the private market and the third term captures the outflow to the rent regulated market. The flow into public housing is given by:

$$
IF_p = k_m \sum_{j=0}^{\infty} p(w_j) \ g_m(w_j) \ IF_{mp}(w_j) \\
+ k_r \sum_{j=0}^{\infty} p(w_j) \ g_r(w_j) \ IF_{rp}(w_j)
$$
where the inflow from the private market conditional on wait time is:

\[ IF_{mp}(w_j) = (1 - q_r) \int_{y \leq \bar{y}} 1\{ v_p(y) \geq v_m(y, 0) \} \ g_m(y|w_j) \ dy \]

(10)

\[ + \ q_r \int_{y \leq \bar{y}} 1\{ v_p(y) \geq \max[v_m(y, 0), v_r(y, 0)] \} \ g_m(y|w_j) \ dy \]

and the inflow from the rent regulated market is given by:

\[ IF_{rp}(w_j) = \int_{y \leq \bar{y}} 1\{ v_p(y) \geq \max[v_m(y, 0), v_r(y, 0)] \} \ g_r(y|w_j) \ dy \]

(11)

Similarly, the voluntary flow of households out of rent regulated housing is given by:

\[ OF_r = k_r \sum_{j=0}^{\infty} p(w_j) \ g_r(w_j) \int_{y \leq \bar{y}} 1\{ v_r(y, 0) \leq \max[v_p(y), v_m(y, 0)] \} \ g_r(y|w_j) \ dy \]

(12)

\[ + \ k_r \sum_{j=0}^{\infty} (1 - p(w_j)) \ g_r(w_j) \int_{y \leq \bar{y}} 1\{ v_m(y, w_j + 1) \geq v_r(y, w_j + 1) \} \ g_r(y|w_j) \ dy \]

\[ + \ k_r \sum_{j=0}^{\infty} g_r(w_j) \int_{y > \bar{y}} 1\{ v_m(y, 0) \geq v_r(y, 0) \} \ g_r(y|w_j) \ dy \]

Note that the first term is the outflow to public housing. The second term is the outflow to private housing if you have an offer to move into public housing. The last term is the outflow to private housing if you do not have an offer to move into public housing. The flow into rent regulated housing is given by:

\[ IF_r = k_p q_r \int 1\{ v_r(y, 0) \geq \max[v_m(y, 0), v_p(y)] \} \ g_p(y) \ dy \]

(13)

\[ + \ k_m \sum_{j=0}^{\infty} p(w_j) q_r \int_{y \leq \bar{y}} 1\{ v_r(y, 0) \geq \max[v_m(y, 0), v_p(y)] \} \ g_m(y|w_j) \ dy \]

\[ + \ k_m \sum_{j=0}^{\infty} (1 - p(w_j)) q_r \int_{y \leq \bar{y}} 1\{ v_r(y, w_j + 1) \geq v_m(y, w_j + 1) \} \ g_m(y|w_j) \ dy \]

\[ + \ k_m \sum_{j=0}^{\infty} q_r \int_{y > \bar{y}} 1\{ v_r(y, 0) \geq v_m(y, 0) \} \ g_m(y|w_j) \ dy \]

In a stationary equilibrium, the inflow has to be equal to the outflow of households for
Definition 1  A stationary equilibrium for this model consists of the following: a) offer probabilities \( p(w) \) and \( q_r \), b) distributions \( g_p(y) \), \( g_m(w) \), \( g_r(w) \), \( g_m(y|w) \), and \( g_r(y|w) \), and c) value functions \( V_p(y) \), \( V_m(y, w) \) and \( V_r(y, w) \), such that:

1. Households behave optimally and value functions satisfy the equations above.
2. The housing authority behaves according the administrative rules described above.
3. The densities are consistent with the laws of motion and optimal household behavior.
4. \( p(w) \) satisfies the market clearing condition for public housing:

\[
OF_p = IF_p
\]  \hspace{1cm} (14)

5. \( q_r \) satisfies the market clearing condition for rent regulated housing:

\[
OF_r = IF_r
\]  \hspace{1cm} (15)

Finally note that we can endogenize the price of housing in the private market by assuming that that there is an upward sloping housing supply function \( H^s_m(p_m) \) and by requiring that the demand for private housing given by

\[
H^d_m = (1 - k_p - k_r) \sum_j g(w_j) \int h(p_m, y) g_m(y|w_j) \, dy
\]  \hspace{1cm} (16)

is equal to the supply.

Figure 3 illustrates the stationary equilibrium densities of income for our estimated model. The top panel compares the income distribution of households in the private market with those that are in public housing. Not surprising we find that the income distribution of

\[^{13}\text{The vacancy rate in NYC has been around 2 percent during the time period of interest. Hence we ignore vacancies.}\]
Figure 3: Stationary Distributions
ineligible households (i.e. households with a priority score of zero) have much higher income than those who live in public housing. More surprising is the result that households with a priority score of years have lower income. The lower panel compares the income distribution of households in the rent stabilized market with those in public housing. Again we find similar qualitative patterns. Households that live in stabilized housing with high priority scores look similar to households in public housing.

3.8 Characterizing Stationary Equilibria

Next we characterize the properties of equilibria. The main result is summarized by the following proposition.

Proposition 1 Any stationary equilibrium with excess demand for public housing has the property that there exists a value $\bar{w} < \infty$ such that: $g(\bar{w} + 1 + j) = 0$, for $j \geq 1$, $p(\bar{w} + 1) = 1$, $0 \leq p(\bar{w}) \leq 1$, and $p(\bar{w} - j) = 0$ for all $j \geq 1$.

Proof:
We use a proof by contradiction. Suppose not, then

$$p(\bar{w} + 1) < 1 \quad (17)$$

and next period there exists some household with priority score $\bar{w} + 2$, hence $g(\bar{w} + 2) > 0$ which violates the stationarity definition and the definition of $\bar{w}$.

Suppose that $p(\bar{w} - j) > 0$ and $p(\bar{w}) \leq 1$. This case violates the assumption that offers to households with lower priority ranks can only be made if all households with higher ranks receive offers.

Suppose that $p(\bar{w} - j) > 0$, $p(\bar{w}) = 1$ and $p(\bar{w} + 1) = 1$, then there will be no household in the next period which priority score $\bar{w} + 1$ which violates the stationarity assumption and that and that $g(\bar{w} + 1) > 0$.

Q.E.D.
The equilibrium thus has the property that everybody in the highest priority group obtains an offer to move into public housing. In addition, a fraction of the households with the second highest priority also gets an offer. Those who do not get an offer, obtain an offer in the next period. The intuition for this result is the following. The wait list partitions the potential demand into $\bar{w} + 2$ cohorts. By adjusting $p(\bar{w})$, we can smooth out the fraction of individuals that obtains an offer. Note that $p(w_j)$ is not uniquely defined for $w_j > \bar{w} + 1$. Since the housing authority makers take-it-or-leave-it offers, there will be no households with wait times larger than $\bar{w} + 1$. Without loss of generality, we can, therefore, set $p(\bar{w} + j) = 1$ for all $j > 1$.

Given this equilibrium offer function, the inflow into public housing has two components and is equal to:

$$IF_p = p(\bar{w}) [k_m g_m(\bar{w}) IF_{mp}(\bar{w}) + k_r g_r(\bar{w}) IF_{rp}(\bar{w})]$$

$$+ [k_m g_m(\bar{w} + 1) IF_{mp}(\bar{w} + 1) + k_r g_r(\bar{w} + 1) IF_{rp}(\bar{w} + 1)]$$

To finish the characterization of the equilibrium, we need to provide the laws of motion for the equilibrium densities. Equations (34) - (42) in Appendix A provide the details.

### 3.9 Extensions: Multiple Household Types

We can allow for different discrete types allowing for differences in family structure. Assume that there are $I$ types of households. Household types are defined by family structure (number of kids, number of adults etc.) Each household has a fixed share denoted by $s_i$, where $\sum_{i=1}^{I} s_i = 1$.

**Assumption 3** The housing authority operates one wait list for all types and all types compete for the same housing units in the private and regulated markets.

Let $(k_{ip}, k_{ir}, k_{im})$ denote the relevant type specific market shares. Let $g_{im}(w)$ $(g_{ir}(w))$ denote the marginal distribution of wait times for households of type $i$ in private (rent regulated)
housing in stationary equilibrium. Let \( g_{ip}(y) \) denote the density of income of households of type \( i \) that are inside public housing at the beginning of each period. Similarly let \( g_{im}(y|w) \) \( (g_{ir}(y|w)) \) denote the stationary density of income conditional on wait time for households in the private (regulated) market.

The voluntary flow of type \( i \) households out of public housing is given by:

\[
OF_{ip} = k_{ip} (1 - q_r) \int 1\{v_{im}(y, 0) > v_{ip}(y)\} g_{ip}(y) \, dy \\
+ k_{ip} q_r \int 1\{v_{im}(y, 0) \geq \max[v_{ip}(y), v_{ir}(y, 0)]\} g_{ip}(y) \, dy \\
+ k_{ip} q_r \int 1\{v_{ir}(y, 0) \geq \max[v_{ip}(y), v_{im}(y, 0)]\} g_{ip}(y) \, dy
\]

Note that the first two terms is the outflow to the private market and the third term captures the outflow to the rent regulated market.

The flow into public housing of type \( i \) households is given by:

\[
IF_{ip} = p(\bar{w})[k_{im} g_{im}(\bar{w}) IF_{imp}(\bar{w}) + k_{ir} g_{ir}(\bar{w}) IF_{irp}(\bar{w})] \\
+ [k_{im} g_{im}(\bar{w} + 1) IF_{imp}(\bar{w} + 1) + k_{ir} g_{ir}(\bar{w} + 1) IF_{irp}(\bar{w} + 1)]
\]

where the inflow from the private market conditional on wait time is:

\[
IF_{imp}(w) = (1 - q_r) \int_{y \leq \bar{y}} 1\{v_{ip}(y) \geq v_{im}(y, 0)\} g_{im}(y|w) \, dy
\]

\[
+ q_r \int_{y \leq \bar{y}} 1\{v_{ip}(y) \geq \max[v_{im}(y, 0), v_{ir}(y, 0)]\} g_{im}(y|w) \, dy
\]

and the inflow from the rent regulated market is given by:

\[
IF_{irp}(w) = \int_{y \leq \bar{y}} 1\{v_{ip}(y) \geq \max[v_{im}(y, 0), v_{ir}(y, 0)]\} g_{ir}(y|w) \, dy
\]

Equilibrium in public housing requires that for each housing type \( i \), we have

\[
IF_p = \sum_{i=1}^{I} IF_{ip} = \sum_{i=1}^{I} OF_{ip} = OF_p
\]
Next consider the market for regulated housing. The voluntary flow of type \( i \) households out of rent regulated housing is given by:

\[
OF_{ir} = k_{ir} \sum_{j=0}^{\infty} p(w_j) g_{ir}(w_j) \int_{y \leq \bar{y}} 1\{v_{ip}(y) \geq \max[v_{im}(y,0), v_{ir}(y,0)]\} g_{ir}(y|w_j) \, dy \tag{24}
\]

\[
+ k_{ir} \sum_{j=0}^{\infty} p(w_j) g_{ir}(w_j) \int_{y \leq \bar{y}} 1\{v_{im}(y,0) \geq \max[v_{ip}(y), v_{ir}(y,0)]\} g_{ir}(y|w_j) \, dy
\]

\[
+ k_{ir} \sum_{j=0}^{\infty} (1 - p(w_j)) g_{ir}(w_j) \int_{y \leq \bar{y}} 1\{v_{im}(y, w_j + 1) \geq \max[v_{ir}(y, w_j + 1)]\} g_{ir}(y|w_j) \, dy
\]

\[
+ k_{ir} \sum_{j=0}^{\infty} g_{ir}(w_j) \int_{y > \bar{y}} 1\{v_{im}(y,0) \geq \max[v_{ir}(y,0)]\} g_{ir}(y|w_j) \, dy
\]

Note that the first term is the outflow to public housing. The second term is the outflow to private housing if you have an offer to move into public housing. The last two terms are the outflow to private housing if you do not have an offer to move into public housing.

The flow into rent regulated housing is given by:

\[
IF_{ir} = k_{im} \sum_{j=0}^{\infty} g_{im}(w_j) IF_{imr}(w_j) + k_{ip} IF_{ipr} \tag{25}
\]

where the inflow from the private market conditional on wait time is:

\[
IF_{imr}(w_j) = q_r p(w_j) \int_{y \leq \bar{y}} 1\{v_{ir}(y,0) \geq \max[v_{im}(y,0), v_{ip}(y)]\} g_{im}(y|w_j) \, dy \tag{26}
\]

and the flow from public housing market to rent regulated housing is given by:

\[
IF_{ipr} = q_r \int_{y \geq \bar{y}} 1\{v_{ir}(y,0) \geq \max[v_{im}(y,0), v_{ip}(y)]\} g_{ip}(y) \, dy \tag{27}
\]

Equilibrium requires that the aggregate outflow equal the aggregate inflow

\[
IF_r = \sum_{i=0}^{l} IF_{ir} = \sum_{i=1}^{l} OF_{ir} = OF_r \tag{28}
\]

As before, we are interested in stationary equilibria:
Definition 2 A stationary equilibrium for the extended model consists of the following: a) market shares \((k_{ip}, k_{ir}, k_{im}) \ i = 1, \ldots, I\), b) offer probability \(p(w)\) and \(q_r\), c) distributions \(g_{ip}(y)\), \(g_{im}(w), g_{ir}(w), g_{im}(y|w)\), and \(g_{ir}(y|w)\), and d) value functions \(V_{ip}(y)\), \(V_{im}(y, w)\) and \(V_{ir}(y, w)\), such that:

1. Households behave optimally and value functions satisfy the equations above.
2. The housing authority behaves according the administrative rules described above.
3. The densities are consistent with the laws of motion and optimal household behavior.
4. \(p(w)\) satisfies the market clearing condition for public housing:

\[
OF_p = IF_p \tag{29}
\]

5. \(q_r\) satisfies the market clearing condition for rent regulated housing:

\[
OF_r = IF_r \tag{30}
\]

6. The following identities hold for the market shares:

\[
\sum_{i=1}^{I} k_{ir} = k_r \\
\sum_{i=1}^{I} k_{im} = k_m \\
k_{ip} + k_{ir} + k_{im} = s_i \quad i = 1, \ldots, I \tag{31}
\]

It is straightforward to extend the law of motions for the equilibrium densities. One just needs to add \(i\) subscripts to all the relevant functions reported in Appendix A.
4 Identification and Estimation

In this section we introduce a parametrization of the model and discuss identification and estimation. We can normalize the price in the private market in a baseline year to be equal to one since the units of housing services are arbitrary. To identify and estimate the price discount in the rent regulated market, we assume that market rents can be decomposed into a price and a quantify index. We assume that the quality index is the same in both markets, but the prices are not. We can, therefore use the techniques discussed in Sieg, Smith, Banzhaf, and Walsh (2002) to identify and estimate the price ratio \( p_t^p/p_t^r \) in the rent regulated market.

We assume that the logarithm of income for each household follows an AR(1) process:

\[
\ln(y_{it}) = \mu + \rho \ln(y_{it-1}) + \epsilon_{it}^y \tag{32}
\]

The mean and the variance of income is identified of the observed income distributions in the data. The autocorrelation parameter is identified of the persistence of housing choices measured by time spent in each housing type.

We also assume that the flow utility functions are given by:

\[
\begin{align*}
    u_p(y, h_p) &= [(1 - \tau)y]^{(1 - \alpha)} h_p^\alpha \\
    u_m(y) &= \alpha^\alpha (1 - \alpha)^{1 - \alpha} y p_m^{-\alpha} \\
    u_r(y, h_r) &= [y - p_r h_r]^{(1 - \alpha)} h_r^\alpha
\end{align*}
\tag{33}
\]

\( \alpha \) is the housing share which is identified of the observed joint distribution of housing and income. The two quality parameters \( h_p \) and \( h_r \) are identified based on the observed market rents given the observed prices in market and regulated housing. It is straight forward to show that the time spent in the unit is a monotonic function of housing quality.

All parameters of the income process and household preferences depend on household type in the extended model. The identification argument extends to that model since all relevant moments are observed conditional on type.
Next consider the tax rate in public housing, denoted by $\tau$. This parameter is determined by the administration of public housing programs. It is a state policy that renters in public housing pay roughly 30 percent of their income in rent.\textsuperscript{14}

The arguments for identification are constructive and suggest that we can estimate the parameters of our model using a methods of moments estimator. We use the following moments in estimation: the fraction of each housing type, the average time spent in unit by housing type, the average income by housing type, the variance of income by type, the autocorrelation of income by type, and the housing expenditure shares by housing type.

5 Empirical Results

Table 2 reports estimated parameters and standard errors for a one-type and a four-type model using data from Manhattan. Overall, we find that all parameter estimates are reasonable. We find that rent stabilized houses rent at a 51 percent discount in Manhattan.\textsuperscript{15} The average quality of public housing is high, but not as high the average quality of rent stabilized unit. Note that quality here is measured as expenditures in the private market, i.e. an average public housing unit in Manhattan provides the same quality as a unit that rents for approximately $26,400 dollars in the private market. The quality for an average rent stabilized apartment is approximately $32,500 indicating that these apartment are of significantly higher quality than units in public housing. In equilibrium, our model generates a wait times of approximately 18 years. The probability of finding a rent stabilized unit is approximately 27 percent. These predictions of our model are robust among the model specifications.

We find that there is a fair bit of heterogeneity in the valuation of public housing among the different household types. Recall that we assume that all household types participate in

\textsuperscript{14}Also note that $k_m$, $k_p$ and $k_r$ are observed in the data.

\textsuperscript{15}See Appendix B for details.
the same wait list and are hence eligible for the same type of apartments.\textsuperscript{16} Difference in the valuation of public housing units may be due to differences in preferences for public housing due to stigma.

\begin{table}[h]
\centering
\begin{tabular}{lcccc}
\hline
 & 1 Type Model & & 4 Type Model & \\
 & all & female & female & male & male \\
 & black & non-black & black & non-black & \\
\hline
$h_p$ & 26,494 & 25,623 & 23,897 & 26,346 & 30,808 \\
$\alpha$ & 0.44 & 0.47 & 0.44 & 0.51 & 0.46 \\
$\rho$ & 0.79 & 0.78 & 0.79 & 0.65 & 0.64 \\
$\mu_y$ & 10.65 & 10.50 & 10.55 & 10.55 & 10.69 \\
$\sigma$ & 0.51 & 0.41 & 0.55 & 0.49 & 0.62 \\
$\tau$ & 0.3 & & 0.3 & & \\
$p_r$ & 0.49 & & 0.49 & & \\
$h_r^1$ & 32,425 & & 30,721 & & \\
$\bar{w}$ & 18 & & 18 & & \\
$p(\bar{w})$ & 0.92 & & 0.63 & & \\
$q_r$ & 0.28 & & 0.27 & & \\
\hline
\end{tabular}
\caption{Estimated Parameters}
\end{table}

The parameter $\alpha$ captures the housing expenditure share for households that rent in the private market. We find that low- and moderate-income households in Manhattan spend approximately 44-51 percent of their income on housing if they have to rent in the private market. Female headed households tend to have lower housing shares than male headed households. The parameters of the income process also depend on the household type. Note surprisingly we find that male headed households tend to have higher and more volatile levels

\textsuperscript{16}Extending the analysis to allow for different wait lists by household size is not difficult.
of income. The autocorrelation coefficient ranges between 0.64 and 0.78 suggesting that income shocks are fairly persistent.

Table 3: Model Fit

<table>
<thead>
<tr>
<th>Housing</th>
<th>Percent</th>
<th>Years</th>
<th>Income</th>
<th>Market Rent</th>
</tr>
</thead>
<tbody>
<tr>
<td>One Type Model</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Public</td>
<td>9.90</td>
<td>9.90</td>
<td>16.18</td>
<td>16.20</td>
</tr>
<tr>
<td>all</td>
<td>57.20</td>
<td>57.20</td>
<td>9.49</td>
<td>8.87</td>
</tr>
<tr>
<td>Market</td>
<td>32.90</td>
<td>32.90</td>
<td>3.85</td>
<td>4.02</td>
</tr>
<tr>
<td>Four Type Model</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>female, black</td>
<td>3.12</td>
<td>2.53</td>
<td>15.65</td>
<td>17.41</td>
</tr>
<tr>
<td>Regulated</td>
<td>5.76</td>
<td>5.53</td>
<td>10.99</td>
<td>11.68</td>
</tr>
<tr>
<td>Market</td>
<td>1.02</td>
<td>1.84</td>
<td>5.18</td>
<td>3.42</td>
</tr>
<tr>
<td>female, non-black</td>
<td>4.14</td>
<td>4.65</td>
<td>15.19</td>
<td>11.92</td>
</tr>
<tr>
<td>Regulated</td>
<td>24.67</td>
<td>24.67</td>
<td>10.06</td>
<td>8.22</td>
</tr>
<tr>
<td>Market</td>
<td>16.69</td>
<td>15.07</td>
<td>3.60</td>
<td>3.97</td>
</tr>
<tr>
<td>male, black</td>
<td>1.08</td>
<td>0.65</td>
<td>20.39</td>
<td>17.39</td>
</tr>
<tr>
<td>Regulated</td>
<td>4.56</td>
<td>4.57</td>
<td>8.99</td>
<td>10.70</td>
</tr>
<tr>
<td>Market</td>
<td>1.20</td>
<td>1.62</td>
<td>6.60</td>
<td>3.29</td>
</tr>
<tr>
<td>male, nonblack</td>
<td>1.56</td>
<td>2.07</td>
<td>16.52</td>
<td>18.21</td>
</tr>
<tr>
<td>Regulated</td>
<td>22.21</td>
<td>21.32</td>
<td>8.58</td>
<td>6.39</td>
</tr>
<tr>
<td>Market</td>
<td>13.99</td>
<td>14.36</td>
<td>3.82</td>
<td>3.72</td>
</tr>
</tbody>
</table>

Tables 3 reports a variety of goodness of fit statistics. Overall, we find that our model fits the key moments used in estimation well.
6 Welfare Analysis

Our estimates imply that public housing is an attractive option for low- and moderate-income households in Manhattan. To gain some additional insights, we compute the welfare of households with and without access to public housing. Figure 4 plots the percentage differences for each of the four types as a function of income.

Figure 4: Difference in Welfare between Public and Private Housing

Figure 4 shows that there are substantial welfare gains for low- and moderate-income households. These gains range between 10 and 16 percent for very poor households and between 8 and 10 percent for households with approximately $25,000 income. These welfare gains primarily arise from two sources. First, public housing provides higher quality housing than housing purchased in equilibrium in the private markets. These gains decrease with income. Second, households in public housing have significant higher levels of non-housing consumption than households that rent in the market. This second effect arises from the fact that households in public housing only pay 30 percent of their income in rent while households that rent in the private market pay between 45 and 50 percent of their income in rent.
Table 4: Difference in Consumption between Public and Private Housing

<table>
<thead>
<tr>
<th>Income</th>
<th>Public Housing</th>
<th>Non-housing</th>
</tr>
</thead>
<tbody>
<tr>
<td>10,000</td>
<td>26,494</td>
<td>4,400</td>
</tr>
<tr>
<td>30,000</td>
<td>26,494</td>
<td>13,200</td>
</tr>
<tr>
<td>50,000</td>
<td>26,494</td>
<td>22,000</td>
</tr>
<tr>
<td>70,000</td>
<td>26,494</td>
<td>30,800</td>
</tr>
</tbody>
</table>

Table 4 illustrates these two effects for households with different income levels. Note that low-income households obtain very large housing subsidies, but only small consumption subsidies. Moderate-income households with incomes ranging between $30,000 and $50,000 obtain significant housing and consumption subsidies. Households with $70,000 income experience a mismatch because public housing quality is too low relative to their preferred housing quality purchased in the private market. Nevertheless, public housing can be attractive since it implies much higher level of consumption.

Figure 5 plots the differences in welfare between households in public housing and households in rent stabilized housing. Overall, we find similar qualitative effects.

One surprising feature of existing public housing policies is that housing authorities rarely ask households to leave public housing once their income exceeds the eligibility threshold. Since exit from public housing is purely voluntary, current policies imply that many eligible households do not have access to public housing, while many ineligible households receive large subsidies. Housing aid is, for all practical purposes, an open-ended entitlement for the lucky few that are supported. At the same time, there are a large number of potentially eligible households that have little hope of receiving assistance.

Tables 5 shows that our model fits the overall fraction of ineligible households in public housing well. Moreover, we also match the average difference between the income threshold and the realized incomes. As a consequence, our model seems to well suited to study the
Figure 5: Difference in Welfare between Public and Rent Stabilized Housing

Table 5: Public Housing Mismatch

<table>
<thead>
<tr>
<th></th>
<th>Fraction</th>
<th>Degree</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Data</td>
<td>Model</td>
</tr>
<tr>
<td>all</td>
<td>0.17</td>
<td>0.19</td>
</tr>
<tr>
<td>female &amp; black</td>
<td>0.19</td>
<td>0.14</td>
</tr>
<tr>
<td>female &amp; non-black</td>
<td>0.07</td>
<td>0.08</td>
</tr>
<tr>
<td>male &amp; black</td>
<td>0.33</td>
<td>0.23</td>
</tr>
<tr>
<td>male &amp; non-black</td>
<td>0.27</td>
<td>0.32</td>
</tr>
</tbody>
</table>
welfare implications of public housing mismatch. We would like to know whether it is possible to design a different housing policy that provides a more equal distribution of the benefits by targeting aid to the most needy part of the population. One way to accomplish this goal is by strictly enforcing the current set of eligibility criteria. Recent reforms of the welfare system have tried to stress the importance of helping individuals to become self-sufficient by imposing strict limits on the eligibility of welfare benefits.\(^\text{17}\) Housing aid is the only major welfare program in the U.S. that does not strictly enforce its eligibility criteria.\(^\text{18}\)

Enforcing eligibility criteria would undoubtedly cause many practical transitional problems, especially in a high rent city such as NYC.\(^\text{19}\) Nevertheless, it is useful to conduct this counterfactual analysis since it provides some interesting insights into the equilibrium properties of our dynamic model.

We evaluate this policy using our estimated estimated four-type model. Figures 6 plots the impact of this policy on wait times as a function of the eviction cut-off. We find that the more restrictive the enforcement policy, the shorter are the average wait times. Consider the case in which the eviction cut-off is set at the median income. Our model suggests that we can reduce average wait times 25 percent without adding to the public housing stock, just by enforcing the eligibility rules. We thus conclude that this policy has the potential to alleviate some of the bottlenecks in the supply of public housing without relying on costly new investments.

We find that such a policy has some interesting distributional welfare effects. Figure 7 plots the welfare impact of this policy as function of income and housing status for a female headed household using median income as the eviction threshold. The main losers of this

\(^{17}\) Self-sufficiency was Johnson’s original goal in launching his War on Poverty. The stated objective was to remove the causes not just the consequences of poverty. (Rektor, 2014)

\(^{18}\) Imposing time limits and requiring work requirements has been another attempt to increase self-sufficiency among welfare recipients.

\(^{19}\) A natural concern would be to minimize potential displacement costs for young children. A full analysis of all its effects is clearly outside the scope of this paper.
Figure 6: Wait Times

Figure 7: Change in Welfare by Income
policy are households that are currently living in public housing. Their welfare is significantly reduced for two reasons. First, households that lose eligibility are evicted from public housing and must rent in the private market. Second, currently eligible households will face possible eviction in the future if they experience a sequence of positive income shocks. The main beneficiaries of this policy are poor and eligible household that are currently on the wait list. Evicting higher income households from public housing creates more vacancies and thus reduces the expected time spent on the wait list. Hence these households are more likely to move into public housing. Welfare also increases for ineligible households outside of public housing, since public housing becomes more readily available and provides limited insurance against negative income shocks.

Figure 8: Change in Welfare by Household Type

Figure 8 illustrates the changes in welfare as a function of the eviction threshold. We find that male headed households tend to lose under the more stringent enforcement policies. This largely follows from the fact that male headed households tend to have higher levels of income. Female headed households tend to gain under this policy independently of their race. As a consequence, the fraction of female headed household in public housing increases.
7 Conclusions

We have developed a new dynamic model that captures search frictions and queuing in the market for affordable housing for low- and moderate-income households. We have characterized the equilibrium of the model. We have shown how to identify and estimate the structural parameters of the model. Our application focus on the housing markets of Manhattan. Overall, our model fits the observed sorting of households well. We have characterized the distribution of welfare that arises in our model and shown that access to public housing can increase welfare by as much as 16 percent. We then turn our attention to policy analysis. We consider the impact of different policies that strictly enforce eligibility criteria. We find that these policies tend to benefit eligible low-income households that are currently on the wait list and hurt households that are currently in public housing. Moreover, these policies significantly lower the average wait time for public housing without requiring costly new investments into the existing public housing stock.

Our research provides ample scope for future work. Current policies may provide disincentives to work. Our data suggest that 70 percent of households that currently life in public housing work, at least, part time defined as making at least $10,000 in labor income in 2011. In contrast, 83 percent of all households in rent stabilized housing and 87 percent of households that rent in the unregulated market are in the work force. Policy makers currently contemplate whether or not to impose work requirements for housing aid recipients. Unfortunately, we do not have detailed data on labor market participation in our data set. But it may be possible to endogenize the income process and study the impact of public housing on work incentives using different data sources.

\footnote{The WSJ reported on May 7, 2013 that the Obama administration was introducing a pilot program to study these changes in the 2014 budget.}
References


A Law of Motions for the Income Distributions

The equilibrium rationing rule then implies the following law of motion for the stationary income distributions:

\[
g_p(y) = k_p (1 - q_r) \int 1\{v_p(x) \geq v_m(x, 0)\} f(y|x) g_p(x) \, dx \\
+ k_p q_r \int 1\{v_p(x) \geq \max[v_m(x, 0), v_r(x, 0)]\} f(y|x) g_p(x) \, dx \\
+ k_m \sum_{j=0}^{\infty} g_m(w_j) p(w_j) (1 - q_r) \int_{x \leq y} 1\{v_p(x) \geq v_m(y, 0)\} f(y|x) g_m(x|w_j) \, dx \\
+ k_m \sum_{j=0}^{\infty} g_m(w_j) p(w_j) q_r \int_{x \leq y} 1\{v_m(x, 0) \geq \max[v_p(x), v_r(x, 0)]\} f(y|x) g_m(x|w_j) \, dx \\
+ k_r \sum_{j=0}^{\infty} g_r(w_j) p(w_j) \int_{x \leq y} 1\{v_p(x) \geq \max[v_m(x, 0), v_r(x, 0)]\} f(y|x) g_r(x|w_j) \, dx
\]

and

\[
g_m(y|0) = k_p (1 - q_r) \int 1\{v_m(x, 0) \geq v_p(x)\} f(y|x) g_p(x) \, dx \\
+ k_p q_r \int 1\{v_m(x, 0) \geq \max[v_p(x), v_r(x, 0)]\} f(y|x) g_p(x) \, dx \\
+ k_m \sum_{j=0}^{\infty} g_m(w_j) p(w_j) (1 - q_r) \int_{x \leq y} 1\{v_m(x, 0) \geq v_p(x)\} f(y|x) g_m(x|w_j) \, dx \\
+ k_m \sum_{j=0}^{\infty} g_m(w_j) p(w_j) q_r \int_{x \leq y} 1\{v_m(x, 0) \geq \max[v_p(x), v_r(x, 0)]\} f(y|x) g_m(x|w_j) \, dx \\
+ k_r \sum_{j=0}^{\infty} g_r(w_j) p(w_j) \int_{x \leq y} 1\{v_p(x) \geq \max[v_m(x, 0), v_r(x, 0)]\} f(y|x) g_r(x|w_j) \, dx \\
+ k_m \sum_{j=0}^{\infty} g_m(w_j) q_r \int_{x > y} f(y|x) g_m(x|w_j) \, dx \\
+ k_m \sum_{j=0}^{\infty} g_m(w_j) q_r \int_{x > y} 1\{v_m(x, 0) \geq v_r(x, 0)\} f(y|x) g_m(x|w_j) \, dx \\
+ k_r \sum_{j=0}^{\infty} g_r(w_j) \int_{x > y} 1\{v_m(x, 0) \geq v_r(x, 0)\} f(y|x) g_r(x|w_j) \, dx
\]
and

\begin{align}
  k_m \, g_m(0) &= k_p \, (1 - q_r) \int 1\{v_m(x, 0) \geq v_p(x)\} \, g_p(x) \, dx \\
  &+ k_p \, q_r \int 1\{v_m(x, 0) \geq \max[v_p(x), v_r(x, 0)]\} \, g_p(x) \, dx \\
  &+ k_m \sum_{j=0}^{\infty} g_m(w_j) \, p(w_j) \, (1 - q_r) \int_{x \leq \bar{y}} 1\{v_m(x, 0) \geq v_p(x)\} \, g_m(x|w_j) \, dx \\
  &+ k_m \sum_{j=0}^{\infty} g_m(w_j) \, p(w_j) \, q_r \int_{x \leq \bar{y}} 1\{v_m(x, 0) \geq \max[v_p(x), v_r(x, 0)]\} \, g_m(x|w_j) \, dx \\
  &+ k_r \sum_{j=0}^{\infty} g_r(w_j) \, p(w_j) \int_{x \leq \bar{y}} 1\{v_m(x, 0) \geq \max[v_p(x), v_r(x, 0)]\} \, g_m(x|w_j) \, dx \\
  &+ k_m \sum_{j=0}^{\infty} g_m(w_j) \, (1 - q_r) \int_{x > \bar{y}} g_m(x|w_j) \, dx \\
  &+ k_m \sum_{j=0}^{\infty} g_m(w_j) \, q_r \int_{x > \bar{y}} 1\{v_m(x, 0) \geq v_r(x, 0)\} \, g_m(x|w_j) \, dx \\
  &+ k_r \sum_{j=0}^{\infty} g_r(w_j) \int_{x > \bar{y}} 1\{v_m(x, 0) \geq v_r(x, 0)\} \, g_r(x|w_j) \, dx
\end{align}

Moreover,

\begin{align}
  g_r(y|0) &= k_p \, q_r \int 1\{v_r(x, 0) \geq \max[v_p(x), v_m(x, 0)]\} \, f(y|x) \, g_p(x) \, dx \\
  &+ k_m \sum_{j=0}^{\infty} g_m(w_j) \, p(w_j) \, q_r \int_{x \leq \bar{y}} 1\{v_r(x, 0) \geq \max[v_p(x), v_m(x, 0)]\} \, f(y|x) \, g_m(x|w_j) \, dx \\
  &+ k_r \sum_{j=0}^{\infty} g_r(w_j) \, p(w_j) \int_{x \leq \bar{y}} 1\{v_r(x, 0) \geq \max[v_p(x), v_m(x, 0)]\} \, f(y|x) \, g_r(x|w_j) \, dx \\
  &+ k_m \sum_{j=0}^{\infty} g_m(w_j) \, q_r \int_{x > \bar{y}} 1\{v_r(x, 0) \geq v_m(x, 0)\} \, f(y|x) \, g_m(x|w_j) \, dx \\
  &+ k_r \sum_{j=0}^{\infty} g_r(w_j) \int_{x > \bar{y}} 1\{v_r(x, 0) \geq v_m(x, 0)\} \, f(y|x) \, g_r(x|w_j) \, dx
\end{align}
and

\[
k_r \, g_r(0) = k_p \, q_r \int 1\{v_r(x, 0) \geq \max[v_p(x), v_m(x, 0)]\} \, g_p(x) \, dx \tag{38}
\]

\[
+ k_m \sum_{j=0}^{\infty} g_m(w_j) \, p(w_j) \, q_r \int_{x \leq y} 1\{v_r(x, 0) \geq \max[v_p(x), v_m(x, 0)]\} \, g_m(x|w_j) \, dx
\]

\[
+ k_r \sum_{j=0}^{\infty} g_r(w_j) \, p(w_j) \int_{x \leq y} 1\{v_r(x, 0) \geq v_m(x, 0)\} \, g_r(x|w_j) \, dx
\]

\[
+ k_m \sum_{j=0}^{\infty} g_m(w_j) \, q_r \int_{x > y} 1\{v_r(x, 0) \geq v_m(x, 0)\} \, g_m(x|w_j) \, dx
\]

\[
+ k_r \sum_{j=0}^{\infty} g_r(w_j) \int_{x > y} 1\{v_r(x, 0) \geq v_m(x, 0)\} \, g_r(x|w_j) \, dx
\]

\[
g_m(y|w_j) = k_m \, g_m(w_j - 1) \, (1 - q_r) \int_{x \leq y} f(y|x) \, g_m(x|w_j - 1) \, dx \tag{39}
\]

\[
+ k_m \, g_m(w_j - 1) \, q_r \int_{x \leq y} 1\{v_m(x, w_j) \geq v_r(x, w_j)\} \, f(y|x) \, g_m(x|w_j - 1) \, dx
\]

\[
+ k_r \, g_r(w_j - 1) \int_{x \leq y} 1\{v_m(y, w_j) \geq v_r(y, w_j)\} \, f(y|x) \, g_r(x|w_j - 1) \, dx
\]

and

\[
g_m(w_j) = g_m(w_j - 1) \, (1 - p(w_j - 1)) \, (1 - q_r) \int_{x \leq y} g_m(x|w_j - 1) \, dx \tag{40}
\]

\[
+ g_m(w_j - 1) \, (1 - p(w_j - 1)) \, q_r \int_{x \leq y} 1\{v_m(x, w_j) \geq v_r(x, w_j)\} \, g_m(x|w_j - 1) \, dx
\]

\[
+ \frac{k_r}{k_m} \, g_r(w_j - 1) \, (1 - p(w_j - 1)) \int_{x \leq y} 1\{v_m(x, w_j) \geq v_r(x, w_j)\} \, g_r(x|w_j - 1) \, dx
\]

\[
g_r(y|w_j) = k_r \, g_r(w_j - 1) \int_{x \leq y} 1\{v_r(x, w_j) \geq v_m(x, w_j)\} \, f(y|x) \, g_r(x|w_j - 1) \, dx \tag{41}
\]

\[
+ k_m \, g_m(w_j - 1) \, q_r \int_{x \leq y} 1\{v_r(x, w_j) \geq v_m(x, w_j)\} \, f(y|x) \, g_m(x|w_j - 1) \, dx
\]
and

\[
g_r(w_j) = g_r(w_j - 1) \left(1 - p(w_j - 1)\right) \int_{x \leq \bar{y}} 1\{v_r(x, w_j) \geq v_m(x, w_j)\} g_r(x|w_j - 1) \, dx \\
+ \frac{k_m}{k_r} g_m(w_j - 1) \left(1 - p(w_j - 1)\right) q_r \int_{x \leq \bar{y}} 1\{v_r(x, w_j) \geq v_m(x, w_j)\} g_m(x|w_j - 1) \, dx
\]  

(42)

B Measuring the Discount in Rent Stabilized Housing

To measure the relative price between unregulated and regulated housing, we estimate a hedonic regression using data on housing units in both markets. As discussed in Section 4 we assume that the quantity index that relates structural and neighborhood characteristics to housing service flows is constant among the two markets. We can, therefore, use these regressions to measure price differences between regulated and unregulated housing markets. Table 6 summarizes our findings. We find that rent regulated units are cheaper by 26.5 (41.7) percent in NYC (Manhattan) compared to the market rated units.
### Table 6: Regression of Log Rent

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<th>NYC</th>
<th>Manhattan</th>
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<tbody>
<tr>
<td>regulated</td>
<td>-0.338***</td>
<td>-0.513***</td>
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<tr>
<td># of bed rooms</td>
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<td>0.124***</td>
</tr>
<tr>
<td># of other rooms</td>
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<td>-0.00249</td>
</tr>
<tr>
<td>complete kitchen</td>
<td>0.271***</td>
<td>0.370**</td>
</tr>
<tr>
<td>complete plumbing</td>
<td>0.112</td>
<td>0.622**</td>
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<td>yes</td>
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<td>yes</td>
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<td>yes</td>
</tr>
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</tr>
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<tr>
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<tr>
<td>unit floor control</td>
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</tr>
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</tr>
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</table>

* p < 0.05, ** p < 0.01, *** p < 0.001