On the Optimality of Financial Repression*

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ABSTRACT

When is financial repression optimal? That is, when is it optimal to have policies that force banks to hold government debt? With commitment, financial repression is not optimal because it crowds out banks’ other productive investments. Without commitment, repression is a costly way of purchasing the credibility of not defaulting. When banks hold government debt, defaults are costly ex post because they reduce bank net worth and, thus, aggregate investment. Forcing banks to hold debt endogenously increases these ex post costs and allows governments to issue more debt credibly, but the policy is also costly because it crowds out investments. Financial repression is optimal when governments need to issue an unusually large amount of domestic debt, such as during wartime or following sudden stops in foreign lending.

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Financial repression refers to a wide array of government policies that restrict the activities of financial intermediaries. In this paper, we focus on one important aspect of such policies: implicit or explicit government policies that require banks and other financial intermediaries to hold more government bonds than they would absent such policies. We refer to these policies as financial repression. We ask when, if ever, financial repression is optimal. We find that under commitment it is never optimal. If, however, a government cannot commit to its policies—in particular, to repaying its debt—then financial repression may be optimal.

We show that our theory implies that governments without commitment are most likely to practice financial repression when, for tax-smoothing purposes, they need to issue an unusually large amount of domestic debt. Such needs can arise either when current government expenditure needs are unusually high, such as during wartime, or when sudden stops in foreign lending occur, that is, times when foreigner lenders suddenly become less willing to lend to domestic governments. After such needs subside, governments run down their accumulated debt and reduce the extent of financial repression.

We study the optimality of financial repression in a model in which financial intermediaries, such as banks, play an essential role in channeling funds from households to firms. Our model is predicated on the idea, dating back to Fisher (1933), that aggregate investment is constrained by the net worth of agents in the economy.

The key idea of our paper is that financial repression is a costly way to purchase credibility to repay debt. The argument that financial repression allows governments to obtain credibility is as follows. Defaulting on the debt held by banks has ex post costs because the default reduces the net worth of banks and thereby reduces future capital accumulation. Forcing banks to hold more government debt than they otherwise would endogenously increases the size of these ex post costs from a default. The increase in ex post costs allows the government to credibly issue more debt. This way of purchasing credibility is costly because it has ex ante crowding-out costs: forcing banks to allocate a greater fraction of their assets to government debt reduces the amount of funding available for productive capital investments.

Obtaining greater credibility is valuable because the increased ability to issue debt allows for greater tax smoothing. The government practices financial repression whenever
the tax-smoothing gains exceed the crowding-out costs. Since these tax-smoothing gains are largest during times when government expenditure needs are unusually high or when sudden stops in foreign lending occur, our model implies that it is during such times that governments practice financial repression.

We develop these ideas in two versions of a standard neoclassical model that is augmented to include banks along the lines of Gertler and Kiyotaki (2010): a closed economy and an open one. Both versions share common features. Banks channel resources from households to firms. Bankers face a collateral constraint that limits the amount of deposits that banks can raise from households, so that the net worth of banks limits the amount of investment. A benevolent government raises revenues to finance government expenditure using proportional taxes to labor income and investment and issues government debt that can be held by households, banks, and in the open economy version, foreign lenders. The government can also regulate the asset holdings of the banks by forcing them to hold a certain fraction of their assets as government bonds. Finally, the government can default on the debt it issues. We assume for the most part that, in its default decision, the government cannot discriminate based on the identity of the debt holder, and briefly show that our main results hold with discriminatory defaults.

With commitment, we show that in both versions of the model, financial repression is not optimal. Specifically, the government can achieve the Ramsey outcome without forcing banks to hold government debt. The key idea is that forcing banks to hold government debt is an inefficient way to raise revenues, because when the collateral constraint is binding, the amount of assets banks can hold is limited by their net worth. Forcing banks to hold government debt reduces the amount of capital that banks can finance, so that financial repression has crowding-out costs. Eliminating financial repression allows the debt to be held directly by households, and possibly by foreign lenders, and avoids the crowding-out costs. Thus, financial repression is a dominated instrument when the government can commit to repaying its debt.

We then turn to an environment in which the government does not have commitment. We begin with the closed economy version. In this version, defaulting on debt held by consumers has a direct tax benefit and an indirect reputational cost. The tax benefit is that
a default reduces the need to levy distorting taxes. The reputational cost is that the default changes the beliefs of households about the likelihood of a future default and thereby reduces the willingness of households to lend to the government in the future. In the model, these standard reputational considerations alone allow a government to issue only a limited amount of debt and, thus, only partially smooth taxes over time.

We show that the government finds it optimal to force banks to hold government debt during times in which its fiscal needs are unusually high. During such times, the gain from issuing debt to smooth taxes is particularly large. Financial repression allows the government to issue more debt than the limited amount of debt sustained by reputation, so that the government will practice repression if the tax-smoothing gains exceed the crowding-out costs. We argue below that, in the data, financial repression tended to occur when government fiscal needs were unusually high.

We go on to show that once fiscal needs have returned to normal levels, the government continues to practice financial repression for some length of time until its debt has fallen to a sufficiently low level, at which time it ceases practicing repression. The basic intuition is that, relative to the Ramsey plan, which has perfect tax smoothing, repression adds extra costs to keeping debt high. These extra costs imply that it is optimal to deviate from perfect tax smoothing and raise taxes early in the plan. This tilting of taxes to be higher earlier in the plan is the force that drives the running down of the debt. This pattern of running down the debt and eventually ceasing financial repression is similar to the patterns after World War II, as we discuss below.

In the open economy version of our model, governments can also borrow from foreign lenders. Following the tradition in the sovereign default literature, we assume that if the government defaults on its debts, foreign lenders impose punishments that result in an output loss. We assume that this output loss is proportional to the size of the foreign debt up to some maximal loss. This maximal loss determines the maximal amount that foreign lenders are willing to lend. We show that economies to which foreign lenders are less willing to lend, in that the maximal loss is smaller, support less tax smoothing and have a greater degree of financial repression.

We then allow the willingness of foreigners’ to lend to fluctuate stochastically. An
abrupt drop in such willingness to lend generates a sudden stop in capital inflows. We show that this sudden stop is associated with a partial default followed by financial repression and limited tax smoothing until the sudden stop ceases and capital inflows resume. As we discuss below, these observations are reminiscent of the experiences of Argentina and other emerging market economies during sudden stop episodes.

Our findings have important policy implications. Several policy makers have argued that financial institutions should be regulated so that they are allowed to hold only small amounts of their own country’s government bonds. (See, for example, Weidmann (2013).) Our analysis emphasizes that such a policy change may not be desirable, because with such a policy change, governments would be more tempted to default and thus the amount of debt could not exceed that supported solely by reputation.

In our paper, we focus attention on public finance considerations in determining the extent of financial repression. Of course, there are other reasons why governments might force banks to hold government debt. A prominent reason is that forcing banks to hold relatively safe assets, such as government debt, might help solve standard moral hazard problems in banking. Our model should be thought of as determining the extent of financial repression above and beyond such considerations. We conjecture that introducing these other considerations will not alter the basic thrust of our results: without commitment financial repression becomes more attractive in times of urgent fiscal needs and, with commitment, financial repression is used solely to address considerations other than those arising from public finance.

Other Related Literature. The idea that shocks that reduce the net worth of agents who make productive investments can induce output downturns dates back to at least Fisher (1933). Bernanke and Gertler (1989), Carlstrom and Fuerst (1997), and Bernanke, Gertler, and Gilchrist (1999) formalized this idea. More recently, Gertler, Kiyotaki, and Queralto (2012) and Gertler and Kiyotaki (2010) showed that shocks that reduce the net worth of financial intermediaries such as banks lead to output downturns by affecting the ability of these intermediaries to intermediate funds.

A related literature develops models in which defaults by governments reduce the net worth of banks and lead to output downturns. The typical mechanism in these papers is that
banks hold government debt as part of their assets, so that a default reduces their net worth. The reduction in net worth reduces the ability of banks to intermediate funds. See, among others, Basu (2009), Bocola (2015), Brutti (2011), Gennaioli, Martin, and Rossi (2014a), and Pei (2014). The main motivation of this literature is to endogenize the output costs of default that is typically taken as exogenous in the literature on sovereign debt default, as in Eaton and Gersovitz (1981), Arellano (2008), Cole and Kehoe (2000), and Aguiar and Gopinath (2006), among many others.

None of this literature addresses financial repression. Our model shares features of the models in this literature in that default leads to output costs by adversely affecting financial intermediation. The main point of our paper, however, is to show that the output costs associated with default can make financial repression optimal without commitment. The most closely related paper to ours is the contemporaneous work of Perez (2015), which focuses on the consequences of the changes of once-and-for-all commitments to a constant degree of repression. In contrast, we let the government choose the degree of repression in each period as a function of the state as part of the equilibrium. We characterize the time path of optimal repression in response to shocks.

Our paper is also related to a literature that studies the transitional dynamics of government debt. Under commitment, the standard result from Lucas and Stokey (1983) is that with complete markets, governments never choose to run down government debt. Under commitment but with incomplete markets, Aiyagari et al. (2002) and Bhandari et al. (2015) show that debt is reduced slowly over time. In our model with complete markets but without commitment, the debt is also reduced over time.

Finally, for related analyses of the behavior of government policy that also introduce political economy considerations, see Ales, Maziero, and Yared (2014) and Yared (2010).

1. Some Supporting Evidence

Here we discuss empirical evidence that supports both the key ingredients and the key implications of our model.

Consider first some evidence that supports the two key ingredients in our model: ex post costs of default and ex ante crowding-out costs of repression. The ex post costs of default
arise because defaults lead to output downturns by reducing the net worth of banks. One piece of evidence comes from Bocola (2015), who estimates a version of the Gertler and Kiyotaki (2010) model without optimizing governments and provides evidence that reductions in the value of government debt induced by expectations of future default lead to output downturns by reducing the net worth of banks. A second piece comes from Gennaioli, Martin, and Rossi (2014a), who, using a panel of developed and emerging economies, show that declines in private credit after a default are stronger in countries where banks hold more public debt. Finally, using data from 20,000 banks in a large panel of countries, Gennaioli, Martin, and Rossi (2014b) show that, even within the same country, banks that hold the most government debt reduce their lending the most following defaults.

In terms of evidence that repression has ex ante crowding-out costs because it crowds out private investment, Becker and Ivashina (2014) provide evidence that in Europe, in the wake of the recent Great Contraction, increased government bond holdings crowded out corporate lending.

Consider next evidence that supports key implications of our model. The first implication is that governments use financial repressions to meet their fiscal needs. This implication is the conventional view among economic historians of banking. Under this conventional view, much of the development of banking is intimately tied to the need to finance government expenditures. Homer and Sylla (1996) and Calomiris and Haber (2014) persuasively argue that the Bank of England was founded specifically to raise funds for the English government to help finance war expenditures. The Bank of England was then governed and regulated to ensure that the government had access to a stable course of funding. Homer and Sylla (1996) also argue that the Banque de France was created to serve a similar need in France.

Bordo, Redish, and Rockoff (2015), Sylla, Legler and Wallis (1987), and Calomiris and Haber (2014) show how the development of state-chartered banks in the United States from 1800 to 1860 was connected to the fiscal needs of state governments. A common device during this period was for individual states to charter banks to issue notes with the proviso that the bank hold an appropriate amount of the debt of the chartering state government. For example, Calomiris and Haber (2014, p. 167) point out that the Pennsylvania Omnibus Charter bill of 1824 required that all state-chartered “banks had to make loans to the state
government at the government’s discretion at an interest rate that could not exceed 5%.” We also find it notable that when the U.S. federal government needed to raise a large amount of funds during the Civil War, one of its first steps was to set up a system of national banks, which were permitted to issue bank notes but were required to hold government debt to back these notes.

The second implication is that once urgent fiscal needs have diminished, debt falls over time and financial repression eventually ceases. Reinhart and Sbrancia (2011) argue that after World War II, many of the Allied countries, facing large government debts, practiced financial repression on a large scale and then ceased to do so once their debts were reduced. Specifically, financial repression took the form of regulatory measures that required financial institutions to hold government debt in their portfolios and restricted international capital flows, thereby limiting the ability of consumers and financial intermediaries to invest in substitutes to their own government’s debt. Reinhart, Kirkegaard, and Sbrancia (2011) have documented that in the wake of the recent Great Contraction, in the face of severe fiscal stress, numerous countries have reinstituted various measures of financial repression.

The third implication is that financial repression is desirable in the face of sudden stops. That governments find financial repression desirable in the face of sudden stops is an idea that is also widely held by many economists. For example, Calvo and Mishkin (2003, p.100) argue that during the 2001 crisis in Argentina, “banks were encouraged and coerced into purchasing Argentine government bonds to fund the fiscal debt.” A related observation by Becker and Ivashina (2014), and Broner et al. (2014), and others is that banks in the periphery countries of the European Union have sharply increased their holdings of their own governments’ debts in the wake of the recent crisis. For example, Becker and Ivashina (2014) point out that by the end of 2013, the share of government debt held by the domestic banking sectors of Eurozone countries was more than twice that held in 2007. A number of authors, including Acharya and Steffen (2015), have also argued that banks have increased their holdings of their own government debt because of increased pressure and portfolio regulations by their own governments. In particular, these authors argue that zero-risk weighting of sovereign debt, even when the spreads of this debt are wide, amounts to a type of financial repression. One view is that this increased pressure is a response by periphery country governments to
the reduced willingness of foreigner lenders to lend to these governments.

2. Environment

Consider an infinite horizon economy that blends elements of Kiyotaki and Moore (1997) and Gertler and Kiyotaki (2010). It is composed of a household that works, saves, consumes, and operates financial intermediaries, referred to as banks, together with firms and a government. Households save by holding deposits in banks and government debt and receive dividends. Banks raise deposits from households and use these deposits plus retained earnings to invest in government debt and capital as well as pay dividends to households. Firms rent capital and labor and produce output. The government finances an exogenous stream of government spending with taxes on labor income and the capital stock, sells government debt, and can practice financial repression by requiring that banks must hold at least a certain fraction of their assets in government debt.

The resource constraint is given by

\[ C_t + K_{t+1} + G_t \leq F(K_t, L_t), \]

where \( C_t \) is aggregate consumption, \( K_{t+1} \) is the capital stock, \( G_t \) is government spending, \( L_t \) is aggregate labor, and \( F \) is a constant returns to scale production function which includes the undepreciated capital stock.

We follow Gertler and Karadi (2011) and Gertler, Kiyotaki, and Queralto (2012) in formulating the problem of households. The representative household is composed of a measure 1 of workers and a measure 1 of bankers. The workers supply labor and return their wages to the household, while each banker manages a bank that transfers nonnegative dividends back to the household. The household has a utility function given by

\[ \sum_{t=0}^{\infty} \beta^t U(C_t, L_t), \]

where \( C_t \) and \( L_t \) denote the household’s consumption and labor supply. Given initial asset holdings \( B_{H0} \) and \( D_0 \), each household maximizes utility by choosing consumption, labor, government debt, and deposits, \( \{C_t, L_t, B_{Ht+1}, D_{t+1}\} \), subject to the budget constraint

\[ C_t + q_{Bt+1}B_{Ht+1} + q_{Dt+1}D_{t+1} \leq (1 - \tau_{Lt})w_tL_t + D_t + \delta_tB_{Ht} + X_t - \frac{1 - \sigma}{\sigma} \bar{n} \]
and

\[ B_{Ht+1} \geq 0, \quad D_{t+1} \geq \bar{D}. \]

In (3), \( q_{Bt+1} \) and \( q_{Dt+1} \) are the prices of government debt and deposits. Buying one unit of government debt at \( t \) entitles the household to \( \delta_{t+1} \) units of goods at \( t + 1 \) where \( \delta_{t+1} = 1 \) signifies that the government repays its debts at \( t + 1 \), and \( \delta_{t+1} = 0 \) signifies that it does not. Buying one unit of deposits at \( t \) entitles the household to one unit of goods paid by the bank at \( t + 1 \). Also, \( w_t \) is the real wage, \( \tau_{Lt} \) is the labor income tax, \( X_t \) are the aggregate dividends paid by banks, and \( \bar{n} \) is the amount of initial equity given to each newly formed bank of which there are a measure \((1 - \sigma)/\sigma\) formed each period. The constraint (4) ensures that no Ponzi schemes are feasible. The nonnegativity constraint on government debt implies that the household cannot borrow from the government. We choose \( \bar{D} \) to be sufficiently negative so that the allocations are interior.

The first order conditions for the household’s problem can be summarized by

\[ \frac{-U_{Lt}}{U_{Ct}} = (1 - \tau_{Lt})w_t \]  \hspace{1cm} (5)

\[ q_{Dt+1} = \beta \frac{U_{Ct+1}}{U_{Ct}} \]  \hspace{1cm} (6)

\[ q_{Bt+1} \geq \beta \frac{U_{Ct+1}}{U_{Ct}} \delta_{t+1} \]  \hspace{1cm} (7)  

with equality if \( B_{Ht+1} > 0 \).

Because households can use deposits to save or borrow but can use government debt only to save, an immediate implication of (6) and (7) is

\[ q_{Bt+1} \geq q_{Dt+1} \delta_{t+1} \]  \hspace{1cm} (8)

with equality if households hold debt. We refer to \( q_{Dt+1} \delta_{t+1} \) as the fair market price of debt because this is the price at which households would willingly hold government debt. The difference between the price of debt and its fair market price, \( q_{Bt+1} - q_{Dt+1} \delta_{t+1} \), measures the extent of the government’s interest rate suppression on its debt. Clearly, if the government suppresses interest rates, households will choose to hold no government debt.

Next consider the banks. In each period \( t \), each bank chooses how much capital, \( k_{t+1} \), and government debt, \( b_{Bt+1} \), to hold, how much deposits, \( d_{t+1} \), to issue, and how much
dividends, $x_t$, to pay out. At the beginning of each period, an idiosyncratic random variable is realized at each existing bank that indicates to the bank whether it will survive to the next period. With probability $\sigma$, the bank will continue in operation until the next period. With probability $1-\sigma$, the bank ceases to exist and, by assumption, pays out all of its accumulated net worth as dividends to the household. Also at the beginning of each period, a measure $(1-\sigma)/\sigma$ of new banks are born, each of which is given an exogenously specified amount of initial equity $\tilde{n}$ from its household. Since only a fraction $\sigma$ of these newborn banks survive until the end of the period, the measure of surviving banks is always constant at 1. This device of having banks die is a simple way to ensure that they do not build up enough equity to make the financial constraints (which we will introduce next) irrelevant.

We now turn to the constraints of an individual bank. Any ongoing bank has a budget constraint

$$(9) \quad x_t + (1 + \tau_{kt})k_{t+1} + q_B b_{Bt+1} - q_B d_{t+1} \leq R_t k_t + \delta_t b_t - d_t,$$

a collateral constraint

$$(10) \quad d_{t+1} \leq \gamma [R_{t+1} k_{t+1} + \delta_{t+1} b_{Bt+1}],$$

where $0 < \gamma < 1$, a regulatory constraint

$$(11) \quad b_{Bt+1} \geq \phi_t (R_{t+1} k_{t+1} + b_{Bt+1}),$$

and nonnegativity constraints on dividends and bond holdings. We let $n_t = R_t k_t + \delta_t b_t - d_t$ denote a bank’s net worth.

The collateral constraint requires that the promised payments on deposits, $d_{t+1}$, be no more than a fraction of the anticipated receipts on assets the bank holds, $R_{t+1} k_{t+1} + \delta_{t+1} b_{Bt+1}$. As we show below, when the rate of return on capital strictly exceeds the rate of return on deposits, this constraint is binding. Banks are unable to arbitrage this difference between a high rate of return on their assets and a low rate of return on their liabilities because they are unable to issue additional liabilities to finance the purchase of additional assets. We provide a motivation for the collateral constraint as arising from limited enforcement below.

The regulatory constraint requires the bank to hold at least a fraction $\phi_t$ of its assets in government debt. Here, $\phi_t$ measures financial repression: whenever $\phi_t > 0$, we say that
the government is practicing financial repression and that the higher the level of $\phi_t$, the greater the degree of financial repression. Note that when this constraint is binding, if the bank desires to acquire an amount of additional capital that generates additional receipts of 1 unit, it is forced to acquire $\phi_t/(1 - \phi_t)$ units of additional government debt.

Note that we have assumed that the tax on capital is levied on the capital stock chosen in period $t$, $k_{t+1}$, rather than income from inherited capital $R_t k_t$. This assumption allows us to focus on the time inconsistency problem associated with default and financial repression rather than the time inconsistency problem arising from the taxation of inherited capital. We also assume that $\tau_K \geq 0$. This assumption restricts the government from having instruments that allow it to effectively subsidize capital accumulation and thereby indirectly circumvent the collateral constraint. Allowing for subsidies to capital accumulation gives the government instruments that allow the government to undermine the financial frictions that are at the heart of our analysis.\textsuperscript{1}

Now consider now the problem of a bank. Recall that a bank that ceases to operate in period $s$ pays out its accumulated net worth $n_s$ as dividends. Banks that continue to operate in $s$ choose the amount of dividends $x_s$ to pay. Thus, the problem of a bank born at $t$ is to maximize

$$v_t(\bar{n}) = \max_{\{k_{s+1}, b_{B,s+1}, d_{s+1}, x_s\}} \sum_{s=t}^{\infty} Q_{t,s} \sigma^{s-t} [\sigma x_s + (1 - \sigma) n_s]$$

subject to (9)–(11) where $n_t = \bar{n}$ and $n_s = R_s k_s + b_{B,s} - d_s$ for $s > t$ where $Q_{s,t} = \beta^{s-t} U_{C,s}/U_{C,t}$ is the bank’s discount factor. Note from the household’s first-order condition (6), that this discount factor $Q_{t,s} = q_{D,t+1} \cdots q_{D,s}$ so that banks and households discount the future in the same way. We can write the continuation value of a surviving bank at $t$ with inherited net worth $n_t$ in a similar fashion.

The following lemma characterizes key features of the solution to a bank’s problem.

Lemma 1. Ongoing banks weakly prefer not to pay dividends and strictly prefer not to do so if the collateral constraint is binding. The value of a newborn bank is linear in initial

\textsuperscript{1}Similarly, we have assumed that banks do not offer deposit contracts with payoffs that depend on whether the government defaults. As we show below, there are no defaults along the equilibrium path, and hence, banks have no incentives to offer such contracts. Thus, if there is any cost to offering such contracts, no bank will do so.
net worth, and the value of a surviving bank is linear in current net worth. If \( k_{t+1} > 0 \), then the after-tax return on capital must be at least as large as the return on deposits in that

\[
(13) \quad \frac{R_{t+1}}{1 + \tau K_t} \geq \frac{1}{q_{Dt+1}}.
\]

Finally, when \( q_{Bt+1} = q_{Dt+1} \) and \( k_{t+1} > 0 \), if (13) holds as a strict inequality, then both the collateral constraint and the regulatory constraint are binding.

The proof of this result and other results, except where noted, are in the Appendix.

First, to see why ongoing banks prefer not to pay dividends, since \( Q_{t,s} = q_{Dt+1} \cdots q_{Ds} \), from the bank’s budget constraint, a bank that is currently paying dividends can finance its purchase of capital by paying fewer dividends and issuing fewer deposits. Since issuing fewer deposits slackens the collateral constraint, whenever the collateral constraint binds, it is optimal to pay no dividends. Thus, it is without loss of generality to assume that banks pay no dividends, except in periods that they die. Second, inspecting the bank’s problem shows that a newborn bank’s policies are linear in \( \bar{n} \) and that an ongoing bank’s policies are also linear in inherited net worth. Third, the cost of investing one unit is the right side of (13), while the benefit of doing so is the left side, so if the bank is investing a positive amount, then (13) must hold.

Next, if the return on debt equals that on deposits, in that \( 1/q_{Bt+1} = 1/q_{Dt+1} \) and the after-tax return on capital is strictly greater than that on deposits, then the collateral constraint must be binding. Suppose, by way of contradiction, that it is not. Then the bank can profitably deviate from the purported allocation. This deviation is to issue deposits, use part of the deposits to hold capital, and use the rest to hold the amount of government debt necessitated by the regulatory constraint. Since the return on capital strictly exceeds that on deposits and the return on debt equals that on deposits, this variation yields strictly higher profits than the purported allocation. This contradiction argument establishes that under these conditions, the collateral constraint must be binding.

Banks will not voluntarily hold government debt if the collateral constraint is binding because holding government debt crowds out capital. To see this crowding-out effect, suppose by way of contradiction that the collateral constraint is binding, the return on debt equals that on deposits, in that \( 1/q_{Bt+1} = 1/q_{Dt+1} \) and \( \delta_{t+1} = 1 \), and that the bank voluntarily
holds government debt. Then, holding fixed the deposits, the bank can reduce its holding of
government debt and increase its holdings of capital and, under (13), both satisfy its collateral
constraint and increase profits. Thus, if banks are forced to hold government debt, capital
must be crowded out.

Here we motivate the collateral constraint as arising from limited enforcement as fol-
lows. First notice that we can write the value function of the bank at $t$ as $v_t(n_t)$, which
just depends on the net worth at $t$. Imagine that at any date $t + 1$ the banker can continue
with the optimal plan of the bank and obtain $v_{t+1}(R_{t+1}k_{t+1} + \delta_{t+1}b_{Bt+1} - d_{t+1})$ or that it
can abscond with the assets of the bank $R_{t+1}k_{t+1} + \delta_{t+1}b_{Bt+1}$ and start a new bank at $t + 1$
with $(1 - \gamma)(R_{t+1}k_{t+1} + \delta_{t+1}b_{Bt+1})$ worth of assets. Here a fraction $\gamma$ of assets are lost in the
process of absconding. Thus, for the banker not to abscond, it must be the case that

\begin{equation}
    v_{t+1}(R_{t+1}k_{t+1} + \delta_{t+1}b_{Bt+1} - d_{t+1}) \geq v_{t+1}((1 - \gamma)(R_{t+1}k_{t+1} + \delta_{t+1}b_{Bt+1})).
\end{equation}

Since $v_{t+1}$ is increasing in its arguments, (14) implies that the argument of the function on
the left side of (14) is greater than the argument of the function of the right side of (14),
which, after rearrangement, implies the collateral constraint.

Finally, consider firms and the government. A representative firm rents capital at rate
$R_t$ from banks and hires $L_t$ units of labor to maximize profits

\begin{equation}
    \max_{K_t, L_t} F(K_t, L_t) - R_t K_t - w_t L_t
\end{equation}

so that

\begin{equation}
    F_{L_t} = w_t \quad \text{and} \quad F_{K_t} = R_t.
\end{equation}

The budget constraint of the government is

\begin{equation}
    G_t + \delta_t B_t \leq \tau_{L_t} w_t L_t + \tau_{K_t} K_{t+1} + q_{Bt+1} B_{t+1},
\end{equation}

where $B_{t+1}$ is bounded by some large positive constant $\bar{B}$. We assume that the levels of
government spending and the initial debt are small enough so that it is always feasible to
finance any government debt by the present discounted value of tax revenues from labor and
capital.
The competitive equilibrium is defined in the standard fashion. In this equilibrium, the policy of the government in period $t$ is $\pi_t = (\delta_t, \tau_{Lt}, \tau_{Kt}, B_{lt+1}, B_{dt+1}, q_{dt+1}, \phi_t)$. We turn next to characterizing the set of allocations and prices that can be implemented as a competitive equilibrium.

Our first result is that interest rate suppression is a redundant instrument in raising revenue to meet the government’s fiscal needs. More precisely, an outcome the government seeks to attain with this instrument can be attained by not suppressing interest rates and adjusting taxes on capital appropriately.

Proposition 1. (Redundancy of interest rate suppression) Any competitive equilibrium with debt prices, deposit prices, and the tax on capital given by \( \{q_{Bt+1}, q_{Dt+1}, \tau_{Kt}\} \) with \( q_{Bt+1} \geq q_{Dt+1}\delta_{t+1} \) can also be supported as a competitive equilibrium with no interest rate suppression, that is, prices and policies given by \( \{q'_{Bt+1}, q_{Dt+1}, \tau'_{Kt}\} \) where \( q'_{Bt+1} = q_{Dt+1}\delta_{t+1} \) and \( \tau'_{Kt} \) are chosen to satisfy the government budget constraint.

The idea behind the proposition is that if the government suppresses interest rates, then banks will hold only the minimum amount required by regulation so that from the regulatory constraint (11), $b_{Bt+1} = \phi_t/(1 - \phi_t)R_{lt+1}k_{lt+1}$. That is, in order to invest $k_{lt+1}$ units of capital, the bank must also hold some debt that earns a lower return. This outcome is equivalent to one in which banks are forced to hold debt but are paid market rates and capital is taxed directly. In this sense, forcing banks to hold government debt at a below-market rate is a form of taxation of capital. This result implies, as we show below, that, with or without commitment, giving the government the power to force banks to hold government debt at below-market rates cannot increase welfare.

In our analyses of government policy with and without commitment, we need a characterization of the set of competitive equilibrium allocations. A key feature of this characterization is that without loss of generality we can restrict ourselves to allocations in which there is no anticipated default, that is, default after period 0. The reason is that any equilibrium with default on debt at period $t + 1$ will have the price of debt $q_{Bt+1} = 0$. Thus, this equilibrium can be equivalently represented as one in which no debt is issued, $B_{t+1} = 0$, and no default occurs, $\delta_{t+1} = 1$. Thus, from now on we will set $\delta_{t+1} = 1$ for $t \geq 0$, and, using Proposition 1, set $q_{Bt+1} = q_{Dt+1}$. Note that in period 0, we need to allow default on the inherited debt
Given this result, we now characterize the set of competitive equilibrium allocations. For any period $t \geq 1$, substituting from the household and firm first order conditions (5), (6), and (16) into the government budget constraint (17) gives that an equilibrium must satisfy an equilibrium version of the aggregate budget constraint of the government,

$$G_t + B_t \leq \left( F_{L_L} + \frac{U_{L_L}}{U_{C_L}} \right) L_t + \tau_{K_t} K_{t+1} + \frac{\beta U_{C_{t+1}}}{U_{C_t}} B_{t+1},$$

along with the bound $B_{t+1} \leq \bar{B}$. Next, adding the budget constraints of newborn and continuing firms, using the consumer first order condition (6) and the result that we can set dividends of ongoing firms to zero, yields an equilibrium version of an aggregate budget constraint for banks,

$$ (1 + \tau_{K_t}) K_{t+1} + \frac{\beta U_{C_{t+1}}}{U_{C_t}} (B_{B_{t+1}} - D_{t+1}) = \sigma (F_{K_t} K_t + B_{B_t} - D_t) + (1 - \sigma)\bar{n}. $$

Adding across the collateral constraints gives the aggregate collateral constraint for banks

$$ D_{t+1} \leq \gamma \left[ F_{K_{t+1}} K_{t+1} + B_{B_{t+1}} \right]. $$

Next, we use two key implications from Lemma 1 and Proposition 1 to establish an inequality and a complementary slackness condition, which we refer to jointly as the capital distortion constraint. Substituting (6) and (16) into (13) gives

$$ F_{K_{t+1}} \geq \frac{(1 + \tau_{K_t}) U_{C_t}}{\beta U_{C_{t+1}}}, $$

and, using Lemma 1 and Proposition 1, yields that whenever (21) is a strict inequality, then the aggregate collateral constraint holds as an equality. At an intuitive level, the constraint (21) captures the idea that a binding collateral constraint distorts capital over and above the distortion that arises from capital taxation. Hence, the least amount of distortion on capital is from the tax on capital alone, and the marginal product of capital can be no lower than the tax-adjusted marginal rate of substitution of the household.

Now consider period 0. Since in this period we allow for default on the inherited debt $B_0 = B_{H_0} + B_{B_0}$, the constraints in period 0 differ from their analog in period $t \geq 1$. In particular, in period 0 the left side of (18) is $G_0 + \delta_0 B_0$, the right side of (19) is $\sigma (F_{K_0} K_0 + \delta_0 B_{B_0} - D_0) + (1 - \sigma)\bar{n}$, and the rest of the constraints are as before.
We refer to the resource constraint (1) together with the constraints (18)–(21) as the implementability constraints. Clearly, the implementability constraints are necessary for a competitive equilibrium. In the next lemma, we show that they are also sufficient.

Lemma 2. (Characterization of Competitive Equilibrium) A set of aggregate allocations and policies is part of a competitive equilibrium if and only if they satisfy the implementability constraints. In particular, these allocations and policies allow for default only in period 0.

To prove sufficiency, we need to show that from the aggregate allocations we can construct profit-maximizing allocations for individual banks. The linearity of the banks’ policy functions makes it possible to construct such individual allocations.

So far we have defined a competitive equilibrium that starts in period 0. To set up our recursive representation that we use later we introduce several other definitions. First, the definition and characterization of a competitive equilibrium that starts in period \( t \) is analogous to one that starts in period 0. In particular, in such an equilibrium we allow for default on the debt inherited in period \( t \), but it is without loss of generality to have no anticipated default in all future periods. Second, for any competitive equilibrium in period \( s < t \), the continuation competitive equilibrium from period \( t \) is characterized by the implementability constraints from \( t \) onward.

3. Equilibrium with Commitment

We now turn to characterizing the best equilibrium under commitment, namely, the Ramsey equilibrium. This equilibrium is defined as the competitive equilibrium that yields the highest utility for households. Our main result here is that, under commitment, financial repression is not optimal. The key idea is that forcing banks to hold government debt tightens collateral constraints and is, therefore, an inefficient way to allocate government debt. We then turn to a detailed recursive characterization of the Ramsey outcomes for a simple version of our model.
A. Characterization for the General Model

The Ramsey problem for this economy is to maximize

\[ (22) \quad \sum_{t=0}^{\infty} \beta^t U(C_t, L_t) \]

subject to the implementability constraints.

Recall that the constraints in period 0 differ from those in any period \( t \geq 1 \), in that in period 0 the government can default on inherited debt, whereas we can assume without loss of generality that it cannot default in any subsequent period. Indeed, if all of the initial debt is held by households, then the government will default on this debt in period 0. If a sufficient amount of the debt is held by banks, repaying this initial debt may be optimal for the government.

Regardless of the optimal default policy in period 0, the key feature of the Ramsey outcome that we emphasize is that it does not have financial repression.

Proposition 2. (Financial Repression Not Optimal with Commitment) The Ramsey outcome can be implemented with no financial repression, that is, \( \phi_t = 0 \) for all \( t \). Moreover, if the aggregate collateral constraint (20) is binding in some period \( t \), then it is strictly optimal not to practice financial repression in that period, in that \( \phi_t = 0 \) and \( B_{Bt+1} = 0 \).

The proposition says that it is always possible to implement the Ramsey outcome with no financial repression. Consider an outcome in which the government is practicing financial repression so that banks are holding government debt. Consider the following variation. Reduce the extent of financial repression so that bank holdings of government debt are reduced by one unit. Also reduce deposits by one unit and increase household holdings of government debt by one unit. This variation leaves the total savings of households unaffected and leaves the total resources of banks available for investment unaffected as well. Hence, the variation supports the original allocations of consumption, labor, and capital. Next, note that if the collateral constraint is binding, this variation relaxes the collateral constraint and thus allows for an improvement in welfare.

The basic idea is that when the collateral constraint is binding, financial repression has crowding-out costs. The crowding-out costs for the economy as a whole are similar to the crowding-out effects on capital for an individual bank. As for an individual bank, forcing
banks as a whole to hold government debt crowds out capital. It is more efficient to have households hold the government debt directly, thus avoiding unnecessary tightening of the collateral constraints and thereby eliminating crowding-out costs.

B. Characterization for the Separable Model

We now turn to an explicit characterization of the commitment equilibrium for a version of our model that imposes several simplifying assumptions. We draw on this characterization when we study the best sustainable equilibrium. The first two assumptions are that the utility function is quasi-linear and the production function is additively separable in that

\[
U(C, L) = C - v(L) \quad \text{and} \quad F(K, L) = \omega_K K + \omega L.
\]

These assumptions eliminate all the cross-partial terms and ensure simple expressions for prices. If consumption and capital are positive in all periods, then \( q_{Dt+1} = \beta \) and \( R_{t+1} = F_{Kt+1} = \omega_K \). Under sufficient conditions, the equilibrium has interior allocations in that consumption and capital are positive in all periods and that the collateral constraint is always binding. These conditions include

\[
1 < \beta \omega_K < 1 / \gamma,
\]

and the rest are provided in the Appendix. Here, the first inequality in (24) implies that investment is worthwhile in that the gross return on investment exceeds the discount rate. The second inequality in (24) implies that the collateral constraint is nontrivial in the sense that increasing revenues from capital and deposits by one unit each tightens the collateral constraint. We refer to the version of our model that satisfies these assumptions as the separable model.

We find it convenient to transform variables and express the capital distortion constraint in a slightly different form. Using \( F_{Kt} = \omega_K \) and letting \( T_K = \tau_k K' \), we can write the capital distortion constraint as

\[
0 \leq T_K \leq (\beta \omega_K - 1)K'.
\]

To conserve on notation, it is convenient to have the government directly choose the revenues from the labor tax rather than labor itself. To make this transformation, note that
labor is determined by the household’s first order condition $v'(L) = (1 - \tau_L)\omega$, so that labor supply depends only on the tax rate on labor. The tax revenue from labor $T = \tau_L\omega L$ satisfies

$$T = (\omega - v'(L))L,$$

so we can let the labor supply $\ell(T)$ associated with labor tax revenues $T$ be implicitly defined by the solution to (26). Here we choose the solution to be on the upward-sloping part of the Laffer curve. We can then let

$$W(T) = \omega\ell(T) - v(\ell(T))$$

denote the net utility from labor, namely, the part of current output that is produced by labor minus the disutility of labor. From now on, we will assume that $W$ is strictly concave, a sufficient condition for which is that the disutility of labor takes the isoelastic form $v(L) = L^{1+\eta}/(1 + \eta)$ for $\eta > 0$. (See Result 1 in the Appendix.) With this notation, utility can be written as

$$\sum_{t=0}^{\infty} \beta^t [C_t - v(L_t)] = \sum_{t=0}^{\infty} \beta^t [W(T_t) + \omega_K K_t - K_{t+1} - G_t]$$

where we have used the resource constraint (1). Throughout, $G_t$ is an additive constant in utility. We drop it from all values both here and in the later sections.

Next, we simplify the expression for utility (28). To do so we substitute out for aggregate deposits using the binding aggregate collateral constraint, $D_{t+1} = \gamma F_{Kt+1}K_{t+1}$ into the aggregate bank budget constraint (19) to obtain the law of motion for the aggregate capital stock, which can be recursively substituted to obtain

$$\sum_{t=0}^{\infty} \beta^t [\omega_K K_t - K_{t+1}] = \omega_K K_0 + A_R + A_N N_0 - \frac{A_N}{\sigma} \sum_{t=0}^{\infty} \beta^t T_{Kt}.$$  

The constant $A_R$ comes from the continual injection of initial net worth to newborn bankers $\bar{n}$, the term $A_N N$ captures the discounted value of returns that emanate from reinvestment of initial net worth, and the last term captures the lost present value of returns that arise because of capital taxation.

Substituting for the capital component of utility (29) into the objective function (28) gives that the present value of utility in a Ramsey equilibrium is

$$V_R(K, D, B_B, B_H) = A_R + \omega_K K + A_N N + \sum_{s=0}^{\infty} \beta^{s-t} \max_{\{T_{Ls}, T_{Ks}\}} \left\{ W(T_{Ls}) - \frac{A_N}{\sigma} T_{Ks} \right\}$$

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subject to the government budget constraint, where \( N = \omega_K K + B_B - D \). Given the form in (30), a natural guess, verified in the Appendix, is that the solution has the form

\[
V_{Rt}(S) = A_R + \omega_K K + A_N N + H_{Rt}(B, G),
\]

where the tax distortion function \( H_R \) is defined as the value of the problem

\[
H_{Rt}(B, G) = \max_{B',T} W(T) - A_N T_K + \beta H_{Rt+1}(B', G')
\]

subject to the government budget constraint (18). The tax distortion function captures the losses from distorting taxes on labor and capital. The value of the Ramsey problem in period 0 is

\[
J_{R0}(S) = \max \{ V_{R0}(S), V_{R0}(K, D, 0, 0, G) \},
\]

where the function \( V_{R0} \) is given by (31). Note that defaulting on the debt in period 0 is equivalent to inheriting zero debt in period 0. Thus, (33) implies that in period 0, the value of the Ramsey equilibrium is the value the government would obtain if we allowed it to explicitly default on the debt. We summarize this discussion as follows.

**Lemma 3.** The value of the Ramsey problem for the separable model is given by (33) where the value function \( V_{R0} \) is given by (31).

Next we characterize the Ramsey allocations and policies, assuming that the non-negativity constraint on debt is not binding. The first order condition for labor tax revenues, together with the envelope theorem, implies that the distortion from labor taxes are equalized in all periods in that

\[
W'(T_t) = W'(T_0),
\]

so that labor tax revenues are constant at a level, say \( T \), in all periods. We now provide sufficient conditions under which it is optimal to set the tax on capital equal to zero in all periods. Since labor taxes are constant, the marginal cost of raising revenues from labor taxes is also constant at the level \(-W'(\hat{T})\) and the marginal cost of raising revenues from capital taxes is \( A_N / \sigma \), so that the tax on investment is zero in all periods if

\[
-W'(\hat{T}) < \frac{A_N}{\sigma},
\]
where \( \hat{T} = (1 - \beta) \left( \sum_{t=0}^{\infty} \beta^t G_t + B_{B0} + B_{H0} \right) \) is the constant labor tax revenue that needs to be collected if no revenue is raised from investment taxes. In what follows, we will assume that the analog of (35) holds so that the tax on investment is zero in all periods.

4. Sustainable Equilibrium

We have shown that financial repression is not useful under commitment. Clearly, since interest rate suppression is feasible only with repression, it follows that interest rate suppression is not useful either. Here we argue that without commitment, financial repression may be beneficial, though interest rate suppression is not.

Here we consider an economy in which standard reputational considerations alone allow a government to issue a positive amount of debt to households. The idea is that as long as the government does not default, households believe it will not in the future, but if it ever defaults then it will revert to the no-reputation equilibrium, which here is the Markov equilibrium. Under these strategies, the government will not default as long as the tax-smoothing gains from now on from not doing so outweigh the one-time gains from defaulting on debt and then reverting to the Markov equilibrium. Clearly, the government will default if the inherited debt is sufficiently large, and, thus, reputation can support only a limited amount of debt. If the government attempts to issue any more than this limited amount, private agents expect the government to default unless it also practices financial repression.

Financial repression turns out to be a mechanism to increase the extent of commitment to not defaulting. If part of the inherited debt is held by banks, then defaulting on the debt has greater ex post costs because such default reduces the net worth of ongoing banks and thereby reduces future capital accumulation. When the government practices financial repression by forcing banks to hold its debt, it endogenously increases the size of these ex post costs it will pay if it defaults on this debt. In this sense, financial repression is effectively a mechanism to increase the extent of commitment to repay debt. Of course, financial repression has ex ante costs, namely, the crowding-out costs we discussed earlier. In equilibrium, the government will choose to practice financial repression if the tax-smoothing gains exceed the crowding-out costs.

We consider an economy that starts in a period of exceptionally large spending needs
and thereafter follows a cyclical pattern. We think of this economy as capturing the effects of transitory periods of high expenditures, such as wartime. We show that the government optimally responds by issuing a large amount of debt in the first period and practices financial repression for some length of time. During the repression phase, governments gradually reduce their debt. Eventually, governments cease practicing financial repression and reputation concerns alone allow the government to sustain tax-smoothing outcomes in the face of cyclical expenditures. As we have argued, this pattern is reminiscent of the behavior of public debt in many of the Allied countries after World War II.

We can also interpret our economy as starting from a stationary equilibrium in which financial repression is not used because reputational concerns sustain adequate tax smoothing and then experiencing a one-time unanticipated shock to government spending. If this shock is sufficiently large, then the government will practice financial repression, issue debt, and then eventually return to a stationary equilibrium with no financial repression. We find these predictions useful in thinking about the recent rise in financial repression in the periphery states in Europe following the Great Recession.

We note that many of the qualitative features of our results continue to hold if reputational considerations play no role. Specifically, if we restrict attention to Markov equilibria, periods of exceptionally large spending needs are followed by financial repression. The key difference is that in a Markov equilibrium, financial repression persists in the long run whereas in the best sustainable equilibrium, financial repression ceases after a finite length of time.

So far we have assumed that the government cannot selectively default on a group of agents. We go on to show that our results are robust to allowing the government to practice discriminatory default, by allowing it to separately choose to default on banks and households.

A. Markov and Sustainable Equilibria

We consider equilibrium outcomes that are sustained by reputational mechanisms. Specifically, we focus on sustainable equilibria with trigger strategies. In these equilibria, if the government deviates from a particular policy path, private agents expect that the government will revert to Markov policies from then on.
We begin by developing the Markov equilibrium. In this equilibrium, outcomes depend only on physical state variables so that reputation plays no role. In particular, policies, decisions, and prices are restricted to depend only on the state of the economy. In each period \( t \), there are two relevant states: a state for the government is \( S_t = (K_t, D_t, B_{Bt}, B_{Ht}, G_t) \) at the beginning of the period and a state for private agents is \((S_t, \pi_t)\) after the policy \( \pi_t \) has been chosen. Let \( Y_t = (C_t, L_t, K_{t+1}, D_{t+1}) \) and \( P_t = (R_t, q_{Dt}) \) denote the resulting allocations and prices.

A Markov equilibrium starting in period 0 consists of a sequence of policy functions, allocation rules, and pricing rules, \{\pi_t(S_t), Y_t(S_t, \pi_t), P_t(S_t, \pi_t)\}_{t=0}^{\infty} \) such that i) the associated outcomes constitute a competitive equilibrium starting in each period \( t \) for all \( S_t \) and \( \pi_t \), and ii) in each period \( t \), given the state \( S_t \) and taking as given future policy functions, allocation rules, and pricing rules, the current policy \( \pi_t(S_t) \) is optimal for the government. The Markov equilibrium starting in period \( t \) is defined analogously.

We now characterize the Markov equilibrium starting in period \( t \) for the general model and then for the separable model. To do so we begin by characterizing the continuation value of the Markov equilibrium for \( r > t \). Since the continuation of a Markov equilibrium must also be the continuation of a competitive equilibrium, in light of the result from Lemma 2 we can restrict ourselves to outcomes that satisfy the implementability constraints and, in particular, have no anticipated default. Thus, this continuation value has the feature that the government is not permitted to default in period \( r \) and has a constraint that guarantees that the government in period \( r + 1 \) does not default either.

Using standard primal logic and suppressing the time subscripts on the states and the choice variables, we can write this continuation value as the value of the problem of choosing the allocations \( Y_r \) to solve

\[
V_{Mr} (S) = \max_Y U(C, L) + \beta V_{Mr+1}(S')
\]

subject to implementability constraints that consist of the resource constraint and the constraints

\[
G + B \leq \left( F_L + \frac{U_L}{U_C} \right) L + q_D(S') B'
\]

\[
K' + q_D(S') (B'_B - D') = \sigma (F_K K + B_B - D) + (1 - \sigma)\bar{n}
\]
\[ D' \leq \gamma [R'(S') K' + B'_B] \]

(40) \[ R(S') \geq \frac{1}{q_D(S')}, \]

where \( q_D(S') = \beta U_C(S')/U_C(S) \) and \( R(S') = F_K(S') \), and the no-default constraint, that is,

(41) \[ V_{Mr+1}(S') \geq V_{Mr+1}(K', D', 0, 0, G'). \]

where the government in period \( r \) takes as given the allocation rules \( C(S') \) and \( L(S') \) that generate \( U_C(S') \) and \( F_K(S') \). To understand the no-default constraint, note that defaulting on inherited debt in the next period has the same continuation value as inheriting zero debt in that period. Thus, this constraint ensures that the government in the next period will not default on the debt. Since this constraint is also present in the government’s problem in future periods, it recursively implies that no future government will ever default.

Given these continuation values, in a Markov equilibrium starting in period \( t \), the government solves a problem similar to that in the continuation problem (36), except that it can default on inherited debt at \( t \). If it chooses to default in period \( t \), its value is the same as if it inherited no government debt at \( t \) and is therefore given by \( V_{Mt}(K, D, 0, 0, G) \). Thus, the value of the Markov equilibrium in period \( t \) is

(42) \[ J_{Mt}(S) = \max \{V_{Mt}(S), V_{Mt}(K, D, 0, 0, G)\} \]

We now turn to the best sustainable equilibrium. We focus on equilibria that can be supported by reversion to Markov equilibria. Let \( H_t = (S_0, \pi_0; \ldots; S_{t-1}, \pi_{t-1}; S_t) \) be the history of allocations and policies up through the beginning of period \( t \). Clearly, \((H_t, \pi_t)\) is the history of allocations at the end of period \( t \) when private agents choose their actions. A \textit{sustainable equilibrium} is a collection of rules for policies, allocations, and prices \( \{\pi_t(H_t), Y_t(H_t, \pi_t), P_t(H_t, \pi_t)\}_{t=0}^{\infty} \) for private agents and the bailout authority such that for all histories \( H_t \), i) the associated outcomes constitute a competitive equilibrium starting in each period \( t \) for all \( H_t \) and \( \pi_t \), and ii) in each period \( t \), given the history \( H_t \) and taking as given future policy functions, allocation rules, and pricing rules, the current policy \( \pi_t(H_t) \) is optimal for the government.
We formalize our restriction to equilibria that can be supported by reversion to Markov equilibria by requiring that the value of the government’s problem for all histories $H_t$, both on and off the equilibrium path, satisfies the *sustainability constraint*

$$\sum_{r=t}^{\infty} \beta^{r-t} U(C_r, L_r) \geq J_{Mt}(S_t),$$  \hfill (43)

where the allocations $\{C_r, L_r\}_{r=t}^{\infty}$ are those induced by the equilibrium strategies from any arbitrary history $H_t$ for all $t \geq 0$.

A *sustainable outcome* $\{\pi_t, Y_t, P_t\}_{t=0}^{\infty}$ is the outcome that arises along the equilibrium path. Standard arguments imply that an outcome $\{\pi_t, Y_t, P_t\}_{t=0}^{\infty}$ is *sustainable* if i) it is a competitive equilibrium in period 0 and ii) the value of the government’s objective along the equilibrium outcome path satisfies the sustainability constraint. Finally, the *best sustainable outcome* maximizes the present value of utility starting in period 0, subject to the implementability constraints and the sustainability constraints for all $t$. Let $J_{S0}(S_0)$ denote the value of the best sustainable equilibrium starting from period 0, and let $V_{St}(S_t)$ denote the associated continuation values from any period $t$. The sustainability constraints (43) can be written in two parts: in period 0 the constraint is

$$J_{S0}(S_0) \geq \max \{V_{M0}(S_0), V_{M0}(K_0, D, 0, 0, G_0)\},$$  \hfill (44)

while in any period $t \geq 1$, in light of Lemma 2, there can be no anticipated default, so that the constraint is

$$V_{St}(S_t) \geq \max \{V_{Mt}(S_t), V_{Mt}(K_t, D_t, 0, 0, G_t)\}.$$  \hfill (45)

Note that, as in the Ramsey outcome, the government’s default policy in period 0 is different from its policy in subsequent periods. In period 0, the government may find it optimal to default, but it is without loss of generality to assume that it never defaults in any period $t \geq 1$.

**B. Characterization of Outcomes**

In order to characterize the best sustainable outcome, we begin by characterizing the Markov equilibrium outcomes.
**Markov Outcomes**

In a Markov equilibrium, there are no costs of defaulting on debt besides those that arise from the government practicing financial repression and forcing the banks to hold debt. Thus, there cannot be an equilibrium in which only households hold the debt so that a positive level of debt can be sustained only if there is financial repression.

*Lemma 4.* The value of debt is positive in a Markov equilibrium only if there is financial repression.

We obtain sharper characterizations of the Markov outcomes for our separable model. To do so, we build on our characterization of the Ramsey equilibrium. As in that equilibrium, a natural conjecture for the separable model is that the value function in the Markov equilibrium is linear in the capital stock and net worth and is separable from a function that captures the distortions arising from labor and capital taxation. We conjecture and verify in the Appendix that our separable economy has a Markov equilibrium with a continuation value given by

\[ V_{Mt}(S) = A_R + \omega_K K + A_N N + H_{Mt}(B, G) \]

where the *tax distortion* function \( H_{Mt} \) satisfies the Bellman equation

\[ H_{Mt}(B, G) = \max_{B'_0, B', T} W(T) - A_B B'_B + \beta H_{Mt+1}(B', G') \]

subject to the government budget constraint \( G + B \leq T + \beta B' \) and the *no-default constraint*, which, given the form of \( V_{Mt}(S) \), can be simplified to

\[ H_{Mt+1}(B', G') + A_N B'_B \geq H_{Mt+1}(0, G') . \]

The value of the Markov equilibrium starting at \( t \) is given by (42).

*Lemma 5.* A Markov equilibrium with values given by (42) and (46) exists.

**Best Sustainable Outcomes**

Here we also conjecture and verify that, in our separable model, the best sustainable equilibrium is linear in the capital stock and net worth and is separable from a function that captures the distortions arising from labor and capital taxation. Under this conjecture,
the continuation values for the best sustainable equilibrium with reversion to the Markov equilibrium can be expressed recursively as

\[ V_{St}(S) = A_R + \omega_K K + A_N N + H_{St}(B, G), \]

where \( H_{St} \) is the largest fixed point of a Bellman equation defined by

\[ H_{St}(B, G) = \max_{B'_0, B'_T, T} W(T) - A_B B'_B + \beta H_{St+1}(B', G') \]

subject to the government budget constraint \( G + B \leq T + \beta B' \) and (45). (Note that the observation that the best sustainable equilibrium is the largest fixed point of the Bellman equation is similar to observations in Abreu, Pearce, and Stacchetti (1990) and Atkeson (1991). Indeed, the smallest fixed point of this operator is the Markov equilibrium value and, at least for sufficiently large \( \beta \), is not the value of the best sustainable equilibrium.)

Consider a relaxed version of this problem in which we replace the right side of the constraint in (45) with \( V_{Mt}(K_t, D_t, 0, 0, G_t) \) so that the constraint becomes \( V_{St}(S_t) \geq V_{Mt}(K_t, D_t, 0, 0, G_t) \). We will show that a solution to the relaxed problem is feasible for the original problem. To see this result, suppose, by way of contradiction, that the solution to the relaxed problem violated the original problem in that \( V_{St}(S_t) < V_{Mt}(S_t) \). But this inequality cannot hold because it is always feasible from any period \( t \) onward to choose the Markov equilibrium outcomes from period \( t \) onwards without affecting outcomes in any previous periods. Thus, it follows that the solution to the relaxed problem coincides with the solution to the original problem. Substituting for \( V_{St} \) and \( V_{Mt} \) from (49) and (46) into the relaxed version of the sustainability constraint, after canceling terms, we can rewrite it as

\[ H_{St+1}(B', G') + A_N B'_B \geq H_{Mt+1}(0, G'). \]

Note that in (50) we look for the largest fixed point associated with the Bellman equation.

We then have the following lemma.

**Lemma 6.** The best sustainable equilibrium has the form (49) where \( H_{St} \) is the largest fixed point of the Bellman equation (50).

Consider the first order conditions for the best sustainable outcome, which we use later. If \( B_{t+1} \) is strictly positive, its first order condition implies that

\[ -\beta W'(T_t) = - (\beta + \mu_t) H'_{St+1}(B', G') = - (\beta + \mu_t) W'(T_{t+1}), \]
where \( \mu_t \) is the multiplier on the sustainability constraint (51) and the second equality follows from the envelope condition on (50). The first order condition for \( B_{t+1} \) implies that

\[
\mu_t \leq A_B / A_N
\]

with equality if \( B_{t+1} > 0 \).

Consider the implications of the first order condition (52). If the sustainability constraint is not binding, so that \( \mu_t = 0 \), then taxes are perfectly smoothed between periods \( t \) and \( t + 1 \) in that \( T_t = T_{t+1} \). If \( \mu_t > 0 \) then taxes are imperfectly smoothed in that \( T_t > T_{t+1} \). This imperfect tax smoothing arises because when the sustainability constraint binds, the amount of debt the government issues is limited by concerns about default. This limit on the amount of debt requires that taxes in the current period be larger than they are with perfect tax smoothing. If \( 0 < \mu_t < A_B / A_N \) then the costs of imperfect tax smoothing are not large enough to warrant financial repression, whereas if \( \mu_t = A_B / A_N \) the costs of imperfect tax smoothing are sufficiently large to warrant financial repression.

**Characterization for a Cyclic Economy**

To make our points in the simplest possible manner, we assume that, after an initial period, government spending deterministically fluctuates between high and low levels over time. In particular, we assume that in period 0 there is an initial level of spending \( G_0 \), zero inherited debt \( B_{H0} = B_{B0} = 0 \), and some given initial deposits \( D_0 \) and capital stock \( K_0 \). In all subsequent periods, government spending has a simple cyclical pattern: in even periods \( G_t = G_L \) and in odd periods \( G_t = G_H \).

In the best sustainable outcome, if \( G_0 \) is sufficiently high, in period 0 the government practices financial repression in order to sell debt into the subsequent low spending period in period 1. In period 1 the government repays part of this debt and continues to practice financial repression in order to roll over some of the debt into the high spending period in period 2. This pattern repeats, but in each subsequent cycle the amount of debt decreases. Eventually, the debt becomes sufficiently small so that reputation alone can sustain it and repression ceases. The outcomes eventually converge to the Ramsey tax-smoothing outcomes.

Proposition 3 below shows that it is optimal to have financial repression in the first period if \( G_0 \) is above a critical value \( G^* \), and Proposition 4 characterizes features of the
dynamic path of repression and debt. To prove the first proposition, first consider the maximal amount of debt inherited in period 1 that can be sustained with trigger strategies alone in period 0. This level of debt $B^*_1$ solves

$$H_{S1}(B^*_1, G_L) = H_{M1}(0, G_L).$$

(54)

The first order conditions in period 0 to problem (50) for $B_1$ and $B_{B1}$ imply that

$$-\beta W'(G_0 - \beta B_1) = - (\beta + \mu_1) H'_{S1}(B_1, G_L)$$

(55) 

$$-A_B + \mu_1 A_N \leq 0,$$

(56)

where (56) holds with equality if $B_{B1}$ is positive. Let $G^*$ be the critical value of $G_0$ at which the government is just indifferent between practicing financial repression and not, given that it issues debt $B^*_1$ in period 0 and does not practice financial repression. Since the government is indifferent between practicing financial repression and not, (56) must hold as an equality. Thus, at these values, the first order conditions (55) and (56) can be combined to yield

$$-\beta W'(G^* - \beta B^*_1) + \left(\beta + \frac{A_B}{A_N}\right) H'_{S1}(B^*_1, G_L) = 0.$$

(57)

Clearly, the equilibrium has financial repression in period 0 if and only if $G_0 > G^*$. Simply stated, increasing initial spending from $G^*$ increases the gains to tax smoothing, which makes the government strictly prefer repression. We summarize these results in the next proposition.

**Proposition 3.** In the best sustainable equilibrium, for any given $\beta$, there is a critical value $G^* = G^*(\beta)$ such that if $G_0 \leq G^*$ there is no financial repression in period 0 and $B_{B1} = 0$, and if $G_0 > G^*$ then there is financial repression in period 0, so that $B_{B1} > 0$.

The intuition is straightforward. Trigger strategies alone can support only a limited amount of debt. When $G_0 < G^*$ at the level of debt supported by trigger strategies, the marginal benefits of tax smoothing are smaller than the crowding-out costs of financial repression. When $G_0 > G^*$, with the limited amount of debt that can be supported by trigger strategies alone, the marginal benefits of tax smoothing are greater than the crowding-out costs of financial repression. The government then finds it optimal to increase its debt issuance by forcing banks to hold some of the debt, thereby achieving greater tax smoothing.
Next, we characterize the transition after period 0. We begin by characterizing the limiting cycle of debt and taxes. To do so, we first consider what limiting cycles are sustainable without any financial repression. To this end, define $\beta^*$ as the smallest discount factor such that a stationary Ramsey outcome in which zero debt is sold from a low spending state to a high spending state is sustainable. Now recall that such a Ramsey outcome has tax revenues in the high spending state equal to the tax revenues in the low spending state and the taxes in the low spending state pay for both the spending in that state and for the debt inherited in that state. Thus, $G_H - \beta B_H = G_L + B_H$ so that solving for the debt gives that 

$$B_H = \frac{(G_H - G_L)}{1 - \beta},$$

and substituting into the government budget constraint gives that the taxes in both states are given by $(\beta G_L + G_H)/(1 + \beta)$. Then $\beta^*$ is the discount factor that solves

$$\frac{1}{1 - \beta} W \left( \frac{\beta G_L + G_H}{1 + \beta} \right) = W(G_L) + \beta H_M(0, G_H).$$

(58)

That is, at $\beta^*$ the sustainability constraint holds with equality in that the value of the Ramsey plan starting from $G_L$ just equals the value of defaulting in $G_L$ and after that following the Markov equilibrium.

Next for any $\beta \geq \beta^*$, define the maximal debt sold from a low spending state, $B_L(\beta)$, such that the associated Ramsey outcome is sustainable, that is, $B_L(\beta)$ solves

$$\frac{1}{1 - \beta} W \left( G_L + B_H(\beta) - \beta B_L(\beta) \right) = W(G_L) + \beta H_M(0, G_H).$$

(59)

where $B_H(\beta) = B_L(\beta) + (G_H - G_L)/(1 - \beta)$. We then have the following proposition.

**Proposition 4.** There is a critical value $\beta^*$ such that for $\beta \geq \beta^*$ there is a finite period $T = T(\beta, G_0)$ such that there is no financial repression for $t \geq T$. Furthermore, if $G_0 > G^*(\beta)$ then $T > 0$ and up until period $T$, government debt is decreasing over each cycle and the fraction of government debt held by banks decreases over the cycle. Moreover, in the limit the economy converges to a Ramsey outcome with perfect tax smoothing and debts $B_L(\beta)$ defined by (59) and its counterpart $B_H(\beta)$.

Combining Propositions 3 and 4, we have that if households are sufficiently patient in that $\beta > \beta^*$ and the spending level in period 0 is sufficiently large in that $G_0 > G^*(\beta)$, the government will practice financial repression for some period of time. Thus, financial
repression is decreasing over time in the sense that the share of total debt held by banks decreases. Eventually the government stops practicing financial repression.

It is easy to show that financial repression plays an essential role in the slow running down of debt. To see this essential role, consider the same economy as in Proposition 4 but suppose that the government is prohibited from practicing repression. In the initial period with \( G_0 \), the government sells a large amount of debt into the subsequent low spending state. After period 0, government debt immediately jumps to the two-period cycle. The sustainability constraint is slack from period 1 on, and government debt and taxes from period 1 on are the Ramsey outcomes for the continuation equilibrium from period 1. In this sense there is no slow running down of debt; rather, debt immediately jumps down to a stationary cycle. Allowing repression allows the government to sell more debt from the initial state \( G_0 \) and also makes it optimal for the government to run down the debt slowly over time. (See Result 2 in the Appendix.)

If \( \beta < \beta^* \), then the outcomes resemble those in the Markov equilibrium. As we discuss in the Appendix, if \( G_0 \) is sufficiently high, in the Markov equilibrium debt also decreases over time. In particular, in the Markov equilibrium in period 0 the government practices financial repression in order to sell debt into the subsequent low spending period in period 1. In period 1 the government repays part of this debt and continues to practice financial repression in order to roll over some of the debt into the high spending period in period 2. This pattern repeats, but in each subsequent cycle the amount of debt decreases. In the limit, the government practices financial repression in the high spending state in order to sell debt in the low spending state and in the low spending state sells zero debt. Thus, in the limit of a Markov equilibrium, the government practices financial repression.

5. Extensions

Here we briefly consider two extensions of our baseline model. The first allows for stochastic government spending. The second allows for the government to choose differential treatment of household and bank debt in terms of default. We show that our key results carry over with both extensions.
**A. Extension to Stochastic Government Spending**

Here we extend our model to have stochastic government spending and show that with complete markets and state-contingent regulation, all of our results go through virtually unchanged. In the Appendix we establish the analogs of Propositions 2, 3, and 4 for the stochastic case.

The most interesting results for the stochastic case can be illustrated with a simple figure. In Figure 1 we graph outcome paths for an economy that starts in period 0 with a large level of spending $G_0$ and after period 0 follows a two-state Markov chain with $G_t \in \{G_L, G_H\}$. We focus on a path in which the realization of spending is always $G_t = G_L$ for $t \geq 1$. The figure shows the Ramsey outcomes, the Markov outcomes, and the best sustainable outcomes. Panel A graphs the value of debt sold at $t$ into the subsequent low and high spending states and Panel B the fraction of total government debt that is held by banks.

First consider the Ramsey outcomes. These outcomes feature no repression and a constant value of debt sold in each period. Next, in the Markov outcomes the value of the debt monotonically declines and financial repression occurs in all periods with positive debt. In the best sustainable outcomes, the value of the debt declines and the government practices financial repression for a finite number of periods. Once the value of the debt has declined to a sufficiently low level, the government stops practicing repression. The patterns of debt and repression in the best sustainable outcomes qualitatively capture those in the United States following World War II.

**B. Discriminatory Default**

In the analysis above we assumed that government default is nondiscriminatory in that in the event of a default the government defaults on both if it defaults on either. This feature is important because it allows the government to credibly commit to repaying debt held by households if a sufficiently large share of debt is held by banks.

Here we briefly consider what happens when the government can choose different default rates on households and banks. We show that if the government can choose one default rate for households, $\delta_H$, and a different default rate for banks, $\delta_B$, then it will always default on households in a Markov equilibrium. In the Appendix we show that, nonetheless,
the government will still practice financial repression by forcing banks to hold government
debt if the tax-smoothing gains are sufficiently large. In the best sustainable equilibrium, all
that happens is that the value of the Markov equilibrium is lower with discriminatory default
than without it, but the qualitative features of the outcomes with and without discriminatory
default are the same. In particular, we establish the following result.

Proposition 5. There is a critical value \(\beta^*\) such that for \(\beta \geq \beta^*\) there is a finite period
\(T = T(\beta, G_0)\) such that there is no financial repression for \(t \geq T\). Furthermore, if \(G_0 > G^*(\beta)\)
then government debt is decreasing over each cycle and the fraction of government debt held
by banks decreases over the cycle. Moreover, in the limit, the economy converges to a limiting
cycle with perfect tax smoothing.

6. Sudden Stops and Repression

In this section we consider an open economy version of the separable model with a
two-period cycle in government spending. We show that a sudden stop, thought of as an
abrupt drop in the willingness of foreigners to lend, can lead the government to partially
default on its inherited debt and then practice financial repression.

Here we extend our environment to include foreign lenders in addition to the domestic
agents. Foreign lenders are risk neutral, their discount factor equals \(\beta\), and they can hold
government debt issued by the government in period \(t\), denoted \(B_{F_{t+1}}\). We require that the
government cannot save by holding foreign assets so that \(B_{F_{t+1}} \geq 0\).

We assume that defaults are nondiscriminatory in the sense that the government can
default either on all of its debt or on none of it. Foreign lenders can impose punishments on
the government in the event of default. We think of these punishments as arising from trade
sanctions and other ways to enforce international contracts. Specifically, we assume that in
the event of default on \(B_F\) units of foreign debt, private consumption is reduced by

\[
\bar{\kappa}(B_F) = \min \{(1 + \kappa)B_F, \chi\} \quad \text{for} \quad \kappa > 0.
\]

We refer to \(\chi\) as the maximal punishment. The idea behind this assumption is that pun-
ishments are proportional to the size of the debt but cannot exceed an upper bound. For
example, we can think of this upper bound as the reduction in domestic output associated
with a complete suspension of international credit. The interesting case we consider below
restricts this upper bound to be suitably low so that the government seeks other sources for credit after it has exhausted its opportunities from foreign borrowing.

Throughout, we assume that $\kappa$ is sufficiently large in the sense that

$$\kappa > A_N, \tag{61}$$

$$W(G_L + B_R) - B_R > W(G_L) - (1 + \kappa)B_R, \tag{62}$$

where $B_R = (G_H - G_L)/(1 + \beta)$ is the Ramsey level of debt. As will become clear, (61) implies that borrowing from foreign lenders is a better way of purchasing commitment than is financial repression. In particular, the net cost of defaulting on one unit of foreign debt is $\kappa$, while the net cost of defaulting on one unit of bank debt is $A_N$. Next, (62) implies that if the maximal punishment $\chi$ is sufficiently high, then the Ramsey outcome is sustainable with just foreign borrowing.

To save on notation, we assume that foreign lenders cannot hold deposits and that domestic private agents cannot hold foreign assets. (It turns out that all the results described here go through whether foreign lenders can make deposits in domestic banks and whether domestic agents can hold foreign assets.)

The problems facing domestic households and domestic banks are identical to those in the closed economy. The only changes are in the government budget constraint and in the incentive for the government to default when it lacks commitment. The government budget constraint is the same as in the baseline model, except that now total government debt, $B_t = B_{Bl} + B_{Bl} + B_{Fr}$, includes the debt to the foreign lenders. Here, instead of a resource constraint there is an economy-wide budget constraint that is obtained by consolidating the budget constraints of all the domestic agents to

$$\delta_t B_{Ft} + C_t + K_{t+1} + G_t = \omega_K K_t + \omega L_t + q_{Bt+1} B_{Ft+1}. \tag{63}$$

The implementability constraints for this economy are the same as those in the closed economy except that the resource constraint (1) is replaced by the consolidated budget constraint (63).

It is immediate that, as in the closed economy, financial repression is not optimal under commitment.
Now consider an economy without commitment. The definitions of Markov and sustainable equilibrium are obvious analogs of those in the closed economy. To characterize the best sustainable equilibrium, we start by setting up and characterizing the primal Markov problem. The aggregate state \( S = (K, D, B_H, B_B, B_F, G) \) confronting the government now records the amount of debt held by foreign lenders in addition to the elements in the state for the closed economy. Using (63) we can write current period utility as

\[
C - v(L) = \omega_K K - G - K' - \delta B_F + \beta B_F' + W(T).
\]

Analogously to the closed economy, the continuation value in a Markov equilibrium is given by

\[
V_{Mt}(S) = A_R + \omega_K K + A_N N - B_F + H_{Mt}(B, G),
\]

where the tax distortion function \( H_{Mt} \) satisfies the Bellman equation

\[
H_M(B, G) = \max_{B_B', B_f', T} W(T) - A_B B_B' + \beta H_M(B', G')
\]

subject to the government budget constraint \( G + B \leq T + \beta B' \) and the no-default constraint, which, given the form of \( V_{Mt}(S) \), can be simplified to

\[
(64) \quad H_M(B', G') - B_F' \geq H_M(0, G') - A_N B_B' - \bar{\kappa}(B_F')
\]

where \( A_B, A_N, \) and \( A_R \) are the same constants as in the closed economy problem. Given this continuation value, the value of a Markov equilibrium in period \( t \) is given by

\[
J_{Mt}(S) = \max \{ V_{Mt}(S), V_{Mt}(K, D, 0, 0, 0, G) - \bar{\kappa}(B_F) \}.
\]

Inspection of the no-default constraint (64) brings out a key difference between debt owed to households and debt owed to foreign lenders. When the government defaults on domestic households, it effectively replaces a distortionary tax with a lump-sum tax. Thus, the gains are only from eliminating a distortion. In contrast, when a government defaults on foreign lenders, it eliminates the need to transfer resources to such lenders. So the gain comes from saving resources. The costs of defaulting on foreign lenders are given by the reduction in private consumption \( \bar{\kappa}(B_F) \).
Now consider the best sustainable equilibrium. Closely following the logic used in the closed economy and that just developed for the Markov equilibrium for the open economy, the best sustainable equilibrium continuation value has the form

\[ V_{St}(S) = A_R + \omega_K K + A_N N - B_F + H_{St}(B, G), \]

where \( H_{St} \) is the largest fixed point of a Bellman equation defined by

\[ H_{St}(B, G) = \max_{B_0, B'} W(T) - A_B B' + \beta H_{St+1}(B', G') \]

subject to the government budget constraint \( G + B \leq T + \beta B' \) and the sustainability constraint

\[ H_{St+1}(B', G') + A_N B' - B'_F \geq H_{Mt+1}(0, G') - \bar{\kappa}(B'_F). \]

Finally, the best sustainable equilibrium has a value given by

\[ J_{S0}(S) = \max \{ V_{S0}(S), V_{S0}(K, D, 0, 0, 0, G) - \bar{\kappa}(B_F) \}. \]

Given this setup, we turn to the analysis of repression. We focus on an economy that has a deterministic cycle in government spending and in period 0 has \( G = G_H \) and no inherited debt. Clearly, this economy has a two-period cycle similar to that in the model without foreign lenders. To characterize this equilibrium, we define \( \chi^* \) to be the smallest maximal punishment imposed by foreign lenders such that the government is indifferent between repressing or not, and we let \( B^* = B(\chi^*) \) be the associated level of debt issued into the low spending state. These two objects are defined by the following two equations:

\[ \beta W'(G_H - \beta B^*) = \left( \beta + \frac{A_B}{A_N} \right) W'(G_L + B^*) \]

\[ \frac{W(G_L + B^*) + \beta W(G_H - \beta B^*)}{1 - \beta^2} - \frac{1}{1 + \kappa} \chi^* = W(G_L) - \chi^* + \beta H_{M}(0, G_H). \]

Note that if the government is just indifferent between repressing and not, then the sustainability constraint must have a positive multiplier and, hence, hold with equality as in (66). In this construction we are assuming that the discount factor is low enough such that \( \chi^* \) is positive. (Clearly, this value of \( \beta \) must be less than the \( \beta^* \) defined for the closed economy.)

We now show that the extent of financial repression depends on the willingness of foreign lenders to lend. To do so consider two economies \( i = 1, 2 \), which have cyclic spending,
the same initial conditions as before, and differ only in the level of the maximal punishment \( \chi_i \) with

\[
\chi_1 > \chi^* > \chi_2.
\] (67)

To determine how the maximal amount of debt that can be supported without repression changes with \( \chi \) we need to determine how the constraint (66) varies with \( \chi \). Since \( \kappa > 0 \), as \( \chi \) is increased the gains from defaulting on debt, \( B_F = \chi/(1 + \kappa) \), rise less than the maximal punishment \( \chi \) from doing so, this *direct effect* alone would lead higher punishments to be associated with higher debt. The subtlety is that as \( \chi \) is increased so is the value of the Markov equilibrium, \( H_M(0,G_H) \). This *indirect effect* captures that idea that the cost of deviating to the Markov equilibrium falls and so, by itself, would lead higher punishments to be associated with lower debt. In Lemma 8 in the Appendix we show that a sufficient condition for the direct effect to outweigh the indirect effect is

\[
\frac{\beta}{1 - \beta^2} \frac{A_B}{A_N} < 1.
\] (68)

Under (68), an increase in \( \chi \) allows more total debt \( B \) to be supported in an equilibrium without repression.

Clearly, in economy 1, foreign lenders lend more than in economy 2 and under (68) the total lending without repression is also higher. For such a cyclic economy we say that repression is *permanent* if each time debt is sold, financial repression is practiced. The following proposition is then immediate given our construction of \( \chi_1 \) and \( \chi_2 \).

**Proposition 6.** (Foreign Lending and Repression) Under (67) and (68), in economy 1 there is no repression and in economy 2 there is permanent repression.

Since in economy 2, the total lending without repression is lower than in economy 1, absent repression, the government is able to achieve less tax smoothing in economy 2 than in economy 1. Under (67), \( \chi_2 \) is sufficiently low so that the gains from tax smoothing outweigh the costs from repression and the government finds it optimal to practice repression. It is worth noting that if \( \chi_2 \) is lowered, to say \( \chi' < \chi_2 \) where \( \chi_2 \) satisfies (67), the amount of tax smoothing stays constant and the government just represses more.

Next we consider sudden stops in a simple stochastic version of this model. Specifically, suppose that \( \chi \) follows a Markov chain \((p_{ij})\) that stochastically fluctuates between two values
\( \chi_1 \) and \( \chi_2 \) that satisfy (67). Government spending follows the same deterministic cycle as before. The government can issue state-contingent debt.

The best sustainable equilibrium in this economy can be implemented as follows. As long as there is no sudden stop, the government practices no repression and perfectly smooths taxes. In each period with high government spending and no sudden stop, the government issues \( B_1 \) into the \( \chi_1 \) state and \( B_{12} \) into the \( \chi_2 \) state with \( B_1 > B_{12} \). This state-contingent debt can be interpreted as debt in which the government partially “defaults” at the onset of the sudden stop. As long as lenders continue to lend small amounts, the government practices financial repression. Once the sudden stop ceases, the government returns to a policy of no repression and smooth taxes.

**Proposition 7.** (Sudden Stops, Default and Repression) Suppose \( \chi_1 \) sufficiently high and \( \chi_2 \) close to \( \chi^* \) and state \( \chi_2 \) is sufficiently persistent. Then whenever foreign lending is high, there is no repression, and when there is a sudden stop, the government partially defaults on inherited debt and practices financial repression until the sudden stop ceases.

The patterns here are reminiscent of those that occurred in countries such as Argentina in the early 2000s. The sudden stop in foreign lending was accompanied by a partial default and then financial repression.

7. **Conclusion**

Financial repression has been widely practiced throughout history. In particular, financial repression has been more likely when government debt was high or when governments wanted to issue a lot of debt. In this paper, we investigate when, if ever, financial repression is optimal. We find that under commitment, financial repression is never optimal because it is at best a redundant instrument. If, however, a government cannot commit to its policies—particularly, if it cannot commit to repaying its debt—then financial repression may be optimal. In particular, when a positive amount of government debt can be sustained with standard reputational arguments, we find that financial repression is practiced only when government spending needs are high. This paper highlights a cost associated with recent proposals to discourage banks from holding domestic government debt. In light of our theory, such proposals may not be a good idea.
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Figure 1. Paths for Ramsey, best sustainable, and Markov outcomes.

Panel A. Value of Debt Issued

Panel B. Fraction of Debt Held by Banks
Appendix (Not for Publication)
On the Optimality of Financial Repression

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ABSTRACT

This document is the Appendix to the paper, On the Optimality of Financial Repression.
A. Omitted Proofs

To set up the proof of Lemma 1, consider the firm maximization problem, with the notation for the multipliers indicated next to the constraints to which they are attached.

\[
\max \sum_{t=0}^{\infty} Q_{0,t} \sigma^t [\sigma x_t + (1 - \sigma)(R_t k_t + \delta_t b_{Bt} - d_t)]
\]

subject to

\[
\begin{align*}
\lambda_t : x_t + (1 + \tau_t K_t)k_{t+1} + \sigma_{t+1} q_{Dt+1}b_{Bt+1} + d_t &\leq R_t k_t + \delta_t b_{Bt} + q_{Dt+1}d_{t+1} \\
\mu_{t+1} : d_{t+1} &\leq \gamma [R_{t+1}k_{t+1} + \delta_{t+1}b_{Bt+1}] \\
\theta_{t+1} : b_{Bt+1} &\geq \phi_t (R_{t+1}k_{t+1} + \delta_{t+1}b_{Bt+1}) \\
\eta_{xt} : x_t &\geq 0, \eta_{Bt+1} : b_{Bt+1} \geq 0
\end{align*}
\]

The first order conditions are

\[
\begin{align*}
x_t : Q_{0,t} \sigma^{t+1} + \eta_{xt} = \lambda_t \text{ so } \lambda_t &\geq Q_{0,t} \sigma^{t+1} \\
k_{t+1} : \lambda_t (1 + \tau_t K_t) = Q_{0,t+1} \sigma^t (1 - \sigma)R_{t+1} + R_{t+1} (\lambda_{t+1} + \gamma \mu_{t+1} - \phi_t \theta_{t+1}) \\
d_{t+1} : -Q_{0,t+1} \sigma^t (1 - \sigma) + \lambda_t q_{Dt+1} = \lambda_{t+1} + \mu_{t+1} \\
b_{Bt+1} : \lambda_t q_{Bt+1} &\leq Q_{0,t+1} \sigma^t (1 - \sigma)\delta_{t+1} + \theta_{t+1} + \delta_{t+1} [\lambda_{t+1} + \gamma \mu_{t+1} - \phi_t \theta_{t+1}]
\end{align*}
\]

(69) (70) (71)

together with the complementary slackness conditions on the inequality constraints.

Proof. We first show that banks weakly prefer not to pay dividends and strictly prefer not to do so when the collateral constraint is binding. We start by showing that if dividends are positive, \( x_t > 0 \), then the collateral constraint is slack, so that \( \mu_{t+1} = 0 \). Using \( Q_{0,t} \sigma^{t+1} = \lambda_t \) and \( Q_{0,t+1} \sigma^{t+2} \leq \lambda_{t+1} \) in (69) gives

\[
(71) -Q_{0,t+1} \sigma^{t+1} (1 - \sigma) + Q_{0,t} \sigma^{t+1} q_{Dt+1} \geq Q_{0,t+1} \sigma^{t+2} + \mu_{t+1}.
\]

Since \( Q_{0,t+1} = Q_{0,t} q_{Dt+1} \), then (71) implies that \( 0 \geq \mu_{t+1} / (Q_{0,t+1} \sigma^{t+1}) \) so \( \mu_{t+1} \leq 0 \). Since the multiplier is nonnegative, we have that \( \mu_{t+1} = 0 \).
That banks are indifferent between paying dividends at \( t \) when the collateral constraint is slack at \( t \) is immediate because households and banks discount payments at the same rate. In particular, if the banks pay dividends at \( t \) of \( \Delta x_t \), the utility to the households from these dividends is \( Q_{0,t} \sigma^t \Delta x_t \). If instead the firm pays zero dividends at \( t \), reduces the deposits it sells at \( t \) so that \( q_{Dt+1} \Delta d_{t+1} = \Delta x_t \), and then tomorrow increases dividends by \( \Delta x_t / q_{Dt+1} \), this variation increases the firm’s value by \( Q_{0,t+1} \sigma^t \Delta x_t / q_{Dt+1} = Q_{0,t} \sigma^t \Delta x_t \) so that the firm is indifferent between these two options.

We now show that if banks hold a positive amount of capital, then (13) holds, which is convenient to write as

\[
q_{Dt+1} \geq \frac{1 + \tau K_t}{R_{t+1}}. \tag{72}
\]

The first order conditions for deposits and capital can be rewritten as

\[
\lambda_t q_{Dt+1} = Q_{0,t+1} \sigma^t (1 - \sigma) + \lambda_{t+1} + \mu_{t+1} \tag{73}
\]

\[
\frac{\lambda_t (1 + \tau K_t)}{R_{t+1}} = Q_{0,t+1} \sigma^t (1 - \sigma) + (\lambda_{t+1} + \gamma \mu_{t+1} - \theta_{t+1} \phi_t). \tag{74}
\]

Subtracting these two gives

\[
\lambda_t \left[ q_{Dt+1} - \frac{1 + \tau K_t}{R_{t+1}} \right] = (1 - \gamma) \mu_{t+1} + \theta_{t+1} \phi_t \geq 0, \tag{75}
\]

which implies (72).

Now, suppose that (72) holds as a strict inequality, \( q_{Bt+1} = q_{Dt+1} \), and \( k_{t+1} > 0 \). We will show that both the regulatory constraint and the collateral constraint bind, in that \( \theta_{t+1} \) and \( \mu_{t+1} \) are both positive. To do so, subtract (74) from the first order condition (70) to get

\[
\lambda_t \left[ q_{Dt+1} - \frac{1 + \tau K_t}{R_{t+1}} \right] = \theta_{t+1}. \tag{76}
\]

Since \( \lambda_t > 0 \) and (72) holds as a strict inequality, \( \theta_{t+1} > 0 \), which implies that the regulatory constraint binds. Then, since the left sides of (75) and (76) are equal, so are the right sides. Since \( \phi_t < 1 \), it follows that \( \mu_{t+1} > 0 \). Moreover, we know that \( \phi_t < 1 \) because if \( \phi_t = 1 \), then \( k_{t+1} = 0 \), which contradicts that \( k_{t+1} > 0 \). Q.E.D.
Proof of Proposition 1. We first show how to choose $\tau'_{Kt}$ so that it raises the same amount of revenues as did the sum of the revenues from the possibly repressed bond price $q_{Bt+1}$ and the original tax on capital income. To that end, define $\tau'_{Kt}$ as follows:

\begin{equation}
\tau'_{Kt}K_{t+1} = \tau_{Kt}K_{t+1} + [q_{Bt+1} - q_{Dt+1}\delta_{t+1}]B_{Bt+1}
\end{equation}

so that if, at the original allocation, debt prices and the tax rate on capital, the bank budget constraint (19) and the government budget constraint (18) hold, then they also hold at the new unrepressed debt prices and the new tax rate on capital.

We will show that with the altered taxes on capital income and return on government debt, the banks optimally choose the same allocation as in the original equilibrium. To do so, notice that if the interest rate on debt is lower than that on deposits, then each bank holds the minimum amount of government debt required by the regulatory constraint. That is, $q_{Bt+1} > q_{Dt+1}\delta_{t+1}$ implies $B_{Bt+1} = \phi_t/(1 - \phi_t)R_{t+1}K_{t+1}$, hence (77) can be rewritten as

\begin{equation}
\tau'_{Kt} = \tau_{Kt} + [q_{Bt+1} - q_{Dt+1}] \frac{R_{t+1}\phi_{t+1}}{1 - \phi_{t+1}}.
\end{equation}

Since the portfolio regulation constraint (11) is binding, we can substitute it into the budget constraint (9) and the collateral constraint (10) to get

\begin{equation}
x_t + (1 + \tau_{Kt})k_{t+1} + \left[ q_{Bt+1} \frac{R_{t+1}\phi_{t+1}}{1 - \phi_{t+1}}k_{t+1} - q_{Dt+1}d_{t+1} \right] \leq n_t
\end{equation}

\begin{equation}
d_{t+1} \leq \gamma \left[ R_{t+1}k_{t+1} + \delta_{t+1} \frac{R_{t+1}\phi_{t+1}}{1 - \phi_{t+1}}k_{t+1} \right]
\end{equation}

We can then think of the bank maximization problem as choosing $k_{t+1}$ and $d_{t+1}$ subject to (79) and (80). But notice that the constraint set defined by these two inequalities is identical to that under the original policies. In particular, under the alternative tax system with $q_{Bt+1} = q_{Dt+1}$ and $\tau'_{Kt}$ given by (77) the budget constraint (9), the collateral constraint (10) and the regulatory constraint (11) holding with equality are equivalent to (79) and (80) so that banks optimally choose the same allocation as in the original equilibrium. Hence, the government can always implement an equilibrium outcome by setting $q'_{Bt+1} = q_{Dt+1}\delta_{t+1}$ and suitably choosing $\tau'_{Kt}$. Q.E.D.

Proof of Lemma 2. Since we already proven necessity in the text, here we prove sufficiency. To do so, consider an allocation and policies that satisfy the properties described
in the statement of the lemma. Let the wage and the rental rate of capital be defined by
\[ w_t = F_{Lt} \] and \[ R_t = F_{Kt} \] so that the firm optimality conditions are satisfied. Use (5) to define \( \tau_{Lt} \) so that the household’s labor first order condition is satisfied. Let \( \phi_t \) be chosen to satisfy (11) with equality.

We are left to show that the bank’s first order conditions are satisfied. If (21) holds with strict inequality, then, as we have shown in Lemma 1, the collateral constraint and the regulatory constraint are binding. Hence, the optimal bank policy is determined by both its collateral constraint and its regulatory constraint holding with equality. Such constraints are met with equality for the proposed allocation since the budget constraint (19) is an equality, the collateral constraint (20) holds with equality when (21) is a strict inequality, and we defined \( \phi_t \) so that the regulatory constraint holds with equality as well. If, instead, the capital distortion constraint (21) is an equality, then the bank is indifferent between all the feasible policies at time \( t \), and so the proposed allocation is trivially optimal. Q.E.D.

**Proof of Proposition 2.** Suppose by way of contradiction that the Ramsey outcome has financial repression in that the government forces the bank to hold some government debt. Specifically, suppose that \( \phi_t > 0 \) so that \( B_{Bt+1} > 0 \). Then consider the following variation on the original allocations and policies. Reduce the amount of deposits the bank obtains from the household at \( t \) by exactly the amount of government debt it holds. That is, set \( \tilde{D}_{t+1} = D_{t+1} - B_{Bt+1} \). Let the households increase their holdings of government debt by \( B_{Bt+1} \) and let the government not require that the bank holds any debt by setting \( \tilde{\phi}_t = 0 \). Let the rest of the allocations and policies be unchanged, in particular \( \{C_t, L_t, K_{t+1}\} \) and \( \{\tau_{Lt}, \tau_{Kt}\} \).

We claim that this variation is feasible, relaxes the collateral constraint, and supports the same allocations. By inspection this variation leaves the constraints on the Ramsey problem (18), (19), and (21) unaffected and relaxes (20). Thus, this variation always weakly raises welfare and does so strictly if the collateral constraint (20) is binding. Q.E.D.

**Result 1.** If \( v(L) = L^{1+\eta}/(1 + \eta) \) for \( \eta > 0 \), then \( W'(0) = 0 \) and \( W'' < 0 \).

**Proof.** To show that \( W'(0) = 0 \) and \( W'' < 0 \), differentiate \( W \) to get that
\[
W'(T) = \left[ \omega - v'(\ell(T)) \right] \ell'(T),
\]
and from $[\omega - v'(\ell(T))] \ell(T) = T$ we have

$$\ell'(T) = -\frac{1}{v'/(\ell(T)) - \omega + v''/(\ell(T))\ell(T)} < 0,$$

which is negative for all $G \in [0, G_{\text{max}}]$ since we have chosen $\ell$ on the side on the Laffer curve in which labor tax revenues are increasing in the rates. Here $G_{\text{max}}$ is the top of the static Laffer curve. Since labor supply is decreasing in the tax rate, this in turn implies $\ell'(T) < 0$ and so $v'(\ell(T)) - \omega + v''/(\ell(T))\ell(T) > 0$.

Using the expression (82) to substitute for $\ell'$ into (81) gives

$$W'(T) = -\frac{\omega - v'(\ell(T))}{v'/(\ell(T)) + v''/(\ell(T))\ell(T) - \omega}.$$

It is then immediate that $W'(0) = 0$. We can further rewrite (83) using the fact that $[\omega - v'(\ell(T))] = T/\ell(T)$ as

$$W'(T) = -\left[\frac{T/\ell(T)}{-T/\ell(T) + v''/(\ell(T))\ell(T)}\right] = \frac{T/\ell(T)}{T/\ell(T) - \eta v'(\ell(T))},$$

where the second equality follows from our functional form assumption. Note that $W'$ is decreasing since the numerator, $T/\ell(T)$, is positive and increasing and the denominator, $T/\ell(T) - \eta v'(\ell(T)) = -v'(\ell(T)) + \omega - v''/(\ell(T))\ell(T)$, is increasing and negative as argued above.\footnote{Here}

Note that an alternative sufficient condition for this lemma is that $v$ is convex and $v''$ is increasing.

**Proof of Lemma 3.** Consider the recursive representation of the Ramsey problem

$$V_{Rt}(S) = \max_{B', T_K, T_L} W(T_L) + \omega K - K' + \beta V_{Rt}(S')$$

\footnote{Here}

$$W''(T_L) = \left[\frac{\partial \text{NUM}(T_L)}{\partial T_L} \frac{\text{DEN}(T_L)}{\text{DEN}(T_L)^2} - \frac{\partial \text{DEN}(T_L)}{\partial T_L} \frac{\text{NUM}(T_L)}{\text{DEN}(T_L)^2}\right],$$

where $\text{NUM}(T_L) = T_L/\ell(T_L) \geq 0$ and $\text{DEN}(T_L) = T_L/\ell(T_L) - \varphi v'(\ell(T_L)) < 0$ and

$$\frac{\partial \text{NUM}(T_L)}{\partial T_L} = \frac{\ell(T_L) - T_L \ell'(T_L)}{\ell(T_L)^2}, \quad \frac{\partial \text{DEN}(T_L)}{\partial T_L} = \frac{\ell(T_L) - T_L \ell'(T_L)}{\ell(T_L)^2} + \varphi v''/(\ell(T_L))\ell'(T_L),$$

where both of these expressions are positive because $\ell' < 0$ and $v'' > 0$. 
subject to the government budget constraint

(85) \( T_L + T_K + \beta B' = G + B. \)

(86) \( (1 - \beta \gamma \omega_K)K' + T_K = \sigma N + (1 - \sigma)\bar{n} \)

where we have substituted in the aggregate bank budget constraint that the collateral constraint binds and that zero debt is held by banks so that \( D' = \gamma \omega_K K' \) and where \( N = \omega_K K + B_B - D \) is the aggregate net worth of continuing banks. Substituting the guess for the value function

\[
V_{Rt}(S) = A_R + \omega_K K + A_N (\omega_K K + B_B - D) + H_{Rt}(B_B + B_H, G)
\]

into the right side of (84) we obtain that \( V_{Rt}(S) \) is the maximum over \( B' \) and \( T_K \) of

(87) \( W (T) + \omega_K K - K' + \beta [A_R + \omega_K K' + A_N (\omega_K K' + B_B - D') + H_{Rt+1}(B', G)] \),

where \( T \) is shorthand notation for \( G + B - \beta B', D' = \gamma \omega_K K' \) and

\[
K' = \frac{\sigma N - T_K + (1 - \sigma)\bar{n}}{(1 - \gamma \beta \omega_K)}.
\]

Substituting for \( K' \) and \( D' \) into (87) we obtain that this value equals

\[
W (T) + \omega_K K - \frac{\sigma N - T_K + (1 - \sigma)\bar{n}}{(1 - \gamma \beta \omega_K)} + \beta \left[ A_R + \omega_K \frac{\sigma N - T_K + (1 - \sigma)\bar{n}}{(1 - \gamma \beta \omega_K)} + A_N \omega_K (1 - \gamma) \frac{\sigma N - T_K + (1 - \sigma)\bar{n}}{(1 - \gamma \beta \omega_K)} + H_{Rt+1}(B', G') \right]
\]

or, after rearranging terms, equals

\[
W (T) + \beta H_{Rt+1}(B', G') - \frac{[\beta A_N \omega_K (1 - \gamma) + \beta \omega_K - 1]}{1 - \gamma \beta \omega_K} T_K
\]

\[
+ \omega_K K + \frac{\sigma [\beta A_N \omega_K (1 - \gamma) + \beta \omega_K - 1]}{1 - \gamma \beta \omega_K} N
\]

\[
+ \beta A_R + (1 - \sigma)\bar{n} \frac{[\beta \omega_K + \beta A_N \omega_K (1 - \gamma) - 1]}{1 - \gamma \beta \omega_K}.
\]

Then the guess is verified if

\[
A_N = \frac{\sigma [\beta A_N \omega_K (1 - \gamma) + \beta \omega_K - 1]}{1 - \gamma \beta \omega_K} = \frac{(\beta \omega_K - 1)\sigma}{1 - \beta \omega_K [\sigma + (1 - \sigma)\gamma]}
\]
\[ A_R = \beta A_R + \frac{\beta \omega_K + \beta A_N \omega_K(1 - \gamma) - 1}{1 - \gamma \beta \omega_K} = \frac{(1 - \sigma)\bar{n}}{(1 - \beta)\sigma} A_N \]

and the function \( H_{Rt} \) solves the functional equation in (32). \( Q.E.D. \)

**Sufficient Conditions for Interior Solutions and a Binding Collateral Constraint.** Our construction of the solution to the Ramsey problem presumed that when the collateral constraint is always binding, capital does not diverge to infinity and that consumption is positive. To provide sufficient conditions for these two requirements, consider the dynamic system governing the evolution of capital and net worth under assumptions that the collateral constraint binds, the tax on investment is zero, and no debt is held by banks. Under these conditions,

(88) \[ K' = \frac{\sigma(1 - \gamma)\omega_K}{1 - \gamma \beta \omega_K} K + \frac{(1 - \sigma)\bar{n}}{1 - \gamma \beta \omega_K} \]

and \( N = (1 - \gamma)\omega_K K \). A sufficient condition for total consumption to be positive is that the capital component of consumption, \( C_{Kt} = \omega_K K_t - K_{t+1} \), is positive. For the capital stock to remain bounded and for the capital component of consumption to be nonnegative, we impose the following two conditions. This first is that

(89) \[ \frac{\sigma(1 - \gamma)\omega_K}{1 - \gamma \beta \omega_K} < 1 \]

so that \( K' \) does not diverge to infinity and, indeed, has a steady state, given by

(90) \[ K_{ss} = \frac{(1 - \sigma)\bar{n}}{(1 - \gamma \beta \omega_K) - \sigma(1 - \gamma)\omega_K}. \]

The second is that the initial capital stock \( K_0 \geq K_{ss}/\omega_K \). Under this assumption on initial capital, the capital component of consumption is nonnegative for all \( t \). We also find it intuitive to assume that the capital stock starts below its steady state value. Under these assumptions, the capital stock, net worth, and consumption all monotonically rise along their transition to their steady state values.

**Proof of Lemma 4.** Suppose, by way of contradiction, that the value of the debt is positive but there is no financial repression. Suppose first that the collateral constraint binds. Then from our earlier results we know that banks hold no debt so that the debt must be held by households. But, since defaulting on debt held by households allows the government to lower distortionary labor taxes and has no cost, the government will default on such debt. Since households anticipate this default, the price of debt will be zero.
Suppose next that the collateral constraint is slack but some combination of banks and households hold debt. Then again, since there is no cost of default, the government will default. Hence, the price of debt will again be zero. \textit{Q.E.D.}

\textit{Proof of Lemma 5.} First, we show that (47) has a solution. Let $T$ be the operator defined by the right side of (47). Let $X$ be the space of continuous, bounded, and concave functions $h$ defined over a compact subset of $(B, G)$. Note that this operator maps this space into itself, it is continuous, and that the family $T(X)$ is equicontinuous. To see the last property, note that for all $h$ in $X$ and for all $B_2 > B_1$, we have that

$$
\| (Th)(B_2) - (Th)(B_1) \| \leq \| (Th)(B_1) - \frac{A_N}{\sigma}(B_2 - B_1) - (Th)(B_1) \| = \frac{A_N}{\sigma}\|B_2 - B_1\|$

since a feasible solution at $B_2$ is to repay the additional debt by choosing the policies that are optimal for $B_1$ and just increasing capital taxes by $B_2 - B_1$. Then every $Th$ in $T(X)$ is Lipschitz with Lipschitz constant $A_N/\sigma$ common to all elements of $T(X)$. We can then apply a version of the Schauder fixed point theorem (Theorem 17.4 in SLP) to conclude that $T$ has a fixed point.

Next, substituting this fixed point $H_M$ into the original problem, we can verify the conjecture and calculate the constants in a similar fashion to that in Lemma 3. \textit{Q.E.D.}

\textit{Proof of Lemma 6.} Consider the problem

$$
V_{St}(S) = \max_{B^*, K, T} W(T) + \omega_K K - K' + \beta V_{St}(S')
$$

subject to the government budget constraint, the bank budget constraint, and the sustainability constraint (51). The right side of (91) defines an operator, $T$. Given the presence of the constraint (51), this operator $T$ is not a contraction mapping. Nevertheless, using a logic similar to Abreu, Pearce, and Stacchetti (1990), if we start with an initial value function $V_0(S, G)$ which is pointwise larger than $V(S, G)$, we can construct a sequence of functions $V_n = T^nV_0$ that converges to $V$ in the sup norm. Recall from our earlier discussion that the Ramsey problem has the form (31)

$$V_R = \omega_K K + A_R + A_N N + H_R(B, G).$$
Since the Ramsey problem has constant taxes, we can simplify (32) to be

\[ H_R(B, G) = \frac{1}{1 - \beta} W \left( (1 - \beta)B + \frac{G + \beta G'}{1 + \beta} \right) \]

so that \( H_R(B, G) \) is a concave function of \( B \). Clearly, since the Ramsey problem solves a less constrained version of the best sustainable problem, \( V_R \) is pointwise larger than \( V \) and we can use \( V_R \) as the initial value function \( V_0 \). We need to show that the sequence of constructed functions \( V_n \) have the form \( V_n = \omega K K + A_R + A_N N + H_n (B, G) \) for some sequence of concave functions \( H_n (B, G) \). To do so, substitute our guess for \( V_n \) in (91) and inspect the resulting problem to conclude that the optimal \( B'_B, B'_H, \) and \( T_K \) are independent of \( K, D \) and the optimal \( K' \) is linear in \( \sigma N + (1 - \sigma)\bar{n} \). Using these two properties, we have that

\[ (92) \quad TV_n (S, G) = \omega K K + A_R + A_N N + H_{n+1} (B, G), \]

where nonnegative \( B'_B, B'_H, \) and \( T_K \) are chosen to solve

\[ H_{n+1} (B, G) = \max_{B'_B, B'_H, T_K} W (T) - \frac{A_N}{\sigma} T_K - A_B B'_B + H_n (B'_B + B'_H, G'), \]

subject to

\[ T = G_t + B - T_K - \beta (B'_B + B'_H) \]

\[ A_N B'_B + H_n (B, G') \geq H_M (0, G'). \]

Concavity of \( H_{n+1} \) follows from concavity of \( H_n \) and \( W \). The result in (92) implies that in the limit,

\[ V (K, N, B, G) = \omega K K + A_R + A_N N + H (B, G), \]

where \( H, \) defined as the limit of \( H_n, \) is also concave. Q.E.D.

**Proof of Proposition 4.** We first show that eventually there is some \( T \) such that there is no financial repression for \( t \geq T \). We do so by showing that eventually the multiplier on the sustainability constraint \( \mu_t < A_B/A_N \).

There are two cases. In the first case, the debt levels \( \{B_t\} \) are bounded away from zero for all \( t \). Since the multiplier on the sustainability constraint \( \mu_t \geq 0 \) for all \( t \), from the first order condition (52) it follows that the tax revenues \( \{T_t\} \) are decreasing. Such a path for tax
revenues implies that debt is decreasing over each cycle. To see why, iterate the government budget constraint forward to write that debt is the discounted value of future government surpluses, which using the cyclical pattern of government spending, gives

\[ B_t = \sum_{s=0}^{\infty} \beta^s T_{t+s} - \frac{G_t + \beta G_{t+1}}{1 - \beta^2}. \]

Next, since the sequence of tax revenues \( \{T_t\} \) is decreasing and bounded below by zero, these tax revenues must converge to a positive level so that taxes are perfectly smoothed in the limit. From (52) it then follows that the multiplier \( \{\mu_t\} \) must converge to zero. Thus, there is a finite \( T(\beta, G_0) \) such that for \( t \geq T(\beta, G_0) \), the multiplier \( \mu_t < A_B/A_N \) so there is no financial repression.

In the second case, there is some finite \( t \) such that \( B_t = 0 \). Consider the interesting case in which the debt level inherited in the high spending state is zero. Then since \( \beta > \beta^* \), the Ramsey plan after this date is sustainable and thus there is no financial repression. A similar argument holds in the uninteresting case in which the inherited debt in the low spending case is zero.

We turn to showing that debt is decreasing over each cycle in that \( B_{t+2} \leq B_t \). To do so, we first show a preliminary result that if \( G_t = G_L \) and \( B_t \geq B_H(\beta) \), then \( B_{t+1} \geq B_L(\beta) \).

Suppose by way of contradiction that \( B_t \geq B_H(\beta) \) but \( B_{t+1} < B_L(\beta) \). By construction, \( B_H(\beta) \) and \( B_L(\beta) \) constitute Ramsey outcomes so that \( H'_S(B_H(\beta), G_L) = H'_S(B_L(\beta), G_H) \) and by concavity of \( H_S \) we have that

\[ H'_S(B_t, G_L) < H'_S(B_H(\beta), G_L) = H'_S(B_L(\beta), G_H) < H'_S(B_{t+1}, G_H). \]

But this leads to a contradiction, since (52) together with the envelope condition implies that \( H'_S(B_t, G_L) > H'_S(B_{t+1}, G_H) \). Thus, \( B_{t+1} \geq B_L(\beta) \). A similar argument implies that if \( G_t = G_H \) and \( B_t \geq B_L(\beta) \), then \( B_{t+1} \geq B_H(\beta) \). Thus, debt levels are strictly positive in all periods.

Using this result, we can now show that \( B_{t+2} \leq B_t \). To do so, combine the first order conditions for debt for period \( t \) and \( t+1 \) and use \( G_t = G_{t+2} \) to get

\[-\beta H'_S(B_t, G_t) = -\frac{1}{\beta} (\beta + \mu_t) (\beta + \mu_{t+1}) H'_S(B_{t+2}, G_t).\]

Since \( H_S \) is strictly concave in \( B \), it follows that \( B_{t+2} \leq B_t \).
We now show that financial repression, measured as the fraction of debt held by banks, is increasing in the level of the debt. Hence, since the debt sequences are decreasing over the cycle, the fraction of debt held by banks must also be decreasing over the cycle. From the sustainability constraint, the share of debt held by banks

\[ r_t = \frac{B_t}{B_t} \max \left\{ \frac{H_M(0, G_t) - H_S(B_t, G_t)}{A_N B_t}, 0 \right\}. \]

Consider the interesting case in which \( r_t \) is strictly positive. Differentiating the expression for \( r_t \) with respect to \( B_t \), we have that

\[
\frac{\partial r_t}{\partial B_t} = -\frac{[H_M(0, G_t) - H_S(B_t, G_t)] - H_S'(B_t, G_t)B_t}{B_t^2} = \frac{[H_S(0, G_t) - H_M(0, G_t)] + [H_S(B_t, G_t) - H_S(0, G_t)] - H_S'(B_t, G_t)B_t}{B_t^2}.
\]

Clearly, \( H_S(0, G_t) > H_M(0, G_t) \). Concavity of \( H_S \) implies that \( [H_S(B_t, G_t) - H_S(0, G_t)] - H_S'(B_t, G_t)B_t > 0 \). The result then follows because this proves that the fraction of debt held by banks is in the level of total debt. \( Q.E.D. \)

**Result 2.** In the cyclic economy in which repression is prohibited, the level of debt from period one onward follows a two-period cycle.

Consider the following outcomes. The debt level sold into period 1, \( B_1 \), is set so that the sustainability constraint in period one holds with equality, that is,

\[ H_S(B_1, G_L) = H_M(0, G_L). \]

The outcomes in all subsequent periods have debt sold into low spending states equal to \( B_1 \) and satisfy perfect tax smoothing in the sense that either taxes are constant in all periods or zero debt is sold into the high spending states. Since the sustainability constraint is relevant only for low spending states and since the debt level in all low spending states is \( B_1 \), this outcome satisfies the sustainability constraint in all periods. To show that this outcome is optimal, note that the maximal amount of debt that can be sold into period 1 is \( B_1 \). Selling less debt than \( B_1 \) gives less tax smoothing and is not optimal. Selling less debt than \( B_1 \) in any subsequent low spending states also leads to less tax smoothing than the candidate allocation and therefore is not optimal. \( Q.E.D. \)
B. Markov Equilibria

Here we characterize the dynamics of the Markov equilibrium outcome.

We first derive sufficient conditions under which it is optimal to practice financial repression. Note that absent financial repression, no debt can be issued by the government in the Markov equilibrium and so $B = 0$. Further notice that the no-default constraint must hold an equality in any Markov equilibrium. Then we use the no-default constraint to substitute for $B$ in the objective function and use the envelope condition $H'_M(B, G) = W'(T_L)$ so that the first order condition for an interior $B_{t+1}$ is

\begin{equation}
-\beta W'(T_L) + (\beta + A_B/A_N) W'(T_{Lt+1}) = 0.
\end{equation}

We then define the cutoff $G^*_M$ as the minimal initial level of government expenditure such that the government is indifferent between practicing financial repression and not, namely,

\begin{equation}
-\beta W'(G^*_M) + (\beta + A_B/A_N) W'(G_L) = 0.
\end{equation}

The following proposition follows immediately.

**Proposition 3M.** In the Markov equilibrium, there is a critical value $G^*_M$ such that if $G_0 \leq G^*_M$, there is no financial repression in period 0 and $B_{B1} = B = 0$, if $G_0 > G^*_M$ then there is financial repression in period 0, $B_{B1} > 0$ and $B_{H1} > B^*_L$.

We now characterize the dynamics of taxes, debt, and the extent of financial repression over time. We consider the case in which the initial $G_0$ is relatively large. In particular, let $G_0 > G_H$ and $G_0 > G^*_M$. We then have the following proposition:

**Proposition 4M.** Government debt is decreasing over each cycle, and the fraction of government debt held by banks decreases over the cycle. Moreover, in the limit the economy converges to a two-period cycle economy in which no government debt is issued in the high spending state. Financial repression persists in the long run if and only if $G_H > G^*_M$.

**Proof.** We start by showing that there is some period $t$ in which $B_t = 0$ and then show that this implies that the economy converges to a two-period cycle. To do so, suppose by way of contradiction that $B_t > 0$ for all $t$. Then, (95) holds for all $t$ and concavity of $W$ implies that $\{T_{Lt}\}$ is decreasing. Then $\{T_{Lt}\}$ must be converging to a limit. Clearly, the limit cannot be strictly positive, otherwise by continuity of $W'$, we have that at the limiting value
\(T_{L,\infty}, -W'(T_{L,\infty}) > -W'(T_{L,\infty})\), which contradicts (95). Then it must be that \(T_{Lt}\) for some \(t\) sufficiently large, \(T_{Lt}\) is arbitrarily close to zero, so that \(\sum_{s=0}^{\infty} \beta^t T_{Lt+s}\) is also arbitrarily close to 0. But then (93) implies that for sufficiently large \(t\), \(B_t = \varepsilon - \frac{G_t + \beta G_{t+1}}{1 - \beta^t} < 0\) where \(\varepsilon\) is some small value that contradicts the hypothesis that \(B_t > 0\) for all \(t\). Then it must be that there exists some finite \(t\) such that \(B_t = 0\).

To see that the debt and, hence, the rest of the economy converges to a two-period cycle once \(B_t = 0\), note that if \(G_t = G_H\) and the inherited debt at \(t\), namely \(B_t = 0\), then at \(t+1\) with \(G_t = 0\) it is optimal to issue \(B_{t+2} = 0\). Suppose by way of contradiction that \(B_{t+2} > 0\); then using the envelope condition at \(t+1\) and the first order condition (95) at \(t+1\) gives that

\[H'_M(B_{t+1}, G_L) = W'(B_{t+1} - \beta B_{t+2}) = \left(1 + \frac{A_B}{A_N \beta} \right) H'_M(B_{t+2}, G_H) < \left(1 + \frac{A_B}{A_N \beta} \right) H'_M(0, G_H),\]

where the inequality follows from the concavity of \(H_M\). Likewise, the envelope condition at \(t\) and the first order condition at \(t\) imply that

\[H'_M(0, G_H) = W'(G_H - \beta B_{t+1}) = \left(1 + \frac{A_B}{A_N \beta} \right) H'_M(0, G_H) .\]

Combining these two equations gives that

\[H'_M(0, G_H) < \left(1 + \frac{A_B}{A_N \beta} \right)^2 H'_M(0, G_H),\]

which is a contradiction since \(1 + A_B/(A_N \beta) > 1\). Thus, eventually the economy follows a two-period cycle in the long run. It is also easy show that in the two-period cycle, the government issues a positive amount of debt in the low state if and only if \(G_H > G'^*_M\).

We then turn to characterizing the transition to the limiting cycle. Using an argument similar to the one in Proposition 4 for the best sustainable equilibrium, we can show that debt is decreasing over each cycle in that \(B_{t+2} \leq B_t\). This follows from the fact that when \(B_t\) is interior, (95) implies that taxes and debt are decreasing.

We now show that \(\{B_{B2t}/B_{2t}\}\) and \(\{B_{B2t+1}/B_{2t+1}\}\) are also decreasing as long as both \(B_{Bt+1}\) and \(B_{t+1}\) are both positive. Since total debt \(\{B_{2t}\}\) is decreasing, the no-default constraint

\[B_{Bt} = \frac{H_M(0, G_t) - H_M(B_t, G_t)}{A_N}\]
implies that the debt held in the banks \( \{ B_{2t} \} \) is also decreasing since \( H_M \) is decreasing in \( B_t \). Moreover, the share of debt held by banks \( r_t = B_{B_t}/B_t \) is such that \( \{ r_{2t} \} \) and \( \{ r_{2t+1} \} \) are decreasing since
\[
 r_t = \frac{B_{B_t}}{B_t} = \frac{H_M(0, G_t) - H_M(B_t, G_t)}{A_N B_t}
\]
is increasing in \( B_t \) and \( \{ B_{2t} \} \) is decreasing over time. That \( r_t \) is increasing in \( B_t \) immediately follows from the concavity of \( H_M \). To see this recall that for any concave function \( f(x) \), the function \( (f(x) - f(0))/x \) is decreasing in \( x \). Q.E.D.

\textbf{C. Discriminatory Default}

Here we assume that \( q_B = \beta \). Allowing the government to set \( q_B < \beta \) and, thereby, offer banks higher interest rates than households receive on deposits allows the government to effectively subsidize capital accumulation and thereby indirectly circumvent the collateral constraint. Allowing for such policies gives the government instruments to undermine the financial frictions that are at the heart of our analysis. This assumption is similar in spirit to our assumption that \( \tau_{K_t} \geq 0 \).

\textbf{A. Markov outcomes with Discriminatory Default}

First consider the case of a Markov equilibrium. The basic idea here is the same as when the government cannot discriminate. The key difference is that with nondiscriminatory default the government could “lever up”: by forcing the banks to hold a relatively small amount of debt, it could make it credible that it would not default on a relatively larger amount of household debt. Now there is no such levering up: the government forces that bank to hold government debt, the private agents hold no government debt, and this arrangement is optimal if the gains from tax smoothing are sufficiently large.

With discrimination, denoted by a superscript \( d \), the value of the continuation of a Markov equilibrium is given by
\[
 V_{Mt}^d(S) = A_R + \omega_K K + A_N N + H_{Mt}^d(B, G),
\]
where the tax distortion function \( H_{Mt}^d \) satisfies the Bellman equation
\[
 H_{Mt}^d(B, G) = \max_{B'B',T} W(T) - A_B B'_B + \beta H_{Mt+1}^d(B', G')
\]
subject to the government budget constraint

\[ T + \beta(B'_B + B'_H) = G + B \]

and the no-default constraint, which can be simplified to be

\[ H^d_{Mt+1}(B', G') \geq H^d_{Mt+1}(0, G') - A_NB'_B, \]

and where \( A_B, A_N, \) and \( A_R \) are the same as those that enter the Ramsey value (31).

It is immediate to show, along the lines of Lemma 5, the following lemma.

**Lemma 7.** A Markov equilibrium with discrimination at \( t \) exists and satisfies

\[ J^d_{Mt}(S) = \max \{ V^d_{Mt}(K, D, B_B, 0, G), V^d_{Mt}(K, D, 0, 0, G) \}, \]

where \( V^d_{Mt} \) is given by (97).

**B. Best Sustainable Outcomes with Discriminatory Default**

With discriminatory default, the continuation values for the best sustainable equilibrium with reversion to the Markov equilibrium can be expressed as

\[ V^d_{St}(S) = \omega_K K + A_R + A_N N + H^d_{St}(B, G) \]

where \( H^d_{St} \) is the largest fixed point of a Bellman equation defined by

\[ H^d_{St}(B, G) = \max_{B_B', B'_T} W(T) - A_B B'_B + \beta H^d_{St+1}(B', G') \]

subject to

\[ G + B \leq T + \beta B', \]

\[ H^d_{St+1}(B', G') \geq \max \{ H^d_{Mt+1}(0, G') - A_NB'_B, H^d_{Mt+1}(B'_B, G') \}. \]

Note that the first term on the right side of (101) is the value to the government if it defaults on both household debt and bank debt, while the second term is the value to the government if it defaults only on household debt.

**Proof of Proposition 5.** The first order conditions for (100) imply

\[ \beta W'(T) = (\beta + \mu) H^d_{St+1}(B', G'). \]
and the envelope theorem gives that $H'(B,G) = W'(T)$. Combining these equations gives that for all $t$,

$$-\beta W'(T_t) = -(\beta + \mu_t) W'(T_{t+1}).$$

Notice the similarity between (102) and (52). Then we can apply the same logic as in Proposition 4 to prove this proposition.

**D. Extension to Stochastic Government Spending**

We assume that government spending $G_t(s_t)$ is stochastic. The probability of a history $s^t = (s_0; s_1, \ldots, s_t)$ is $\pi(s^t|s_0)$.

An individual bank’s budget constraint is now given by

$$x_t(s^t) + (1 + \tau_K(t)k_{t+1}(s^t) + \sum_{s^{t+1}} [q_{Bt+1}(s^{t+1})b_{Bt+1}(s^{t+1}) - q_{Dt+1}(s^{t+1})d_{t+1}(s^{t+1})]$$

$$\leq R_t(s^t)k_t(s^{t-1}) + \delta_t(s^t)b_{Bt}(s^t) - d_t(s^t),$$

where $b_{Bt+1}(s^{t+1})$ and $d_{t+1}(s^{t+1})$ are state-contingent government debt and deposits purchased at state $s^t$ at prices $q_{Bt+1}(s^{t+1})$ and $q_{Dt+1}(s^{t+1})$. For each $s^{t+1}$ the collateral constraint is

$$d_{t+1}(s^{t+1}) \leq \gamma [R_{t+1}(s^{t+1})k_{t+1}(s^{t}) + \delta_{t+1}(s^{t+1})b_{Bt+1}(s^{t+1})]$$

and the regulatory constraint is

$$b_{Bt+1}(s^{t+1}) \geq \phi_t(s^{t+1}) [R_{t+1}(s^{t+1})k_{t+1}(s^{t}) + b_{Bt+1}(s^{t+1})].$$

The household problem is extended in a similar fashion.

The following extensions of our earlier propositions are immediate.

**Proposition 2’. (Financial Repression Not Optimal with Commitment)** The Ramsey outcome can be implemented with no financial repression, that is, $\phi_t(s^{t+1}) = 0$ for all $s^{t+1}$. Moreover, if the aggregate collateral constraint (20) is binding at $s^t$, then it is strictly optimal not to practice financial repression at that state, in that $\phi_t(s^{t+1}) = 0$ and $B_{Bt+1}(s^{t+1}) = 0$.

Here we specialize the stochastic process as follows. The initial state $s_0$ is drawn from a distribution $\pi_0(s_0)$, and the subsequent states follow a stochastic process with probability $\pi_t(s_1, \ldots, s_t)$. This captures the idea that the initial period is one with exceptional fiscal needs.
such as wartime or during a depression. After that initial period, the economy returns to normal times and the fiscal needs during those times are independent of those during the exceptional time at date zero.

**Proposition 3’.** In the best sustainable equilibrium, for any given $\beta$, there is a critical value $G^* = G^*(\beta)$ such that if $G_0 \leq G^*$, there is no financial repression and $B_{B1}(s^1) = 0$ for all $s^1 = (s_0, s_1)$, and if $G_0 > G^*$, then there is financial repression and $B_{B1}(s^1) > 0$ for at least one state.

We further specialize our stochastic process so that $\pi_t(s_1, ..., s_t)$ can be represented by a Markov chain $\pi(s_1|s)$ where the initial state $s_1$ is drawn from $\pi_1(s_1)$.

**Proposition 4’.** There is a critical value $\beta^*$ such that for $\beta \geq \beta^*$ there is a finite period $T = T(\beta, G_0)$ such that there is no financial repression for $t \geq T$ for all histories $s^T$. Furthermore, if $G_0 > G^*(\beta)$, then both the value of the government debt and the fraction of government debt held by banks decrease over the cycle. Moreover, in the limit the economy converges to a limiting cycle with perfect tax smoothing.

**Proof of Proposition 3’.** First write the programming problem as $H$:

$$H_{S0}(0, G_0) = \max_{B_{B1}} W(T) - A_B \sum_{G'} \pi_1(G') B_B'(G') + \beta \sum_{G'} \pi_1(G') H_S(B', G')$$

subject to

$$\mu_1(G') : H_S(B'(G'), G') \geq H_M(0, G') - A_N B_B'(G')$$

and the nonnegativity constraints on $B'(G')$ and $B_B'(G')$ where $T = G_0 - \beta \sum_{G'} \pi_1(G') B'(G')$.

The first order conditions for each $B'(G')$ and $B_B'(G')$ are

$$(103) \quad -\beta W'(T) + (\beta + \mu_1(G')) H_S'(B'(G'), G') \leq 0$$

$$(104) \quad -A_B + \mu_1(G') A_N \leq 0,$$

where (103) and (104) hold with equality if $B(G')$ and $B_{B1}(G')$ are positive, respectively. Let $G^*$ be the critical value of $G_0$ at which the government is just indifferent between practicing financial repression in at least one state and not. To define this value, note that the maximal amount of debt that the government can issue in period 0 without practicing financial repression is $B_1^* = \sum_{G'} \pi_1(G') B_1^*(G')$ where $B_1^*(G')$ solves

$$(105) \quad H_S(B_1^*(G'), G') = H_M(0, G').$$
At this level of debt, the first order conditions (103) and (104) can be combined to yield

\[-\beta W'(G^* - \beta B_1^*) + \left( \beta + \frac{A_B}{A_N} \right) H_S'(B_1^* (G'), G') \leq 0.\]

Moreover, if the equation above holds with equality then $B_{B1} > 0$. We can define the lowest $G^*$ such that there is repression as the $G^*$ that satisfies

\[-\beta W'(G^* - \beta B_1^*) + \left( \beta + \frac{A_B}{A_N} \right) \min_{G'} \{ H_S'(B_1^* (G'), G') \} = 0.\]

Clearly, the equilibrium has financial repression in period 0 if and only if $G_0 > G^*$. Q.E.D.

**Proof of Proposition 4’**. Here we characterize the transition after period 0. We begin by characterizing the limiting cycle of debt and taxes. To do so, we first consider what limiting cycles are sustainable without any financial repression. To this end, define $\beta^*$ as the smallest discount factor such that a stationary Ramsey outcome with positive debt in all states is sustainable. Let

\[\hat{G} (s) = G (s) + \beta \sum_{s'} \pi (s'|s) G (s')\]

be the net present discounted value of government expenditure when today’s state is $s$. Without loss of generality, order the states $s_1, ..., s_N$ so that if $s_2 > s_1$, then $\hat{G} (s_2) > \hat{G} (s_1)$. Then let $B_{RR} (s_N) = 0$, and $B_{RR} (s_n) = \hat{G} (s_N) - \hat{G} (s_n)$ so that there is perfect tax smoothing. Let the associated tax rate be $T_{R} = (1 - \beta) \hat{G} (s_N)$. Let $\beta^*$ be the discount factor that makes the government indifferent between continuing the Ramsey plan with inherited debt $\{B_{RR}\}$ and a plan of defaulting today and then pursuing the Markov policy from then onward. That is:

\[
\frac{W (T_{R})}{1 - \beta^*} = \max_s \left\{ W(G (s)) + \beta \sum_{s'} \pi (s'|s) H_M(0, G (s')) \right\}.
\]

Throughout, we use the first order conditions for the best sustainable outcome. The best sustainable outcome for $t \geq 1$ onward solves the following programming problem:

(106) $H_S(B, s) = \max_{B_{B},B',T} W (T) - A_B \sum_{s'} \pi_1 (s') B_B' (s') + \beta \sum_{s'} \pi (s'|s) H_S(B' (s'), s')$

subject to

(107) $G (s) + B = T + \beta \sum_{s'} \pi (s'|s) B' (s')$
\[ H_s(B'(s'), s') \geq H_M(0, s') - A_N B'_B(s'). \]

Letting \( \mu(s') \) denote the multiplier on the sustainability constraint (108) in state \( s' \), the first order conditions for the problem (106) if \( B(s^{t+1}) \) is strictly positive is

\[ -\beta W'(T(s')) \]
\[ = - [\beta + \mu(s'^{t+1})] H'_s(B(s^{t+1}), G(s_{t+1})) = - [\beta + \mu(s'^{t+1})] W'(T(s'^{t+1})) , \]

where the second equality follows from the envelope condition on (106), and the first order condition for \( B_B(s^{t+1}) \) implies that

\[ \mu(s'^{t+1}) \leq A_B/A_N . \]

Moreover, there is financial repression, in that \( B_B(s^{t+1}) > 0 \), only if \( \mu(s'^{t+1}) = A_B/A_N . \)

Suppose first that the debt levels \( \{B(s'^{t+1})\} \) are bounded away from zero for all \( s'^{t+1} \).

Since the multiplier on the sustainability constraint \( \mu(s'^{t+1}) \geq 0 \) for all \( s'^{t+1} \), from (109) it follows that the tax revenues \( \{T(s')\} \) are decreasing. Such a path for tax revenues implies that debt is decreasing over each cycle. To see why, iterate the government budget constraint forward to write that debt is the discounted value of future government surpluses, which, using the cyclical pattern of government spending, gives

\[ B(s^t) = \sum_{r=0}^{\infty} \sum_{s^{t+r}} \beta^r \pi(s^{t+r}|s^t) T(s^{t+r}) - \hat{G}(s_t) . \]

Next, since the sequence of tax revenues \( \{T(s')\} \) is decreasing and bounded below by zero, these tax revenues must converge to a positive level so that taxes are perfectly smoothed in the limit. From (109) it then follows that the multiplier \( \{\mu(s'^{t+1})\} \) must converge to zero. Thus, there is a finite \( T(\beta, G_0) \) such that for \( t+1 \geq T(\beta, G_0) \), the multiplier \( \mu(s'^{t+1}) < A_B/A_N \) so there is no financial repression.

Suppose next that at some finite \( t \) such that \( B(s^t) = 0 \). Then since \( \beta > \beta^* \), the Ramsey plan after this date is sustainable and thus there is no financial repression.

We turn to showing that debt is decreasing over each cycle in that \( B(s^t, s_A) \geq B(s^t, s_A, s_A) \) for all \( s^{t+1} \) and all \( s_A \in \{s_1, ..., s_N\} \). To do so we first show a preliminary result that if \( B(s^t) \geq B_R(s_t) \) then \( B(s^t, s_{t+1}) \geq B_R(s_{t+1}) \) for all \( s_{t+1} \). Suppose by way of
contradiction that $B(s^t) \geq B_R(s_t)$, but $B(s^t, s_{t+1}) < B_R(s_{t+1})$. By construction, $\{B_R(s)\}$ constitute Ramsey outcomes so that for all $s, s'$ $H'_S(B_R(s), s) = H'_S(B_R(s'), s')$ and by concavity of $H_S$ we have that

$$H'_S(B(s^t), s_t) \leq H'_S(B_R(s_t), s_t) = H'_S(B_R(s_{t+1}), s_{t+1}) < H'_S(B(s^t, s_{t+1}), s_{t+1}).$$

This leads to a contradiction since (109), together with the envelope condition, implies that $H'_S(B(s^t), s_t) \geq H'_S(B(s^t, s_{t+1}), s_{t+1})$. Thus, $B(s^t, s_{t+1}) \geq B_R(s_{t+1})$. Thus, debt levels are strictly positive in all periods if $G_0 > G^*(\beta)$.

Using this result, we can now show that $B(s^t, s_A) \geq B(s^t, s_A, s_A)$ for all $s^{t+1}$ and all $s_A \in \{s_1, ..., s_N\}$. In fact, since $\{B(s^t)\}$ is bounded away from zero, we can combine the first order conditions for debt issued at $(s^t, s_A)$ in state $s_{t+2} = s_A$ with the envelope condition to get

$$-\beta H'_S(B(s^t, s_A), s_A) = (\beta + \mu(s^t, s_A, s_A)) H'_S(B(s^t, s_A, s_A), s_A).$$

Since $H_s$ is strictly concave in $B$, it follows that $B(s^t, s_A) \geq B(s^t, s_A, s_A)$, strictly whenever $\mu(s^t, s_A, s_A) > 0$.

We now show that financial repression, measured as the fraction of debt held by banks, is increasing in the level of the debt. Hence, since the debt sequences are decreasing over the cycle, the fraction of debt held by banks must also be decreasing over the cycle. From the sustainability constraint, the share of debt held by banks $r(s^t) = B_B(s^t) / B(s^t)$ satisfies

$$r(s^t) = \frac{B_B(s^t)}{B(s^t)} = \max \left\{ \frac{H_M(0, s_t) - H_S(B(s^t), s_t)}{A_N B(s^t)}, 0 \right\}.$$

Using an argument similar argument to that in Proposition 4, we have the fraction of debt held by banks is in the level of total debt. Hence, since $B(s^t, s_A) \geq B(s^t, s_A, s_A)$, then $r(s^t, s_A) \geq r(s^t, s_A, s_A)$, strictly whenever $\mu(s^t, s_A, s_A) > 0$. Q.E.D.

E. Sudden Stops and Repression

We first show that the sustainability constraint tightens as $\chi$ decreases. Here we use the notation $H_M(0, G_L|\chi)$ to make explicit that the value of the Markov equilibrium depends on the maximal punishment $\chi$. It turns out that when there is no financial repression along
the equilibrium path and the government borrows the maximal possible amount from foreign lenders, the sustainability constraint can be written as

\[ H_S(B, G) \geq H_M(0, G_L|\chi) - \chi \left(1 - \frac{1}{1 + \kappa}\right). \]

(111)

where from now on we use the notation \( H_M(0, G_L|\chi) \) to make clear that the value of the Markov equilibrium depends on \( \chi \). The following lemma, used in the proof of Proposition 6, shows that the right side of this constraint, which is interpreted as the net benefit from default is decreasing in the maximal punishment.

**Lemma 8.** Under the assumption in Proposition 5, if

\[ \frac{\beta A_B}{1 - \beta^2 A_N} < 1, \]

(112)

then the net benefit of default (111) is decreasing in \( \chi \) for all \( \chi < \chi^* \).

**Proof.** The idea of the proof is that under these assumptions, if the maximal punishment for defaulting on foreign debt is decreased from \( \chi^* \), it is optimal to keep taxes on labor as they were, issue less debt to foreign lenders, \( \chi/(1 + \kappa) \) instead of \( \chi^*/(1 + \kappa) \), and repress banks enough so that domestic banks and households together hold the difference in debt held by foreign lenders at \( \chi^* \) and at \( \chi \).

Consider an economy with maximal punishment \( \chi^* \). Recall that in period 0 the economy begins in a high spending state with zero debt. Thus, the economy has a cyclic pattern: zero debt is issued in a low spending period, a total debt of \( B^* = B^*(\chi^*) \) is issued in a high spending period, foreign debt is at its limit \( B_F^* = \chi^*/(1 + \kappa) \), and, by definition of \( \chi^* \) the government is just indifferent to repressing so that

\[ W' (G_H - \beta B^*) = W' (G_L + B^*) (1 + A_B/\beta A_N). \]

(113)

The government sets \( B_F^* = 0 \) and let the tax on labor in the low and high spending periods be \( T_L^* \) and \( T_H^* \). Since the government is indifferent to repressing, the sustainability constraint must hold with equality and thus these allocations satisfy

\[ \frac{W (G_L + B^*) + \beta W (G_H - \beta B^*)}{1 - \beta^2} = W(G_L) - \chi^* (1 - \frac{1}{1 + \kappa}) + \beta H_M(0, G_H|\chi^*) \]

(114)
In (114) we have used a property of the Markov strategies, referred to as the restart property: after a default in the low spending period it is optimal to not issue any debt into the subsequent high spending period and then from that high spending state on, follow the original plan. This restart property implies that

\[ H_M(0, G_L | \chi^*) = W(G_L) - \chi^*(1 - \frac{1}{1 + \kappa}) + \beta H_M(0, G_H | \chi^*). \]

The conjectured allocations for the economy with \( \chi < \chi^* \) have the same taxes, so that the labor taxes in the high and low spending periods are \( T_L = T_L^*, T_H = T_H^* \), zero debt is issued when spending is low, the same level of debt is issued when spending is high, \( B = B^* \), and the foreign debt is at its new maximal level, so that \( B_F = \chi/(1 + \kappa) \). Moreover, at these allocations, the government represses \( (B_B > 0) \) until the point where the sum of private debt in the hands of households and banks is increased so as to exactly offset the reduction in foreign debt and the amount of debt held by banks \( B_B \) is chosen so that at these allocations the government is indifferent between defaulting and not. Thus, at a level of total debt \( B^* \) with \( B_B \) held by domestic banks, the sustainability constraint holds so that

\[ W(0) (T_H) = W(0) (T_L) (1 + A_B/\beta A_N). \]

Note that since the conjectured policies satisfy this first order condition, the conjectured policies are optimal.

We now turn to developing an explicit expression for the amount of repression captured by \( B_B \) and then use this value to calculate the value in the lemma. Subtracting (114) from (116) gives

\[ A_NB_B = (\chi^* - \chi)(1 - \frac{1}{1 + \kappa}) + \beta [H_M(0, G_H | \chi) - H_M(0, G_H | \chi^*)] \]
Under the conjectured plans the values for the Markov plans under $\chi$ and $\chi^*$ are given by

\begin{align}
(118) \quad H_M(0, G_H | \chi) &= \frac{W(T_H) + \beta W(T_L)}{1 - \beta^2} - \frac{A_B}{1 - \beta^2} B_B \\
(119) \quad H_M(0, G_H | \chi^*) &= \frac{W(T_H) + \beta W(T_L)}{1 - \beta^2}.
\end{align}

Notice that in (118), we have subtracted the cost of repression $A_B B_B$ that is borne in every high spending period from the net utility from the labor taxes. To solve for $B_B$ in terms of primitives, substitute (118) and (119) into (117) to get

\begin{equation}
(120) \quad B_B = \frac{1}{A_N} \left(1 - \frac{1}{1 + \kappa}\right) (\chi^* - \chi)
\end{equation}

Substituting $B_B$ from (120) into (118) gives

\begin{equation}
(121) \quad H_M(0, G_H | \chi) = \frac{W(T_H) + \beta W(T_L)}{1 - \beta^2} - \frac{\chi^* - \chi}{1 - \beta^2} A_B \left(1 - \frac{1}{1 + \kappa}\right)
\end{equation}

We want to show that the right-side of (111) is decreasing in $\chi$. Using (121) the right-side of (111) can be written

\begin{equation}
(122) \quad H_M(0, G_L | \chi) - \chi \left(1 - \frac{1}{1 + \kappa}\right) = D + \left(1 - \frac{1}{1 + \kappa}\right) \left[\frac{\beta}{1 - \beta^2} A_N - 1\right] \chi
\end{equation}

where $D$ is a constant that does not vary with $\chi$. Clearly, under (112), the right side of (122) is decreasing in $\chi$. Q.E.D.

*Proof of Proposition 6.* The proof follows easily from our construction of $\chi^*$ and $B^*$. We first show that if $\chi > \chi^*$, there is no repression. To see why, note that since the left side of (66) is increasing in $B^*$ and, by Lemma 8, the right side of (66) is decreasing in $\chi$, then for $\chi > \chi^*$, the level of debt that can be supported without repression is greater than $B^*$. Since, by definition, $B^*$ is such that the government is indifferent between repressing and not, then for $B > B^*$ the government prefers not to repress. Formally, since $W'$ is decreasing, then

$$-\beta W'(G_H - \beta B) + \left(\beta + \frac{A_B}{A_N}\right) W'(G_L + B) < 0.$$ 

The intuition is that the higher $\chi$ allows more debt to be sold to foreign lenders, so that even without repression, taxes can be smoothed better than at $\chi^*$. Hence, the costs of practicing repression stay the same, but the benefits fall, so repression is strictly not optimal.
Next, the argument that there is repression when $\chi < \chi^*$ is exactly analogous. The intuition is, for $\chi < \chi^*$, less debt can be supported with repression, the gains to tax smoothing rise while the costs of repression stay the same. Hence, it is optimal to practice repression. Q.E.D.

Setup for Proposition 7.

We conjecture the following patterns of taxes and debt and then describe sufficient conditions for these patterns to be optimal. Throughout, the government sells the maximal amount of debt to foreign lenders $B_{Fi} = \chi_i/(1 + \kappa)$. In the high $\chi$ state, $\chi_1$, the government does not practice financial repression. In a period with high spending and $\chi = \chi_1$, the government sells contingent debt $(B_{11}, B_{12})$ with the interpretation that $B_{1j}$ is the amount of debt owed in the following period in state $j$. In a period with high spending and $\chi = \chi_2$, the government sells contingent debt $(B_{21}, B_{22})$ with the interpretation that $B_{2j}$ is the amount of debt owed in the following period in state $j$. We will construct policies so that

$$B_{11} = B_{21} = B_1$$

so that the amount of debt sold into a low spending state is the same regardless of whether or not the economy is in a sudden stop. Also, in a period with low spending, the government sells zero state contingent debt, regardless of the current state or the subsequent state.

In terms of taxes, we assume that absent a sudden stop, that is in state $\chi_1$, there is perfect tax smoothing. We assume the sustainability constraint is slack from one high lending state to another, that is $\mu_{11} = 0$, and that the sustainability constraint is binding from the high lending state to the low lending state, $\mu_{12} > 0$, and from the low lending state to either the low or the high lending state, $\mu_{21}$ and $\mu_{22} > 0$. Finally, we assume that there is repression from the low lending state to the low lending state, so that $\mu_{22} = A_B/A_N$.

For these conjectured allocations and multipliers to be an equilibrium, we need the following conditions to be satisfied. First there is perfect tax smoothing from one high lending state to another:

$$(123) \quad -\beta W'(G_H - \beta [p_{11}B_1 + p_{12}B_{12}]) = -\beta W'(G_L + B_1).$$

Next, the choice of contingent debt $B_{12}$ sold from $\chi_1$ to $\chi_2$ is consistent with no repression, $\mu_{12} < A_B/A_N$, the sustainability constraint binding, $\mu_{12} > 0$, and partial default so that,
Thus,

\[ -\beta W'(G_H - \beta [p_{11}B_1 + p_{12}B_{12}]) = - (\beta + \mu_{12}) W'(G_L + B_{12}) \text{ with } \mu_{12} \in [0, A_B/A_N) \]  

(124)

\[ H_{S2}(B_{12}, G_L) = H_{M2}(0, G_L) - \tilde{\chi}_2, \]

where \( \tilde{\chi}_i = \chi_i (1 - 1/(1 + \kappa)) \). Notice that since the left-sides of (123) and (124) are the same so must be the right-sides, that is

\[ -\beta W'(G_L + B_{12}) = - (\beta + \mu_{12}) W'(G_L + B_{12}) \]

(126)

since \( \mu_{12} > 0 \) and \( W' \) is decreasing, (126) requires that \( B_{12} < B_1 \).

Next, that there is repression from a low lending state to another requires a tax-smoothing condition and that the sustainability constraint binds, that is,

\[ -\beta W'(G_L + B_1) = - (\beta + \mu_{12}) W'(G_L + B_{12}) \]

(127)

\[ H_{S2}(B_{22}, G_L) = H_{M2}(0, G_L) - A_N B_{B_{22}} - \tilde{\chi}_2 \]

(128)

with \( B_{B_{22}} > 0 \). Finally that the optimal choice of debt from a low lending state to a high lending state, \( B_{21} \), is consistent with \( B_{21} = B_1 \), no repression and a binding sustainability constraint requires

\[ -\beta W'(G_H - \beta [p_{21}B_1 + p_{22}B_{22}]) = - (\beta + A_B/A_N) W'(G_L + B_{22}) \]

(129)

\[ \mu_{21} \in (0, A_B/A_N) \]

\[ H_{S1}(B_1, G_L) = H_{M1}(0, G_L) - \tilde{\chi}_1 \]

(130)

Consider first the case where \( p_{21} = 0 \), so that there is a permanent switch from state 1 to state 2. After the switch \( B_{22} \) and \( B_{B_{22}} \) are given by those constructed in Proposition 6 and \( B_{21} \) is irrelevant. Note that we can solve for \( B_{12} \) by using that after the switch, the sustainability constraint holds with equality so that subtracting (128) from (125) and using

\[ H_{S2}(B_{12}, G_L) = W(G_L + B_{12}) + \beta H_{S2}(0, G_H), \]

(131)

\[ H_{S2}(B_{22}, G_L) = W(G_L + B_{22}) + \beta H_{S2}(0, G_H) \]

(132)

gives

\[ W(G_L + B_{12}) - W(G_L + B_{22}) = A_N B_{B_{22}} \]

(133)
Then, given the resulting $B_{12}$, (123) defines $B_1$. In particular, (133) defines $B_{12}$ given $\chi_2$ which in turn pins down $B_{22}$ and $B_{B22}$. Here, from the proof of Lemma 7, $B_{B22}$ is given by

$$B_{B22} = \frac{1}{A_N} \left( 1 - \frac{1}{1 + \kappa} \right) (\chi^* - \chi_2)$$

Clearly $B_{B22}$ is decreasing in $\chi_2$. To see how $B_{12}$ varies with $\chi_2$ we use the following: $B_{22} = B^*$ and neither vary with $\chi$, $B_{B22}$ is decreasing in $\chi_2$, so from (133), $B_{12}$ is increasing in $\chi_2$ since $W$ is decreasing.

We now turn to showing that these are optimal and feasible. For this we have to verify that three conditions hold. First, before the switch the sustainability constraint is slack, that is,

$$H_{S1}(B_1, G_L) \geq H_{M1}(0, G_L) - \tilde{\chi}_1.$$ 

Second, it is not optimal to repress when selling $B_{12}$,

$$-\beta W' \left( G_H - \beta \sum_i p_{1i} B_{1i} \right) \leq - \left[ \beta + \frac{A_B}{A_N} \right] W' (G_L + B_{12}).$$

Third, at $B_{12}$ the sustainability constraint binds. When the multiplier on the sustainability constraint is binding, the multiplier $\mu_{12} > 0$ and the first order conditions, which are the analog of (52) implies that

$$-\beta W' \left( G_H - \beta \sum_i p_{1i} B_{1i} \right) > -\beta W' (G_L + B_{12}).$$

Under the conjecture that we have perfect tax smoothing before the switch so that (137) implies

$$W' (G_L + B_1) > -W' (G_L + B_{12}).$$

We now argue that if $\chi_1$ is sufficiently large and $\chi_2 < \chi^*$ but close to $\chi^*$ then these three conditions are satisfied. To see this, let $\chi_2 = \chi^*$ then $B^*_B = 0$ and so $B_{12} = B^*$. Let $\chi_1 = \chi_1(\chi^*)$ defined as the smallest punishment before the switch to $\chi^*$ such that there is perfect tax smoothing before the switch. Thus, $\chi_1(\chi^*)$ is such that $B_1$ defined in (123) is sustainable, in that

$$H_{S1}(B_1, G_L) = H_{M1}(0, G_L) - \tilde{\chi}_1.$$
where $H_{S1}$ and $H_{M1}$ both vary with $\chi_1$ and $\chi_2$ and here we are evaluating them at $\chi_1 = \chi_1(\chi^*)$ and $\chi_2 = \chi^*$. We now show that the construction works for all $\chi_1 > \chi_1(\chi^*)$. Clearly, the first-order condition, (135), holds by definition of $\chi_1$. If $B_1 > B^*$ then

$$T_H = G_H - \beta p_{11} B_1 - \beta p_{12} B^* < G_H - \beta B^*$$

and so the associated marginal cost of taxes satisfies

$$-\beta W'(T_H) < -\beta W'(G_H - \beta B^*) = -\left[\beta + \frac{A_B}{A_N}\right] W'(G_L + B^*)$$

and so (136) is satisfied. We are left to show (137). Suppose by way of contradiction that $B_1 \leq B^* = B_{12}$, then

$$-\beta W'(G_H - \beta B^*) \leq -\beta W'(G_H - \beta p_{11} B_1 - \beta p_{12} B^*)$$

$$= -\beta W'(G_L + B_1) \leq -\beta W'(G_L + B^*) < -\left[\beta + \frac{A_B}{A_N}\right] W'(G_L + B^*)$$

where the first and third (weak) inequalities follows from the contradiction hypothesis and concavity of $W$, the equality follows from (123), and the strict inequality follows since $A_B/A_N$ is positive. Clearly, this equation contradicts the definition of $B^*$. Hence $B_1 > B^* = B_{12}$. Therefore by continuity if $\chi_2$ is reduced to be marginally below $\chi^*$ then it is optimal to repress after the switch but not before. Moreover, $B_{12} < B_1$ so we can interpret the equilibrium as having partial default after the switch and then repression. This completes the proof when $p_{21} = 0$.

Now consider the case when $p_{21}$ is small but positive. By continuity, all the conditions above are still satisfied and we only need to check that the optimality condition for $B_{21} = B_1$, (129), holds. To check this one around $p_{21} = 0$, consider (129) and (127) evaluated at $p_{21} = 0$:

(138) $-\beta W'(G_H - p_{22} \beta B_{22}) = -(\beta + \mu_{21}) W'(G_L + B_1)$

(139) $-\beta W'(G_H - p_{22} \beta B_{22}) = -(\beta + A_B/A_N) W'(G_L + B_{22})$

Since $\chi_2$ is close to $\chi^*$ then $B_{22}$, defined in (134) is close to zero, so comparing the sustainability constraints (125) and (128) $B_{22}$ is close to $B_{12} < B_1$ then $B_{22}$ is also less than $B_1$. This implies that $-W'(G_L + B_1) > -W'(G_L + B_{22})$ and so (138) and (139) imply that $\mu_{21} < A_B/A_N$ and so (129), holds. Q.E.D.