Unemployment Insurance in Macroeconomic Stabilization

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Abstract

Does the generosity of unemployment insurance (UI) have a role to play in macroeconomic stabilization? When inefficient fluctuations arise from nominal rigidities and constraints on monetary policy, I demonstrate that it does, owing to the interaction between UI and aggregate demand. From a positive perspective, a marginal increase in UI generosity affects output and employment through a redistribution effect on aggregate demand. From a normative perspective, two forces determine optimal generosity beyond the classic trade-off between insurance and incentives: an aggregate demand externality and an effect of low aggregate demand on the social cost of disincentives. The aggregate demand externality summarizes the welfare impact of the redistribution effect when the economy is slack; both are governed by the difference in marginal propensities to consume between the unemployed and employed. Quantitatively, a calibrated model with search frictions, incomplete markets, and a binding zero lower bound suggests that the 2008–13 UI benefit extensions in the U.S. had important stabilization effects through these channels. Compared to counterfactual benefit durations capped at 9 months in the calibrated model, the extensions to 22 months prevent a 2–5 percentage point rise in the unemployment rate and generate a strict Pareto improvement.

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†Online appendix material is on my website http://scholar.harvard.edu/rkekre.
1 Introduction

Economists have long viewed unemployment insurance (UI) as an important automatic stabilizer — but should it also serve as a discretionary tool in the stabilization of short-run fluctuations? Since the 1950s, policymakers in the United States have treated UI generosity as precisely such an instrument, routinely extending benefits in recessions. This practice was expanded in unprecedented and controversial fashion during the Great Recession, when benefit durations were raised almost four-fold at the depth of the downturn. Supporters of increased generosity have pointed to the stimulus benefits of transfers to the unemployed, many of whom face binding liquidity constraints. Critics have emphasized the counterproductive supply-side effects of more generous UI.\footnote{See Summers [2010], Congressional Budget Office [2012], and Blanchard et al. [2013] for support of more generous UI. See Barro [2010] and Mulligan [2012] for more critical commentary on its effects.}

The standard analysis of UI in the economics literature is largely silent on this debate because it has been developed in a partial equilibrium setting which assumes away interactions between UI and macroeconomic slackness.\footnote{There is a growing literature developed since the Great Recession which studies optimal UI over the business cycle, which my paper joins. My analysis is the first to account for interactions between UI and aggregate demand. I discuss this related work later in this section.} This paper embeds UI in a macroeconomic framework with search frictions and incomplete markets, and where inefficient fluctuations arise from nominal rigidities and constraints on monetary policy. In this setting, I theoretically characterize the effects of UI generosity on equilibrium output, employment, and welfare. I study these effects quantitatively in a dynamic simulation of the 2008-13 benefit extensions in the U.S. I conclude that expanded UI generosity can, and should, play an important role in the policy response to economic slackness.

Theoretically, the interaction between UI and aggregate demand naturally motivates a role for higher generosity when the economy is slack. Nominal rigidities imply that an increase in UI generosity affects output through a redistribution effect on aggregate demand, governed by the difference in marginal propensities to consume (MPCs) between the unemployed and employed. In terms of social welfare, this generates an aggregate demand externality from transfers when the economy is inefficiently slack, as when monetary policy is constrained. In addition, low aggregate demand in such a recession itself changes the social cost of disincentivizing labor supply, the sign of which depends on whether a reduction in employment changes goods supply or demand by more in the short run. When the unemployed have a higher MPC than the employed and are net debtors, these channels imply a positive effect of redistribution on output, and optimal generosity which is higher than the classic partial equilibrium formula would imply.

Quantitatively, I find that the 2008–13 benefit extensions in the U.S. indeed had large, positive effects on employment and welfare operating through these channels. I extend the framework used in the theoretical analysis to simulate an infinite-horizon model with finite-duration UI. Relative to a counterfactual path of benefits capped at 9 months of duration in the calibrated model, I find that the observed extensions to 22 months prevent a further rise in the unemployment rate of 2–5 percentage points, depending on the value of calibrated parameters. Across calibrations, I further find that the observed extensions generate a strict Pareto improvement relative to the
counterfactual. Remarkably, even the employed gain from the policy change: while they expect to finance incremental transfers to the unemployed, the stronger labor market induced by transfers more than compensates by raising their continuation utility should they lose their jobs.

My paper is the first to analytically characterize the role of UI generosity alongside monetary policy in stabilizing an economy with nominal rigidities. Three frictions on the real side of the economy, and my characterization of optimal policy absent nominal rigidities, set the stage for this stabilization problem. First, search and matching frictions in the tradition of Diamond [1981], Mortensen [1982], and Pissarides [1984] (hereafter, DMP) give rise to involuntary unemployment. Second, market incompleteness with respect to unemployment risk generates an efficient role for the public provision of UI. Third, unobservable worker search intensity in the matching process leads to a moral hazard cost in the provision of UI. In this environment, I obtain closed form formulas by specializing to the case of a two-period economy with a “short run” and “long run”. Absent nominal rigidities, the efficient level of UI generosity is characterized by a general equilibrium version of the benchmark Baily [1978]-Chetty [2006] formula from public finance. In terms of sufficient statistics, this formula demonstrates that optimal UI balances the welfare gain from consumption insurance with the disincentive cost from moral hazard.

My first main result is a generalized Baily-Chetty formula characterizing optimal UI generosity in the presence of macroeconomic slackness. Slackness arises from the combination of nominal rigidity and an inability of monetary policy to perfectly stabilize the economy on its own, owing to a zero lower bound on the nominal interest rate. There are two distinct channels through which slackness affects the optimal generosity of UI. First, an aggregate demand externality in the class identified by Farhi and Werning [2015] generates a distinction between the private and social value of transfers. If unemployed workers’ MPC out of income exceeds that of employed workers in the short run, the social value of transfers exceeds the private gains from consumption insurance because UI can raise aggregate demand and thus economic activity. Second, low aggregate demand itself changes the social cost of disincentivizing labor supply. The disincentive effect of UI is welfare-relevant because lower employment affects economy-wide resources available for consumption. At each date, the resulting social cost depends on the shadow price of aggregate resources and the effect of lower employment on the net supply of goods, keeping in mind that lower employment not only reduces production but also changes the composition of demand. In the short-run, if unemployed workers are net debtors, lower employment unambiguously reduces net supply. In a recession caused by low aggregate demand, a low shadow price on resources makes this less costly.

My second main result is a formula for the UI multiplier characterizing the effect of an increase...
in unemployed workers’ income on equilibrium output. A marginal increase in their income affects output through a redistribution effect on aggregate demand, governed by the difference in MPCs between the unemployed and employed. This effect underlies the aggregate demand externality in the normative analysis, and is absent in prior analyses of the positive effects of UI in partial equilibrium or flexible price settings. It only relies on nominal rigidities, though constraints on monetary policy motivate why there may be no endogenous policy response to the stance in fiscal policy. There is a subtle difference from the standard Keynesian intuition, however: supply-side elasticities still matter even when production adjusts to meet desired demand. In particular, when vacancies adjust to offset the reduction in search effort induced by higher UI, this raises recruiting intensity, which is costly in a frictional labor market. On the margin, this effect may be small relative to the aggregate demand effects of transfers. But it underscores why, at the extreme, complete insurance need not maximize output even in a demand-determined world. It also explains why disincentives still matter in determining the optimal generosity of UI.

In an enriched quantitative model, I then assess the effects of the unprecedented expansion of UI generosity in the U.S. during the Great Recession. The federal-state Extended Benefits program and the federal government’s Emergency Unemployment Compensation Act of 2008 together raised benefit durations from an average of 26 weeks across states to 99 weeks at the depth of the recession. To study these policies, I generalize the model to an infinite horizon setting which combines the dynamics of the DMP labor market with rich consumption and savings decisions in the tradition of Bewley [1983], Huggett [1993], and Aiyagari [1994] (hereafter, BHA). This allows me to capture important dynamic issues introduced by policy affecting the duration of benefits, such as effects on precautionary saving. It also allows me to accommodate the endogenously changing employment and wealth distribution over the course of the Great Recession in shaping the effects of UI.

In the steady-state of the calibrated model, the MPCs of unemployed agents rise sharply with duration of unemployment — an endogenous outcome, not a calibrated target. In my benchmark calibration, the long-term unemployed (those unemployed for at least 6 months, and who thus exhaust benefits in steady-state) have a monthly MPC which exceeds that of the employed by 0.24. This arises from the combination of two forces operating as an agent proceeds through an unemployment spell: (i) precautionary behavior which tends to raise the MPC at any given level of wealth, and (ii) the endogenous decision to draw down on assets, pushing the agent closer to the borrowing constraint. To use the language of public finance, it means that in a BHA framework with DMP labor market dynamics calibrated to match the incidence and duration of unemployment in the U.S. economy, the long-term unemployed are an extremely promising “tag” for high MPCs.\(^5\)

In response to a shock which pushes the economy with fully sticky prices against the zero lower bound, I find that the 2008-13 benefit extensions have large, positive effects on employment and welfare through aggregate demand. At date 1 of the simulation, which corresponds to July 2008, there is an unanticipated shock to preferences which raises desired saving, and thus reduces the

\(^5\)I use “tag” as defined by Akerlof [1978]. It means that long-term unemployment is an observable characteristic which correlates strongly with the MPC, a characteristic which is less easily observable but is what fundamentally matters for stabilization policy according to my theoretical formulas.
natural rate of interest. The path of fundamentals and the assumed monetary policy rule imply a binding zero lower bound on the interest rate for 90 months (consistent with “lift-off” from the zero lower bound in December 2015). In my baseline experiment, I compare outcomes under the observed extensions to counterfactual benefits capped at 9 months of duration, consistent with the greatest average degree of generosity across U.S. states at any point prior to the Great Recession. Relative to this counterfactual, the observed extensions prevent a further rise in the unemployment rate of 2–5 percentage points and generate a strict Pareto improvement. Even employed agents gain because of the stronger labor market induced by transfers to the long-term unemployed. Notably, these results are reversed when prices are flexible, where disincentives dominate and motivate a reduction in utilitarian social welfare.

Consistent with the theoretical analysis, the effects of the benefit extensions on employment and welfare appear driven by the high MPCs of the marginal recipients of transfers, the long-term unemployed. In sensitivity analysis, I demonstrate that the implied pattern of MPCs in the calibrated model indeed matters more than other economic moments in determining the magnitude of the employment and welfare results. A broader set of counterfactual policy experiments further suggests the importance of this moment. For instance, I find that the observed extensions deliver greater employment and welfare than any of the counterfactuals I explore with durations of 6 months, but more generous replacement rates. Even though these replacement rate policies increase generosity for any agent who becomes unemployed, the high MPCs of the long-term unemployed render the observed extensions more powerful in stimulating aggregate demand.

As such, my results suggest the importance of future empirical work focused on estimating the profile of MPCs by duration of unemployment. An active research area in macroeconomics has advanced our understanding of the level and heterogeneity of MPCs across various groups. But we know little about heterogeneity in MPCs by employment status and duration of unemployment.\(^6\) Estimating this heterogeneity could help narrow down the range of effects obtained in this paper, which accounts for the substantial uncertainty there currently is regarding the MPC profile. This profile should join the consumption drop upon unemployment, the disincentive elasticity to benefit changes, and the wage elasticity to benefit changes as a key object of interest in the applied microeconomics literature on UI.

At the same time, my analysis motivates further theoretical work on the stabilization effects of changes in precautionary saving alongside heterogeneity in MPCs. My analytical formulas abstract from precautionary saving, as they are derived in a setting with perfect foresight after the initial resolution of uncertainty. In a dynamic environment with continued separation and job-finding risk, as in the calibration, I conjecture that more generous UI further raises aggregate demand by reducing agents’ need to precautionary save. This channel would complement the redistribution channel emphasized by my analytical formulas. In ongoing work, I develop a multi-period extension of my analytical framework to study this channel and isolate sufficient statistics governing its size.

\(^6\)To my knowledge, Japelli and Pistaferri [2014] is the only study speaking to this dimension of heterogeneity, in a survey of Italian households.
**Relation to literature.** My paper is the first to analytically characterize the role of UI generosity as a second-best policy instrument when monetary policy is constrained. It joins an emerging literature inspired by the Great Recession focused on other second-best instruments such as conventional government spending (Woodford [2010], Eggertsson and Krugman [2012], Werning [2012]), macroprudential regulation (Farhi and Werning [2015], Korinek and Simsek [forthcoming]), tax policy (Correia et al. [2013], Farhi et al. [2013]), capital controls (Schmitt-Grohe and Uribe [2012], Farhi and Werning [2013]), and cross-border transfers (Farhi and Werning [2014]).

In characterizing the optimal use of UI generosity in stabilization, I integrate Baily [1978] and Chetty [2006]’s classic analysis of optimal UI with Farhi and Werning [2015]’s recent framework for second-best macroeconomic stabilization. The latter authors build a general theory of macroprudential policies in the presence of nominal rigidities and constraints on monetary policy. My analysis demonstrates that the aggregate demand externalities which motivate their theory of macroprudential policy similarly motivate a role for social insurance in macroeconomic stabilization, manifest in optimal UI which departs from the Baily-Chetty benchmark.

Relative to other recent efforts studying the Baily-Chetty formula over the business cycle, mine is uniquely focused on nominal rigidities, accommodating a role for aggregate demand effects of UI. Jung and Kuester [2015] instead study departures from the Baily-Chetty benchmark as part of a broader analysis of optimal labor market policy in a real business cycle model. Landais et al. [2015a,b] study departures from the Baily-Chetty benchmark using a sufficient statistic approach which does not need to take a stand on the source of inefficiency, but does rule out any aggregate demand effects of UI. These authors’ “rat race effect” is, however, similar to the intuition behind low aggregate demand changing the social cost of disincentives in my optimal UI formula.\(^7\) Kroft and Notowidigdo [2015] explore how the elasticities in the standard Baily-Chetty formula vary over the business cycle, and Schmieder et al. [2012] do the same in an analogous formula developed for benefit extensions. The present analysis identifies an important setting when these elasticities are no longer sufficient; the difference in MPCs enters in a distinct, important role.

Quantitatively, my paper is the first to study the effects of UI extensions in a calibrated model with search frictions, incomplete markets, and nominal rigidities. Krusell et al. [2010], Nakajima [2012], and Mitman and Rabinovich [2015] analyze UI in calibrated models merging the DMP and BHA traditions, but with business cycle dynamics in the real business cycle tradition. As I show in my analysis, moving from flexible to sticky prices reverses the employment and welfare effects of UI. Christiano et al. [2015] and den Haan et al. [2015] account for nominal rigidities in their simulations, but the former do not model incomplete markets, and the latter focus on changes in replacement rate generosity in a model with infinite duration UI. den Haan et al. [2015] further emphasize the demand for real money balances and its impact on costly deflation, which is shut down here as I study the cashless limit with fully sticky prices. My framework instead emphasizes

\(^7\) In Landais et al. [2015a,b], the “rat race effect” reduces the cost of disincentivizing labor supply when tightness is inefficiently low, naturally motivating optimally countercyclical UI generosity. My analysis formalizes why the economy can find itself in such a situation of inefficient tightness, owing to nominal rigidity, a binding zero lower bound, and thus low aggregate demand.
heterogeneity in MPCs, which loom large because I study policies on the duration margin.

Relative to a growing literature exploring the policy consequences of MPC heterogeneity in incomplete markets New Keynesian economies, my paper adds DMP dynamics to the labor market. Oh and Reis [2012] and Guerrieri and Lorenzoni [2015] combined the incomplete markets and New Keynesian traditions, and in such an environment researchers have studied general transfers (Bilbiie et al. [2013], Mehrotra [2014], Giambattista and Pennings [2015]), automatic stabilizers (McKay and Reis [2015]), and monetary policy itself (Auclert [2015], Kaplan et al. [2015]). All of these have featured a neoclassical labor market. In my setting, DMP dynamics interact with a collapse in aggregate demand to generate an endogenous rise in the high-MPC long-term unemployed. The rise in the long-term unemployed was a salient feature of the Great Recession, and is an important driver of the scale of stimulus I find from the benefit extensions.\(^8\)

Finally, my calibration approach complements a separate strand of the literature using quasi-experimental research designs to estimate the stabilization effects of UI generosity. The evidence is mixed. di Maggio and Kermani [2015] find that more generous UI tends to have a stabilizing impact on business cycle fluctuations through its effect on aggregate demand. Hagedorn et al. [2015a] and Hagedorn et al. [2015b] instead estimate negative effects of the 2008-13 UI benefit extensions on employment, while Coglianese [2015] estimates positive effects of the same extensions. Such quasi-experimental studies face the difficulty of controlling for the endogeneity of labor market policy in response to fluctuations in economic activity, which may explain the differing results. My approach does not resolve this debate. Instead, I ask what micro-level behavioral moments imply regarding the effects of UI extensions in the laboratory of a rational expectations, general equilibrium model of the U.S. economy at the zero lower bound.

Outline. I begin in section 2 by characterizing the theoretical role of UI in a short-run/long-run economy featuring constrained efficient UI, nominal rigidities, and potential constraints on monetary policy. I extend this framework in section 3 to conduct a quantitative evaluation of the benefit extensions in the U.S. during the Great Recession. Finally, in section 4 I conclude.

2 Theory

In this section I embed UI in a macroeconomic model with inefficient fluctuations to study the role of UI in stabilization. I begin by characterizing an economy with search frictions, incomplete markets, and moral hazard in which the government provision of UI is constrained efficient. I then examine the normative and positive consequences of changes in UI generosity in the presence of short-run fluctuations arising from macroeconomic shocks and nominal rigidities.

I find that general equilibrium interactions between UI and aggregate demand motivate a role for UI in stabilization when monetary policy is constrained. Normatively, optimal UI generosity

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\(^8\)An interesting question, related to but not the focus of the present paper, is the extent to which such a calibrated model can explain the joint dynamics of vacancies, unemployment, and the Beveridge curve during the Great Recession. In a similar general equilibrium framework, Ravn and Sterk [2014] study this question in depth.
departs from the classic partial equilibrium trade-off between insurance and incentives due to an aggregate demand externality and an effect of low aggregate demand on the social cost of disincentives. Positively, a marginal increase in UI generosity affects output and employment through a redistribution effect on aggregate demand. The aggregate demand externality summarizes the welfare impact of the redistribution effect when the economy is slack. Both are governed by the difference in MPCs between the unemployed and employed.

2.1 Economic environment

Consider a closed economy, where the treatment of dynamics is simplified by considering a “short run” with endogenous production at date 1 and “long run” with traded endowments at date 2. Three departures from the neoclassical benchmark are sufficient to motivate a constrained efficient role for UI in the short run, as characterized in the next subsection: a static version of search and matching, an absence of private markets to insure against idiosyncratic unemployment risk, and worker search intensity which is unobserved by the government.

Technologies, tastes, and endowments. There are two classes of firms in this economy: measure one producers of a homogenous intermediate good, and measure one retailers who purchase this input and use a pure pass-through technology to sell a differentiated variety to consumers. Producers are perfectly competitive, while retailers are monopolistically competitive.

Producers engage in two classes of activities: recruiting and production. Recruiting proceeds according to a static version of Diamond-Mortensen-Pissarides search and matching frictions, in which workers actively search for jobs and firms post vacancies to hire them. Given aggregate worker search effort \( \bar{s} \) and firm vacancies \( \bar{\nu} \), the economy sees an aggregate number of matches

\[
m(\bar{s}, \bar{\nu}) = \bar{m} s^{1-\eta} \bar{\nu}^\eta
\]

in the short run. Defining labor market tightness \( \theta \equiv \bar{\nu} \bar{s} \), this implies a vacancy-filling probability \( q(\theta) \) and job-finding probability per unit effort \( p(\theta) \) defined as

\[
q(\theta) \equiv \bar{m} \theta^{n-1} = \frac{m(\bar{s}, \bar{\nu})}{\bar{\nu}}, \quad p(\theta) \equiv \bar{m} \theta^n = \frac{m(\bar{s}, \bar{\nu})}{\bar{s}}.
\]

Assuming rational expectations, a given producer thus expects that by posting \( \nu \) vacancies it will yield \( q(\theta) \nu \) workers. Recruiting has a cost \( k \) per vacancy, which I express in units of foregone worker time spent producing. As such, this producer has \( q(\theta) \nu - k \nu \) units of labor spent in production, which yields

\[
f(q(\theta) \nu - k \nu) \equiv a(q(\theta) \nu - k \nu)^{\alpha}
\]

9The distinction between these two classes of firms, common in the New Keynesian literature, is simply made for tractability in separating the problems of production and recruiting on the one hand, and price-setting on the other. I follow Shimer [2010] in making this modeling choice. In a static framework without incumbent workers, it is of course a theoretical abstraction; in the dynamic framework I study in section 3, it more directly maps to the time spent by incumbent workers on recruiting new hires in practice.
units of output given productivity $a$ and returns to scale $\alpha \in (0, 1]$ common to all firms.

On the worker side, there are measure one agents who are ex-ante identical. In the short-run, these agents are endowed with one indivisible unit of labor. Putting in search effort $s$, a particular agent becomes employed with probability $p(\theta)s$, generating ex-ante utility

$$U = (p(\theta)s) u^e(c_1^e, c_2^e) + (1 - p(\theta)s) u^u(c_1^u, c_2^u) - \psi(s).$$

(1)

I allow arbitrary non-separabilities between short-run and long-run consumption $\{c_1, c_2\}$ in each employment state, though of course time-separable consumption with discount factor $\beta$ is a special case.\(^{11}\) I further assume arbitrarily different subutility functions in each employment state $\{u^e(\cdot), u^u(\cdot)\}$, which can accommodate non-pecuniary costs of employment and/or unemployment. Short-run consumption is itself a CES aggregator over differentiated varieties

$$c_i^e = \left[ \int_0^1 c_i^e(j) \frac{j^{-1}}{\varepsilon} \, dj \right]^{\frac{\varepsilon - 1}{\varepsilon}}, \quad i \in \{e, u\}.$$  

(2)

The cost of search effort $\psi(s)$ is assumed to be separable from consumption for expositional simplicity, and satisfies standard regularity conditions.\(^{12}\) Besides their short-run labor endowment, I finally assume that workers are endowed with equal equity shares in the economy’s firms, and receive a long-run goods endowment $y^e_2$ or $y^u_2$ which depends on their short-run employment status, capturing permanent income differences between the employed and unemployed.

**Policy, markets, and equilibrium.** I focus on five instruments available to policymakers: three instruments to intervene in the labor market, and two instruments to conduct monetary policy. In the labor market, they can assess a lump-sum tax $t$ and ad-valorem payroll tax $\tau$ on employed workers, and provide a lump-sum payment $b$ to unemployed workers. To conduct monetary policy, they can set the nominal interest rate $i$ between the short- and long-run, as well as directly choose the long-run price level $P_2$.\(^ {13,14}\) Beyond these instruments, I assume for convenience that policymakers can assess an ad-valorem tax on retailers $\tau^r$ and lump-sum tax on all agents $T^r$ which will only be used to offset the monopolistic competition distortions introduced by retailers. Throughout the paper, I will assume that $\tau^r = -\frac{1}{\varepsilon}$ and that $T^r$ is set to finance these subsidies, so that they play no stabilization role in my analysis.\(^{15}\)

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\(^{11}\) Time separable preferences are standard in dynamic macroeconomic models. I am deliberately more general here, not just because the results do not require time separability, but also because non-separability affords another useful interpretation of my results as pertaining to a small open economy with non-traded and traded consumption (as I discuss in section 2.6). In that case, non-separability between non-traded and traded consumption is standard in the open economy macroeconomics literature.

\(^{12}\) It is positive ($\psi(s) \geq 0$, strictly for $s > 0$), increasing ($\psi'(s) \geq 0$, strictly for $s > 0$) and convex ($\psi''(s) > 0$).

\(^{13}\) Throughout the paper, I will denote all nominal objects in upper case, and real objects in lower case.

\(^{14}\) Note that in my specification of worker utility in (1), I assumed no utility from holdings of money balances. Following Woodford [2003], I model money at the “cashless limit” where it serves as only a unit of account. The assumption that policymakers can directly control the long-run price level along with the nominal interest rate is needed to ensure price level determinacy in this finite horizon environment.

\(^{15}\) Given the other tax instruments I have assumed, these do not change the set of implementable allocations. I
We can now characterize the structure of particular markets in this economy and, given the policy instruments above, the implied optimization problems faced by firms and workers. I first discuss wage determination in the labor market. As is well known, the presence of search frictions generates a continuum of wages which are bilaterally efficient. To resolve this indeterminacy, three wage determination processes are widely used in the search and matching literature: Nash bargaining, wage posting, and rigid real wages. My framework can accommodate all three cases, with no changes to the main results on UI in macroeconomic stabilization.

For concreteness, I present here the case where the labor market is characterized by wage posting by producers. Following Acemoglu and Shimer [1999], all producers anticipate a tightness schedule \( \theta(W) \) defined over submarkets indexed by their prevailing wage. Since there is a representative producer and worker ex-ante, there will only be one prevailing wage rate in equilibrium; the schedule thus reflects a form of subgame perfection, in which all producers act optimally given shared off-path beliefs. In equilibrium, the schedule \( \theta(W) \) will be defined to be that which makes workers indifferent to applying to alternative submarkets offering lower or higher wages.

Given this labor market structure, the representative producer faces

\[
\Pi = \max_{\nu, W} P^I f(q(\theta(W))\nu - k\nu) - Wq(\theta(W))\nu
\]

given a price of intermediate goods \( P^I \).

I turn now to asset markets, which I assume are incomplete in one important way: agents cannot buy private insurance against their idiosyncratic risk of unemployment. Beyond that, markets are complete. In particular, uncertainty over macroeconomic aggregates — such as the structure of preferences or the value of long-run endowments \( \{y^e_2, y^u_2\} \) — is resolved at date 1. In principle, agents can trade securities indexed to such aggregate sources of risk, but we can ignore this since there is a representative worker ex-ante. Ex-post, agents can trade a bond paying the riskless nominal rate \( 1 + i \) at date 2.

Given this asset market structure, ex-post, worker \( i \in \{e, u\} \) faces

\[
v^i = \max_{\{c^i_1(j)\}, c^i_2, z^i_1} u^i(c^i_1, c^i_2) \text{ s.t.} \]

\[
(RC)^i_1 : \int_0^1 P_1(j)c^i_1(j)dz^i_1 + P_1z^i_1 \leq Y^i_1,
\]

\[
(RC)^i_2 : P_2c^i_2 \leq P_2y^i_2 + (1 + i)P_1z^i_1,
\]

assume them only to eliminate monopolistic competition in a transparent way consistent with the literature.

\[16\] Hall and Krueger [2012] provide evidence on the prevalence of Nash bargaining and wage posting in the U.S. labor market. Shimer [2005] and Hall [2005] argue that rigidity in real wages is needed to explain the magnitude of observed economic fluctuations.

\[17\] I discuss my results under these alternative assumptions on wages in more depth in Online Appendix B.

\[18\] In part, I lead with this case because it applies to arbitrary returns to scale in the production function \( \alpha \). Nash bargaining is well-defined when there are constant returns to scale in production, but not when decreasing returns to scale generate a distinction between the marginal product of the “first” and “last” worker. Alternative bargaining protocols such as the one proposed by Stole and Zwiebel [1996] could be considered in that case.
where \( z^t_i \) is \( i \)'s net asset position in the short-run, and short-run disposable incomes are

\[
Y^e_1 = (1 - \tau)W - P_1t + (\Pi + \Pi^r - T^r),
\]
\[
Y^u_1 = P_1b + (\Pi + \Pi^r - T^r),
\]
given aggregate retailer profits \( \Pi^r \equiv \int_1^1 \Pi^r(j) dj \) defined below.\(^{19} \)

Ex-ante, the representative worker faces

\[
v = \max_s (p(\theta)s)v^e + (1 - p(\theta)s)v^u - \psi(s)
\]
given the specification of ex-ante utility in \( (1) \).

I now turn to the problem of retailers, which is more standard. Exploiting the standard solution to workers’ lower-stage optimization problem in \( (4) \), retailer \( j \) faces

\[
\Pi^r(j) = \max_{P_1(j),y_1(j),x(j)} P_1(j)y_1(j) - (1 + \tau^r)P^I x(j) \text{ s.t.}
\]
\[
(Tech)(j): x(j) = y_1(j),
\]
\[
(Demand)(j): y_1(j) = \left( \frac{P_1(j)}{P_1} \right)^{-\epsilon} ((p(\theta)s)c^e_1 + (1 - p(\theta)s)c^u_1).
\]

where \( y_1(j) \) and \( x_1(j) \) are final goods produced and intermediate goods purchased by \( j \), respectively.

I finally summarize market clearing and the government’s budget constraint in this economy. Final goods market clearing at each date is given by

\[
(p(\theta)s)c^e_1(j) + (1 - p(\theta)s)c^u_1(j) = y_1(j) \forall j,
\]
\[
(p(\theta)s)c^e_2 + (1 - p(\theta)s)c^u_2 = (p(\theta)s)y^e_2 + (1 - p(\theta)s)y^u_2,
\]

while in the short-run intermediate goods market clearing is given by

\[
\int_0^1 x_1(j) dj = f(q(\theta)v - kv)
\]

and bond market clearing is given by

\[
(p(\theta)s)z^e_1 + (1 - p(\theta)s)z^u_1 = 0.
\]

Assuming that the government separately balances its budgets for policy in the labor market and policy targeted at retailers, we lastly have

\[
p(\theta)s [P_1t + \tau W] = (1 - p(\theta)s)P_1b,
\]
\[
T^r + \tau^r P^I \int_0^1 x(j) dj = 0.
\]

\(^{19}\text{I have not yet defined the price index } P_1; \text{ in equilibrium this will take on the standard CES form, as given in (13). Anticipating this, I use it here so I can directly work with the real net asset positions } z^t_i \text{ and level of UI } b.\)
We are now ready to define an equilibrium in the present environment.

**Definition 1.** A flexible price and wage equilibrium is an allocation \( \{ \{ c^e_1(j) \}, \{ c^e_2 \}, \{ z^e_1(j) \}, s, \nu, \{ x(j) \}, \{ y_1(j) \} \} \), set of nominal prices, wages, and profits \( \{ P^l, \{ P_1(j) \}, P_2, W, \Pi, \{ \Pi'(j) \} \} \), and tightness schedule \( \theta(W) \) such that, given policy \( \{ b, t, \tau; i, P_2; T^r, \tau^r \} \):

1. producers solve (3);
2. workers solve (4) and (5);
3. retailers solve (6);
4. \( \theta(W) \) is consistent with worker indifference across submarkets;
5. equilibrium tightness is consistent with worker and firm behavior (\( \theta = \frac{\nu}{s} \));
6. goods and bond markets clear at each date according to (7)-(10); and
7. the government’s budget is balanced according to (11) and (12).

Assuming that the allocation is interior and that the first order conditions of agents’ problems are necessary to characterize an optimum, Online Appendix A characterizes optimality and worker indifference across submarkets in the flexible price and wage equilibrium.

### 2.2 Optimal policy in the flexible price and wage benchmark

With the economic environment in place, I now seek to characterize optimal government policy given flexible prices and wages. The key result is that optimal UI remains characterized by the classic partial equilibrium trade-off between insurance and incentives from public finance: a Baily [1978]-Chetty [2006] formula. This provides a useful benchmark against which I can study the role of UI in stabilization in the following subsections.

The approach I use to characterize optimal UI is a methodological contribution which can be used in the analysis of other social insurance programs in general equilibrium. Most prior work on optimal UI is in partial equilibrium, which allows the researcher to ignore the effects of UI on macroeconomic aggregates. In the present setting, we do not have this luxury: were we to optimize among the set of competitive equilibria with respect to \( b \) and other instruments, the analysis would immediately become complex as wages \( W \), profits \( \Pi + \Pi' \), and tightness \( \theta \) respond to these changes.

Inspired by Farhi and Werning [2015]’s approach to the study of macroprudential regulation, the innovation is to solve an equivalent problem in which the planner’s controls are instead the distribution of wealth across agents and tightness itself. This remains a dual approach to the Ramsey policy problem, but amounts to a reparameterization of the controls. It is distinct from the primal approach to Ramsey policy analysis, more common in macroeconomics, in which the researcher directly optimizes over the set of implementable allocations in terms of consumption and labor supply (see, e.g., Chari and Kehoe [1999]).
This has two advantages. First, it allows us to use the tools of price theory to relate optimal policy prescriptions to behavioral elasticities which are easily interpretable and measurable, despite the complexities of working in general equilibrium with many frictions. This is particularly powerful once nominal rigidities and constraints on monetary policy are added to the environment, complicating the policy problem. Second, like the primal approach to Ramsey policy analysis, the resulting characterization of optimal policy can be implemented with a broader set of instruments than I have assumed so far, since these leave the set of implementable allocations unchanged. I discuss these implementation issues in more depth in Online Appendix B.

Formally, we make progress as follows. Standard two-stage budgeting given CES preferences over varieties implies a short-run price index

\[ P_1 = \left[ \int_0^1 P_1(j)^{1-\varepsilon} dj \right]^{\frac{1}{1-\varepsilon}} \]  

(13)

It follows that \( \{c^i_1, c^i_2\} \) which solves the ex-post problem (4) for worker \( i \in \{e, u\} \) solves the equivalent problem

\[ v^i = \max_{c^i_1, c^i_2} \left( u^i(c^i_1, c^i_2) \right) \text{ s.t.} \]

\[ (RC)^i: c^i_1 + p_2 c^i_2 = w^i_1 \]  

(14)

where real wealth levels are given by

\[ w^e_1 = \left[ \frac{1}{P_1} ((1 - \tau) W - P_1 t + (\Pi + \Pi^r - T^r)) \right] + p_2 [y^e_2], \]  

(15)

\[ w^u_1 = \left[ \frac{1}{P_1} (P_1 b + (\Pi + \Pi^r - T^r)) \right] + p_2 [y^u_2], \]  

(16)

and \( p_2 \), the relative price of long-run consumption in terms of short-run consumption, is simply the inverse of the real interest rate \( 1 + r \) defined using the standard Fisher equation:

\[ p_2 \equiv \frac{1}{1 + r}, \text{ where } 1 + r = \left( \frac{P_1}{P_2} \right) (1 + i). \]  

(17)

With this reformulation, (14) generates standard Marshallian demand functions \( c^i_1(w^1_i, p_2) \) and \( c^i_2(w^1_i, p_2) \), and indirect utility \( v^i(w^1_i, p_2) \). Then, the representative worker’s ex-ante problem (5) can be written

\[ v = \max_s \left( p(\theta)s \right) v^e(w^e_1, p_2) + (1 - p(\theta)s) v^u(w^u_1, p_2) - \psi(s), \]  

(18)

generating a labor supply function \( s(w^e_1, w^u_1, p_2, \theta) \).

With these reformulations of workers’ problems in hand, the following result greatly simplifies the characterization of the Ramsey optimal allocation.

**Proposition 1.** An allocation \( \{c^e_1, c^e_2, c^u_2, s, \theta\} \) and relative price \( p_2 \) \((\Leftrightarrow \text{ real interest rate } 1 + r)\)
form part of a flexible price and wage equilibrium if and only if there exist wealth levels \( \{ w_1^c, w_1^u \} \) satisfying the economy-wide resource constraints

\[
(p(\theta)s)c_1^c + (1 - p(\theta)s)c_1^u = f(p(\theta)s - k\theta s), \\
(p(\theta)s)c_2^c + (1 - p(\theta)s)c_2^u = (p(\theta)s)y_2^c + (1 - p(\theta)s)y_2^u,
\]

given implementability constraints \( c_1^c = c_1^c(w_1^c, p_2), c_1^u = c_1^u(w_1^u, p_2), c_2^c = c_2^c(w_1^c, p_2), c_2^u = c_2^u(w_1^u, p_2), \) and \( s = s(w_1^c, w_1^u, p_2, \theta) \) as defined in (14) and (18).

Given a utilitarian social welfare function, the Ramsey planning problem can be now stated as

\[
\max_{w_1^c, w_1^u, p_2} (p(\theta)s)(v^c(w_1^c, p_2) + (1 - p(\theta)s)v^u(w_1^u, p_2) - \psi(s(\cdot))) \text{ s.t.} \\
(RC)_1 : (p(\theta)s)c_1^c(\cdot) + (1 - p(\theta)s)c_1^u(\cdot) = f(p(\theta)s(\cdot) - k\theta s(\cdot)),
\]

\[
(RC)_2 : (p(\theta)s)c_2^c(\cdot) + (1 - p(\theta)s)c_2^u(\cdot) = (p(\theta)s(\cdot)y_2^c + (1 - p(\theta)s(\cdot)y_2^u,
\]

where I integrate the implementability constraints into the resource constraints, and suppress the dependence of the Marshallian demand and labor supply functions on controls, for brevity.

As is evident from this formulation of the problem, we can immediately use standard results from price theory to characterize agents’ behavioral responses given changes in the controls. Using this approach, we can manipulate the first order conditions of (21) with respect to \( w_1^c, w_1^u, \) and \( p_2 \) to obtain the following key result:

**Proposition 2.** Ramsey optimal risk-sharing is characterized by

\[
\frac{\partial v^c}{\partial w_1^c} \left|_{\text{GE fiscal externality}} \right. = \frac{\partial v^u}{\partial w_1^u} \left|_{\text{GE fiscal externality}} \right. \\
1 - \frac{1}{p(\theta)s} \left( \frac{\partial x_1}{\partial s} + p_2 \frac{\partial x_2}{\partial s} \right) \frac{\partial s}{\partial w_1^c} \\
1 - \frac{1}{1 - p(\theta)s} \left( \frac{\partial x_1}{\partial s} + p_2 \frac{\partial x_2}{\partial s} \right) \frac{\partial s}{\partial w_1^u}
\]

where the production-inclusive excess supply functions \( x_1 \) and \( x_2 \) are defined as functions of \( s \) and \( \theta \), holding consumption levels fixed:

\[
x_1(s, \theta) \equiv f(p(\theta)s - k\theta s) - [(p(\theta)s)c_1^c + (1 - p(\theta)s)c_1^u)] \bigg|_{c_1^c, c_1^u},
\]

\[
x_2(s, \theta) \equiv [(p(\theta)s)y_2^c + (1 - p(\theta)s)y_2^u] - [(p(\theta)s)c_2^c + (1 - p(\theta)s)c_2^u)] \bigg|_{c_2^c, c_2^u}.
\]

In (22), the Ramsey planner would like to allocate wealth levels to equate agents’ private marginal utilities of income \( \frac{\partial v^c}{\partial w_1^c} \) and \( \frac{\partial v^u}{\partial w_1^u} \), but is limited by moral hazard manifesting itself in a general equilibrium analog of a fiscal externality. In public finance, the fiscal externality reflects the impact of agents’ behavioral responses to a policy change on the government’s budget constraint. Here, the general equilibrium analog is the impact of agents’ behavioral responses on the economy’s resource constraints — more precisely, the production-inclusive excess supply functions \( \{x_1, x_2\} \). If \( \frac{\partial x_1}{\partial s} + p_2 \frac{\partial x_2}{\partial s} > 0 \), lower search reduces the net present value of resources in the economy, which is
socially costly. Since transfers to the unemployed reduce incentives to search (that is, $\frac{\partial s}{\partial w_u} < 0$ and $\frac{\partial s}{\partial w_e} > 0$), the Ramsey planner provides incomplete insurance at the optimum.

As suggested by the connection to the standard fiscal externality in public finance, we can manipulate (22) to obtain a characterization of optimal UI consistent with the classic Baily [1978]-Chetty [2006] formula. A key step is to recognize that (23) and (24) in fact imply

$$\frac{\partial x_1}{\partial s} + p_2 \frac{\partial x_2}{\partial s} = p(\theta) \left[ f'(p(\theta)s - k\theta s) \left(1 - \frac{k}{q(\theta)}\right) + p_2(y_e^u - y_e^u) \right] \equiv p(\theta) \left[ \omega \right],$$

(25)

where I refer to $\omega$, defined at a particular allocation $\{c_e^1, c_u^1, c_e^2, c_u^2, s, \theta\}$ and relative price $p_2$, as the size of transfers. The connection between $\omega$ and transfers arises because, in a competitive equilibrium, employed agents will be paid their marginal product. If the difference between employed and unemployed consumption is less than their difference in marginal product, this must be induced by a wealth transfer between these agents in equilibrium. This is formalized by the following lemma.

**Lemma 1.** In a flexible price and wage equilibrium, a particular size of transfers $\omega$ (as defined in (25)) is implemented by UI benefits $b$ satisfying

$$\omega = \frac{1}{p(\theta)s} b.$$  

Combining Proposition 2 with Lemma 1, we obtain the following implementation result:

**Proposition 3.** Ramsey optimal risk-sharing is implemented by UI benefits $b$ satisfying a general equilibrium version of the Baily-Chetty formula

$$\left(\frac{1}{p(\theta)s}\right)^2 \varepsilon_b^P(\text{unemp}) = \frac{\frac{\partial u^u}{\partial w_u} - \frac{\partial u^u}{\partial w_e}}{\frac{\partial u^u}{\partial w_e}},$$

(26)

where $\varepsilon_b^P(\text{unemp})$, the micro elasticity of the probability of being unemployed at date 1 with respect to an increase in $b$, is given by

$$\varepsilon_b^P(\text{unemp}) = -\left(\frac{p(\theta)s}{1 - p(\theta)s}\right) \frac{\partial s}{\partial w_u} b > 0.$$

(27)

Hence, as in the classic partial equilibrium analysis, optimal UI trades off the private gains from consumption insurance against the (micro) disincentive effect of UI. Why does this partial equilibrium trade-off carry over to the general equilibrium case? Intuitively, we have another supply-side instrument — the payroll tax $\tau$ — which can be used to offset any general equilibrium effects of UI on firms’ labor demand. Consistent with the intuition behind the Diamond and Mirrlees [1971] result on production efficiency, UI is then left to solve the partial equilibrium problem.
Two additional results on the Ramsey optimum are outlined in Online Appendix B. First, I characterize the Ramsey optimality condition with respect to tightness $\theta$, and show that the optimal payroll tax $\tau$ implements a generalized Hosios condition in the labor market. Hosios [1990] demonstrates that a particular surplus-sharing rule among firms and workers will induce efficiency in models with search and matching frictions, as it induces both sides to internalize the search externalities they impose on others in the labor market. In the present setting, there are additional general equilibrium fiscal externalities from search behavior. I demonstrate that the resulting optimal surplus-sharing rule requires a worker surplus share less than that implied by the Hosios condition. In a competitive equilibrium characterized by wage posting, this is implemented by a positive payroll tax, or more precisely, a progressive income tax schedule.\textsuperscript{20}

Second, I demonstrate that the optimal set of monetary policy instruments $\{i, P_2\}$ is indeterminate. This arises from a real/nominal dichotomy with flexible prices and wages, consistent with more standard models in monetary economics. The choice of the nominal rate and long-run price level serves to pin down the level of prices $P_1$ by the Fisher equation in (17), but this has no effect on the real allocation $\{c^e_1, c^u_1, c^e_2, c^u_2, s, \theta\}$ and relative price $p_2$.

Two other sets of authors, Landais et al. [2015a,b] and Jung and Kuester [2015], have previously derived a Baily-Chetty formula for optimal UI in general equilibrium. The dual approach I have developed here differs from their solution approaches.\textsuperscript{21} The difference matters in allowing me to transparently study the case with both nominal rigidities and heterogeneity in MPCs, which they do not consider — and to which I now turn.

### 2.3 Introducing nominal rigidities and inefficient fluctuations

I now motivate the study of UI in stabilization by adding two ingredients familiar from the monetary economics tradition: nominal rigidity and potential constraints on monetary policy, such as a zero lower bound on the nominal interest rate.

In particular, suppose that all retailers have posted prices in advance of date 1 at some level:

$$P_1(j) = \bar{P}_1 \forall j$$

With identical pre-set prices, each retailer $j$ facing problem (6) will simply produce and accommodate desired demand, provided it can earn non-negative profits:

$$x(j) = y_1(j) = \begin{cases} 
(p(\theta)s)c^e_1 + (1-p(\theta)s)c^u_1 & \text{provided } \bar{P}_1 \geq (1 + \tau^r)P^I, \\
0 & \text{otherwise.}
\end{cases}$$

\textsuperscript{20}To my knowledge, this result is novel. Lehmann and van der Linden [2007] demonstrate that a payroll tax may be needed alongside a UI benefit to implement the constrained efficient allocation, but do not sign it. Landais et al. [2015a] note that their “efficiency term” is related to the Hosios condition in ensuring efficiency in the labor market, but do not specify how surplus shares should precisely differ from the Hosios condition.

\textsuperscript{21}Landais et al. [2015a,b] use a dual approach, but with a different parameterization of the controls. Jung and Kuester [2015] use the primal approach.
We thus obtain the following definition of equilibrium.

**Definition 2.** A fully sticky price equilibrium is an allocation, set of nominal prices, wages, and profits, and tightness schedule such that, given policy, conditions 1-2 and 4-7 of Definition 1 are satisfied, and condition 3 is replaced by (28) and (29).

In the usual way, nominal rigidities break the real/nominal dichotomy characterized in the last subsection and enable monetary policy to be used in macroeconomic stabilization. In particular, revisiting (17), the real interest rate can now directly be controlled by policymakers through their choice of \( \{i, P_2\} \):

\[
1 + r = \left( \frac{\bar{P}_1}{P_2} \right) (1 + i). \tag{30}
\]

But monetary policy may face constraints. I consider two constraints observed in practice:

\[
\begin{align*}
  i &\geq 0 \\
  P_2 &\leq \bar{P}_2
\end{align*} \tag{31}
\]

First, the nominal interest rate is bounded below by zero, the assumed nominal rate of return on cash.\(^{22}\) Second, the long-run price level is constrained above by \( \bar{P}_2 \), equivalent to a constraint on expected inflation in view of sticky short-run prices \( \bar{P}_1 \). This can be motivated as a political economy constraint on policymakers, or the result of (unmodeled) costs of inflation between the short- and long-run. It can also reflect the stickiness of inflation expectations among the public.

The constraints in (31) can be combined with the Fisher equation in (30) to generate a lower bound on the economy’s real interest rate:

\[
1 + r \geq \left( \frac{\bar{P}_1}{P_2} \right) . \tag{32}
\]

This can become an important source of inefficiency, the consequences of which I trace out in the next subsection for the determination of optimal UI. It also motivates why the real interest rate may remain unchanged in response to a marginal change in UI generosity, generating results for the positive effects of UI in the following subsection which reverse the conclusions of prior work.

### 2.4 Normative role of UI: a generalized Baily-Chetty formula

I begin by revisiting the Ramsey optimal generosity of UI. The core result is a generalization of the Baily-Chetty formula when monetary policy is constrained, providing a precise characterization of the channels through which optimal UI should vary for macroeconomic stabilization purposes.

Characterizing optimal UI in this even more complex environment is again made tractable using the following equivalence result on the set of implementable allocations:

\(^{22}\)As argued by Eggertsson and Woodford [2004], this constraint holds even in an economy at the “cashless limit” such as the one under present study, provided that agents have the option of holding currency.
Proposition 4. An allocation \( \{c^e_1, c^u_1, c^e_2, c^u_2, s, \theta\} \) and relative price \( p_2 \) \((\Leftrightarrow \text{real interest rate } 1 + r)\) form part of a fully sticky price equilibrium if and only if there exist wealth levels \( \{w^e_1, w^u_1\} \) satisfying the economy-wide resource constraints

\[
(p(\theta)s)c^e_1 + (1-p(\theta)s)c^u_1 = f(p(\theta)s - k\theta s),
\]
\[
(p(\theta)s)c^e_2 + (1-p(\theta)s)c^u_2 = (p(\theta)s)y^e_2 + (1-p(\theta)s)y^u_2,
\]
and the ZLB implementability constraint

\[
p_2 \leq \bar{p}_2 \equiv \frac{\bar{P}_2}{\bar{P}_1}, \tag{33}
\]
given implementability constraints \( c^e_1 = c^e_1(w^e_1, p_2), \ c^u_1 = c^u_1(w^u_1, p_2), \ c^e_2 = c^e_2(w^e_1, p_2), \ c^u_2 = c^u_2(w^u_1, p_2), \) and \( s = s(w^e_1, w^u_1, p_2, \theta) \) as defined in (14) and (18).

Intuitively, the ZLB implementability constraint reflects the fact that a lower bound on the real interest rate amounts to an upper bound on the relative price of long-run consumption in terms of short-run consumption. When the real interest rate is up against the bound, long-run consumption is cheap relative to short-run consumption, reducing aggregate demand and thus output in the short-run — in a sense I make more precise below.

Comparing Proposition 4 to Proposition 1, it follows that the planning problem is identical to that under flexible prices and wages, except with (33) as an additional constraint:

\[
\max_{w^e_1, w^u_1, \theta, p_2} (p(\theta)s)(v^e(w^e_1, p_2) + (1-p(\theta)s)v^u(w^u_1, p_2) - \psi(s(\cdot))) \text{ s.t.}
\]
\[
(RC)_1 : (p(\theta)s(\cdot))c^e_1(\cdot) + (1-p(\theta)s(\cdot))c^u_1(\cdot) = f(p(\theta)s(\cdot) - k\theta s(\cdot)),
\]
\[
(RC)_2 : (p(\theta)s(\cdot))c^e_2(\cdot) + (1-p(\theta)s(\cdot))c^u_2(\cdot) = (p(\theta)s(\cdot))y^e_2 + (1-p(\theta)s(\cdot))y^u_2,
\]
\[
(ZLB) : p_2 \leq \bar{p}_2
\]

The close relationship between (34) and (21) made apparent with this reformulation of the problem will allow us to tightly compare optimal UI to its benchmark level under flexible prices and wages.

I now characterize the solution to this planning problem separately for the cases where the zero lower bound does not, and does, bind at the optimum.

Slack zero lower bound. When the zero lower bound does not bind, planning problems (34) and (21) are identical. We immediately obtain the following result:

Proposition 5. If the zero lower bound is slack, the Ramsey optimal allocation is identical to that under flexible prices and wages.

The key implication of this result is that Ramsey optimal risk-sharing characterized in Proposition 2 does not change despite the presence of nominal rigidities. The rationale is that, while there is a new distortion in nominal rigidity, risk-sharing across employed and unemployed agents need
not be affected, for policymakers have the right tool to offset the price distortion using monetary policy. Indeed, defining the natural rate of interest $1 + r^n$ to be the real interest rate in the Ramsey optimal allocation under flexible prices and wages, monetary policy can be used to target precisely this natural rate via the Fisher equation in (30).

The implementation of Ramsey optimal risk-sharing in the labor market is more subtle, however, as implied by the following lemma regarding the size of transfers $\omega$ as defined in (25).

**Lemma 2.** In a fully sticky price equilibrium, a particular size of transfers $\omega$ is implemented by UI benefits $b$ satisfying

$$\omega = \frac{1}{p(\theta)s} b + (\mu - 1) w,$$

where $\mu \equiv \frac{\bar{P}}{P_I}$ is the gross retailer mark-up and $w \equiv \frac{W}{P_I}$ is the real wage.

Comparing this to the implementation of transfers $\omega$ in Lemma 1, we see that markup variation under sticky prices can generate an additional wealth transfer across employed and unemployed agents beyond that induced by government UI policy. The sign and direction of this transfer depends on the allocation of equity shares across agents. In the present environment, employed and unemployed agents have identical equity shares since they are ex-ante identical. Since only the employed have a claim on labor income, an increase in markups amounts to a reallocation of wealth from labor to profit income, and thus a transfer to the unemployed.

The behavior of mark-ups in the Ramsey optimum is indeterminate, however. As is standard in models with sticky prices, and as is discussed further in Online Appendix B, implementing the Ramsey optimal level of production in the economy pins down the after-tax real wage $(1 - \tau)\frac{W}{P_I}$ which induces the necessary worker labor supply. However it is that the payroll tax $\tau$ is set, the pre-tax real wage $\frac{W}{P_I}$ and thus the inverse gross mark-up $\frac{P_I}{P_1}$ will adjust to induce the necessary labor supply to satisfy Ramsey optimal consumption demand.\(^\text{23}\)

Given this indeterminacy, I assume that the payroll tax is used to keep markups unchanged from the flexible price and wage allocation (i.e., at $\mu = 1$). This allows me to focus attention on redistribution through UI alone, particularly since the redistributive role of mark-ups is likely small in practice.\(^\text{24}\) Furthermore, any other choice would lead to costly price dispersion if some retailers could adjust their prices. We thus obtain the following implementation of the Ramsey optimum.

**Proposition 6.** The Ramsey optimum is implemented by monetary policy $\{i, P_2\}$ targeting the natural rate of interest

$$(1 + i)\frac{P_1}{P_2} = 1 + r^n,$$

\(^{23}\)Notably, this means that the Ramsey optimal allocation can be achieved even if no payroll tax is available ($\tau = 0$). This illustrates a general principle true in other models of nominal rigidities (e.g., Farhi and Werning [2013]): policymakers can face a relaxed planning problem as compared to the problem they face with flexible prices and wages, given the additional margin of control they gain from monetary policy.

\(^{24}\)The redistributive role of mark-ups will be small if the employed are also disproportionate owners of firm equity, as is true in practice.
and, assuming the payroll tax $\tau$ is used to eliminate any markups ($\mu = 1$), optimal UI benefits $b$ continuing to satisfy the general equilibrium Baily-Chetty formula

$$\left(\frac{1}{p(\theta)s}\right)^2 P(\text{unemp}) = \frac{\partial u^x}{\partial x_1} - \frac{\partial u^c}{\partial x_1} \frac{u^c}{\partial x_1}.$$

Incentives

Insurance

Hence, I conclude that optimal UI may fluctuate in response to macroeconomic shocks, as these generically would cause the elasticities in the Baily-Chetty formula to vary — but it remains characterized by the simple trade-off between incentives and insurance. In this sense, UI is a second-best instrument in stabilization: it is not the first tool policymakers should turn to for macroeconomic stabilization purposes. I find a stabilization role for UI in the next case, when monetary policy is unable to stabilize the economy on its own.

**Binding zero lower bound.** In this case, let us first define the relative price wedge $\tau_{1,2}$.

**Definition 3.** At the Ramsey optimum, the relative price wedge

$$\tau_{1,2} \equiv 1 - \frac{\lambda_{RC1}}{\lambda_{RC2}} p_2,$$

where $\lambda_{RC1}$ and $\lambda_{RC2}$ are the multipliers on the resource constraints in planning problem (34).

The relative price wedge captures the wedge between the social marginal rate of transformation across dates, given by the ratio of shadow prices $\frac{\lambda_{RC2}}{\lambda_{RC1}}$, and the marginal rate of substitution across dates perceived by private agents, given by the relative price $p_2$ (i.e., the real interest rate). At a first best allocation, these will be equalized — but with nominal rigidity and a zero lower bound, they may not, as formalized in the following result.

**Proposition 7.** At the Ramsey optimum,

$$\tau_{1,2} \propto \lambda_{ZLB},$$

where $\lambda_{ZLB}$ is the multiplier on the zero lower bound in planning problem (34).\footnote{I characterize the exact relationship between $\tau_{1,2}$ and $\lambda_{ZLB}$ in the proof of this proposition.} Hence,

$$\tau_{1,2} \begin{cases} = 0 & \text{if (ZLB) is slack,} \\ \geq 0 & \text{if (ZLB) binds.} \end{cases}$$

As such, the relative price wedge emerges as a sufficient statistic for the distortion created by a binding zero lower bound at the Ramsey optimum. When the zero lower bound binds and the real interest rate is “too high”, the relative price wedge makes this statement precise: the real interest rate is too high relative to the social marginal rate of transformation across dates. In this situation, the Ramsey planner would like private agents to reallocate consumption from the future
to the present, but cannot induce them to do so in view of a binding zero lower bound. This role of a relative price wedge in summarizing the consequences of constraints on monetary policy is introduced in the theory of macroprudential regulation advanced by Farhi and Werning [2015]; as I show here, it similarly motivates a stabilization role for UI.

In particular, a positive relative price wedge has two effects on the Ramsey optimal risk-sharing condition from the flexible price and wage benchmark:

**Proposition 8.** *Ramsey optimal risk-sharing is characterized by*

$$\frac{\partial v_e}{\partial w_1} = 1 - \tau_{1,2} \frac{\partial x_1}{\partial w_1} - \frac{1}{p(\theta)s} \left( (1 - \tau_{1,2}) \frac{\partial x_1}{\partial s} + P_2 \frac{\partial x_2}{\partial s} \right) \frac{\partial s}{\partial w_1},$$

$$\Delta \text{ cost of fiscal ext.} \quad \Delta \text{ cost of AD ext.}$$

$$= \frac{\partial v_u}{\partial w_1} = 1 - \tau_{1,2} \frac{\partial x_1}{\partial w_1} - \frac{1}{1 - p(\theta)s} \left( (1 - \tau_{1,2}) \frac{\partial x_1}{\partial s} + P_2 \frac{\partial x_2}{\partial s} \right) \frac{\partial s}{\partial w_1},$$

where the production-inclusive excess supply functions $x_1$ and $x_2$ are as defined in (23) and (24).

These two effects on Ramsey optimal risk-sharing directly imply two corresponding changes in the formula for optimal UI, as formalized in the following implementation result:

**Proposition 9.** *The Ramsey optimum is implemented by monetary policy \{i, P_2\} up against its constraints*

$$i = 0, P_2 = \bar{P}_2$$

and, assuming the payroll tax $\tau$ is used to eliminate any markups ($\mu = 1$), optimal UI benefits $b$ satisfying a generalized Baily-Chetty formula

$$\left(\frac{1}{p(\theta)s}\right)^2 \varepsilon b^{P\text{(unemp)}} \left( 1 - \tau_{1,2} \left( 1 - \frac{\partial c_u}{\partial w_1} - \frac{z_u}{b} \right) \right) = \frac{\partial u_u}{\partial c_1} - \frac{\partial u_e}{\partial c_1} + \tau_{1,2} \left( \frac{\partial c_u}{\partial w_1} - \frac{\partial c_e}{\partial w_1} \right),$$

(36)

for $\tau_{1,2}$ small, where $-\frac{z_u}{b}$ is the debt / UI ratio for unemployed agents in the short-run.\(^{26}\)

First, in (35) a novel aggregate demand externality distorts optimal risk-sharing across the employed and unemployed. Given short-run macroeconomic slackness summarized by a positive relative price wedge $\tau_{1,2}$, there is a difference between the social and private valuation of income arising from an agent’s propensity to spend, and thus help stabilize the economy, in the short-run.

In (36), the aggregate demand externality means that the social value of UI differs from its private insurance value in the standard Baily-Chetty formula. The wedge is governed by the

\(^{26}\)To ease interpretation of the formula, I take a first order Taylor approximation of both sides around $\tau_{1,2} = 0$. The exact formula for general $\tau_{1,2}$ is presented in the proof of this proposition.
difference in MPCs between the unemployed and employed. When this difference is positive, the social value of transfers to the unemployed through the UI system exceeds the private value of consumption insurance. This distinction between the private and social valuation of transfers in the presence of slackness is an application of Farhi and Werning [2015]’s core insight, developed in the context of macroprudential regulation, to the present framework focused on social insurance.

Second, in (35) short-run macroeconomic slackness changes the welfare cost of the general equilibrium fiscal externality. This further distorts optimal risk-sharing across the employed and unemployed. Consider the case when \( \frac{\partial x_1}{\partial s} > 0 \). This means that reduced search, and thus employment, tends to reduce the supply of resources by more in the short-run than it does the demand on those resources. The more depressed is short-run demand, as summarized by a higher relative price wedge \( \tau_{1,2} \), the less socially costly is the reduction in production induced by reduced search.

In (36), this changes the social cost of disincentives in the Baily-Chetty formula. Slackness unambiguously reduces the disincentive cost of UI when, as is empirically realistic, the unemployed are net borrowers in the short-run, which ensures that \( \frac{\partial x_1}{\partial s} > 0 \). This reduction in the social cost of disincentives shares intuition with the “rat race” effect of Landais et al. [2015a,b], who argue that inefficiently low tightness reduces the cost of disincentivizing labor supply and thus motivates optimally higher UI in a general matching framework. The present setting formalizes why the economy can find itself in a situation of inefficiently low tightness, owing to nominal rigidity, a binding zero lower bound, and thus low aggregate demand.

Taken together, in the empirically realistic case, a two-way interaction between UI and aggregate demand motivates optimally higher UI benefits than the Baily-Chetty benchmark when the economy is slack. On the one hand, UI has a novel macroeconomic stabilization role to play in view of a positive aggregate demand externality from transfers to unemployed. On the other hand, macroeconomic slackness itself reduces the social cost of disincentives due to the provision of UI.

In Online Appendix B, I complete the description of the Ramsey optimum by characterizing the optimality condition with respect to tightness \( \theta \). I use this to demonstrate that the relative price wedge \( \tau_{1,2} \) is itself revealed in a deviation of the labor wedge (as studied by Chari et al. [2007], Shimer [2009], and many others) from its natural level. This formalizes the intuition that a binding zero lower bound, reflected in a positive relative price wedge, leads to inefficient employment in the competitive equilibrium.

---

27 In the present environment, this difference in marginal propensities to consume arises from differences in the utility functions \( u^e(c_1^e, c_2^e) \) and \( u^u(c_1^u, c_2^u) \), to illustrate these ideas in the simplest way. In a robustness exercise in section 2.6, I demonstrate that this formula continues to hold when agents are hand-to-mouth, generating more realistic sources of MPC variation. And in my infinite horizon analysis studied numerically in section 3, precautionary behavior generates differences in MPCs which are sizeable and drive the aggregate demand effects of UI.

28 When the unemployed are net debtors, it must be that the difference in the level of employed and unemployed consumption is smaller than their difference in labor income, and thus marginal product. In this case, incrementally lower employment will reduce the economy’s supply of resources by more than it does the demand on those resources in the short-run — that is, \( \frac{\partial x_1}{\partial s} > 0 \).

29 By natural level, I mean the labor wedge at the Ramsey optimum with flexible prices and wages. As I show in Online Appendix B, the natural level is not zero, owing to the presence of search frictions and heterogeneity (induced by incomplete markets and moral hazard) which distinguish my environment from the neoclassical benchmark.


2.5 Positive impact of UI: the *UI multiplier*

I now move beyond welfare and evaluate how higher UI generosity affects output and employment. I define the *UI multiplier* to be the marginal impact of an increase in unemployed agents’ income on output, as an analog to the standard government spending multiplier. With fully sticky prices, the core channel is an effect of redistribution on aggregate demand, which indeed forms the basis for the aggregate demand externality identified in the normative analysis. A more subtle result is that supply-side parameters also matter, however, contrary to the standard Keynesian intuition.

**Notation and policy of interest.** I first introduce new notation which simplifies the exposition of these positive results. Define the short-run real disposable income of agents

\[
    y^e_i = \frac{1}{P_1} \left( (1 - \tau)W - P_1 t + (\Pi + \Pi' - T) \right), \\
    y^u_i = \frac{1}{P_1} \left( P_1 b + (\Pi + \Pi' - T) \right),
\](37)

so that each agent’s real wealth levels defined in (15) and (16) can be summarized as

\[
    w^e_i = y^e_i + p_t y^e_t, \\
    w^u_i = y^u_i + p_t y^u_t.
\]

In the sticky price equilibrium, the income of unemployed agents \( y^u_i \) in (37) simplifies to

\[
y^u_i = b + (1 - \mu^{-1} \alpha) f(p(\theta)s - k\theta s)
\]

where, recall, \( \alpha \) is the return to scale in production and \( \mu \equiv \bar{P}_1 P_I \) is the retailer mark-up.

In this section, I will treat the income of unemployed agents \( y^u_i \) rather than UI benefits \( b \) as the primary policy instrument of interest. As is evident from (38), \( y^u_i \) differs from \( b \) due to unemployed agents’ claim on firm equity. In response to variation in UI benefits \( b \), there will be changes in firm profits which further change the level and distribution of agents’ income, generating “second-round feedback” effects which are sensitive to the fact that all agents hold symmetric equity shares in this short-run/long-run environment. To avoid complicating the formulas based on such second-round effects, and since in practice unemployed workers likely hold smaller equity shares than employed workers, I instead directly treat the disposable income \( y^u_i \) as the policy instrument of interest. One interpretation of the resulting multipliers is that they correspond to an increase in \( b \), assuming that the payroll tax \( \tau \) is used to keep markups unchanged at \( \mu = 1 \) (as I assume in section 2.3) and that the economy’s profit share is zero (\( \alpha = 1 \)). In that case, (38) implies \( y^u_i = b \).

Now let employment and output in the short-run be \( n_1 \) and \( y_1 \), respectively:

\[
    n_1 \equiv p(\theta)s, \\
    y_1 \equiv f(p(\theta)s - k\theta s).
\]

23
The goal of this section will be to characterize the impact of changes in unemployed workers’ income $y_u$ and monetary policy $\{i, P_2\}$ on these short-run aggregates.

**Three aggregate relations.** I derive the marginal impact on employment and the UI multiplier by characterizing the micro-level responses to changes in policy, and then aggregating up to general equilibrium. After doing so, I find that three aggregate relations jointly summarize the response of output $y$, employment $n$, and search $s$ to policy:

**Aggregate demand:**
\[
\frac{dy_1}{y_1} = \mu_{AD}^y \frac{dy_1^u}{y_1} - \mu_{P_2}^y \left( \frac{di}{P_2} - \frac{dP_2}{P_2} \right) + \mu_n^y \frac{dn_1}{n_1},
\]
(41)

**Technology:**
\[
\frac{dy_1}{y_1} = \mu_{tech}^n \frac{dn_1}{n_1} + \mu_s^s \frac{ds}{s},
\]
(42)

**Labor supply:**
\[
\frac{ds}{s} = -\mu_{LS}^y \frac{dy_1^u}{y_1} + \mu_{P_2}^s \left( \frac{di}{P_2} - \frac{dP_2}{P_2} \right).
\]
(43)

In Online Appendix C, I formally derive these relations. Here, I discuss in detail four of the coefficients which are most important to build intuition: $\{\mu_{AD}^y, \mu_{P_2}^y, \mu_{tech}^s, \mu_{LS}^y\}$. I then combine these relations to characterize the effects on employment and output.

**Aggregate demand.** The coefficient $\mu_{AD}^y$ captures the redistribution effect on aggregate demand:

\[
\mu_{AD}^y = \frac{1}{1 - \frac{\partial c_e^1}{\partial w_1^1}} \left[ \frac{(1-n_1)y_1^u}{y_1} \left( \frac{\partial c_u^1}{\partial w_1^1} - \frac{\partial c_e^1}{\partial w_1^1} \right) \right].
\]

This is the key coefficient of interest. It means that a marginal increase in the income of the unemployed affects output according to the difference in MPCs between the unemployed and employed. This effect underlies the aggregate demand externality in the normative analysis. In terms of affecting output, it is scaled up by the relative fraction of aggregate income accruing to the unemployed, which has the intuitive implication that transfers to the unemployed will have greater stimulus in times of high unemployment. It is further scaled by $(1 - \frac{\partial c_e^1}{\partial w_1^1})^{-1}$, reflecting a standard Keynesian cross, wherein higher output raises disposable income, demand, and thus output, and so on.

A coefficient of secondary interest is $\mu_{P_2}^y$, which captures the aggregate demand stimulus from monetary policy:

\[
\mu_{P_2}^y = \frac{1}{1 - \frac{\partial c_e^1}{\partial w_1^1}} \left[ \frac{n_1}{y_1} \left( \frac{\partial c_e^{e,h}}{\partial P_2} - \frac{\partial c_e^1}{\partial w_1^1} \right) \right].
\]

Each agent’s interest rate sensitivity is governed by a substitution effect, which depends on the compensated demand sensitivity $\frac{\partial c^{e,h}}{\partial P_2}$ for Hicksian demand $c^{e,h}(v^1(w_1, P_2), P_2)$, as well as an income-
cum-wealth effect, which depends on a given agent’s net asset position $z_i$. The latter reflects the redistribution channel of monetary policy, as in Auclert [2015]’s analysis. The economy’s interest rate sensitivity is intuitively governed by a weighted average of these effects across agents. In times of high unemployment, economy-wide sensitivity will thus tilt towards that of unemployed agents.

These aggregate demand effects do not fully describe the equilibrium effects on output, however, owing to the presence of employment $n_1$ in the relation. This term, not present in representative agent models, captures the effect of extensive margin changes in employment on the level of aggregate demand since employed and unemployed agents consume different amounts. We thus exploit the technological relationship between output and employment to make further progress.

**Technology.** Output $y_1$, employment $n_1$, and search effort $s$ are related by (39) and (40), which implies (42). In particular, the effect of search on production (holding fixed employment) is

$$\mu_{tech}^s \equiv \alpha \left( \frac{\frac{1}{\eta} - 1}{1 - \frac{k}{q(\theta)}} \right),$$

where $\alpha$ is the return to scale in production, $\eta - 1$ is the elasticity of the vacancy-filling rate with respect to tightness, and $\frac{k}{q(\theta)}$ summarizes recruiting costs per hire. Given positive recruiting costs $\frac{k}{q(\theta)}$, a decrease in search effort $s$ will thus reduce equilibrium output.\(^{30}\) Intuitively, a reduction in search acts like a decrease in the productivity of labor, as firms need to engage in more, costly recruiting to hire the same number of workers.\(^{31}\) Contrary to the standard Keynesian intuition, the labor supply response to policy thus matters even in this demand-determined world.

**Labor supply.** The general equilibrium behavior of labor supply could potentially be quite complex, owing to the endogenous response of the wealth distribution to changes in policy. However, in Online Appendix C I show that the labor supply response is considerably simplified when considering a local change around the flexible price and wage Ramsey optimum. In that case

$$\mu_{LS}^y \equiv \frac{1 - n_1}{(n_1)^2} \left( \frac{y_1^u}{b} \right) \varepsilon_\text{b}^{P(\text{unemp})},$$

where $\varepsilon_\text{b}^{P(\text{unemp})}$, the micro-elasticity of the probability of unemployment with respect to an increase in UI, is defined in (27). Hence, the micro-elasticity tightly characterizes the general equilibrium response of search to changes in income $y_1^u$, similar to its sufficiency in the normative Baily-Chetty formula. A rise in unemployed agents’ income $y_1^u$ unambiguously reduces equilibrium search.

**UI multiplier and discussion.** We now can characterize the UI multiplier:

\(^{30}\)This requires $1 - \frac{k}{q(\theta)} > 0$, which must be true in a well-defined equilibrium since $1 - \frac{k}{q(\theta)} = \frac{1}{p(\theta)}(p(\theta)s - k\theta s)$.

\(^{31}\)This effect shares intuition with models in the “labor hoarding” literature, in which firm productivity changes with labor utilization. The difference here is that productivity-augmenting effort of workers occurs on the extensive margin, rather than the intensive margin.
Proposition 10. Around the flexible price and wage Ramsey optimum, the marginal effects of policy on employment and output are given by

\[
\frac{dn_1}{n_1} = -\mu_{p_1}^{n_1} \left( \frac{di - dP_2}{P_2} \right) + \mu_{y_1}^{n_1} \frac{dy_1}{y_1},
\]

\[
\frac{dy_1}{y_1} = -\mu_{p_2}^{y_1} \left( \frac{di - dP_2}{P_2} \right) + \mu_{y_1}^{y_1} \frac{dy_1}{y_1},
\]

where

\[
\mu_{y_1}^{n_1} = \frac{\mu_{y_1}^{AD} + \mu_{y_1}^{tech} \mu_{y_1}^{LS}}{\mu_{p_2}^{AD} + \mu_{p_2}^{tech} \mu_{p_2}^{LS}},
\] (44)

and the UI multiplier is given by

\[
\mu_{y_1}^{y_1} = \mu_{y_1}^{tech} \mu_{y_1}^{n_1} - \mu_{y_1}^{tech} \mu_{y_1}^{LS},
\] (45)

for aggregate demand coefficients in (41), technological coefficients in (42), and labor supply coefficients in (43).

We can understand these results in two steps. First consider the case when hiring costs per hire \( \frac{k}{q(\theta)} \) are small, which implies \( \mu_{tech}^{s} \) is small and also can be used to show \( \mu_{tech}^{n_1} \approx \alpha. \)

If the labor supply elasticity \( \varepsilon_b^{P(unemp)} \) is not too big, we have \( \mu_{tech}^{tech} \mu_{y_1}^{n_1} \approx 0 \) and can also show \( \mu_{tech}^{tech} \mu_{y_1}^{LS} \approx 0. \)

In that case, the UI multiplier in (45) becomes

\[
\mu_{y_1}^{y_1} \approx \alpha \mu_{y_1}^{n_1} \approx \alpha \mu_{p_2}^{n_1} \left( \frac{\mu_{y_1}^{AD}}{\mu_{p_2}^{AD}} \right) = \alpha \mu_{p_2}^{n_1} \left( \frac{1 - n_1) y_1 \left( \frac{\partial c_b}{\partial w_1} - \frac{\partial e_b}{\partial w_1} \right)}{n_1 \left( \frac{\partial c_b}{\partial p_2} - \frac{\partial e_b}{\partial p_2} \right) + (1 - n_1) \left( \frac{\partial c_b}{\partial p_2} - \frac{\partial e_b}{\partial p_2} \right)} \right). \]

Hence, the redistribution effect on aggregate demand \( \mu_{y_1}^{AD} \) is approximately sufficient to characterize the multiplier on output. Normalizing by the effect of monetary stimulus is a convenient way to collect general equilibrium effects (the Keynesian cross and the effect of extensive margin changes in employment on the level of demand) so that we can focus on the core mechanism.

Why do supply-side elasticities of UI emphasized in prior work, such as a disincentive effect or wage elasticity, not appear in this approximate formula? First, the disincentive effect of UI on search effort — which indeed survives in general equilibrium, as implied by the labor supply relation (43) — is swamped by a general equilibrium response of labor demand to the change in

\[ \text{Relative AD stimulus from redistribution} \]

\[ \text{Employment stimulus} \]

\[ \text{Loss from recruiting} \]

---

\[ \text{\textsuperscript{32}} \]If we take this short-run/long-run model literally, \( \frac{k}{q(\theta)} = \frac{k}{q(\theta)} = \frac{k}{q(\theta)} \) is the ratio of recruiters to employment. Citing the 1997 National Employer Survey, Landais et al. [2015b] report this at 2.5%, while the calibration of Shimer [2010] implies this is 0.5%. Of course, the absence of any incumbent workers makes this mapping inappropriate — and, indeed, is an important reason why I move to an infinite horizon model when I consider the quantitative application in section 3 — but it suggests that small hiring costs are an important benchmark case of interest.
aggregate demand. Since firms post more vacancies to offset any reduction in search among workers, measuring only the disincentive effect of UI will miss its aggregate effects. Second, the effect on wages is irrelevant, conditional on overall disposable income for each agent. While changes in wages may affect the source of income earned by agents (labor vs. profit), the overall level of disposable income is what matters for aggregate demand.\footnote{Aggregate demand, in turn, drives labor demand for standard Keynesian reasons, with no role played by wages.} 

At the same time, the UI multiplier is not completely determined by the redistribution effect on aggregate demand for positive recruiting costs \(\frac{k}{q(\theta)}\). In (45), the general equilibrium reduction in search (through \(\mu_{L}^{S}\)) raises recruiting costs and thus output (through \(\mu_{tech}^{s}\)). This reveals a subtle difference from the standard Keynesian intuition: while firm labor demand will endogenously offset a reduction in worker search effort due to higher UI, this is not a costless one-for-one switch. In a frictional labor market, the increase in recruiting costs tends to reduce output.

Indeed, recruiting costs rise sharply as tightness rises, and thus the vacancy-filling rate collapses:

\[
\frac{k}{q(\theta)} \to \infty \text{ as } \theta \to \infty.
\]

This underscores the limits of the earlier approximation result for small hiring costs. It suggests why, at the extreme, complete insurance need not maximize output even in a demand-determined world. As such, it provides intuition for why disincentives still matter in my generalized Baily-Chetty formula for optimal UI.

### 2.6 Robustness of the formulas

The preceding subsections characterized two key results: optimal UI in the presence of inefficient fluctuations, and the effect of a marginal increase in unemployed workers’ income on equilibrium output. In this subsection, I demonstrate that these results are robust to a number of alternative settings: sticky wages rather than prices; hand-to-mouth rather than unconstrained unemployed agents; and a small open economy facing a fixed exchange rate. Here, I provide intuition on what changes from the baseline case. The formal analysis of each case is in Online Appendix D.

#### 2.6.1 Sticky wages

With sticky wages, producers’ optimal vacancy posting condition in (3) and retailers’ optimal price-setting in (6) leads to labor demand relation

\[
f'(p(\theta)s - k\theta s) \left(1 - \frac{k}{q(\theta)}\right) = \frac{\bar{W}}{P_1}. \tag{46}
\]

\footnote{The wage response may change the distribution of disposable income across unemployed and employed agents through a redistributive role of markup variation. This depends on the allocation of equity shares across agents. As noted in footnote 24, this redistributive role of markups is likely to be small in practice. As discussed earlier, to avoid focusing on this effect, I treat the disposable income of the unemployed as the policy instrument of interest.}
The left-hand side is the marginal product of labor, accounting for hiring costs. The right-hand side is the real wage. Re-arranging to solve for $P_1$ and combining with the Fisher equation in (17), we find that the real interest rate in this economy is

$$1 + r = \left( \frac{\bar{W}}{P_2} \left( f'(p(\theta)s - k\theta s) \left(1 - \frac{k}{q(\theta)}\right) \right) \right) (1 + i)$$

$$\geq \left( \frac{\bar{W}}{P_2} \left( f'(p(\theta)s - k\theta s) \left(1 - \frac{k}{q(\theta)}\right) \right) \right),$$

where the inequality accounts for the constraints on monetary policy. Unlike the sticky price case, the real interest rate is not fully determined by monetary policy: a reduction in the marginal product of labor (accounting for hiring costs) raises producers’ real marginal cost and drives up the price level, raising the real interest rate. For the same reason, the constraint on the real interest rate posed by the zero lower bound is now endogenous.

Normatively, this leads to an additional change in the disincentive cost of UI when monetary policy is constrained. If production is characterized by decreasing returns to scale ($\alpha < 1$), the disincentive effect of UI raises the marginal product of labor by reducing employment, which relaxes the binding constraint on the real interest rate in (47). This effect further pushes towards higher optimal UI relative to the Baily-Chetty benchmark.

Positively, this generates scope for the supply-side effects of UI to partially choke off aggregate demand. In particular, the economy can be characterized by an upward-sloping aggregate supply curve relating employment and the price level, as is standard in models with sticky wages. Conditional on employment, a reduction in search effort amounts to an upward shift in the aggregate supply curve, as it raises recruiting costs and thus firm marginal costs. For small hiring frictions, this effect may again be second-order relative to the aggregate demand impact of transfers; but it again highlights why disincentives continue to matter in the determination of optimal UI.

### 2.6.2 Hand-to-mouth unemployed

Let us return to the sticky price case, but now suppose unemployed agents are characterized by hand-to-mouth behavior owing to a loss of access to credit markets. In this case, their optimization problem (14) is replaced by

$$v^u = \max_{c_1^u, c_2^u} u^u(c_1^u, c_2^u) \text{ s.t.}$$

$$(RC)^u : c_1^u + p_2 c_2^u = w_1^u,$$

$$(HTM)^u : c_2^u = y_2^u.$$

\[ 48 \]
where the \((HTM)^u\) constraint is new. This generates indirect utility \(u^u(w^u_1, p_2, y^u_2)\) and consumption demands \(c^u_1(w^u_1, p_2, y^u_2), c^u_2(w^u_1, p_2, y^u_2)\). It leads to three economic implications in equilibrium:

\[
\frac{\partial c^u_1}{\partial w^u_1} = 1, \tag{49}
\]

\[
\frac{\partial c^u_1}{\partial p_2} = 0, \tag{50}
\]

\[
z^e_1 = z^u_1 = 0. \tag{51}
\]

The first two reflect the fact that without access to credit markets, the consumption of the unemployed perfectly tracks short-run income, but is fully insensitive to the real interest rate. The last is an implication of equilibrium in the asset market: since the unemployed cannot access credit markets, the net asset position of the employed must be zero.

The resulting formulas for optimal UI and the UI multiplier are unchanged from the benchmark case, except with (49)-(51) plugged in everywhere. The hand-to-mouth behavior of the unemployed makes the aggregate demand externality and relative stimulus from redistribution unequivocally positive. Now it is unambiguous that in the presence of a binding zero lower bound, optimal UI exceeds the Baily-Chetty benchmark.

### 2.6.3 Small open economy with a fixed exchange rate

I now consider a very different environment: a small open economy (SOE) of the sort studied by Obstfeld and Rogoff [2000], Schmitt-Grohe and Uribe [2012], and Farhi and Werning [2014], with applications to peripheral European economies in the recent Eurozone downturn. In a static setting, this SOE has two classes of goods: non-traded goods produced by a domestic sector, and traded goods which are treated as endowments. There is nominal wage rigidity in the non-traded sector coupled with a fixed exchange rate and a price of traded goods (in foreign currency) which is fixed on world markets. This combination prevents adjustment in the relative price of non-tradeables, which can lead to real distortions in the presence of macroeconomic shocks.

Unlike the above authors, I enrich the production of non-traded goods with search and matching frictions, giving rise to involuntary unemployment.\(^{34}\) I further assume the presence of a government-run UI scheme, owing to the absence of private markets to insure against unemployment risk, but limited by moral hazard. The formal characterization of the economic environment and equilibrium is provided in Online Appendix D.3. In this context, does UI have any macroeconomic stabilization role to play when the SOE is buffeted by macroeconomic shocks which cannot be addressed by domestic monetary policy, owing to the fixed exchange rate?

It does — and despite the considerable differences from the benchmark model, in fact all of the earlier results apply with a simple relabeling. Short-run and long-run consumption are replaced by non-traded and traded consumption. The relative price wedge which motivates a macroeconomic

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\(^{34}\)Schmitt-Grohe and Uribe [2012] provide an alternative way of capturing unemployment in such a SOE, assuming a rationing rule with downward nominal wage rigidity in an otherwise Walrasian labor market.
stabilization role for UI concerns the relative price of non-traded and traded goods, rather than the relative price of consumption across dates (the real interest rate). And the aggregate demand effects of UI are governed by the difference in marginal propensities to spend on domestic non-tradeables, rather than on short-run consumption.\textsuperscript{35}

**Summing up.** Across the extensions considered here, the core results of my benchmark model regarding the normative and positive role of UI in stabilization remain robust. First, in the presence of nominal rigidities (either in prices or wages) and constraints on monetary policy (due to a zero lower bound or fixed exchange rate), optimal UI departs from the classic insurance-incentives trade-off familiar from public finance. An aggregate demand externality gives UI a role to play in macroeconomic stabilization, while macroeconomic slackness itself changes the relative cost of disincentives. These effects become sharply signed when unemployed agents are hand-to-mouth, unambiguously motivating higher optimal UI generosity relative to the Baily-Chetty benchmark in the presence of macroeconomic slackness.

Second, under the same nominal rigidities, a marginal increase in the income of the unemployed affects employment and output through the redistribution effect on aggregate demand. This is governed by the difference in MPCs between the unemployed and employed, and underlies the aggregate demand externality in the normative analysis. The disincentive effect of UI matters through its effect on recruiting costs; while this effect may be small, its presence suggests why disincentives still matter in the formula for optimal UI in a demand-determined world.

### 3 Quantitative application: U.S. during the Great Recession

I turn now to an analysis of the benefit extensions in the U.S. during the Great Recession – an expansion of generosity unprecedented in the history of the UI program. From a base of 26 weeks of benefit duration in almost all states, benefits were substantially extended from August 2008 through December 2013, reaching 99 weeks in some states at the depth of the recession. What effect did these extensions have on equilibrium employment and welfare?

To study these policies, I generalize the model from the theoretical analysis to an infinite horizon setting, which allows me to study the effects of policies affecting the duration of benefits in a long-lasting recession. An important implication of the calibrated model in steady-state is that the MPC of unemployed agents rises sharply with duration of unemployment. This means that in a Bewley-Huggett-Aiyagari (BHA) framework with Diamond-Mortensen-Pissarides (DMP) labor market dynamics calibrated to match salient features of the U.S. economy, the long-term unemployed are an extremely promising “tag” for high MPCs.

My simulation of transitional dynamics suggests that the benefit extensions had quantitatively...
large, positive effects on aggregate demand. Relative to a less generous counterfactual path of benefits capped at 9 months of duration in the calibrated model, the observed extensions prevent a further rise in the unemployment rate of 2–5 percentage points, and generate a strict Pareto improvement. Consistent with the theoretical analysis, these effects appear driven by the high MPCs of the marginal recipients of transfers, the long-term unemployed.

3.1 Overview of 2008–2013 benefit extensions

I first provide background on the 2008–2013 benefit extensions before moving forward with the quantitative analysis.

Almost all states had maximum benefit durations of 26 weeks prior to the Great Recession.36 Other aspects of the programs, such as eligibility for UI benefits, the replacement rate (fraction of pre-unemployment wages paid in benefits), and funding structures, differed more substantially across states (United States Department of Labor [2008]).

Over the period 2008–2013, two programs together led to a dramatic expansion in benefit durations across the country: the Extended Benefits program (EB), and the Emergency Unemployment Compensation Act of 2008 (EUC08). EB, which has been in place since 1970, automatically extends the duration of benefits in a particular state when its unemployment rate rises above a particular threshold. Over 2009-2012, EB led to an increase in benefit durations of up to 20 weeks in some states. EUC08, which was signed into federal law on June 30, 2008, substantially extended these benefit durations specifically in response to the Great Recession. A first tier of benefits provided 13 — and then, starting in November 2008, 20 — additional weeks of benefits to workers in all states. A further three tiers provided up to 33 additional weeks of benefits for states with high unemployment rates. In total, EUC08 led to an increase in benefit durations of up to 53 weeks.

Taken together, benefit durations were extended up to 99 weeks in some states at the depth of the Great Recession. Figure 1 illustrates the median benefit duration across all U.S. states over time.37 For visual comparison, the figure extends backwards to incorporate the UI response to the 2001-02 recession. As is suggested by the figure, and confirmed by examination of the historical record extending back to the creation of the UI program during the Great Depression, the benefit extensions over 2008–2013 marked an unprecedented expansion of generosity.

3.2 Economic environment

I now enrich the environment from section 2.1 to enable a more realistic evaluation of the 2008-13 benefit extensions. The primary change is the move to an infinite horizon, which allows me to more fully explore the rich interplay between incomplete markets (à la BHA) and search and matching (à la DMP) in determining the effects of changes in finite duration UI policy. In addition, I add

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36 Two states offered higher potential durations: Montana (28 weeks) and Massachusetts (30 weeks).

37 This is the median across all U.S. states as reported by Farber and Valetta [2013], weighting each state by its population as measured by BLS’ monthly estimates of the civilian noninstitutional population. I thank John Coglianese for providing me with this data.

31
three other elements which allow me to obtain a more realistic calibration of the U.S. economy: (i) duration-dependent match efficiency, (ii) an explicit borrowing constraint, and (iii) sources of non-own-labor income (such as spousal income in practice) which cushion the effects of job loss.

In this dynamic framework, a given period can be broken down into four phases: search, matching, consumption and production, and separation. I characterize the search and matching processes in more detail below. Exogenous separations, standard in such models, occur at the end of the period, and ensure that there is a positive unemployment rate in steady-state.

The economic actors remain the same as before: measure one each of workers, intermediate-good producers engaged in recruiting and production, and retailers who can accommodate demand-driven output if prices are sticky. I revisit their optimization problems, and then the policy and general equilibrium structure, before defining a flexible price equilibrium in this setting.

Agent optimization revisited. There is no longer a representative ex-ante worker; instead, workers at the beginning of each period are heterogeneous on three dimensions: the real value of assets \(z\), employment status \((e \text{ or } u)\), and the number of prior periods the worker has been unemployed in the current spell \(d\) if she is unemployed. These, correspondingly, become state variables in a recursive representation of workers’ problem. At the beginning of period \(t\), employed and unemployed workers respectively face

\[
\begin{align*}
\bar{v}_t^e(z_t) &= v_t^e(z_t), \\
\bar{v}_t^u(z_t, d_t) &= \max_{s_t} (p(\theta_t; d)s_t)v_t^e(z_t) + (1 - p(\theta_t; d)s_t)v_t^u(z_t, d_t) - \psi(s_t),
\end{align*}
\]

where \(v_t^e(z)\) and \(v_t^u(z, d)\) are value functions for workers in the middle of the period, and are conditional on employment status in the middle of period \(t\). Hence, at the beginning of the period, employed workers are already secured in their employment, while unemployed workers need to search, facing job-finding probability per unit effort \(p(\theta; d)\) which I characterize in more detail.
where $q$ denotes the exogenous real wage without commitment to long-run wage contracts. Denoting the exogenous real wage $w$ as a subscript, to avoid confusion with the state variables in parenthesis which identify a particular agent.

39 Agents can continue to trade (only) a one-period riskless nominal bond at price $M_t \equiv \frac{1}{1 + t}$ — but now, their assets are bounded below owing to a real borrowing constraint of $\bar{z}_t$ in period $t$. I defer a specification of agents’ per-period incomes $Y_t^e$ and $Y_t^u(d)$ until I discuss government policy below.

Intermediate good producers face a richer dynamic problem in view of the stock of incumbent workers they carry forward from one period to the next, and imperfect substitutability of incumbent and new workers owing to hiring costs. I specialize here to the case with exogenous real wages within the per-period bargaining set between workers and firms. In the present environment with heterogeneity across workers owing to incomplete markets, this greatly simplifies the characterization of producer optimization (and equilibrium) relative to the alternatives.\footnote{Under Nash bargaining between workers and firms, heterogeneity across workers gives them heterogeneous outside options, generating an equilibrium wage schedule as characterized in Krusell et al. [2010]. Under competitive search, workers are further heterogeneous in their attitudes towards to risk, leading to sorting across a continuum of submarkets indexed by differing degrees of labor market tightness, as pointed out by Acemoglu and Shimer [1999].}

At the same time, building on the point of Hall [2005], it remains consistent with bilateral efficiency in an environment without commitment to long-run wage contracts. Denoting the exogenous real wage $w_t$, the representative producer with $n_t$ incumbent workers then faces

\[
J_t(n_t) = \max_{\nu_t} P_t^I f(n_t + q(\theta_t)\nu_t - k\nu_t) - P_t w_t(n_t + q(\theta_t)\nu_t) + M_t J_{t+1}(n_{t+1}) \quad \text{s.t.}
\]

\[
(Evolution)_t(n_t) : n_{t+1} = (1 - \delta)(n_t + q(\theta_t)\nu_t),
\]

where $q(\theta_t)$ is the vacancy-filling rate as before, and $\nu_t$ is the number of vacancies posted. I assume

Note that this formulation of the problem already embeds an implicit assumption that incumbent workers will prefer employment to unemployment (i.e., will not voluntarily quit). Since I assume there is no fixed disutility from employment, this will of course be true in equilibrium, so for brevity I directly assume it here.

I now index consumption of a particular variety $j$ as a subscript, to avoid confusion with the state variables in parenthesis which identify a particular agent.
that producers discount future profits at the nominal discount factor $M_t$.\footnote{The proper specification of the firm’s objective in such incomplete markets environments is not obvious, as agents’ imperfectly insured idiosyncratic risk means that they will value future profit streams differently (see, e.g., Geanakoplos et al. [1990]). I make the present assumption for simplicity.}

Finally, consider retailers: since they still operate with a pure pass-through technology and can update prices each period, they continue to simply face a sequence of static problems. In particular, retailer $j$ faces

$$
\Pi_{tj} = \max_{P_{tj}, y_{tj}, x_{tj}} P_{tj} y_{tj} - (1 + \tau_r) P_{tj} x_{tj} \quad \text{s.t.} \quad 
(Tech)_{tj} : x_{tj} = y_{tj}, \\
(Demand)_{tj} : y_{tj} = \left(\frac{P_{tj}}{P_t}\right)^{-\varepsilon} c_t.
$$

(57)

Relative to (6), the only change beyond notation is that aggregate consumption $c_t$, defined below, accounts for the rich heterogeneity across agents in employment status, duration of unemployment, and asset holdings in the present setting.

**Policy and general equilibrium revisited.** I begin with labor market policy. I enrich the modeling of UI to account for duration-dependent UI benefits, as observed in practice. In particular, at time $t$, I assume a generic schedule of real benefits $b_t(d)$, financed by real lump-sum taxes on employed agents $t_t$. I no longer model additional intervention via payroll taxes.\footnote{As noted by Chetty [2006], the payroll taxes which finance the UI program in the U.S. are assessed on an earnings base which renders them inframarginal for most workers. As a result, I model such taxes as lump-sum via the tax $t_t$, and leave the analysis of other, income-contingent taxes to future work. The lump-sum tax financing UI remains distortionary through its effect on labor supply, owing to incomplete markets.}

Given the rich heterogeneity across workers in employment status, duration of unemployment, and assets in this enriched framework, we need new notation to capture the distribution of workers across the idiosyncratic state space. At the beginning of period $t$, I let $\tilde{\varphi}_e^t(z)$ and $\tilde{\varphi}_u^t(z, d)$ denote the conditional distribution of employed and unemployed agents across assets and duration of unemployment, respectively, and $\tilde{\lambda}_e^t$ denote the fraction of employed agents. I let \{\varphi}_e^t(z), \varphi}_u^t(z, d), \lambda_e^t\} denote the analogs in the middle of period $t$. Given this distribution of workers, government budget balance in its UI program at date $t$ is given by

$$
\lambda_e^t t_t = (1 - \lambda_e^t) \left[ \sum_{d=0}^{\infty} \left( \int z \varphi_u^t(z, d) dz \right) b_t(d) \right].
$$

(58)

Beyond government policy in the labor market, I assume instruments for monetary policy and policy targeted at retailers which are analogous to those in the short-run/long-run case. Monetary policy in this dynamic environment can be specified with a nominal interest rate rule in the usual way (see, e.g., Woodford [2003]). An invariant tax on retailers $\tau_r = -\frac{1}{2}$ coupled with lump-sum financing $T_t$ on all agents continues to be a passive instrument which offsets the distortion from
monopolistic competition. Government budget balance in policy targeted at retailers is then

\[ T_t^r + \tau^r P_t^l \int_0^1 x_{tj}dj = 0. \]  

(59)

In view of the above government policy, I can now characterize agents’ per-period incomes \( Y_t^e \) and \( Y_t^u(d) \). I add an ingredient not present in the short-run/long-run model: a uniform, non-random endowment \( \omega_t \) earned by all agents, capturing spousal and other household income as in the data. Then, agents’ incomes are

\[ Y_t^e = W_t - P_t b_t + (\Pi_t + \Pi_t^r - T_t^r) + P_t \omega_t, \]

\[ Y_t^u(d) = P_t b_t(d) + (\Pi_t + \Pi_t^r - T_t^r) + P_t \omega_t, \]

where \( \Pi_t \) is the per-period profit earned by the representative producer in (56).43

I now complete the general equilibrium structure of this economy by detailing the matching process and market clearing conditions. Matching differs from the short-run/long-run model owing to the assumption that match efficiencies are duration-specific. This feature of the matching process allows me to capture the real-world phenomena of employer screening and a loss of human capital as agents proceed through an unemployment spell, as implied by recent resume audit studies (Ghayad [2013], Kroft et al. [2013], and Eriksson and Rooth [2014]). When I study the UI extensions shortly, this feature also allows me to accommodate the possibility of potential hysteresis effects of extended joblessness and extended benefits (see, e.g., Ball [2009]). I assume in particular a general job-finding probability per unit effort for unemployed agents at duration \( d \) of

\[ p(\theta_t; d) = (\bar{m}(0))^{1-\eta} \bar{m}(d) \rho_t^\eta, \]  

(60)

where tightness \( \theta_t \equiv \frac{s_t}{\bar{s}_t} \) and aggregate (weighted) search

\[ \bar{s}_t \equiv 1 - \bar{\lambda}_t \sum_{d=0}^{\infty} \frac{\bar{m}(d)}{\bar{m}(0)} \int_z s_t(z,d) \phi_t^\eta(z,d) dz. \]  

(61)

In this formulation, the parameters \( \{\bar{m}(1), \bar{m}(2), \ldots\} \) control the relative efficiency of matching, as (60) implies

\[ \frac{p(\theta_t; d'')}{p(\theta_t; d')} = \frac{\bar{m}(d'')}{\bar{m}(d')} , \]

while \( \bar{m}(0) \) controls the overall level of match efficiency in the economy. Indeed, the aggregate

\[43\]I assume for simplicity that agents cannot trade equity securities, resulting in an invariant, uniform distribution of equity shares across the population. An alternative approach is to allow agents to trade these securities. While in principle this would require an additional state variable in equity holdings, I could follow Krusell et al. [2010] and take advantage of a no-arbitrage equilibrium relationship linking the nominal interest rate to the nominal return on equity shares. This would allow me to model asset holdings \( z \) as bonds plus equity. Since I calibrate the model with constant returns to scale in production in section 3.3, profit income in the calibrated economy is roughly proportional to hiring costs and is very small. I thus expect the approaches to yield virtually identical simulation results.
number of matches implied by the above formulation is
\[ m(\tilde{s}_t, \tilde{\nu}_t) = (\tilde{m}(0))^{1-\eta} \tilde{s}_t^{1-\eta} \tilde{\nu}_t^\eta, \]
and the vacancy-filling probability facing firms is given by
\[ q(\theta_t) = (\tilde{m}(0))^{1-\eta} \theta_t^{\eta-1}. \]
When \( \tilde{m}(d) = \tilde{m} \) for all \( d \), this matching process collapses to that in the short-run/long-run model.
Lastly, final goods and intermediate goods market clearing at date \( t \) are given by
\[ \begin{align*}
\lambda_t^e \left( \int_z \varphi_t^e(z) c_{ij}^e(z) dz \right) + (1 - \lambda_t^e) \left( \sum_{d=0}^\infty \int_z \varphi_t^u(z, d) c_{ij}^u(z, d) dz \right) &= y_{ij} + \omega_t \forall j, \quad (62) \\
\int_0^1 x_{ij} dj &= f(n_t + q(\theta_t) \nu_t - k \nu_t), \quad (63)
\end{align*} \]
respectively. And bond market clearing at date \( t \) is given by
\[ \begin{align*}
\lambda_t^e \left( \int_z \varphi_t^e(z) z_{t+1}^e(z) dz \right) + (1 - \lambda_t^e) \left( \sum_{d=0}^\infty \int_z \varphi_t^u(z, d) z_{t+1}^u(z, d) dz \right) &= 0. \quad (64)
\end{align*} \]
We are now ready to characterize a flexible price equilibrium in the present dynamic environment with search frictions and heterogeneous agents.

**Definition 4.** A flexible price equilibrium is a sequence of
- worker value functions \( \{\tilde{v}_t^e, \tilde{v}_t^u, v_t^e, v_t^u\} \) and policies \( \{s_t, \{c_{ij}^e, z_{t+1}^e, \{c_{ij}^u, z_{t+1}^u\}\} \);
- representative producer value functions \( J_t \) and policies \( \nu_t \);
- retailer profit functions \( \Pi_t \) and policies \( \{P_{tj}, y_{tj}, x_{tj}\} \);
- labor market tightness \( \theta_t \), employment \( n_t \), and nominal prices and profits \( \{P_t^I, \Pi_t\} \);
- and probability measures characterized by \( \{\tilde{\lambda}_t^e, \tilde{\varphi}_t^e, \tilde{\varphi}_t^u, \lambda_t^e, \varphi_t^e, \varphi_t^u\} \)

such that, given policy \( \{b_t(d), t; i_t; \tau_t; T_t^r\} \) and exogenous real wages \( \{w_t\} \):

1. workers solve (52)-(55);
2. producers solve (56);
3. retailers solve (57);
4. tightness is consistent with worker and firm behavior \( \theta_t = \frac{\lambda_t^u}{s_t}, \) given \( \bar{s}_t \) in (61);
5. goods and bond markets clear at each date according to (62)-(64);
6. the government’s budget is balanced according to (58) and (59);

7. and the probability measures characterized by \( \tilde{\lambda}^e_t, \tilde{\varphi}_t^e, \tilde{\varphi}_t^u, \lambda_t^e, \varphi_t^e, \varphi_t^u \) are consistent with labor market clearing \( \tilde{\lambda}_0^e = n_0 \) at date 0, and consistent with the above policies and stochastic elements of the model for all future dates.

### 3.3 Calibration and properties of the stationary RCE

I will assume that the economy starts in a stationary recursive competitive equilibrium (RCE), wherein macroeconomic policy and aggregates are constant. In this subsection, I numerically solve and characterize the stationary RCE as calibrated to match salient features of the U.S. economy prior to the onset of the Great Recession. The key feature of the stationary RCE is sharply rising MPCs by duration of unemployment — an endogenous outcome, not a calibrated target.

#### 3.3.1 Calibrating the stationary RCE

The definition of a stationary RCE is standard and identical to Definition 4, without time subscripts.

I first specify functional forms and certain assumed parameter values for use in the calibration, as summarized in Table 1. I assume isoelastic forms for search costs and flow utility from consumption for workers. Consistent with the approach and empirical evidence of Kroft et al. [forthcoming], I assume that relative match efficiencies by unemployment duration are related by an exponential function. The elasticity of job-finding with respect to tightness is taken from the central estimate of Petrongolo and Pissarides [2001]. For simplicity, I assume constant returns to scale in production for intermediate good firms. Finally, I assume an exogenous separation rate as calculated by Chodorow-Reich and Wieland [2015].

I assume a stepwise UI benefit schedule which is consistent with regular UI benefits in the U.S. Through the first 6 months of an unemployment spell, agents receive a UI benefit which is 50% of the prevailing wage rate in the economy. After that point, agents have exhausted their UI benefits, but receive other social assistance (welfare, food stamps, etc.) which I assume to be 10% of the wage rate.\(^{44}\) In practice of course UI and social assistance are distinct programs; for parsimony, I model them here together, since they represent fungible uses of the fiscal authority’s tax revenue.

I then solve for the stationary RCE at a monthly frequency, calibrating the remaining parameters to target six macroeconomic aggregates characterizing the U.S. economy before the Great Recession, and two behavioral elasticities highlighted as important by the earlier theoretical analysis. The targeted and simulated moments are summarized in Table 2. I furthermore include the value of the economic parameter which I primarily vary in order to target the given moment.\(^{45}\)

\(^{44}\)The UI replacement rate is consistent with the replacement rate for the average worker as reported by the Department of Labor’s Employment and Training Administration, after accounting for additional savings in taxes. The assumption for social assistance is set to be roughly consistent with the fall in household income upon UI exhaustion reported by Rothstein and Valletta [2014] in Table 3. In practice social assistance is not indexed to wages, but I simply present it this way for ease of interpretation; it has no effects on my simulation results.

\(^{45}\)In the usual way, changing any one parameter affects all of the moments in question, so the mapping is not one-to-one. Nonetheless, this mapping provides a useful guide to the underlying parameters which vary most when I
### Table 1: Functional forms and Assumed Parameters

<table>
<thead>
<tr>
<th>Side of economy</th>
<th>Functional form</th>
<th>Assumed parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>Search</td>
<td>$\psi(s) = s^{\xi+1}$</td>
<td></td>
</tr>
<tr>
<td>Matching</td>
<td>$p(\theta; d) = (\bar{m}(0))^{1-\eta} \frac{\bar{m}(d)}{\bar{m}(0)}^{\eta}$</td>
<td>$\eta = 0.7$ (PP [2001])</td>
</tr>
<tr>
<td>Consumption</td>
<td>$u(c) = \frac{c^{1-\sigma} - 1}{1-\sigma}$</td>
<td>$\sigma = 4$</td>
</tr>
<tr>
<td>Production</td>
<td>$f(n + q(\theta)\nu - k\nu) = a(n + q(\theta)\nu - k\nu)^{\alpha}$</td>
<td>$a = 0.5, \alpha = 1$</td>
</tr>
<tr>
<td>Separation</td>
<td>exogenous at rate $\delta$</td>
<td>$\delta = 0.066$ (CRW [2015])</td>
</tr>
<tr>
<td>UI and social assistance</td>
<td>$b(d) = \begin{cases} b^UI &amp; \text{for } d &lt; 6 \ b^{SA} &amp; \text{for } d \geq 6 \end{cases}$</td>
<td>$b^UI = 0.5w$ \hspace{1cm} $b^{SA} = 0.1w$ (RV [2014])</td>
</tr>
</tbody>
</table>

Among macroeconomic aggregates, I use the discount factor $\beta$ to target a real interest rate of 2%, characterizing the steady-state of the U.S. economy prior to 2008. Consistent with the long-run patterns presented in Kroft et al. [forthcoming], I target an unemployment rate of 5.5% and long-term unemployment rate of 1%, where the latter captures those who are unemployed for at least 6 months (roughly 26 weeks, in practice). I use the exogenous real wage $w$ to target the former — through its impact on firm labor demand — and use the decay of match efficiencies by duration $\lambda$ to target the latter. Importantly, I find that match efficiencies must fall through an unemployment spell to rationalize the observed fraction of long-term unemployed in the U.S. economy, consistent with empirical evidence on negative duration dependence in job-finding rates.\(^{46}\)

Defining conventional market tightness to be the ratio of aggregate vacancies and aggregate unemployment at the beginning of each period, I use the level of match efficiency $\bar{m}(0)$ to target conventional tightness of 0.634 as reported by Hagedorn and Manovskii [2008].\(^{47}\)

Recognizing that household income — at which level consumption decisions are made — does not fall as sharply as individual income upon job loss, I use the exogenous endowment $\omega$ earned by all agents to target an initial drop in household income of 25%, consistent with the evidence of Rothstein and Valletta [2014]. Finally, I use the hiring costs on the firm side, $k$, to target a recruiting to employment ratio of 2.5% as reported by Landais et al. [2015b].

In terms of behavioral elasticities, I focus on targeting the key supply- and demand-side statistics which the theoretical analysis in section 2 suggests are important. On the supply-side, the micro-level elasticity of job-finding with respect to an increase in UI generosity was the key parameter in the general equilibrium Baily-Chetty, and generalized Baily-Chetty, formulas. In the present pursuit sensitivity analysis in the next section.

\(^{46}\)In an alternative calibration where I eliminate duration dependence in match efficiencies ($\lambda = 0$) and use the remaining parameters to target the remaining moments, the calibrated model implies a long-term unemployment rate of 0.3%. Intuitively, agents search considerably harder as they proceed through an unemployment spell, leading to (counterfactual) positive duration dependence in job-finding. I discuss this and other features of labor market dynamics in the calibrated stationary RCE in Online Appendix E.

\(^{47}\)This is distinct from the concept of tightness $\theta$ which matters for job-finding and vacancy-filling in the model. $\theta$ accounts for the search effort and match-efficiencies of the unemployed, and is thus unobservable in the data.
setting, as in the real world, UI generosity involves both the level (replacement rate) and duration of benefits. Since the duration of benefits is the policy instrument which varied during the Great Recession, I focus on targeting the observed elasticity of unemployment duration to potential duration of benefits, reported by Krueger and Meyer [2002] to be 0.1. I use the elasticity of workers' disutility to marginal increases in search, $\xi$, to target this moment.

On the demand-side, the difference in MPCs between the unemployed and employed was the key statistic driving the aggregate demand externality in the generalized Baily-Chetty formula, and the redistribution effect in the UI multiplier. There is not much evidence on heterogeneous MPCs by employment status. To make progress, I use the borrowing limit $z$ to target a difference in average marginal propensities to consume between the unemployed and employed of 0.06 monthly, or 0.18 quarterly. A difference in quarterly MPCs of 0.18 is roughly halfway between the point estimates of Johnson et al. [2006] and Parker et al. [2013] concerning the difference in non-durable MPCs between medium- and low-income households, where I proxy for the unemployed with low-income.\footnote{Differences in observed income are a very imperfect proxy for employment status; the former is likely to be driven by differences in permanent income, whereas job loss is primarily a loss in temporary income.}

Given the considerable uncertainty regarding the true value of the difference in MPCs between the unemployed and employed, I consider a range of alternative targets in sensitivity analysis in the next section. In Online Appendix H, I pursue an alternative approach of calibrating the model to match other consumption moments about which we have more evidence, such as the path of consumption levels upon unemployment (e.g., Gruber [1997], Chodorow-Reich and Karabarounis [forthcoming], and Ganong and Noel [2015]). The resulting calibrations imply differences in MPCs consistent with the range I study here.

### 3.3.2 Characterizing the stationary RCE

The consumption dynamics of the calibrated stationary RCE endogenously generate sharply rising MPCs as agents proceed through an unemployment spell. Coupled with the insights of the theoretical analysis from prior sections, this previews why I find large, positive aggregate demand effects of the UI benefit extensions in the next section.
Rising MPCs through an unemployment spell are an outcome of the consumption policy functions for agents as a function of their assets, coupled with the evolution of agents’ asset holdings through an unemployment spell. Figure 2 illustrates these features of the stationary RCE by comparing the consumption policy functions and marginal distributions over asset holdings for employed agents and unemployed agents in their first month of unemployment ($d = 0$), in their fourth month ($d = 3$), and after the expiration of unemployment benefits ($d = 6$).

First, at a given level of wealth, the consumption policy functions imply a rising MPC the longer an agent remains unemployed. At high levels of wealth, the difference in consumption between the short-run and long-run unemployed is small. At low levels of wealth, though, precautionary behavior reduces the consumption of the long-run unemployed by considerably more than it does the short-run unemployed. Loosely speaking, it follows that consumption as a function of wealth will grow more concave as an agent proceeds through an unemployment spell. Since wealth and current income enter symmetrically into cash on hand, it follows that the MPC out of income tends to rise with unemployment duration, holding wealth constant.\footnote{One region where this does not hold is at the very lowest levels of wealth, when comparing unemployed agents in their first month of unemployment ($d = 0$) with those a few months into it ($d = 3$). This has to do with the path of job-finding rates per unit effort and income through an unemployment spell. At low levels of wealth, unemployed agents at $d = 0$ are optimally up against the borrowing constraint since they expect to exit unemployment fairly soon, and thus borrow considerably against future income. As an unemployment spell drags on, however, job-finding rates fall, and benefit exhaustion draws near. Hence, unemployed agents at $d = 3$ no longer borrow so aggressively, so in the lowest region of wealth their MPC will be lower than that of agents at $d = 0$.}

Second, the marginal distributions of agents reveal that they optimally decumulate assets through an unemployment spell, pushing them deeper into the region of the state space with higher marginal propensities to consume. Intuitively, an unemployment spell corresponds to a loss in temporary income relative to permanent income. Agents thus tend to borrow against future income while unemployed, despite the precautionary concerns highlighted above.

Together, these forces mean that in a BHA framework with DMP labor market dynamics calibrated to match the incidence and duration of unemployment in the U.S. economy, the long-
term unemployed are an extremely promising “tag” for high marginal propensities to consume. Table 3 summarizes the one-month and implied three-month MPCs out of unexpected, transitory income for unemployed agents by duration, as compared to those of employed agents. The model implies that relative to the MPC of the employed, the MPC of the long-term unemployed is 24% higher at a 1-month horizon, and 56% higher at a 3-month horizon. Later, I demonstrate that these differences remain large across calibrations I study in the sensitivity analysis.

I conjecture that two changes to the model could change this result. First, additional heterogeneity among workers could render the long-term unemployed a fundamentally different population than the employed or short-term unemployed. This is intimately related to the reason for duration dependence in job-finding rates: I assume structural duration dependence, whereas an alternative literature argues that heterogeneity among job-seekers is more important in explaining this phenomenon (e.g., Ahn and Hamilton [2015] and Alvarez et al. [2015]). If heterogeneity is important, and the long-term unemployed are characterized by low temporary and permanent income, then their MPC may not be so much higher than that of the short-run unemployed or employed.

Second, additional sources of idiosyncratic risk could change the dynamic evolution of wealth among the unemployed. In the present setting, long-term unemployment is “as bad as it gets”. As such, a long-term unemployed agent will borrow heavily against future income, pushing her closer to the borrowing constraint and driving up her MPC. In practice, additional risks, such as health risks or the loss of spousal income, may change this outlook for the long-term unemployed. The desire to maintain a buffer stock of savings could mitigate borrowing, and thus reduce the MPC.

Empirical work on the MPC profile by unemployment duration could help us identify whether adding such model ingredients to the present framework is first-order. Future applied work on unemployment should focus on measurement of this MPC profile, given its important role in driving the aggregate demand effects of UI extensions in the simulations I turn to next.

### 3.4 Transitional dynamics and evaluation of UI policy

I now characterize the economy’s transitional dynamics in response to a macroeconomic shock causing a large recession in view of sticky prices and a binding zero lower bound. I calibrate the shock to induce a binding zero lower bound for 90 months and rise in the unemployment rate of 4.5 percentage points, as has been roughly true in the U.S. during the Great Recession. I then compare outcomes under the observed path of UI benefit extensions with those under counterfactual paths.

Relative to less generous policies, I find large, positive effects of the benefit extensions on employment and welfare operating through aggregate demand. The result is traced to this paper’s...
unique combination of DMP labor market dynamics, heterogeneity in MPCs by unemployment duration, and collapse in aggregate demand owing to sticky prices and a binding zero lower bound.

3.4.1 Equilibrium given a macroeconomic shock

I assume that an unexpected macroeconomic shock occurs at date 1, which I associate with July 2008. Once the shock is realized, I assume there is no remaining uncertainty over the future path of macroeconomic aggregates and economic policy, as in similar experiments performed in Guerrieri and Lorenzoni [2015], Auclert [2015], and Kaplan et al. [2015].

Like the first two papers, I assume a straightforward form of nominal rigidity under which I can evaluate stabilization policy in this context: retailer prices are permanently fixed at

\[ P_{tj} = \bar{P} \forall j, t. \]  

(65)

Each retailer \( j \) facing problem (57) will then simply produce and accommodate desired demand, provided it can earn non-negative profits:

\[ x_{tj} = y_{tj} = \begin{cases} c_t & \text{provided } \bar{P}_t \geq (1 + \tau^r)P_t^I, \\ 0 & \text{otherwise}. \end{cases} \]  

(66)

While this is of course an extreme treatment of nominal rigidity, it can be justified on three grounds. First, the shock modeled below will be temporary; since the economy will converge to the original steady-state in the long-run, retailers’ incentive to update prices will eventually vanish. Second, in environments with a binding zero lower bound, it is well known that policies with ordinarily contractionary supply-side effects can become expansionary, owing to the creation of inflation expectations which serve to lower the real interest rate. This has led to a vigorous debate over the plausibility of this channel and the implicit assumptions on which it is based (see, e.g., Kiley [2014], Wieland [2014], and Cochrane [2015]). Fully sticky prices ensure that all of my results below are not generated through this channel, and instead allow me to focus on the other more transparent effects on aggregate demand. Third and finally, inflation has indeed been quite low during the Great Recession.

We thus obtain the following definition of equilibrium.

**Definition 5.** A fully sticky price equilibrium is a set of value functions, policies, tightness, employment, nominal prices and profits, and probability measures such that, given policy and exogenous real wages, conditions 1-2 and 4-7 of Definition 4 are satisfied, and condition 3 is replaced by (65) and (66).

I will study this equilibrium with an initial distribution of agents by employment status, unemployment duration, and assets inherited from the stationary RCE characterized earlier. I now discuss the exogenous shock and policy response which drive dynamics in this economy.

In my benchmark analysis, I assume that the relevant macroeconomic shock at date 1 is to preferences: a rise in the discount factor raises desired saving in the economy, driving down the real
interest rate. The entire path of the discount factor after the initial shock is depicted in the first panel of Figure 3.\footnote{I choose the path of the discount factor to match two features of the observed macroeconomic dynamics during the Great Recession. First, the initial rise in the discount factor is chosen to match the observed 4.5% increase in the unemployment rate in the equilibrium with sticky prices and a binding zero lower bound. Second, the speed with which the discount factor returns to steady-state is chosen to match a binding zero lower bound for roughly 90 months, consistent with exit in December 2015 (given date 1 corresponding to July 2008).} Such a shock is easy to work with and has also been used by other researchers studying the zero lower bound (e.g., Krugman [1998], Eggertsson and Woodford [2003]).

The path of UI policy from period 1 onwards matches that observed in the U.S. from July 2008 onwards. The second panel of Figure 3 summarizes the simulated duration of benefits in months, which is simply identical to Figure 1 after assuming 4.5 weeks per month.

Given this shock and policy, the endogenous behavior of macroeconomic aggregates in the natural allocation — the equilibrium under flexible prices — is summarized by the dotted lines in Figure 4. Except for two brief periods when the natural rate exceeds zero, around 3 and then 5 years after the initial shock, the natural rate of interest in the first panel is negative for roughly 90 months.\footnote{The fluctuations in the natural rate in these two episodes result from the significant change in the generosity of UI around these times: the triggering off of benefits provided by the EB program in the first half of 2012, and termination of the EUC program in December 2013.} The shock, coupled with the dramatic expansion in generosity of UI, has implications for the aggregate unemployment rate in the second panel, but it remains within roughly one percentage point of its steady-state level.

With fully sticky prices, I assume that the monetary authority follows an interest rate rule which implies that the nominal rate targets the natural rate of interest once the economy is permanently away from a binding zero lower bound, and is at the zero lower bound otherwise.\footnote{This path of the nominal rate may not be optimal — indeed, in a representative agent framework Krugman [1998], Eggertsson and Woodford [2003], and Werning [2012] demonstrate the gains from policy commitments beyond the liquidity trap, such as maintaining a zero nominal rate. Characterizing optimal monetary policy in the present heterogeneous agent environment remains an open question for future work. I choose the present path for simplicity.} The resulting path of the real interest rate is summarized by the solid line in the first panel of Figure 4.
In view of the resulting binding zero lower bound for 90 months, the economy undergoes a severe recession, as indicated by the dramatic rise in the unemployment rate in the second panel of Figure 4.\footnote{Note that labor market variables exhibit damped oscillations in response to shocks in my simulation. For ease in visualizing outcomes, I smooth the labor market series by plotting every other point. This damped oscillatory property of the model appears related to the value of hiring costs $k$. In alternative calibrations where this is reduced, such behavior disappears.} This headline rate rises to roughly 10\% in view of the collapse in aggregate demand, consistent with the maximum unemployment rate in the U.S. during the Great Recession. Of course, the path of the unemployment rate differs from that observed in practice; whereas perfect foresight of a binding zero lower bound implies peak unemployment at month 1 in the simulated model, in practice the unemployment rate did not peak until late 2009, well after the downturn had begun.\footnote{It would be interesting to understand the consequences of uncertainty in the duration of the zero lower bound on the time-path of the simulated unemployment rate, and the analysis of counterfactuals I turn to next. Accounting for this source of aggregate risk in heterogeneous agent environments is computationally demanding for well known reasons, though approximation algorithms like those in Krusell et al. [2010] and Carroll et al. [2015] may be helpful in this regard. I leave this extension of the model to future research.}

Beyond the headline behavior in the unemployment rate, low demand induced by a binding zero lower bound has consequences for a number of additional labor market aggregates whose behavior was not targeted in calibrating the discount factor shock. I discuss these in Online Appendix F. Notably, vacancies, and thus a conventional measure of tightness, collapse relative to the natural allocation. This occurs for two reasons. First, a collapse in aggregate demand reduces labor demand through standard Keynesian channels. Second, similar to a point made by Hall [2015], a real interest rate which exceeds that in the natural allocation makes hiring, a form of investment in this frictional labor market, more costly.

With the transitional dynamics under the observed 2008-13 UI policy in place, we can now turn to an evaluation of counterfactual paths.
3.4.2 2008-13 extensions vs. counterfactual capped at 9 months

I begin by comparing the observed path of extensions to counterfactual UI policy capped at 9 months of duration. The latter is consistent with the greatest degree of average generosity across U.S. states prior to the Great Recession. An alternative interpretation of this counterfactual is that (roughly speaking) it would have allowed for the duration extensions provided by the EB program, which is legislated into law, but would have eliminated the additional emergency measures taken by the federal government through their EUC08 program. Figure 5 compares the observed and counterfactual path of benefit durations.

To evaluate outcomes under the counterfactual policy, I first re-solve for the natural allocation under this fiscal policy stance, and then apply the nominal interest rate policy rule discussed in the prior subsection to characterize a counterfactual path for the nominal interest rate. Figure 6 illustrates the path of the natural rate and nominal interest rate given sticky prices under this
Figure 7 summarizes the key quantitative, positive takeaway of my analysis: under sticky prices and a binding zero lower bound, the greater generosity of UI under the observed path prevents a substantial rise in the unemployment rate. In this benchmark calibration, I find the observed extensions prevent an initial rise in the unemployment rate of 5 percentage points, and that this difference in simulated unemployment rates remains positive for over 3 years after the initial shock.\textsuperscript{55} This is consistent with large, positive effects on aggregate demand.

Notably, the unemployment differences between the actual and counterfactual UI scenarios are reversed from those in a flexible price environment, where the effect of UI on disincentives dominates. In particular, in the flexible price case, the unemployment rate reaches a level over 1 percentage point \textit{higher} under the more generous, actual UI policy. Prior analyses of unemployment insurance in calibrated macroeconomic models have focused on such flexible price frameworks (e.g., Krusell et al. [2010], Nakajima [2012], and Mitman and Rabinovich [2015]). In this sense, accounting for the consequences of nominal rigidities and constraints on monetary policy during the Great Recession reverses our understanding of the general equilibrium effects of UI.

Beyond the unemployment comparison, we can further compare welfare under the two policy regimes. For each agent, we can compare indirect utility at the beginning of date 1, after realization

\textsuperscript{55}Note that the initial increase in unemployment under the counterfactual path exceeds the fraction who would exogenously separate from their firms in month 1. This reflects the collapse in aggregate demand, and thus desired hiring, under the counterfactual policy. Firms in fact would want to shed some workers in this case. To accommodate this and thus obtain a well-defined interior equilibrium, I enrich the model to allow firms to costlessly lay off workers, as described in Online Appendix G. Layoffs only occur in month 1, owing to my assumption of perfect foresight.
of the macroeconomic shock, under the actual and counterfactual policy. Formally, we define

\[ dW^e(z) = \tilde{v}^e_1(z)|_{\text{actual}} - \tilde{v}^e_1(z)|_{\text{counterfactual}}, \]
\[ dW^u(z,d) = \tilde{v}^u_1(z,d)|_{\text{actual}} - \tilde{v}^u_1(z,d)|_{\text{counterfactual}}, \]

where the first corresponds to employed agents and the second to unemployed agents at the beginning of date 1. We can then aggregate up using utilitarian weights for agents within each employment and duration category to compute

\[ dW^e \equiv \int_z dW^e(z) \hat{\varphi}^e(z) dz, \]
\[ dW^u(d) \equiv \frac{1}{\int_z \hat{\varphi}^u(z,d) dz} \int_z dW^u(z,d) \hat{\varphi}^u(z,d) dz. \]

Plotting these aggregates, Figure 8 demonstrates that the greater generosity of UI under the actual path leads to welfare gains for the average agent across all categories. Remarkably, even initially employed agents who are likely to bear the brunt of paying for the benefit extensions gain from greater generosity. Intuitively, the effects of greater redistribution on aggregate demand raise currently these agents’ job finding rates should they lose their job in the future, raising their continuation utility by enough that they gain from the policy change — consistent with the aggregate demand externality emphasized by the earlier theoretical analysis.

In fact, a closer examination of welfare changes within each employment and duration category reveals that the actual policy leads to a strict Pareto improvement as compared to the counterfactual policy under sticky prices. Even the wealthiest, employed agents gain from the observed extensions, owing to strong aggregate demand externalities from transfers to the unemployed.

Aggregating these welfare changes with utilitarian weights across all agents, the greater generosity of UI under the actual path leads to a 3.2% increase in social welfare. In money metric
terms, this is equivalent to the welfare gain provided by a 0.4% per-period rise in consumption for all agents in all states, at all dates, as summarized in the second column of Table 4.\textsuperscript{56}

Again, these welfare results are reversed from the case with flexible prices, highlighting the importance of nominal rigidities and constraints on monetary policy to these results. With flexible prices, in Figure 8, only the short-run unemployed who will directly gain from the benefit extensions beyond 9 months see their welfare rise from the actual policy. The initially employed, as well as long-term unemployed, see considerable welfare losses, owing to the direct cost of the policy and the reduction in job-finding rates due to general equilibrium effects on labor demand. On balance, as shown in the first column of Table 4, utilitarian social welfare falls under the actual benefit extensions relative to the counterfactual.

**Sensitivity.** I now characterize the sensitivity of these employment and welfare results to alternative calibrations. I focus on two simulated moments highlighted above: the change in peak unemployment and the change in social welfare between the observed extensions and counterfactual policy. I define these moments so that a positive value is consistent with a stabilizing effect of greater generosity provided by the observed extensions. In the baseline case characterized above, these were simulated to be 5 percentage points and 3.2%, respectively.

I focus on varying the calibration to target alternative values for three empirical moments about which there is uncertainty. First, as noted earlier, we know least about the heterogeneity in MPCs by employment status. While in the baseline I targeted a difference in average MPCs between the unemployed and employed of 0.06, here I consider smaller differences of 0.02 and 0.04 to assess whether the sizable effects of the extensions on aggregate demand survive.\textsuperscript{57} Second, applied researchers have obtained a range of estimates of the unemployment duration elasticity to the potential duration of UI benefits. While in the baseline I targeted an elasticity of 0.1, consistent with the central tendency of the estimates surveyed by Krueger and Meyer \cite{2002}, here I consider alternatives of 0.05 and 0.15.\textsuperscript{58} Third, estimates of the magnitude of recruiting costs in the economy

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|}
\hline
Metric & Flexible prices & Sticky prices + ZLB \\
\hline
\%\(\Delta SW\) & -0.3\% & +3.2\% \\
Equiv \%\(\Delta c\) & -0.03\% & +0.4\% \\
\hline
\end{tabular}
\caption{Utilitarian social welfare under actual policy relative to counterfactual terms, this is equivalent to the welfare gain provided by a 0.4\% per-period rise in consumption for all agents in all states, at all dates, as summarized in the second column of Table 4.\textsuperscript{56}}
\end{table}

\textsuperscript{56}This is a simple money metric to compute numerically, as it keeps the labor supply allocation unchanged. Of course, for the same reason, it is not an implementable (incentive compatible) counterfactual.

\textsuperscript{57}One motivation for these bounds comes from Japelli and Pistaferri \cite{2014}, which to my knowledge is the only paper measuring heterogeneity in MPCs by unemployment status. In a survey of Italian households, these authors find that unemployed households report an MPC which exceeds that of employed households by 0.07. Survey respondents were not asked over what horizon they intended to spend a temporary income windfall. While we must be cautious about extrapolating from Italy to the United States, the bounds I consider here are roughly consistent with the evidence from this paper, accounting for uncertainty in the horizon of reported spending.

\textsuperscript{58}This is a smaller range than I would like; Meyer \cite{1990} and Katz and Meyer \cite{1990} report a duration elasticity with respect to potential duration as high as 0.5. I am unable to solve for a stationary RCE with a duration elasticity exceeding 0.18, suggesting that there is a restriction on equilibrium existence given the particular functional forms and other parameter values I have assumed.
vary depending on the source and definition used. While in the baseline I targeted a fraction of recruiters to employment of 2.5%, here I consider alternatives of 1.5% and 0.5%. The latter, while quite low, is consistent with the calibration of Shimer [2010]. Online Appendix I summarizes the calibrations in each case.

Consistent with the theoretical analysis, Figures 9 and 10 demonstrate that the targeted difference in average MPCs has a very significant impact on the simulated effect of benefit generosity on employment and welfare, while the targeted duration elasticity to benefit duration, and fraction of employment spent on recruiting, play smaller roles. Moreover, in all cases the extensions to 22 months prevent a further deterioration in employment, and raise social welfare, relative to the case with benefits capped at 9 months.

The one exception is that the fraction of employment spent on recruiting appears to have a fairly big impact on the welfare results. When I reduce hiring costs $k$ to vary this moment, the equilibrium labor market dynamics exhibit smaller oscillatory behavior in view of the point made in footnote 53. I conjecture that this is responsible for the sensitivity of the welfare results to this parameter, since welfare will clearly be affected by idiosyncratic employment volatility for individual agents.
Given the lack of substantial empirical evidence regarding the heterogeneity in MPCs by employment status, I use the full range of estimates in Figures 9 and 10 to conclude that the observed benefit extensions prevent a further rise in the unemployment rate by 2–5 percentage points, and raise social welfare by 1-4%, relative to the counterfactual. Notably, across this range of calibrations, I still find that the observed policy generates a strict Pareto improvement relative to the counterfactual, owing to the aggregate demand externality from marginal extensions covering the long-term unemployed.

**Discussion.** I now discuss the interpretation and implications of these results.

*MPCs vs. precautionary saving.* My theoretical formulas and the above sensitivity analysis demonstrates the importance of MPC differences in driving aggregate demand effects of the benefit extensions. But it is important to point out that there exists another, distinct effect of the benefit extensions on aggregate demand operating through precautionary saving. This effect was not present in my short-run/long-run model, owing to perfect foresight after the resolution of initial unemployment uncertainty, but is present in the calibration — and surely in the real world.

In particular, an expansion in generosity for the long-term unemployed will reduce the idiosyncratic downside risk faced by the short-term unemployed, and even the employed. This can be expected to reduce their incentives to precautionary save. In Online Appendix J, I make this precise in a partial equilibrium analysis of a short-term unemployed agent in a dynamic setting. I focus on an experiment in which benefit generosity is raised should she remain unemployed in the following period, while taxes financing the policy change are raised should she gain employment in the following period. This renders her expected future income unchanged; yet, when her flow utility is consistent with a positive coefficient of prudence, her present consumption will rise.

In general equilibrium, we can expect that the reduction in precautionary saving due to this channel will further boost aggregate demand. Relative to other forms of fiscal stimulus, this may render UI extensions a particularly powerful tool in stabilization owing to the considerable downside risk posed by job loss. In ongoing research, I am extending the theoretical analysis to account for this precautionary savings channel and identify statistics which may be sufficient to characterize its relative importance compared to heterogeneity in MPCs.60

**Wages.** In my counterfactual analysis, I keep real wages unchanged from their (flat) path assumed in the simulation of the observed benefit extensions. This is consistent with available micro evidence that the re-employment wage elasticity to changes in benefit generosity is indistinguishable

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60Note that my sensitivity analysis does not quite speak to the relative roles played by MPC differences and this precautionary channel. The primitive which I primarily vary in order to target a particular MPC difference is the borrowing constraint $z$. But this should tend to affect the precautionary saving channel in the same direction as it affects the MPC channel. Varying the coefficient of relative risk aversion, related to the coefficient of prudence with CRRA preferences, could be one way of distinguishing between these channels. However, this would also have a direct impact on the economy’s response to primitive shocks owing to the tight link with the intertemporal elasticity of substitution given CRRA preferences, again making the interpretation of the sensitivity results challenging.
from zero (e.g., Card et al. [2007], Lalive [2007], and van Ours and Vodopivec [2008]). In recent work, however, Hagedorn et al. [2015a] have estimated large and positive effects of the 2008-13 benefit extensions on local labor market wages.

Allowing wages to respond to changes in UI policy would not have any direct effect on labor demand in my simulation. With fully sticky prices, nominal rigidity renders output demand-determined through standard Keynesian channels. The wage response to benefits would affect output only through implied redistribution from markup variation, which depends on the allocation of equity shares across agents, and which in practice is small when firm profits are small. I demonstrate this formally in Online Appendix K, where a real wage elasticity of the order of magnitude in Hagedorn et al. [2015a] has a very minor effect on the comparison between the observed UI extensions and counterfactual durations capped at 9 months under sticky prices.

Accommodating partial price stickiness would change this result, but I conjecture would only increase the stimulative impact of the benefit extensions. When prices can adjust, upward pressure on wages would raise firms’ marginal cost — but, in an economy at the zero lower bound, would create inflation expectations which reduce the real interest rate and stimulate aggregate demand. This “paradox of toil” is well established in the New Keynesian literature at the zero lower bound (see, e.g., Eggertsson and Krugman [2012]), and has indeed been found by other researchers to motivate higher stimulus from transfers (Christiano et al. [2015] and Giambattista and Pennings [2015]). I conclude that an equilibrium wage response, if one exists, is unlikely to reverse the stabilizing role played by UI benefit extensions in the present framework.

**Magnitudes and measurement.** In addition to perfect foresight regarding the path of the nominal interest rate, discussed earlier, my calibration assumes perfect foresight over the path of UI policy. This is a very helpful simplification for numerical purposes, but surely misses some degree of uncertainty regarding the government’s policy response in practice. Uncertainty over the path of policy will likely mitigate the initial stabilization gains from the extensions in my simulated model, as the permanent income gains from future stimulus will not be fully anticipated by agents.

Conversely, my simulation does not account for the fact that the observed UI extensions were primarily debt financed, rather than balanced-budget increases. Since the model is non-Ricardian, debt finance is likely to increase the aggregate demand effects of the benefit extensions, rendering my estimates an underestimate.

Finally, given the importance of MPCs in driving these general equilibrium results, my analysis highlights the importance of future empirical work measuring the consumption response of unemployed agents to transitory income shocks at various durations of unemployment. Such estimates could narrow down the range of effects I find in this paper. As discussed earlier, such estimates could also inform whether other sources of heterogeneity or idiosyncratic risk are important features to be added to the modeling framework. Measurement of this MPC profile should join the consumption drop upon unemployment, the disincentive elasticity to benefit changes, and the wage elasticity to benefit changes as a key object of interest in the applied literature on UI.
3.4.3 Additional counterfactuals on the duration and replacement rate margin

We can use the simulated model to consider two final policy experiments of interest. First, we can compare the observed extensions to a broader set of counterfactuals operating on the duration margin: to what extent were the observed extensions optimal? Second, we can compare the observed extensions to counterfactuals keeping durations at the regular 6 months, but varying the replacement rate of benefits: how would these policies compare?

**Broader set of counterfactual durations.** I first consider a broader set of counterfactual paths operating along the duration margin. At the extremes, benefits could have been limited to the regular duration of 6 months, or could have been extended to cover any unemployed worker at the depth of the Great Recession. A less extreme set of changes would have been to cap benefits at 16 months, or extend them to 28 months. Along with the baseline counterfactual of durations capped at 9 months, I summarize the full set of counterfactual durations in Figure 11. Unemployment and
welfare under this class of counterfactual policies is summarized in Figure 12.

The analysis of these counterfactuals reveals that within this restricted class, indefinite benefit extensions are optimal, while the absence of any extensions make an already bad recession considerably worse. Importantly, it is worth remembering that indefinite benefit extensions for a finite period of calendar time does not eliminate the incentives to search: since the replacement rate remains unchanged, and the extensions are only temporary, the value from employment still exceeds that from unemployment.

A subtle implication of these counterfactuals is an asymmetry result: relative to doing nothing, the observed policy delivers very significant employment and welfare gains; but relative to extending benefits indefinitely, the observed policy achieves almost all of the gains in employment and welfare. This result follows from a rich interplay between the pattern of unemployment rates and MPCs by duration during the simulated recession. First, while there are many agents who become unemployed for 6 to 22 months during the simulated downturn, there are considerably fewer who remain unemployed for more than 22 months. Second, while the MPC tends to rise through an unemployment spell, it is bounded above by one. Taken together, the aggregate demand effects of extensions from 6 to 22 months are considerably larger than those from 22 to indefinite duration. This interaction between the fraction of agents covered by the marginal policy change, and the MPC of the marginal recipients of transfers, is precisely what is suggested by my theoretical result on the size of the UI multiplier in the simpler short-run/long-run model.

**Duration vs. level of benefits.** I now compare the observed policy to a class of counterfactuals where the level, rather than duration, of benefits is made more generous. In particular, I consider five counterfactual policies in which the duration of benefits is the regular 6 months, while the replacement rate rises from 50% to reach 60%, 70%, 80%, 90%, or 100%. For each counterfactual, I assume that the time-path of replacement rates mirrors that of the benefit durations under the observed policy. I summarize the counterfactual replacement rates in Figure 13. Unemployment and welfare under this class of counterfactual policies is summarized in Figure 14.

The observed duration extensions deliver greater employment and utilitarian social welfare than each of these policy changes on the level margin. This result is consistent with a dominant role for the differences in MPCs: while the replacement rate increases apply to any agent who becomes unemployed, rather than only the long-term unemployed, the duration extensions target an endogenously high-MPC population. This result is particularly notable because the calibration has “stacked the deck” against the duration extension: in the stationary RCE, the simulated duration elasticity to a change in the replacement rate is 0.16, whereas the central tendency from empirical estimates is 0.5 (Krueger and Meyer [2002]). Thus, even with a disincentive response to replacement rate changes which may be too low relative to that observed in the real world, the simulated model still implies that the observed duration extensions had larger, positive effects on employment and welfare due to their effects on aggregate demand.

While further research on the optimal policy is surely needed, my results suggest that policy-
makers should continue to focus on duration extensions rather than replacement rate generosity as the primary lever within the UI system to vary in macroeconomic stabilization, given the high MPCs of the long-term unemployed.

4 Conclusion

This paper contributes to our understanding of the role that the UI system, a key part of the social safety net in advanced economies, can play in macroeconomic stabilization of short-run fluctuations.

Building on the classic analysis of optimal UI in public finance, I theoretically demonstrate that the interaction between UI and aggregate demand naturally motivates higher generosity when the economy is slack. In the presence of nominal rigidities, a redistribution effect on aggregate demand drives the marginal impact of higher UI on output. When the economy is slack, this means that UI takes on a novel macroeconomic stabilization role arising from the aggregate demand externality.
caused by transfers to the unemployed. Moreover, low aggregate demand itself changes the social cost of disincentivizing labor supply. When the unemployed have a higher MPC than the employed and are net debtors, these channels imply a positive effect of redistribution on output, and optimal generosity which is higher than the classic public finance formula would imply.

In a calibrated infinite-horizon generalization of this model, I apply these insights to speak directly to the policy debate over the unprecedented extension of UI generosity in the U.S. during the Great Recession. My simulations suggest that the observed path of extensions substantially helped to prevent a deeper recession and raise welfare. Relative to a counterfactual path capped at 9 months of duration, I find that the observed extensions to 22 months prevent a further rise in the unemployment rate of 2–5 percentage points and generate a strict Pareto improvement. Consistent with the theoretical analysis, these effects appear driven by the fact that the benefit extensions targeted transfers to the high-MPC long-term unemployed.

My characterization of UI in stabilization can guide research on the role of other social insurance and cash transfer programs over the business cycle. The analysis of these programs will add considerable color to our understanding of fiscal policy in macroeconomic stabilization — a welcome development, in view of the very significant role that these social programs play in actual government budgets (McKay and Reis [2015]). I leave the analysis of these other programs, and the comparison of UI and these programs to more standard government purchases, to future research.

References


Camille Landais, Pascal Michaillat, and Emmanuel Saez. A macroeconomic approach to optimal unemployment insurance: Theory. 2015a.

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Proofs

For section 2.2: optimal policy in the flexible price and wage benchmark

Proposition 1: equivalence of implementable allocations

Proof. The text almost completely proves the $\Rightarrow$ direction. Suppose there exists an allocation $\{c_1^r, c_1^i, c_2^r, c_2^i, s, \theta\}$ and relative price $p_2$ as part of a flexible price and wage equilibrium. Then in the same competitive equilibrium, define $w_2^r$ and $w_1^i$ as in (15) and (16). By the definition of worker optimization in the competitive equilibrium, $c_i^r = c_i^r(w_i^1, p_2)$ and $s = s(w_1^c, w_1^i, p_2, \theta)$ for Marshallian demand and labor supply functions defined in (14) and (18), respectively.

It only remains to prove that the resource constraints (19) and (20) are indeed satisfied in the above equilibrium. Optimal price-setting by retailers will clearly lead to identical posted prices $P_1(j) = P_1$, which implies that lower-stage worker optimization will imply $c_1^r(j) = c_1^r$. Hence, date 1 final goods market clearing in (7), the pass-through technology of retailers in (6), and intermediate goods market clearing in (9) imply that (19) must be satisfied. And date 2 final goods market clearing in (8) immediately implies that (20) is satisfied.

I turn to the $\Leftarrow$ direction. Suppose there exist $\{w_1^c, w_2^r\}$ such that, for a particular allocation $\{c_1^r, c_1^i, c_2^r, c_2^i, s, \theta\}$ and relative price $p_2$, resource constraints (19) and (20) are satisfied and the stated implementability constraints are also satisfied. The goal is to show that this allocation and relative price form part of a flexible price and wage equilibrium for some prices, wages, and profits $\{\{P_1(j)\}, P^f, P_2, W, \Pi, \{\Pi'(j)\}\}$, tightness schedule $\theta(W)$, and policy $\{b, t, \tau; i, P_2, T^r, \tau^r\}$ per Definition 1.

The proof is constructive, demonstrating that appropriate values of prices and policies ensure that agent first-order conditions, agent resource constraints, and market clearing conditions are satisfied at the given allocation and relative price. The first two imply that agents are solving their appropriate optimization problem, given my maintained assumption that their first-order conditions are sufficient to characterize optimality. I will refer to first-order conditions obtained in my analysis of equilibrium in Online Appendix A.

I start with firms. Given $s$ and $\theta$, define $\nu = \theta s$. Then, producers’ first-order condition with respect to $\nu$ pins down $W_{p^f}$. Producers’ first-order condition with respect to $W$ pins down $\frac{\theta(W)}{\theta'(W)}$ (locally). Without loss of generality, I then set $\tau^r = -\frac{1}{2}$. Retailers’ first-order condition with respect to $P_1(j)$ pins down $\frac{P_1(j)}{p^f_1} = 1 \forall j$. Defining the CES aggregator in (13), this implies that $P_1(j) = P_1 \forall j$ and that $\frac{P^f_1}{p^f_1} = 1$. The constraints of problem (6) then pin down $x(j) = y_1(j) = (p(\theta)s)c_1^r + (1 - p(\theta)s)c_1^i$. Given $\tau^r$, government budget balance in policy targeted at retailers (12) pins down $\frac{T^r}{\theta'(W)}$.

There is now a standard indeterminacy with flexible prices and wages: we can arbitrarily pick $P_1$. Given relative price (inverse real interest rate) $p_2$, this requires monetary policy $\{i, P_2\}$ satisfying the Fisher equation in (17). Having picked $P_1$, the above results pin down $P^f, W$, and $T^r$. Moreover, the objective functions of producers and retailers pin down profits $\Pi$ and $\Pi'(j)$.

I turn now to worker optimization. Given $\frac{\theta(W)}{\theta'(W)}$ (locally), local worker indifference across
submarkets pins down \( \tau \). Then, given wealth levels \( w^t_i \) and \( w^u_i \), \( t \) and \( b \) are pinned down by (15) and (16), respectively. Worker optimization of the second-stage problem is satisfied with \( c^t_i(j) = c^t_1 \). And since the implementability constraints \( c^t_i = c^t_i(w^t_i, p_2) \) and \( s = s(w^t_i, w^u_i, p_2, \theta) \) are assumed satisfied at the given allocation, it follows that \( \{c^t_1, c^t_2\} \) and \( s \) solve (14) and (18). Given that \( t \) and \( b \) were constructed to satisfy (15) and (16), it follows that \( \{c^t_1, c^t_2\} \) and \( s \) solve the original problems (4) and (5). Agents’ net asset positions in the short-run \( z^t_i \) and \( z^u_i \) are then pinned down by the date 2 budget constraints in (4). As all macroeconomic aggregates are pinned down at this point, the full schedule \( \theta(W) \) can be defined by worker indifference across submarkets indexed by alternative \( W \).

Since the given allocation satisfies (19) and (20), goods market clearing at each date is ensured. All that remains is to check that agents’ date 1 budget constraints in (4) are satisfied, that bond market clearing (10) is satisfied, and that government budget-balance in the labor-market (11) is satisfied. The first two are implied by Walras’ law for agents’ utility maximization problem. The second two are implied by Walras’ law holding for the economy’s excess demand function. \( \square \)

**Proposition 2: Ramsey optimal risk-sharing**

**Proof.** The first-order conditions to the Ramsey planning problem with respect to \( w^t_i \) and \( w^u_i \) are

\[
\begin{align*}
 p(\theta)s \frac{\partial v^e}{\partial w^t_i} + \lambda_{RC1} \left( \frac{\partial x_1}{\partial s} \frac{s}{\partial w^t_i} - p(\theta)s \frac{\partial c^e_1}{\partial w^t_i} \right) + \lambda_{RC2} \left( \frac{\partial x_2}{\partial s} \frac{s}{\partial w^t_i} - p(\theta)s \frac{\partial c^e_2}{\partial w^t_i} \right) &= 0, \\
(1 - p(\theta)s) \frac{\partial v^u}{\partial w^u_i} + \lambda_{RC1} \left( \frac{\partial x_1}{\partial s} \frac{s}{\partial w^u_i} - (1 - p(\theta)s) \frac{\partial c^u_1}{\partial w^u_i} \right) + \lambda_{RC2} \left( \frac{\partial x_2}{\partial s} \frac{s}{\partial w^u_i} - (1 - p(\theta)s) \frac{\partial c^u_2}{\partial w^u_i} \right) &= 0,
\end{align*}
\]

given multipliers on the resource constraints \( \lambda_{RC1} \) and \( \lambda_{RC2} \), and the definition of the production-inclusive excess supply functions \( x_1(s, \theta) \) and \( x_2(s, \theta) \) in (23) and (24). As in the standard partial equilibrium analysis, these first-order conditions make use of the Envelope theorem to ignore the change in the planner’s objective from the labor supply response to marginal changes in wealth. Straightforward algebraic manipulation then yields:

\[
\begin{align*}
\frac{\partial v^e}{\partial w^t_i} &= \frac{\lambda_{RC2}}{p_2} \left( \frac{\lambda_{RC1}}{\lambda_{RC2}} p_2 \frac{\partial c^e_1}{\partial w^t_i} + p_2 \frac{\partial c^e_2}{\partial w^t_i} - \frac{1}{p(\theta)s} \left( \frac{\lambda_{RC1}}{\lambda_{RC2}} \frac{\partial x_1}{\partial s} + p_2 \frac{\partial x_2}{\partial s} \right) \frac{\partial s}{\partial w^t_i} \right), \\
\frac{\partial v^u}{\partial w^u_i} &= \frac{\lambda_{RC2}}{p_2} \left( \frac{\lambda_{RC1}}{\lambda_{RC2}} p_2 \frac{\partial c^u_1}{\partial w^u_i} + p_2 \frac{\partial c^u_2}{\partial w^u_i} - \frac{1}{1 - p(\theta)s} \left( \frac{\lambda_{RC1}}{\lambda_{RC2}} \frac{\partial x_1}{\partial s} + p_2 \frac{\partial x_2}{\partial s} \right) \frac{\partial s}{\partial w^u_i} \right).
\end{align*}
\]

Now, define the relative price wedge \( \tau_{1,2} \equiv 1 - \frac{\lambda_{RC1}}{\lambda_{RC2}} p_2 \) as in Definition 3. Then

\[
\left( \frac{\lambda_{RC1}}{\lambda_{RC2}} \right) p_2 \frac{\partial c^i_1}{\partial w^i} + p_2 \frac{\partial c^i_2}{\partial w^i} = 1 - \left( 1 - \left( \frac{\lambda_{RC1}}{\lambda_{RC2}} \right) p_2 \right) \frac{\partial c^i_1}{\partial w^i},
\]

where the first line uses the identity \( \frac{\partial c^i_1}{\partial w^i} + p_2 \frac{\partial c^i_2}{\partial w^i} = 1 \) implied by partial differentiation of the budget.
constraint in agent $i$’s ex-post problem (14). We can plug this result and the definition of $\tau_{1,2}$ into (67) and (68), and then solve out for $\frac{\lambda_{RC2}}{p_2}$ to obtain

$$1 - \tau_{1,2} \frac{\partial c^*_1}{\partial w_1} - \frac{1}{p(\theta)s} \left((1 - \tau_{1,2}) \frac{\partial x_1}{\partial s} + p_2 \frac{\partial x_2}{\partial s}\right) \frac{\partial s}{\partial w_1} = \frac{\lambda_{RC2}}{p_2},$$

which is

$$= 1 - \tau_{1,2} \frac{\partial c^u_1}{\partial w_1} - \frac{1}{1 - p(\theta)s} \left((1 - \tau_{1,2}) \frac{\partial x_1}{\partial s} + p_2 \frac{\partial x_2}{\partial s}\right) \frac{\partial s}{\partial w_1}.$$  \hspace{1cm} (69)

Finally, using the result of Proposition 7, $\tau_{1,2} = 0$ in the present planning problem without any ZLB constraint, for clearly this is equivalent to the solution of the problem with a slack constraint. Plugging this in above, the Ramsey optimal risk-sharing result with flexible prices and wages immediately follows.

Lemma 1: implementing the size of transfers

Proof. By producer optimization in vacancy posting and a zero equilibrium markup in the flexible price and wage equilibrium, as detailed in Online Appendix A, we have

$$w = f'(p(\theta)s - k\theta s) \left(1 - \frac{k}{q(\theta)}\right),$$

where $w$ is the real wage. It follows that

$$\omega = (w + p_2(y^e_2 - y^u_2)) - ((c^e_1 + p_2 c^e_2) - (c^u_1 + p_2 c^u_2)),
= ((w + \pi + p_2 y^e_2) - (c^u_1 + p_2 c^u_2)) - ((\pi + p_2 y^u_2) - (c^u_1 + p_2 c^u_2)),
= ((w + \pi + p_2 y^e_2) - ((1 - \tau)w - t + \pi + p_2 y^u_2)) - ((\pi + p_2 y^u_2) - (b + \pi + p_2 y^u_2)),
= \tau w + t + b,
= \frac{1}{p(\theta)s} b,$$

where $\pi \equiv \frac{1}{p^2} (\Pi + \Pi' - T')$, the third line uses agents’ budget constraints in (15) and (16) accompanying optimization problem (14), and the final line uses government budget balance in (11).

Proposition 3: a general equilibrium Baily-Chetty formula

Proof. We make use of two facts at the micro level. First, workers’ ex-post problem (14) implies

$$\frac{\partial v^i}{\partial w^i_1} = \frac{\partial u^i}{\partial c^i_1},$$  \hspace{1cm} (70)
a standard application of the Envelope theorem. Second, the first-order condition of workers’
ex-ante problem (18) implies that

$$\frac{\partial s}{\partial w_1^e} = - \frac{\partial v^e}{\partial w_1^u} \frac{\partial s}{\partial u_1},$$

(71)

which captures the tight link between the labor supply response to changes in wealth when employed
versus unemployed.

Then, manipulating the Ramsey optimal risk-sharing condition from Proposition 2, we obtain:

$$\frac{\partial v^e}{\partial w_1^u} = \frac{\partial v^u}{\partial w_1^u} - \frac{\partial v^e}{\partial w_1^u} \frac{\partial s}{\partial u_1},$$

(70)

$$\Rightarrow - \left( \frac{1}{1 - p(\theta)s} + \frac{1}{p(\theta)s} \right) \left( \frac{\partial x_1}{\partial s} + p_2 \frac{\partial x_2}{\partial s} \right) \frac{\partial s}{\partial w_1^u} = - \left( \frac{1}{1 - p(\theta)s} + \frac{1}{p(\theta)s} \right) \left( \frac{p(\theta)\omega}{s} \right) \frac{\partial s}{\partial w_1^u},$$

(71)

where the third line uses (71) and the last line uses (70).

Now, on the left-hand side,

$$- \left( \frac{1}{1 - p(\theta)s} + \frac{1}{p(\theta)s} \right) \left( \frac{\partial x_1}{\partial s} + p_2 \frac{\partial x_2}{\partial s} \right) \frac{\partial s}{\partial w_1^u} = - \left( \frac{1}{1 - p(\theta)s} + \frac{1}{p(\theta)s} \right) \left( \frac{p(\theta)\omega}{s} \right) \frac{\partial s}{\partial w_1^u},$$

$$= - \left( \frac{1}{1 - p(\theta)s} + \frac{1}{p(\theta)s} \right) \left( \frac{1}{s} \right) \frac{\partial s}{\partial w_1^u},$$

$$= - \left( \frac{1}{p(\theta)s} \right)^2 \frac{p(\theta)\omega}{s} \frac{\partial s}{\partial w_1^u},$$

$$= \left( \frac{1}{p(\theta)s} \right)^2 \varepsilon_{b(\text{unemp})},$$

where the first line uses the definition of the size of transfers $\omega$ in (25), the second line uses the
implementation of transfers in Lemma 1, and the last line uses the definition of the micro-elasticity
$\varepsilon_{b(\text{unemp})}$ in (27). The general equilibrium Baily-Chetty formula follows. $\Box$
Proposition 4: equivalence of implementable allocations

Proof. As with the proof of Proposition 1, the text almost completely proves the $\Rightarrow$ direction. Given an allocation and relative price as part of a fully sticky price equilibrium, then in the same equilibrium define $w^e_1$ and $w^u_1$ as in (15) and (16). By the definition of worker optimization in the competitive equilibrium, $c^e_i = c^e_i(w^e_1, p_2)$ and $s = s(w^e_1, w^u_1, p_2, \theta)$ for Marshallian demand and labor supply functions defined in (14) and (18), respectively.

Unlike the flexible price and wage case, retailers no longer update prices; nonetheless, the assumption of identical pre-set prices $P_1(j) = \bar{P}_1$ ensures that lower-stage worker optimization still implies $c^e_1(j) = e^e_1$. Hence, date 1 final goods market clearing in (7), the pass-through technology of retailers in (6), and intermediate goods market clearing in (9) again imply that the resource constraint (19) must be satisfied. And date 2 final goods market clearing in (8) again implies that the resource constraint (20) is satisfied.

Finally, as described in the text, the real interest rate must be bound below by (32) in the given competitive equilibrium. It follows that the ZLB implementability constraint (33) is satisfied.

I turn to the $\Leftarrow$ direction.

As in the constructive proof developed for Proposition 1, $s$ and $\theta$ imply $\nu \equiv \theta s$. Producers’ first-order condition with respect to $\nu$ pins down $W_{\theta s}$ and their first-order condition with respect to $W$ pins down $\frac{\theta(W)}{\theta(W)}$ (locally). Without loss of generality, I again set $\tau^r = -\frac{1}{2}$. Now there is no retailer optimality condition to consider; instead, they simply satisfy demand at posted, identical prices $\bar{P}_1$. Since prices are identical, we still have $x(j) = y_1(j) = (p(\theta)s)c^e_1 + (1 - p(\theta)s)c^u_1$. Given $\tau^r$, government budget balance in policy targeted at retailers (12) still pins down $T_r$.

There is no longer any nominal indeterminacy. Given $\bar{P}_1$, the relative price (inverse real interest rate) $p_2$ must be implemented by a particular monetary policy $\{i, P_2\}$ satisfying the Fisher equation in (17). Since the ZLB implementability constraint (33) holds, there must exist at least one such combination of $i$ and $P_2$ which also satisfy the constraints on monetary policy in (31).

I turn now to worker optimization. Given $\theta(W)/\theta(W)$ (locally), local worker indifference across submarkets pins down $W_{\bar{P}_1}(1 - \tau)$. But importantly, while the post-tax real wage is pinned down, the decomposition between the gross wage and payroll taxes is not (as in the standard Keynesian analysis with sticky prices). Equivalently, $\tau$ can be used to target any gross real wage $W_{\bar{P}_1}$ and thus the inverse gross markup $\frac{P^I_{\bar{P}_1}}{P^I_{\bar{P}_1}}$. Conditional on picking $\tau$, we have $P^I$, $W$, and $T^r$, and then the objective functions of producers and retailers again pin down profits $\Pi$ and $\Pi^r$.

The remainder of the proof proceeds exactly as in that for Proposition 1. Importantly, once the redistribution from markups is pinned down by the choice of $\tau$ above, the given wealth levels $w^e_1$ and $w^u_1$ again pin down $t$ and $b$ using (15) and (16), respectively. 

\[\square\]
Proposition 5: optimality of the flexible price and wage allocation

Proof. The result follows immediately from the equivalence of Ramsey planning problems under flexible prices (21) and sticky prices (34), when the (ZLB) constraint is slack. □

Lemma 2: implementing the size of transfers

Proof. By producer optimization in vacancy posting as detailed in Online Appendix A, we have

\[ w = \mu^{-1} f'(p(\theta)s - k\theta s) \left( 1 - \frac{k}{q(\theta)} \right), \]

where \( w \) is the real wage and \( \mu^{-1} \equiv \frac{\delta l}{\delta P} \) is the inverse gross markup of retailers. It follows that

\[ \omega = \left( \mu w + p_2(y_e^2 - y_u^2) \right) - ((c_1^e + p_2c_2^e) - (c_1^u + p_2c_2^u)) + (\mu - 1)w, \]

\[ = \frac{1}{p(\theta)s} b + (\mu - 1)w, \]

where the last line uses the same steps as in the proof of Lemma 1. □

Proposition 7: a sufficient statistic for the ZLB constraint

Proof. The first-order condition of the Ramsey planning problem with respect to \( p_2 \) is

\[
\begin{align*}
& p(\theta)s \frac{\partial v^e}{\partial p_2} + (1 - p(\theta)s) \frac{\partial v^u}{\partial p_2} \\
& + \lambda_{RC1} \left( \frac{\partial x_1}{\partial s} \frac{\partial v^e}{\partial p_2} - p(\theta)s \frac{\partial c_1^e}{\partial p_2} - (1 - p(\theta)s) \frac{\partial c_1^u}{\partial p_2} \right) \\
& + \lambda_{RC2} \left( \frac{\partial x_2}{\partial s} \frac{\partial v^e}{\partial w_1^e} - p(\theta)s \frac{\partial c_2^e}{\partial p_2} - (1 - p(\theta)s) \frac{\partial c_2^u}{\partial p_2} \right) = \lambda_{ZLB}. \tag{72}
\end{align*}
\]

I focus on simplifying the left-hand side. To do so, it is helpful to use Roy’s identity

\[ \frac{\partial v}{\partial p_2} = - \frac{\partial v}{\partial w_1^e} c_2, \]

as well as an implication of Roy’s identity on the first-order condition of workers’ ex-ante problem (18)

\[ \frac{\partial s}{\partial p_2} = - \frac{\partial s}{\partial w_1^e} c_2 - \frac{\partial s}{\partial w_1^u} c_2, \]
which captures the tight link between the labor supply response to changes in prices and wealth. Using the above two identities, (72) can be re-expressed as

\[
- c_2^e \left( p(\theta) s \frac{\partial v^e}{\partial w_1^i} + \left( \lambda_{RC1} \frac{\partial x_1}{\partial s} + \lambda_{RC2} \frac{\partial x_2}{\partial s} \right) \frac{\partial s}{\partial w_1^i} \right) \\
- c_2^e \left( (1 - p(\theta)s) \frac{\partial v^u}{\partial w_1^i} + \left( \lambda_{RC1} \frac{\partial x_1}{\partial s} + \lambda_{RC2} \frac{\partial x_2}{\partial s} \right) \frac{\partial s}{\partial w_1^i} \right) \\
- \lambda_{RC1} \left( p(\theta)s \frac{\partial c_i^{e,h}}{\partial p_2} + (1 - p(\theta)s) \frac{\partial c_i^{u,h}}{\partial p_2} \right) - \lambda_{RC2} \left( p(\theta)s \frac{\partial c_i^{e,h}}{\partial p_2} + (1 - p(\theta)s) \frac{\partial c_i^{u,h}}{\partial p_2} \right) = \lambda_{ZLB}.
\]

Then, the first-order conditions for the planning problem with respect to \( w_i^e \) and \( w_i^u \) (detailed in the proof of Proposition 2) can be used to simplify the left-hand side, implying

\[
- c_2^e \left( \lambda_{RC1} p(\theta)s \frac{\partial c_i^{e,h}}{\partial w_1^i} + \lambda_{RC2} p(\theta)s \frac{\partial c_i^{u,h}}{\partial w_1^i} \right) \\
- c_2^e \left( \lambda_{RC1} (1 - p(\theta)s) \frac{\partial c_i^{e,h}}{\partial w_1^i} + \lambda_{RC2} (1 - p(\theta)s) \frac{\partial c_i^{u,h}}{\partial w_1^i} \right) \\
- \lambda_{RC1} \left( p(\theta)s \frac{\partial c_i^{e,h}}{\partial p_2} + (1 - p(\theta)s) \frac{\partial c_i^{u,h}}{\partial p_2} \right) - \lambda_{RC2} \left( p(\theta)s \frac{\partial c_i^{e,h}}{\partial p_2} + (1 - p(\theta)s) \frac{\partial c_i^{u,h}}{\partial p_2} \right) = \lambda_{ZLB}.
\]

The Slutsky equation can be used to collect terms on the left-hand side, substantially simplifying the identity to

\[
- \lambda_{RC1} \left( p(\theta)s \frac{\partial c_i^{e,h}}{\partial p_2} + (1 - p(\theta)s) \frac{\partial c_i^{u,h}}{\partial p_2} \right) - \lambda_{RC2} \left( p(\theta)s \frac{\partial c_i^{e,h}}{\partial p_2} + (1 - p(\theta)s) \frac{\partial c_i^{u,h}}{\partial p_2} \right) = \lambda_{ZLB}
\]
given Hicksian demand functions \( c_i^{e,h}(v^i(w_1^i, p_2), p_2) \). Finally, we can use the identity for compensated price derivatives

\[
\frac{\partial c_i^{e,h}}{\partial p_2} + p_2 \frac{\partial c_i^{u,h}}{\partial p_2} = 0
\]

to further simplify the above as

\[
\frac{\lambda_{RC2}}{p_2} \left( 1 - \frac{\lambda_{RC1}}{\lambda_{RC2}} p_2 \right) \left( p(\theta)s \frac{\partial c_i^{e,h}}{\partial p_2} + (1 - p(\theta)s) \frac{\partial c_i^{u,h}}{\partial p_2} \right) = \lambda_{ZLB},
\]

\[
\Rightarrow \frac{\lambda_{RC2}}{p_2} \tau_{1,2} \left( p(\theta)s \frac{\partial c_i^{e,h}}{\partial p_2} + (1 - p(\theta)s) \frac{\partial c_i^{u,h}}{\partial p_2} \right) = \lambda_{ZLB},
\]

\[
\Rightarrow \tau_{1,2} = \frac{\lambda_{ZLB}}{\frac{\lambda_{RC2}}{p_2} \left( p(\theta)s \frac{\partial c_i^{e,h}}{\partial p_2} + (1 - p(\theta)s) \frac{\partial c_i^{u,h}}{\partial p_2} \right)},
\]

where the second line uses the definition of the relative price wedge in Definition 3. Since these compensated cross-price derivatives must be non-negative, we obtain \( \tau_{1,2} \preceq \lambda_{ZLB} \).  

\(\square\)
Proposition 8: Ramsey optimal risk-sharing

Proof. The proof proceeds exactly like that for Proposition 2. Ramsey optimal risk-sharing with a general relative price wedge $\tau_{1.2}$ is characterized in (69).

\[\square\]

Proposition 9: a generalized Baily-Chetty formula

Proof. We again make use of two facts at the micro level, (70) and (71), described in the proof of Proposition 3.

Manipulating the Ramsey optimal risk-sharing condition from Proposition 8, we obtain:

\[
1 - \tau_{1.2} \frac{\partial c^e}{\partial w_1^u} - \frac{1}{p(\theta)s} \left( (1 - \tau_{1.2}) \frac{\partial x_1}{\partial s} + p_2 \frac{\partial x_2}{\partial s} \right) \frac{\partial s}{\partial w_1^u} = 1 - \tau_{1.2} \frac{\partial c^e}{\partial w_1^u} - \frac{1}{1 - p(\theta)s} \left( (1 - \tau_{1.2}) \frac{\partial x_1}{\partial s} + p_2 \frac{\partial x_2}{\partial s} \right) \frac{\partial s}{\partial w_1^u},
\]

\[
\Rightarrow 1 - \tau_{1.2} \frac{\partial c^e}{\partial w_1^u} - \frac{1}{1 - p(\theta)s} \left( (1 - \tau_{1.2}) \frac{\partial x_1}{\partial s} + p_2 \frac{\partial x_2}{\partial s} \right) \frac{\partial s}{\partial w_1^u} = 1 - \tau_{1.2} \frac{\partial c^e}{\partial w_1^u} - \frac{1}{p(\theta)s} \left( (1 - \tau_{1.2}) \frac{\partial x_1}{\partial s} + p_2 \frac{\partial x_2}{\partial s} \right) \frac{\partial s}{\partial w_1^u},
\]

\[
\Rightarrow 1 - \tau_{1.2} \frac{\partial c^e}{\partial w_1^u} - \frac{1}{1 - p(\theta)s} \left( (1 - \tau_{1.2}) \frac{\partial x_1}{\partial s} + p_2 \frac{\partial x_2}{\partial s} \right) \frac{\partial s}{\partial w_1^u} = 1 - \tau_{1.2} \frac{\partial c^e}{\partial w_1^u} - \frac{1}{p(\theta)s} \left( (1 - \tau_{1.2}) \frac{\partial x_1}{\partial s} + p_2 \frac{\partial x_2}{\partial s} \right) \frac{\partial s}{\partial w_1^u},
\]

where the third line uses (71) and the last line uses (70).

On the right-hand side,

\[
\frac{\partial u^u}{\partial c^e} - \frac{1}{1 - \tau_{1.2} \frac{\partial c^e}{\partial w_1^u}} = \frac{\partial u^u}{\partial c^e} - \frac{1}{1 - \tau_{1.2} \frac{\partial c^e}{\partial w_1^u}} + \tau_{1.2} \frac{\partial c^e}{\partial w_1^u} \left( \frac{\partial c^e}{\partial w_1^u} - \frac{\partial c^e}{\partial w_1^u} \right).
\]
And on the left-hand side,

\[- \left( \frac{1}{1 - p(\theta)s} + \frac{1}{p(\theta)s} \right) \left( \frac{1 - \tau_{1,2}}{1 - \tau_{1,2}} \frac{\partial x_1}{\partial s} + p_2 \frac{\partial x_2}{\partial s} \right) \frac{\partial s}{\partial w_1^u} \]

\[= - \left( \frac{1}{1 - p(\theta)s} + \frac{1}{p(\theta)s} \right) \left( \frac{\partial x_1}{\partial s} + \frac{1}{1 - \tau_{1,2}} p_2 \frac{\partial x_2}{\partial s} \right) \left( \frac{1 - \tau_{1,2}}{1 - \tau_{1,2}} \frac{\partial c_1}{\partial w_1^u} \right) \frac{\partial s}{\partial w_1^u}, \]

\[= - \left( \frac{1}{1 - p(\theta)s} + \frac{1}{p(\theta)s} \right) \left( p(\theta)\omega + \frac{\tau_{1,2}}{1 - \tau_{1,2}} p_2 \frac{\partial x_2}{\partial s} \right) \left( \frac{1 - \tau_{1,2}}{1 - \tau_{1,2}} \frac{\partial c_1}{\partial w_1^u} \right) \frac{\partial s}{\partial w_1^u}, \]

\[= - \left( \frac{1}{1 - p(\theta)s} + \frac{1}{p(\theta)s} \right) \frac{1}{s} \frac{\partial s}{\partial w_1^u} \left( \frac{1 + \frac{\tau_{1,2}}{1 - \tau_{1,2}} \frac{p_2}{b} \frac{\partial x_2}{\partial s}}{1 - \tau_{1,2}} + \frac{\tau_{1,2}}{b} \frac{\partial c_1}{\partial w_1^u} \right), \]

\[= \left( \frac{1}{p(\theta)s} \right)^2 \varepsilon_b^2 \frac{p_{\text{unemp}}}{b} \left( \frac{1 - \tau_{1,2}}{1 - \tau_{1,2}} \frac{\partial c_1}{\partial w_1^u} + \frac{\tau_{1,2}}{b} \frac{\partial c_1}{\partial w_1^u} \right), \]

where the second equality uses the definition of the size of transfers \( \omega \) in (25), the third equality uses the implementation of transfers in Lemma 2 assuming that the payroll tax is used to keep markups at \( \mu = 1 \), and the final equality uses the same steps as in the proof of Proposition 3 to collect terms into the micro-elasticity \( \varepsilon_b^2 \frac{p_{\text{unemp}}}{b} \). Moreover, we have

\[
p_2 \frac{\partial x_2}{\partial s} = \frac{p_2 p(\theta)s}{b} \left( (y_2^e - y_2^u) - (c_2^e - c_2^u) \right),
\]

\[
= \frac{p_2 p(\theta)s}{b} \left( (y_2^e - c_2^e) - (y_2^u - c_2^u) \right),
\]

\[
= \frac{p_2 p(\theta)s}{b} \left( \frac{1}{p_2} \frac{1}{w_1} + \frac{1}{p_2} \frac{1}{w_1} \right),
\]

\[
= \frac{p_2 p(\theta)s}{b} \left( \frac{1}{p_2} \frac{1}{w_1^u} \right),
\]

\[
= \frac{z_1^u}{b},
\]

where the first line uses the definition of the long-run production-inclusive excess supply function \( x_2(s, \theta) \) in (24), the third line uses agents’ second period resource constraints in the competitive equilibrium, and the fourth line uses bond market clearing (10).

We thus obtain the exact generalized Baliy-Chetty formula

\[
\left( \frac{1}{p(\theta)s} \right)^2 \varepsilon_b^2 \frac{p_{\text{unemp}}}{b} \left( \frac{1 - \tau_{1,2}}{1 - \tau_{1,2}} \frac{\partial c_1}{\partial w_1^u} + \frac{\tau_{1,2}}{1 - \tau_{1,2}} \frac{\partial c_1}{\partial w_1^u} \right) = \frac{\partial w_1^u}{\partial \varepsilon_b^2} - \frac{\partial w_1^u}{\partial \varepsilon_b^2} \frac{\partial w_1^u}{\partial \varepsilon_b^2} + \frac{\tau_{1,2}}{1 - \tau_{1,2}} \frac{\partial c_1}{\partial w_1^u} - \frac{\partial c_1}{\partial w_1^u} \frac{\partial c_1}{\partial w_1^u}. \quad (73)
\]
Using the Taylor approximations for a small relative price wedge

\[
\frac{1 - \tau_{1,2}}{1 - \tau_{1,2} \frac{\partial c_1^e}{\partial w_1^e}} + \frac{\tau_{1,2}}{1 - \tau_{1,2} \frac{\partial c_1^e}{\partial w_1^e}} \frac{z_1^u}{b} = 1 - \tau_{1,2} \left( 1 - \frac{\partial c_1^e}{\partial w_1^e} - \frac{z_1^u}{b} \right) + o(||\tau_{1,2}||^2),
\]

\[
\frac{\tau_{1,2}}{1 - \tau_{1,2} \frac{\partial c_1^e}{\partial w_1^e}} = \tau_{1,2} + o(||\tau_{1,2}||^2),
\]

we obtain the approximate formula in the text.

\[\square\]

**For section 2.5: positive impact of UI in stabilization**

**Proposition 10: the UI multiplier**

**Proof.** I first summarize the arguments used in Online Appendix C which result in aggregate relations (41)-(43).

**Micro-level responses.** Standard results in price theory can be used to obtain the micro-level responses

\[
dc_1^i = \frac{\partial c_1^i}{\partial w_1^i} dy_1^i + \left( \frac{\partial c_1^{i,h}}{\partial p_2} - \frac{\partial c_1^i}{\partial w_1^i} \frac{z_1^i}{p_2} \right) dp_2,
\]

\[
ds = \frac{ds}{dw_1^e} \left( dy_1^e - \frac{z_1^e}{p_2} dp_2 \right) + \frac{ds}{dw_1^u} \left( dy_1^u - \frac{z_1^u}{p_2} dp_2 \right) + \frac{ds}{d\theta} d\theta,
\]

given Marshallian demand function \( c_1^i(w_1^i, p_2) \) in (14), Hicksian demand function \( c_1^{i,h}(v^i(w_1^i, p_2), p_2) \) implied by (14), and labor supply function \( s(w_1^e, w_1^u, p_2, \theta) \) in (18).

**Aggregation.** Two identities facilitate aggregation:

\[
(p(\theta)s)c_1^e + (1 - p(\theta)s)c_1^u = f(p(\theta)s - k\theta s),
\]

\[
(p(\theta)s)y_1^e + (1 - p(\theta)s)y_1^u = f(p(\theta)s - k\theta s).
\]

The first simply reflects goods market clearing. The second says that aggregate income must equal aggregate resources.

Total differentiation of these conditions, combined with the micro-level consumption response in (A.14), gives the aggregate demand relation in (41). The micro-level labor supply response in (A.15) alongside (A.17) and the Ramsey optimality conditions from section 2.2 give the labor supply relation in (43). Finally, the specification of technology gives the technological relation in (42).

With (41)-(43) in hand, the remaining results are straightforward. Plugging the technological
relation in (42) into the aggregate demand relation in (41), we obtain an equation in \{n_1, s, y_u^0, p_2\}. Substituting in for \(s\) using the labor supply relation in (43) and collecting terms, we obtain the marginal effect on employment in (44). Substituting this back into (42) and using (43) to substitute in for \(s\), we obtain the UI multiplier in (45).