The Limits to Partial Banking Unions:
A Political Economy Approach

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Abstract

Will a banking union increase welfare in all Eurozone countries? This paper studies the desirability of a banking union given political economy frictions in the member countries. Government funds to distressed banks are allocated by policymakers who can receive lobbying rents from the financial sector. In equilibrium, a banking union increases public funding to the banking sector, but it also increases rent seeking. This can result in lower household welfare than under no banking union. The negative welfare effects can be partially reversed through fiscal rules, but a Pareto improvement may not be achieved without domestic institutional reforms.

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1 Introduction

The recent banking and sovereign debt crises have renewed interest in creating common cross-country rules for government interventions in the banking sector. This has been particularly relevant for the Eurozone, given the large cross-border spillovers from public bailouts. Naturally, the presence of such spillovers suggests that a banking union may deliver a Pareto improvement for all member countries. Domestic political economy constraints may, however, interfere with the functioning of such a supranational institution. First, policymakers may want to keep certain policy decisions under their direct control rather than delegating them to the supranational level. Second, once a banking union is in place, policymakers may divert resources towards socially inefficient rents. This raises the question of whether such a banking union can improve consumer welfare and achieve a more efficient supranational coordination of government interventions in the banking sector.

The policy debates surrounding the creation of a European Banking Union highlight how domestic political economy considerations affect the supranational coordination of banking policy. In drafting the plan for a banking union, the Eurozone countries have reached agreement on a unique supranational supervisory authority (the Single Supervisory Mechanism), in charge of coordinating bank supervision and regulation. Yet, another key component of the banking union is the Single Resolution Mechanism that would coordinate responses to a banking crisis. In its current form, this institution leaves national authorities in each country with significant decision powers over the management of a banking crisis and the funding of bailouts.\(^1\) This creates a system which falls short of a fully centralized mechanism, and therefore it is referred to as a ‘partial banking union.’ This paper considers a supranational arrangement in the form of a partial banking union and studies the welfare

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\(^1\)A summary of the progress towards implementing the European Banking Union is available on the European Commission website ec.europa.eu (accessed October 21, 2015). State sovereignty over decisions that affect national banks, voter concerns over the use of public funds towards bank bailouts, and domestic political rent-seeking have been indicated as major factors driving the proposed banking union structure.
implications of domestic political economy distortions.

The case of the Spanish savings and loan sector (the ‘cajas’) provides an illustration of the type of political economy distortion analyzed in this paper and of the role it played in the recent banking crisis.\(^2\) The Spanish cajas were led by politically appointed executives, and the political connections of these executives influenced the types of loans that they extended in the pre-crisis period. For example, regional governments used the cajas to fund projects that had little social benefit, but served political interests (e.g., airports with no flights, unused theme parks).\(^3\) During the crisis, local policymakers decided to rescue undercapitalized cajas by merging them, and these mergers were based on political and regional motives rather than economic efficiency. These inefficient mergers led to the creation of larger troubled entities, increasing the cost of public bailouts and the pressure on public finances in Spain and in the Eurozone. Given the lessons from this episode, one of the major concerns for creditor countries in the partial banking union arrangement is that their public funds will be used towards inefficient or politically motivated recapitalizations. Concerns also exist on the side of citizens in the countries receiving funds through the partial banking union, since access to these funds - which in the end might be used for inefficient bailouts - might be conditioned on unpopular reforms or changes in their government’s public spending.

The features of a partial banking union and of domestic politics are built into a model that sheds light on the welfare effects of such a supranational arrangement, in which financial integration is not accompanied by political integration. I model a union of governments that are electorally accountable to voters within their own country, but that may also be susceptible to political influence through lobbying from financial institutions requiring bailouts. Each government can provide public funds to distressed banks in its country through a process of bank recapitalizations, and these policies have cross-country spillover effects. The spillovers provide a motivation for cross-country

\(^2\)Discussed in greater detail in Garicano (2012) and Cuñat and Garicano (2009).
transfers and for the supranational coordination of recapitalization policies. The model considers a partial banking union, in which the funds for recapitalizations and cross-country transfers are determined at the supranational level, but each country’s government can decide how to allocate the recapitalization funds, which gives rise to rent-seeking opportunities. The governments also face a trade-off when determining how many funds to allocate to recapitalizations versus other spending on non-financial public goods (e.g., infrastructure projects).

The paper’s main result shows that creating a partial banking union under the above domestic political economy constraints can reduce consumer welfare in the country that receives supranational transfers. The mechanism for this result relies on the following. A partial banking union takes into account for the positive cross-border spillovers from public recapitalizations in one country and will implement higher recapitalizations than those chosen by governments independently. Due to lobbying, an increase in the funds allocated to recapitalizations will also lead to an increase in the funds directed towards political rents. The key determinant of whether more recapitalizations are welfare-increasing for households is the way in which the spending on additional recapitalizations is split between the countries that form the union. This split depends on the bargaining weight that each country has in the union, as well as on the outside option of the politicians in each country. A lower bargaining weight means that the country will receive fewer transfers and will be conditioned to cut public spending to non-financial areas in order to fund higher recapitalizations. Yet, whether such an agreement is implementable depends on the domestic political distortion - the degree of rent seeking that the politician can engage in. The more the politician has access to rents, the more likely the politician is to implement the supranational agreement, since the terms of the agreement always favor an increase in rents. Therefore, a partial banking union which comes with low transfers and high conditionality is unfavorable to the receiving country’s households, but can still be accepted by a politician who benefits from higher rents.

The main result hinges on the distribution of bargaining weights between
the countries that form the partial banking union. A hallmark of policy co-
ordination without political integration is that bargaining weights depend on
the relative weights of the countries in the financial sector, and not on their
relative weights in terms of overall public finances. The model shows that,
if the receiving country’s relative financial assets are sufficiently low, then a
partial banking union results in a loss in welfare for the receiving country.
This result emerges even in the cases in which bargaining weights reflecting
overall relative public finances would guarantee a Pareto improvement. This
difference in outcomes reflects the costs of the lack of political integration,
which can undermine the benefits of financial integration.

The political economy distortions are central to the negative welfare out-
come, so an implication of the above results would be to find policies that
proxy for the lack of political integration of the countries. One such proposal
has been to implement fiscal rules that constrain overspending and debt accu-
mulation. Such fiscal rules have the effect of reducing both overall spending
and rents. The reason for this is that fiscal rules alone cannot restrict rents
without also restricting spending in general. In the case of large negative
shocks to the banking sector, this results in both insufficient recapitalizations
and insufficient public good provision. Yet, if a partial banking union is imple-
mented with fiscal rules in place, their constraining effects may end up being
beneficial to the country receiving transfers. Fiscal rules that limit debt-taking
make it costlier for the supranational agreement to impose conditionality on
an already fiscally constrained country. This may result in less conditionality
and larger transfers to the receiving country.

Another potential solution to the lack of political integration could be to
focus on the domestic political economy distortions, and engage in electoral
reforms which reduce the influence of campaign contributions and political
lobbying. Such reforms reduce the rent seeking incentives of politicians and
ensure that a partial banking union leads to welfare improvements for both
countries. Also, domestic electoral reforms can augment the effects of fiscal
rules by ensuring that a welfare-improving supranational agreement can be
achieved even with smaller bargaining weights for the country receiving trans-
fers. Therefore, fiscal rules and electoral reforms together can deliver a Pareto improvement over having no banking union in cases in which fiscal rules alone cannot deliver a Pareto improvement.

The above results highlight how domestic political economy distortions have welfare implications at the supranational level, and they may make it costlier to implement cross-country policies. This is particularly relevant for the Eurozone, where banking policies are only partially centralized, and significant decision power still lies with each country’s government. The results suggest that supranational policies must be complemented by domestic policies in order to overcome the spillovers generated by domestic political economy distortions.

**Related Literature.** The interplay between fiscal policy and financial integration has been vastly studied in the literature. Yet, the main focus for most of the work in this area has been on optimal policy design with a benevolent government. This includes the study of optimal fiscal policy coordination (Kehoe 1987, Chari and Kehoe 1990, Beetsma and Lans Bovenberg 1998, Halac and Yared 2015), fiscal rules in currency unions (Von Hagen and Eichengreen 1996, Ferrero 2009), or the role of fiscal transfers in providing efficient insurance within a currency union (Farhi and Werning 2012). All these papers abstract from the effects of political economy distortions or political decision-making. By contrast, this paper considers the issue of policy coordination with financial integration, taking into account the political economy issues that emerge when there is no political integration and policymakers are partially self-interested. Therefore, this paper is most closely related to the political economy work that considers the effects of different political institutions in the context of fiscal or financial integration (Tabellini 1990, Lohmann 1993, Persson and Tabellini 1996a, Persson and Tabellini 1996b). Whereas this literature focuses mainly on the effects of different electoral institutions and the aggregation of voter preferences, this paper considers the issue of political rent seeking and examines the distortion to supranational policies due to domestic rent seeking, and political lobbying.

The link between financial integration and domestic public debt in the
presence of political economy constraints has also been studied by Tabellini (1990) and Azzimonti, de Francisco, and Quadrini (2014), who show how fiscal or financial integration can lead to higher public debt due to political economy biases. This paper, however, highlights a different channel for the increase in public debt. Debt does not increase due to lower costs of borrowing (as in Tabellini 1990) or the aggregation of heterogeneous voter preferences (as in Azzimonti, de Francisco, and Quadrini 2014), but rather because cross-country transfers create higher incentives for current spending and rent-seeking. The increase in debt is directly linked to the existence of supranational agreements in the absence of political integration. In a set of papers also motivated by the European supranational institutions, Persson and Tabellini (1996a) and Persson and Tabellini (1996b) study cross-country insurance and the effect of fiscal transfers on welfare under different political decision-making institutions, specifically direct voting versus bargaining. This paper provides a complement to their results. While their papers highlight the inefficiencies that emerge under various institutions of collective choice – voting versus bargaining–, this paper considers inefficiencies rooted in domestic institutions – rent-seeking and lobbying. Moreover, it presents another channel through which domestic institutions affect supranational agreements: that of rule implementation (the allocation of transfers by the local policymaker) rather than rule selection (the collective choice of transfers).

The modeling approach in this paper uses a principal-agent framework similar to those developed in Acemoglu (2005) and Acemoglu and Robinson (2006), and in Yared (2010), which considers political rent seeking and models electoral incentives as voters’ demand for a minimal utility level each period. The model also builds on the framework developed in Acemoglu, Golosov, and Tsyvinski (2008) and Acemoglu, Golosov, and Tsyvinski (2011) but differs from these models in two main ways. First, it focuses on a two-period game as opposed to the best Subgame Perfect Equilibrium in an infinite horizon model; and second, it considers an endowment economy without capital, but with public debt, supranational transfers and limits on spending and debt. Finally, this model links rent-seeking to recapitalizations using an approach
similar to that of Milesi-Ferretti (2004), which models ‘creative accounting’ as the difference between a true fiscal variable and its corresponding ‘measured’ variable, then uses this difference over the business cycle to infer the effect of budget rules on fiscal policy. In this paper, supranational rules are imposed on spending measures which can differ from the true spending on recapitalizations, due to the presence of rents. The desirability of supranational controls over domestic spending has also been examined in Dewatripont and Seabright (2006), but in the context of a politician whose type is unknown to voters, and who uses domestic spending to signal his type. This paper considers the role of supranational controls in a model without private information, where the politician has a direct preference for rent-seeking.

The rest of the paper is organized as follows. Section 2 presents the setup of the model. Section 3 presents the benchmark case with benevolent governments. Section 4 analyzes the implications of a partial banking union when political economy distortions are considered. Section 5 considers the role of fiscal rules in reducing rent-seeking and correcting the inefficiency generated by it. Section 6 concludes, and the Appendix contains the proofs.

2 Environment

Consider a two-period economy, with periods $t = 0, 1$. The economy consists of two countries, a donor country, referred to as Foreign, and a receiving country, referred to as Home. An independent supranational authority plays the role of a Principal who proposes the terms of a partial banking union between countries. Each of the two countries is made up of a continuum of mass 1 of identical households.

2.1 Households

At date 0, all households from both countries start with a perfectly diversified portfolio of risky projects, in the form of deposits in banks.\footnote{A more detailed description of the banks is provided in the Appendix.} Home
households hold total deposits $z^H$, a fraction $\alpha^H$ of which is deposited in Home banks, while the remaining fraction is deposited in Foreign banks.\footnote{The assumption that households hold deposits in banks outside their country’s borders is a simplification meant to capture the loans made by local banks to banks outside their country. Specifically for the case of the Eurozone, direct deposits by households in foreign banks represent a negligible fraction of cross-border banking compared to the substantial cross-border loans made between banks.} Similarly, Foreign households hold total deposits $z^F$, a fraction $\alpha^F$ of them is deposited in Foreign banks and the remaining fraction is deposited in Home banks.

Households derive utility from private consumption equal to their deposit returns and from a domestic public good $g^H$ provided by the government. Their preferences are given by\footnote{For ease of notation, I omit the subscripts for the period 0 variables and keep only the subscripts for the period 1 policies.}

\[
U^i(x^i, x^j, g^i, g^j) = u(c^i(x^i, x^j, \theta)) + w(g^i) + \beta (u(z^i) + w(g^j)), \tag{1}
\]

where

\[
c^i = R(1 - \theta) \left( \alpha^i z^i + (1 - \alpha^i) z^j \right) + R\sigma^i x^i + R \left( 1 - \sigma^i \right) x^j,
\]

$i, j = \{H, F\}$, $i \neq j$, $\sigma^i = \alpha^i z^i / (\alpha^i z^i + (1 - \alpha^i) z^j)$ and $u(\cdot)$ and $w(\cdot)$ are strictly concave, increasing, $0 < u'(0) < \infty$, $0 < w'(0) < \infty$, $\lim_{g \to \infty} w'(g) = 0$. Notice that household utility depends on the recapitalization decided by its government and also on the recapitalization performed by the other country’s government.

\subsection{2.2 Banks}

In each country, there is a mass 1 of ex-ante identical banks. These banks hold ex-ante identical, risky investment projects which pay off at the end of period 0. They do not have any equity and can fund projects exclusively using household deposits. Moreover, I make the simplifying assumption that the banks are owned by the same households that hold deposits in them, so that the objective of the bank is to maximize the expected household utility
from private consumption. The initial investment made by banks in country \( i \in \{H, F\} \) consist of the deposits of Home and Foreign households in that country:

\[
\begin{align*}
I^H &= \alpha^H z^H + (1 - \alpha^F) z^F, \\
I^F &= (1 - \alpha^H) z^H + \alpha^F z^F.
\end{align*}
\]

The project return is subject to uncertainty. Following investment, an aggregated i.i.d. shock \( \theta \in \Theta \) is realized. After the shock, a fraction \( \theta \) of the investment is lost, while the remaining \( (1 - \theta) \) fraction of the project is intact. The intact portion of the project has a rate of return \( R > 1 \) in the next period. The distressed portion does not produce any returns, unless additional funds are reinvested. After observing of \( \theta \) and prior to the investment project completion, the banks in country \( i \) can reinvest \( x^i \) new funds into their projects, such that the total size of the projects is at most equal to the initial investment size: \( x^i \leq \theta I^i \).

Since the households and banks do not have access to any storage technology and there is no outside loan market for the banks to access reinvestment funds, all reinvestment funds \( x^i \) must be provided by the government of country \( i \), the country in which the banks are located. Another key assumption is that reinvestment funds cannot be targeted towards the deposits of a particular household. This ensures that both the home and donor households benefit from the reinvestment in proportion to their contribution to the total investment. At the end of period 0, the project is completed and returns \( R((1 - \theta)I^i + x^i) \) consumption units, where \( R > 1/\mathbb{E}_\theta[1 - \theta] \).

In the second period, banks hold safe projects with rate of return \( R = 1 \) and receive deposits from Home and Foreign households, in the same proportions as in period 0. The assumption of a second period without aggregate shocks is made for simplicity. It creates a role for public debt in smoothing public good provision over time, as further discussed below.

\[\text{7The liquidity shock is modeled as a simplified version of the one in Holmström and Tirole (1998).}\]
3 A Partial Banking Union with Benevolent Governments

We begin with the case in which a partial banking union is established between two benevolent governments. This provides a baseline scenario from which we can analyze the effects of political economy distortions. Each government $i$ maximizes the utility of its households, as given in (1). The government of each country receives an endowment $e^i$, $i = H, F$ at the beginning of each period. In period 0, each government $i \in \{H, F\}$ can also take on one-period debt $b^i$ at the risk-free rate $1/\beta$, with an exogenous lower limit $b \leq 0$ and upper limit $e^i$. The government budget can be used for the provision of the public good each period and for public recapitalizations in period 0.

The supranational authority seeks to maximize a weighted sum of Home and Foreign household utilities. It can set the terms of the partial banking union between the countries. Specifically, a partial banking union consists of a transfer $\tau$ from the Foreign country to the Home country and a minimum recapitalization expense $x$ that the Home country must commit to. Any pair $(\tau, x)$ proposed by the supranational authority must be preferred by each country government over the outside option of no agreement. The problem for the supranational authority is therefore given by

$$\max_{\tau, x}\{\eta U^H(x^H, x^F, g^H, g^H_1) + (1 - \eta)U^F(x^H, x^F, g^F, g^F_1)\}$$

subject to

$$U^H(x^H, x^F, g^H, g^H_1) \geq U^H(x^{H0}, x^{F0}, g^{H0}, g^{H0}_1), \quad (2)$$

$$U^F(x^H, x^F, g^F, g^F_1) \geq U^F(x^{H0}, x^{F0}, g^{F0}, g^{F0}_1), \quad (3)$$

where $\{x^{H0}, g^{H0}, g^{H0}_1\}$ are the policies chosen by the Home government and $\{x^{F0}, g^{F0}, g^{F0}_1\}$ are the policies chosen by the Foreign government when no partial banking union is in place. I assume that $e^H$ is sufficiently small such that the Home government does not fully recapitalize banks if no par-
tial banking union is in place, i.e., $x^{H0} < \theta \left( \alpha^H z^H + (1 - \alpha^F)z^F \right)$. Moreover, I also assume that the Foreign government’s endowment $e^F$ is sufficiently large so that full recapitalizations are provided to the Foreign banks, $x^{F0} = \theta \left( \alpha^F z^F + (1 - \alpha^H)z^H \right)$ and a transfer to Home country is feasible, i.e.,

$$
\eta u_x^H (c^H (x^{H0}, x^{F, MAX^0}, \theta)) + (1 - \eta) u_x^H (c^F (x^{F, MAX}, x^{H0}, \theta)) > (1 - \eta) w^0 (g^{F0}),
$$

where $u_x^H$ and $w^0$ denote the first derivative with respect to $x^H$ and $g^{F0}$, respectively. In this case, a partial banking union will be implemented by the supranational authority, with $\tau > 0$ and $x \geq 0$.

The policies under the banking union, $\{x^H, g^H, g^H_1, b^H_1\}$, are chosen by the Home government to maximize utility $U^H(x^H, x^H, g^H, g^H_1)$, as given in (1), subject to the constraints

$$
\begin{align*}
x^H + g^H &\leq e^H + \beta b^H_1 + \tau, \\
x^H &\geq \bar{x}, \\
g^H_1 &\leq e^H - b^H_1, \\
b^H_1 &\leq e^H, \\
b^H_1 &\geq \underline{b}, \\
x^H &\leq R \theta \left( \alpha^H z^H + (1 - \alpha^F)z^F \right).
\end{align*}
$$

Constraints (4a) and (4c) are the budget constraints of the Home government in periods 0 and 1, respectively. Constraint (4b) specifies the minimum required spending on recapitalizations under the partial banking union, constraints (4d) and (4e) set the bounds on debt, and constraint (4f) gives the maximum level of recapitalizations given the shock $\theta$.

Similarly, the policies in the Foreign country $\{g^F, g^F_1, x^F\}$ are chosen by
the Foreign government maximizing (1) given the constraints

\[ x^F + g^F + \tau \leq e^F + \beta b^F_1, \tag{5} \]
\[ g^F_1 \leq e^F - b^F_1, \tag{6} \]
\[ b^F_1 \leq e^F, \tag{7} \]
\[ b^F_1 \geq \bar{b}, \tag{8} \]
\[ x^F \leq R\theta \left( (1 - \alpha^H) z^H + \alpha^F z^F \right). \tag{9} \]

The constraints are equivalent to the Home government’s constraints, with the exception of the minimum required spending on recapitalizations, which does not apply to the country that provides transfers. Moreover, the assumption of a sufficiently large government budget \( e^F \) ensures that constraint (9) binds.

A partial banking union requires that the participation constraints of the two governments, (2) and (3), are satisfied. Therefore, household utility in either the Home or the Foreign country will never decrease under a partial banking union. The households in at least one of the two countries will be made better off with the partial banking union, so a Pareto improvement will always be achieved.

4 A Partial Banking Union with Domestic Political Economy Constraints

The baseline model presented above is enriched to incorporate political economy distortions in both countries. We consider the following political economy mechanism in each country. The government can use the public budget in order to provide public goods, recapitalizations and also to take on rents. The rent-seeking process is as follows. The government can choose the degree of efficiency of its reinvestment in projects. The most socially efficient intervention provides reinvestment funds \( x^i \) for the distressed projects. The politician can also choose less efficient interventions. In this type of interven-
tions, he provides reinvestment funds $x^i$ but can also decide to expand the capacity of the project. Only the original project returns rate $R$ to households. The expansion of the project has a rate of return of 1 for the politician, in the form of rents. The degree of spending towards political rents is determined by as in (Grossman and Helpman 1994), since the politician weights both household utility and the benefits coming from political rents. Therefore, the total spending towards recapitalizations will be equal to $x + r$, but only $x$ are recapitalizations that provide returns to households. The allocation of funds between rents versus recapitalizations cannot be verified by the supranational authority. Therefore, the intervention level $x$ encompasses both rents and recapitalizations, and the required intervention level is satisfied as long as

$$x^H + r^H \geq x.$$  

Government $i$’s utility is then given by

$$V^i(r^i, x^i, x^j, g^i, g^j_1) = (1 - \gamma^i)v(r^i) + \gamma^i U^i(x^i, x^j, g^i, g^j_1),$$

where $i, j \in \{H, F\}, i \neq j, \gamma^i \in (0, 1)$ represents the weight placed on household utility relative to rents by government $i$ and $v(\cdot)$ is weakly concave and increasing, $v' < \infty$.

Since rents are extracted as part of the recapitalization process, they can only be extracted in the first period. A dynamic extension of the model presented in the online Appendix considers the case of future rents and discusses the implications of changes in debt on expected rent-seeking.

### 4.1 Timing

The timing of the model is as follows. In period 0, the supranational authority proposes a transfer $\tau$ and intervention level $x$. The Foreign government decides whether to accept the proposed agreement and make transfer $\tau$, and the Home government decides whether to accept the transfer and provide total intervention $x$. Finally, recapitalizations $x^i$, rents $r^i$, the domestic public good
$g_i$, and debt $b_i$ are decided by each government. In the second period, the
governments provide the domestic public good $g^*_i$, given the available budget
after any debt repayments, and households consume the returns from their
second period deposits.

4.2 Recapitalization Policies and Rent Seeking

To shorten notation in the rest of the analysis, define

$$u^i(x^i, x^j, \theta) \equiv u(R(1 - \theta) (\alpha^i z^i + (1 - \alpha^j) z^j) + R \sigma^i x^i + R(1 - \sigma^j)x^j),$$

and

$$x^{i, \text{MAX}} \equiv \theta \left( \alpha^i z^i + (1 - \alpha^j) z^j \right), \quad (12)$$

$i, j \in \{H, F\}, i \neq j$.

The Home country government chooses $\{r^H, x^H, g^H, b^H_1\}$ to maximize ob-
jective (11) subject to the following constraints:

$$r^H + x^H + g^H \leq e^H + \beta b^H_1 + \tau, \quad (13a)$$

$$r^H + x^H \geq x, \quad (13b)$$

$$g^H \leq e^H - b^H_1, \quad (13c)$$

$$b^H_1 \leq e^H, \quad (13d)$$

$$b^H_1 \geq b, \quad (13e)$$

$$x^H \leq R\theta \left( \alpha^H z^H + (1 - \alpha^F) z^F \right), \quad (13f)$$

which are the analogous constraints to constraints (4a)-(4f), with the addition
of the political rents in period 0. A key relationship coming out of the gov-
ernment’s maximization problem is that an increase in recapitalizations $x^H$
cannot be achieved without an increase in rents $r^H$. The result comes from
considering the first-order conditions to the politician’s problem (given the
assumption of an interior solution):

$$\left(1 - \gamma^H\right) v_r(r^H) = \gamma^H u_{x^H}^H(x^H, x^F, \theta).$$

15
The politician’s utility is concave in both rents and recapitalizations, so any incentive to increase recapitalizations will also give the politician the incentive to increase rents. The only way for the supranational authority to increase recapitalizations is to increase transfers \( \tau \) or the required intervention level \( x \), and to accept an increase in both rents and recapitalizations.

### 4.3 The Supranational Authority’s Problem

With domestic political economy constraints, the supranational authority must now propose a transfer \( \tau \) and minimum intervention level \( x \) such that the Home and Foreign governments both agree to participate in the partial banking union. The problem for the supranational authority is

\[
\max_{\tau, x} \{ \eta U^H(x^H, x^F, g^H, g_1^H) + (1 - \eta) U^F(x^H, x^F, g^F, g_1^F) \} \tag{14}
\]

subject to

\[
V^H(r^H, x^H, x^F, g^H, g_1^H) \geq V^H(r^{H0}, x^{H0}, x^{F0}, g^{H0}, g_1^{H0}), \tag{15}
\]

\[
V^F(r^F, x^H, x^F, g^F, g_1^F) \geq V^F(r^{F0}, x^{F0}, x^{H0}, g^{F0}, g_1^{F0}). \tag{16}
\]

Constraints (15) and (16) denote the participation constraints for the Home and Foreign governments, respectively. The supranational authority must ensure that the Home government’s utility in the partial banking union is as least as high as the Home government’s utility without a banking union. This allows for the possibility that the Home household utility is not higher in the partial banking union compared to the no banking union case. Similarly, the Foreign government must agree to the participation in the banking union, given the utility of the Foreign politician.

### 4.4 Household Welfare under a Partial Banking Union

Given the above setup, the following result captures the welfare effects of a partial banking union.
Proposition 1 A partial banking union does not achieve a Pareto improvement if \( \eta < \eta^* \) where \( \eta^* \in (0, 1) \): it increases Foreign household welfare, but it lowers Home household welfare.

Proof. In the Appendix. ■

The reasoning for the result is as follows. In a partial banking union, the supranational authority sets \( \tau > 0 \) and an intervention level \( \bar{x} \) in order to maximize the weighted sum of household utilities (14) while ensuring that both governments agree to participate. The Home government will participate in the banking union if

\[
(1 - \gamma^H)v(r^H) + \gamma^H U^H(x^H, x^F, g^H, g_1^H) \geq (1 - \gamma^H)v(r^{H0}) + \gamma^H U^H(x^{H0}, x^{F0}, g^{H0}, g_1^{H0})
\]

An intervention level \( \bar{x} \) implies that \( r^H + x^H \geq r^{H0} + x^{H0} \). Since the politician’s utility from rents \( r^H \) and recapitalizations \( x^H \) is concave and additive, the intervention requirement \( \bar{x} \) induces the politician to increase both rents and recapitalizations, so \( r^H \geq r^{H0} \) and \( x^H \geq x^{H0} \). This implies \( v(r^H) \geq v(r^{H0}) \). Therefore, (17) is satisfied as long as

\[
U^H(x^{H0}, x^{F0}, g^{H0}, g_1^{H0}) - U^H(x^H, x^F, g^H, g_1^H) \leq \frac{(1 - \gamma^H)}{\gamma^H} \left[ v(r^H) - v(r^{H0}) \right],
\]

and this upper bound on the change in Home household utilities is positive whenever \( v(r^H) > v(r^{H0}) \).

From the supranational authority’s maximization problem, it follows that the Home household utility \( U^H(x^H, x^F, g^H, g_1^H) \) is decreasing as a function of \( \eta \). At \( \eta = 0 \), \( U^H(x^H, x^F, g^H, g_1^H) < U^H(x^{H0}, x^{F0}, g^{H0}, g_1^{H0}) \), while at \( \eta = 1 \), \( U^H(x^H, x^F, g^H, g_1^H) > U^H(x^{H0}, x^{F0}, g^{H0}, g_1^{H0}) \). Then, \( \eta^* \in (0, 1) \) denotes the value at which

\[
U^H(x^{H0}, x^{F0}, g^{H0}, g_1^{H0}) - U^H(x^H(\eta^*), x^F(\eta^*), g^H(\eta^*), g_1^H(\eta^*)) = 0.
\]

\(^8\)The details are provided in the Appendix.
The intuition for this result is that two different forces are at work: one domestic and one supranational. On the domestic side, participation in the partial banking union requires the governments of the two countries to be at least as well off as under no banking union. For the Home government, this means that entering a partial banking union should not decrease the utility of the Home government. Entering a partial banking union brings the Home government the benefit of transfers from the Foreign government. The cost of entering the partial banking union is that the agreement forces the Home government to provide a minimum intervention \( x \) in the financial sector, and therefore less spending on non-financial public goods. This cost in terms of fewer public goods is incurred by both the households and the politician. The Home households only receive the benefit from higher recapitalizations, while the benefit from higher spending on political rents accrues only to the politician. Therefore, any offer \((\tau, x)\) carries a higher relative benefit for the politician compared to the households, and the Home politician would prefer to accept the partial banking union terms even when households would not.

At the supranational level, the cost of additional recapitalizations is less public good provision in the Home or the Foreign country. The supranational authority divides this cost between countries given the relative weight \( \eta \). A lower \( \eta \) implies that the Foreign country has a higher bargaining weight over the splitting of the recapitalization costs. Since the benefits of the partial banking union relative to its costs are higher for the Home politician than for the Home households, the Home politician is willing to participate in a partial banking union for values of \( \eta < \eta^* \), for which the costs that the Home households must bear exceed the benefits they receive from higher recapitalizations.

An immediate step in studying the implications of Proposition 1 is to look at the possible determinants of \( \eta \) and how they affect the probability of \( \eta \) being lower than \( \eta^* \). Consider the case in which \( \eta \) reflects the proportion of Home country assets in the banking system, \( \eta = \frac{z^H}{z^H + z^F} \).
Let $r^{H,\text{MAX}}(\theta)$ be defined as the value of rents when full recapitalizations are performed by the Home government:

$$(1 - \gamma^H)w'(r^{H,\text{MAX}}) = \gamma^H w^H(x^{H,\text{MAX}}, x^{F,\text{MAX}}, \theta).$$

Let $\left(1 - \delta^H\right)$ denote the minimum value that the marginal propensity to rent-seek can take in the Home country given the feasible range of recapitalization values, and $\left(1 - \delta^F\right)$ denote the maximum value that the marginal propensity to rent-seek can take in the Foreign country, given the feasible values of transfers.

**Condition 1** The following inequality holds:

$$\frac{z^F}{z^H} \geq \frac{w'(g^{H0} - \delta^H \cdot \theta(\alpha^H z^H + (1 - \alpha^F)z^F))}{w'(g^{F0} - (1 - \delta^H) \delta^F \theta(\alpha^H z^H + (1 - \alpha^F)z^F))}.$$  

where $w'(\cdot)$ denotes the first derivative for the function $w(\cdot)$.

Condition 1 sets a lower bound on the relative financial assets of Foreign versus Home households by considering the case in which both the Home government and the Foreign government provide full recapitalizations of their respective banks.

**Proposition 2** If $\eta = \frac{z^H}{z^H + z^F}$ and Condition 1 holds, then a partial banking union lowers Home household welfare.

**Proof.** In the Appendix. ■

Proposition 2 provides the condition under which supranational bargaining weights equal to the relative financial assets of the countries will always lead to a decrease in Home household welfare. The intuition for this result comes from the constraint imposed in Condition 1. A larger ratio of Foreign to Home financial assets increases the relative benefits to the Foreign country from the recapitalization of Home banks. It increases the relative stake that the Foreign households have in the Home country’s banks. This, in turn,
increases the returns to the supranational authority from demanding higher recapitalizations in the Home country. The right-hand side of the inequality in Condition 1 denotes the ratio of marginal utilities from public goods in the Home versus the Foreign country. It captures the relative cost of financing additional recapitalizations using funds from the Home country versus using funds from the Foreign country’s budget. Condition 1 requires that the relative benefits to the Foreign country from the recapitalization of Home banks are higher than the relative costs to the Home country from providing these additional recapitalizations. Then, the supranational authority will find it optimal to require the Home country to spend more on the financial sector. Even if this decreases Home household welfare, the relative benefit to the Foreign households is larger.

The choice of $\eta$ in Proposition 2 captures the case in which the supranational authority represents an institutional arrangement created within the financial regulatory sphere, without accounting for government resources and public spending outside of the financial sector. It shows that, in this case, countries with relatively few financial assets and relatively high government spending on public goods can suffer a welfare loss from entering partial banking union. Since their weight at the supranational level only reflects their relative position in terms of financial assets, it underweights the costs they face in terms of public resources from shifting spending more towards the financial sector. A corollary of the above result is that for $\frac{e^H}{e^H + e^F} > \frac{z^H}{z^H + z^F}$, a weight $\eta$ that reflects relative government resources, $\eta = \frac{e^H}{e^H + e^F}$, overturns the welfare loss result in all but the most extreme of rent-seeking cases. Therefore, the welfare loss to the Home country emerges primarily from the lack of political integration to accompany financial integration.

4.4.1 Comparative Statics on $\eta^*$

Proposition 1 shows that a partial banking union decreases household utility in the country receiving transfers when the weight that the country has at the supranational level is below a threshold $\eta^*$, where the value of $\eta^*$ is derived from (20). The following results place bounds on $\eta^*$ and describe its
relationship to relative government incomes and rent-seeking incentives.

**Result 1** The value of $\eta^*$ can be bounded from below by

$$
\eta^* = \frac{1}{1 + \frac{w'(\delta H H + (1-\delta F)z_F)}{w'(\delta H H + (1-\delta F)z_F)}}
$$

where $\delta H$ and $\delta F$ are rent-seeking propensities defined above.

**Proof.** In the Appendix. ■

As (21) makes it clear, $\eta^*$ is determined by the difference between the two countries in the provision of public goods under full recapitalization of banks. Since the countries are hit by the same shock, this difference is ultimately driven by the difference in government revenues between the two countries.

We can also derive comparative statics on the effect of government revenue on $\eta^*$.

**Result 2** The threshold $\eta^*$ is an increasing function of $e^H / e^F$, it is increasing in $e^H$ and decreasing in $e^F$.

As the Home country’s government revenue is closer to the Foreign country’s government revenue, the bargaining weight that the Home country must have in order for household utility to increase is also higher. The intuition for this result is that a lower government revenue in the Home country generates a redistributive motive for the supranational authority as it is allocating the costs of recapitalizations between the two countries. As the income of the Home country increases, the redistributive motive becomes weaker, and the supranational authority assigns a higher share of the costs of recapitalizations to the Home country. The implication of this result is that, from an institutional design perspective, the bargaining weights assigned countries in a partial banking union must be increasing as a function of government revenue in order to avoid a decrease in household utility.

**Result 3** The threshold $\eta^*$ decreases as rent-seeking incentives in the Home country increase relative to the Foreign country, i.e., as $\frac{1-\gamma_H}{1-\gamma_F}$ increases.
Higher rent-seeking incentives for the Home politician imply a higher relative benefit to the politician from the banking union transfers relative to the cost of satisfying the higher intervention requirement through less public good provision. This amounts to a higher willingness of the Home politician to participate in the partial banking union, even when Home household utility decreases. For the Foreign country, a lower rent-seeking incentive means that more of the cost to the Foreign politician of making transfer $\tau$ will come from less public good provision. Since public good provision is accounted for in the supranational authority’s objective function, this means that the share of the recapitalization costs that the supranational authority assigns to the Foreign country is lower.

5 Solutions to the Inefficiency

5.1 The Role of Fiscal Rules

The banking crisis in period 0 increases the government’s need to take on debt in order to smooth the cost of recapitalizations over time. Access to public debt allows for more spending on interventions in the banking sector, since more funds can be made available in the current period at the cost of lower future public good provision. Yet, some of the interventions in the banking sector are political rents that do not provide any benefit to households. Given this political friction, a policy that could increase household utility is to limit how much the government can borrow, and consequently spend on interventions in the banking sector. In line with (Milesi-Ferretti 2004), (Corsetti and Roubini 1997), and (Halac and Yared 2015), consider the policy of each country setting a limit on debt at the beginning of period 0; before the shock $\theta$ is realized. As in the above analysis, we first look at the benchmark case without a banking union, and then analyze the effect of introducing a partial banking union once the fiscal rules are in place.
5.1.1 Fiscal Rules

In the absence of a partial banking union, countries set their debt limits at the beginning of period 0, before the shock \( \theta \) is realized. With separate debt limits for each country, this is equivalent to the decentralized fiscal rules system in (Halac and Yared 2015).\(^9\) As before, we assume that the Foreign country has sufficient funds in order to fully recapitalize it’s banks: \( x^F(\theta) = x^{F,MAX}(\theta) \). The debt limit \( \bar{b}^H \) for the Home country is set so as to maximize expected Home household utility:

\[
\max_{\bar{b}^H} \mathbb{E}_\theta [u^H(x^H(\theta), x^{F,MAX}(\theta), \theta)) + w(g^H(\theta)) + \beta w(e^H - b^H(\theta))] \tag{22}
\]

subject to

\[
u^Hr(x^H, x^{F,MAX}, \theta)) = w^H(g^H(\theta)), \tag{23}
\]
\[
\gamma^H w^H(g^H(\theta)) = (1 - \gamma^H) v^H(r^H(\theta)), \tag{24}
\]
\[
w^H(g^H(\theta)) = w^H(e^H - b^H(\theta)), \tag{25}
\]
\[
r^H(\theta) + x^H(\theta) + g^H(\theta) \leq e^H + \beta b^H(\theta), \tag{26}
\]
\[
b^H(\theta) \leq \bar{b}^H. \tag{27}
\]

Constraints (23)-(25) are the equilibrium conditions that come out of the Home government’s maximization problem with debt limit \( \bar{b}^H \). Constraint (26) represents the budget constraint of the Home government, and condition (27) represents the upper limit on public debt.

The above problem can be simplified by noticing that if \( \bar{b}^H \) binds for some \( \hat{\theta} \in \Theta \), then it binds for all \( \theta > \hat{\theta} \). For \( \Theta = [\bar{\theta}, \tilde{\theta}] \subset \mathbb{R} \), problem (22) can then be expressed as a maximization problem in which for any \( \bar{b}^H \), there exists a

\(^9\)In the context of this model, having a centralized mechanism for fiscal rules would lead to no fiscal rules (non-binding debt limits), as the supranational authority internalizes the externality coming from more spending on recapitalizations, and this leads it to prefer the same debt levels as those preferred by politicians. This analysis is available upon request.
\( \tilde{\theta}(\tilde{b}^H) \in \Theta \) such that the debt limit binds for \( \forall \theta \geq \tilde{\theta}(\tilde{b}^H) \):

\[
\max_{\theta \geq \tilde{\theta}(\tilde{b}^H)} \mathbb{E}[u(c^H(x^H(\theta), x^{F, MAX}(\theta), \theta)) + w(g^H(\theta)) + \beta w(e^H - \tilde{b}^H)] + \\
\mathbb{E}_{\theta < \tilde{\theta}(\tilde{b}^H)}[u(c^H(x^H(\theta), x^{F, MAX}(\theta), \theta)) + w(g^H(\theta)) + \beta w(e^H - b^H(\theta))],
\]

subject to (23)-(26).

In order to ensure that the objective in program (28) is concave in \( \tilde{b}^H \), we make the following assumption about the government’s utility from rent-seeking:

**Assumption 1** For any set of feasible policies \( \{x^H, g^H, r^H\} \) and \( \theta \in \Theta \) such that

\[
\frac{\partial u(x^H, x^{F, MAX}, \theta)}{\partial x} = \frac{1 - \gamma}{\gamma} \frac{\partial v(r^H)}{\partial r^H}, \\
\frac{\partial w(g^H)}{\partial g^H} = \frac{1 - \gamma}{\gamma} \frac{\partial v(r^H)}{\partial r^H},
\]

the following conditions are satisfied

\[
\frac{1 - \gamma}{\gamma} \frac{\partial}{\partial x} \left( \frac{1}{u^{HH}(x^H, x^{F, MAX}, \theta)} \right) / \partial x^H \geq \frac{\partial}{\partial r^H} \left( \frac{1}{v''(r^H)} \right) \quad (29)
\]

\[
\frac{1 - \gamma}{\gamma} \frac{\partial}{\partial g^H} \left( \frac{1}{w''(g^H)} \right) / \partial g^H \geq \frac{\partial}{\partial r^H} \left( \frac{1}{v''(r^H)} \right) \quad (30)
\]

where \( u'', v'', w'' \) denote the second derivative of functions \( u^H(x^H, x^{F, MAX}, \theta) \), \( w(g^H) \) and \( v(r^H) \), respectively.

Assumption 1 sets conditions on the shape of the government’s utility from rent-seeking relative to other uses of public funds, such that the marginal propensity to rent-seek is increasing in the size of the available government budget that period.

**Lemma 1** The objective function (28) is strictly concave in \( \tilde{b}^H \) and the maximization problem has a unique solution \( \tilde{b}^{H*} \in [b, e^H] \).
Proof. In the Appendix.

We proceed to analyze the effect of this fiscal rule on household welfare in the case without a partial banking union.

**Proposition 3** There exists $\theta^G \in \Theta$, $\theta^G < \overline{\theta}$ such that the debt limit is binding for the Home government if $\theta \geq \theta^G$. Moreover, there exists a minimum skewness of the distribution of $\theta$, $\theta_{\text{MIN}}^1(\theta)$, such that for $\theta_1(\theta) > \theta_{\text{MIN}}^1(\theta)$, fiscal rules reduce Home household utility whenever $\theta > \theta^H$, where $\theta^H > \theta^G \in \Theta$.

Proof. In the Appendix.

Proposition (3) captures the main trade-off of fiscal rules: on the one hand, they limit the government’s ability to engage in excessive spending in the first period; since part of first-period spending goes towards rents, the debt limit is beneficial to households because it reduces rents; on the other hand, the fiscal rules limit government’s ability to borrow in order to recapitalize banks in period 1. If the probability of a large shock is small, then the first effect dominates the second, and stricter debt limits are preferred; yet, if the large shock is realized, the debt limit hinders government borrowing beyond what households would prefer, and this leads to a decrease in household welfare.

For the Foreign country, we assume the analogous decision problem to (22), such that debt limit $\overline{\theta}^F$ is set, but we continue to assume that the government’s budget each period, $e^F$, is sufficiently high to ensure full recapitalization of foreign banks ($x^F(\theta) = x^{F,\text{MAX}}(\theta)$).  

### 5.2 A Partial Banking Union with Fiscal Rules

Consider the supranational authority’s problem when each country sets a debt limit as described above. Denote by $U^H(\theta, \overline{\theta}^H, \tau, x)$ the household utility in the Home country after shock $\theta$ is realized, the debt limit is $\overline{\theta}^H$, and the terms of the partial banking union are $(\tau, x)$. The problem faced by the supranational authority can then be expressed as

$$\max_{\tau, x} \{ \eta U^H(\theta, \overline{\theta}^H, \tau, x) + (1 - \eta) U^F(\theta, \overline{\theta}^F, \tau, x) \}$$

(31)
subject to

\[
(1 - \gamma)v(r^H) + \gamma U^H(\theta, \bar{b}^H, \tau, \bar{x}) \geq (1 - \gamma)v(r^{H0}) + \gamma U^H(\theta, \bar{b}^H, 0, 0), \quad (32)
\]

\[
(1 - \gamma)v(r^F) + \gamma U^F(\theta, \bar{b}^F, \tau, \bar{x}) \geq (1 - \gamma)v(r^{F0}) + \gamma U^F(\theta, \bar{b}^F, 0, 0). \quad (33)
\]

Constraints (32) and (33) represent the participation constraints for the Home and Foreign governments, respectively, given the domestically chosen debt limits. Analyzing problem (31) and comparing it to problem (14) we obtain the following result.

**Proposition 4** In the presence of fiscal rules, there exists a threshold level \( \eta^{**} \in (0, \eta^*] \) such that a partial banking union does not achieve a Pareto improvement compared to no banking union whenever \( \eta < \eta^{**} \).

**Proof.** In the Appendix.

Proposition 4 shows that domestically set fiscal rules can partially offset the Home households’ loss in welfare under a partial banking union. The intuition for this result is that domestic fiscal rules limit how much the cost of recapitalizations can be spread over the two periods. If Home government is unable to borrow more in order to finance interventions in the banking sector, then any additional spending must come from less public good provision in the current period and from outside transfers. The burden of financing banking sector interventions is therefore partially shifted towards the Foreign country.

### 5.3 The Power of Domestic Political Mechanisms along with Fiscal Rules

The previous sections have shown that an obstacle in increasing the welfare gains from partial banking union is that the composition of government spending within each period cannot be fully controlled by the supranational authority. Domestic policies can help overcome this obstacle and achieve a Pareto improvement beyond the reach of supranational rules. First, notice that any domestic institutional changes that affect the relative weight of the
political rents—for example campaign finance reforms or the change in the rules of appointment of board members in banks with public capital—increase household welfare without the banking union as well as the welfare gains from the partial banking union.

**Proposition 5** The cutoff value $\eta^{**}(\theta)$ decreases in $\gamma^H$.

**Proof.** In the Appendix. ■

Proposition 5 captures the effect of domestic institutional reforms. As the benefits from rents decrease, household utility weighs more in the participation constraint of the politician. This will limit the welfare losses to households that would be acceptable to the politician.

Another observation regarding the effects of institutional reforms is that a reduction in $\gamma^H$ can achieve a Pareto improvement even in the absence of fiscal rules, since $\eta^*$ is also a decreasing function of $\gamma^H$.

**Corollary 1** The difference $(\eta^* - \eta^{**})$ decreases in $\gamma^H$.

**Proof.** In the Appendix. ■

Corollary 1 shows that the welfare gains from fiscal rules are higher when rent-seeking incentives are higher. Even though they are not targeting rent-seeking directly, fiscal rules limit the extent to which Home public goods can be reduced in order to increase rents.

The power of domestic institutions is not limited to major institutional reforms though. Even in the absence of such reforms, domestic political forces can play a crucial role in changing the welfare gains from a partial banking union. When major reforms that change the influence of political rents $\gamma^H$ are not feasible, the political process can still play a crucial role in changing the welfare gains from a partial banking union. Moreover, together with fiscal rules, even a simplistic, myopic electoral control mechanism can ensure a Pareto improvement is achieved under the partial banking union. Consider the case in which voters remove the politician from office if household utility in the first period is below a threshold $u$ at least as large as the utility of households without any voting mechanism in place. In this case, we obtain the following result.
Proposition 6  With binding fiscal rules and a retrospective voting rule in the Home country, a partial banking union achieves a Pareto improvement in household welfare $\forall \eta \geq 0$.

Proof. In the Appendix. 

Proposition 6 shows that myopic voters are able to impose sufficient constraints on rent-seeking so as to guarantee that a Pareto improvement can be achieved whenever fiscal rules are in place. This happens because fiscal rules restrict borrowing, while the retrospective voting rule restricts any intra-period distortions of spending that could decrease household utility. Together, these two policies guarantee that a partial banking union will deliver a Pareto improvement.

5.4 Numerical Example

To illustrate the range of values that $\eta^*$ takes on as a function of the relative government revenues, consider the following example which assumes logarithmic utility functions and symmetry between the countries on all dimensions except for government revenue:

$$V^i = (1 - \gamma^i)\rho \log(1 + r^i) + \gamma^i\zeta \log(1 + c^i((x^i, x^i)) + \gamma^i \vartheta \log(1 + g^i)$$

$$+ \gamma^i \beta \vartheta \log(1 + g^i),$$

where $i \in \{H, F\}$, $\rho = 1.1$, $\zeta = 4.5$, $\vartheta = 1$, $z^H = z^F = 10$, and the values of $\gamma^i$ are given below.

The following graph shows that value of $\eta^*$ for different values of $e^H/e^F$. The black line illustrates the case in which $\gamma^H = \gamma^F = 0.8$, while the red line illustrates the case when $\gamma^H = 0.8$ and $\gamma^F = 1$. The value of $\eta^*$ increases with $e^H/e^F$, up until the value of $e^H/e^F$ at which the difference in government budgets between the two countries is sufficiently low that no transfers are optimal for the supranational authority to implement, and the optimal solution is to have no partial banking union.
6 Conclusion

This paper presented a model of a partial banking union with domestic rent seeking and showed that implementing such a banking union can reduce household welfare in the country receiving transfers. The result emerges when the bargaining weights of countries at the supranational level are linked to their relative financial assets and not to their overall public finances. This points out to the inefficiency of a partial banking union emerges from a system in which financial integration is not accompanied by political integration. Fiscal rules meant to reduce rent-seeking may do so, but at the cost of lower welfare in both the donor and receiving country compared to the case without fiscal rules. These implications seem relevant for the proposed banking union in the Eurozone, in which not all decisions regarding interventions in the banking sector can be centralized.
References


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A Appendix

A.1 The Home and Foreign Government Maximization Problem

Without the banking union, the Home government chooses policies \( \alpha^{H0} \equiv \{ r^{H0}, x^{H0}, g^{H0}, g_1^{H0}, b_1^{H0} \} \) in order to maximize

\[
\max_{\alpha^{H0}} (1 - \gamma^H) v(r^{H0}) + \gamma^H u^H(x^{H0}, x^{F0}, \theta) + \gamma^H w(g^{H0}) + \beta \gamma^H w(g_1^{H0}),
\]

subject to

\[
\begin{align*}
    r^{H0} + x^{H0} + g^{H0} &\leq e^H + \beta b_1^{H0}, \\
g_1^{H0} &\leq e^H - b_1^{H0}, \\
b_1^{H0} &\leq e^H, \\
b_1^{H0} &\geq b, \\
x^{H0} &\leq R\theta (\alpha^H z^H + (1 - \alpha^F)z^F).
\end{align*}
\]

Let be the \( \lambda^{H0} \) and \( \beta \mu^{H0} \) be the Lagrange multipliers on constraints (34a) and (34b), respectively. The first-order conditions for an interior solution to the above problem are

\[
\begin{align*}
    (1 - \gamma^H) v_r(r^{H0}) &= \lambda^{H0}, \\
    \gamma^H u_{x^{H0}}^H(x^{H0}, x^{F0}, \theta) &= \lambda^{H0}, \\
    w_g(g^{H0}) &= \lambda^{H0}, \\
    w_g(g_1^{H0}) &= \mu^{H0}, \\
    \lambda^{H0} &= \mu^{H0}.
\end{align*}
\]

Therefore, the above conditions imply

\[
\begin{align*}
    (1 - \gamma^H) v_{r^{H0}}(r^{H0}) &= \gamma^H u_{x^{H0}}^H(x^{H0}, x^{F0}, \theta) = w_{g^{H0}}(g^{H0}) = w_{g^{H0}}(g_1^{H0}), \\
    g^{H0} &= g_1^{H0}.
\end{align*}
\]
In a partial banking union, the Home government chooses policies $\alpha^H \equiv \{r^H, x^H, g^H, g_1^H, b_1^H\}$ given $\{\tau, x\}$ in order to maximize

$$\max_{\alpha^H} (1 - \gamma^H) v(r^H) + \gamma^H u^H(x^H, x^F, \theta) + \gamma^H w(g^H) + \beta \gamma^H w(g_1^H),$$

subject to

$$r^H + x^H + g^H \leq e^H + \beta b_1^H + \tau, \quad (35a)$$
$$x^H + r^H \geq x, \quad (35b)$$
$$g_1^H \leq e^H - b_1^H, \quad (35c)$$
$$b_1^H \leq e^H, \quad (35d)$$
$$b_1^H \geq \underline{b}, \quad (35e)$$
$$x^H \leq R \theta (\alpha^H z^H + (1 - \alpha^F) z^F). \quad (35f)$$

Let be the $\lambda^H$, $\vartheta^H$, and $\beta \mu^H$ be the Lagrange multipliers on constraints (35a), (35b), and (35c), respectively. The first-order conditions for an interior solution to the above problem are

$$(1 - \gamma^H) v_r(r^H) = \lambda^H - \vartheta^H,$$
$$\gamma^H u_{x^H}(x^H, x^F, \theta) = \lambda^H - \vartheta^H,$$
$$w_g(g^H) = \lambda^H,$$
$$w_g(g_1^H) = \mu^H,$$
$$\lambda^H = \mu^H.$$ 

Therefore, the above conditions imply

$$(1 - \gamma^H) v_{r^H}(r^H) = \gamma^H u_{x^H}^H(x^H, x^F, \theta),$$
$$g_1^H = g^H.$$ 

A binding constraint (35b) implies that $r^H > r^{H0}$, $x^H > x^{H0}$, and $g^{H0} > g^H$. 

35
For the Foreign government, we focus on the case in which the government has sufficient revenue in the first period in order to provide full recapitalizations to its banks: \( x^F = x^{F,\text{MAX}} \). Without a baking union, the problem for the Foreign government is to choose policies \( F_0 \) in order to maximize

\[
\max_{\alpha^{F,0}} (1 - \gamma^F) v(r^{F,0}) + \gamma^F u^{F}(x^{F,0}, x^{H,0}, \theta) + \gamma^F w(g^{F,0}) + \beta^F w(\beta^{F,0}),
\]

subject to

\[
\begin{align*}
    r^{F,0} + x^{F,0} + g^{F,0} & \leq e^F + \beta^{F,0}, \\
    g^{F,0}_{1,0} & \leq e^F - b^{F,0}, \\
    b^{F,0} & \leq e^F, \\
    b^{F,0}_{1,0} & \geq b, \\
    x^{F,0} & \leq R\theta \left( \alpha^F z^F + (1 - \alpha^H)z^H \right).
\end{align*}
\]

Let be the \( \lambda^{F,0}, \beta^{F,0}, \) and \( \xi^{F,0} \) be the Lagrange multipliers on constraints (36a), (36b), and (36e), respectively. The first-order conditions for a solution in which constraint (36e) binds are

\[
\begin{align*}
    (1 - \gamma^F) v_r(r^{F,0}) & = \lambda^{F,0}, \\
    \gamma^F u_{x^{F,0}}(x^{F,0}, x^{H,0}, \theta) & = \lambda^{F,0} + \xi^{F,0}, \\
    w_g(g^{F,0}) & = \lambda^{F,0}, \\
    w_{g_1}(g^{F,0}_{1,0}) & = \mu^{F,0}, \\
    \lambda^{F,0} & = \mu^{F,0}.
\end{align*}
\]

Given \( x^{F,0} = R\theta \left( \alpha^F z^F + (1 - \alpha^H)z^H \right) \), let \( r^{F,\text{MAX}} \) and \( g^{F,\text{MAX}} \) be the values at which

\[
\gamma^F w_{g^{F,\text{MAX}}}(g^{F,\text{MAX}}) = (1 - \gamma^F) v_r(r^{F,\text{MAX}}) = \gamma^F u_{x^{F,\text{MAX}}}(x^{F,\text{MAX}}, 0, \theta).
\]
Then, a sufficient condition for full recapitalization to be achieved is that
\[ e^F \geq \frac{r^F,M\!AX + x^F,M\!AX}{1 + \beta} + g^F,M\!AX. \]

The above condition ensures that the Foreign government finds it optimal to finance the full recapitalization need of its banks even when the Home government provides no recapitalization funds.

With a partial banking union, the problem for the Foreign government is to choose policies \( \alpha^F \equiv \{ r^F, x^F, g^F, g_1^F, b_1^F \} \) given \( \{ \tau, \varepsilon \} \) in order to maximize
\[
\max_{\alpha^F} (1 - \gamma^F) v_r(r^F) + \gamma^F u^F(x^F, x^H(\tau, \varepsilon), \theta) + \gamma^F w(g^F) + \beta \gamma^F w(g_1^F),
\]
subject to
\[
\begin{align*}
& r^F + x^F + g^F \leq e^F + \beta b_1^F - \tau, \quad (37a) \\
& g_1^F \leq e^F - b_1^F, \quad (37b) \\
& b_1^F \leq e^F, \quad (37c) \\
& b_1^F \geq b_2, \quad (37d) \\
& x^F \leq R\theta \left( \alpha^F z^F + (1 - \alpha^H) z^H \right). \quad (37e)
\end{align*}
\]

Let be the \( \lambda^F, \beta^F, \xi^F \) be the Lagrange multipliers on constraints (36a), (36b), and (36e), respectively. The first-order conditions for a solution in which constraint (36e) binds are
\[
\begin{align*}
(1 - \gamma^F) v_r(r^F) &= \lambda^F, \\
\gamma^F u^F_{xx}(x^F, x^H, \theta) &= \lambda^F + \xi^F, \\
\lambda^F &= \mu^F,
\end{align*}
\]
\[
\begin{align*}
w_g(g^F) &= \lambda^F, \\
w_{g_1}(g_1^F) &= \mu^F,
\end{align*}
\]

Then, \( x^F = R\theta \left( \alpha^F z^F + (1 - \alpha^H) z^H \right) \), and \( r^F,M\!AX \) and \( g^F,M\!AX \) be the
values at which
\[ \gamma^F w_{g^F,\text{MAX}}(g^F) = (1 - \gamma^F)v_r(r^F). \]

A.2 The Supranational Authority’s Maximization Problem

The supranational authority chooses \{\tau, \underline{x}\} maximizes
\[
\max_{\tau, \underline{x}} \eta \left[ u^H(x^H, x^F, \theta) + w(g^H) + \beta w(g^H_1) \right] \\
+ (1 - \eta)\left[ u^F(x^F, x^H, \theta) + w(g^F) + \beta w(g^F_1) \right]
\]
subject to
\[
V^H(r^H, x^H, x^F, g^H, g^H_1) \geq V^H(r^{H0}, x^{H0}, x^{F0}, g^{H0}, g^{H0}_1), \\
V^F(r^F, x^H, x^F, g^F, g^F_1) \geq V^F(r^{F0}, x^{F0}, x^{H0}, g^{F0}, g^{F0}_1).
\]

For an internal solution, the first-order conditions for \tau and \underline{x}, with \underline{x} binding for the Home government, are
\[
\eta w_g(g^H) = (1 - \eta)w_g(g^F)\frac{\partial g^F}{\partial \tau}, \\
[\eta u^H_{x^H}(x^H, x^F, \theta) + (1 - \eta)u^F_{x^H}(x^F, x^H, \theta)] \frac{\partial x^H}{\partial \underline{x}} = \eta w_g(g^H),
\]
where \(g^H(\tau, \underline{x}, \theta), x^H(\tau, \underline{x}, \theta), g^F(\tau, \underline{x}, \theta), x^F(\tau, \underline{x}, \theta)\) are the solutions that come out of the governments’ maximization problems.

A.3 The Planning Problem with Fiscal rules

The planner’s two-step problem for choosing the debt limit \(b^H(\tilde{\theta})\) can be expressed as:
$$\max_{\theta \in \Theta} \mathbb{E}_{\theta \geq \tilde{\theta}} [u(c^H(x^H(\theta), x^{F,\text{MAX}}(\theta), \theta)) + w(g^H(\theta)) + \beta w(e^H - \tilde{b}^H(\tilde{\theta}))] +$$
$$\mathbb{E}_{\theta \leq \tilde{\theta}} [u(c^H(x^H(\theta), x^{F,\text{MAX}}(\theta), \theta)) + w(g^H(\theta)) + \beta w(e^H - b^H(\theta))], \quad (43)$$

with

$$\tilde{b}^H(\tilde{\theta}) = \arg \max_{b^H \geq b^H(\tilde{\theta})} \mathbb{E}_{\theta \geq \tilde{\theta}} [u(c^H(x^H(\theta), x^F, \theta)) + w(g^H(\theta)) + \beta w(e^H - \tilde{b}^H)], \quad (44)$$

subject to

$$x^H + g^H \leq e^H + \beta \tilde{b}^H + \tau, \quad (45a)$$
$$g^H_1 \leq e^H - \tilde{b}^H, \quad (45b)$$
$$x^H \leq R\tilde{\theta} \left( \alpha^H z^H + (1 - \alpha^F) z^F \right). \quad (45c)$$

The value of $b^H(\tilde{\theta})$ is obtained from the Home government’s unconstrained maximization problem when $\theta = \tilde{\theta}$:

$$b^H(\tilde{\theta}) = \arg \max_{r^H, x^H, g^H, b^H} [(1 - \gamma^H)v(r^H) + \gamma^H u(c^H(x^H, x^F, \tilde{\theta})) + \gamma^H w(g^H) + \gamma^H \beta w(e^H - b^H)],$$

subject to

$$r^H + x^H + g^H \leq e^H + \beta b^H + \tau, \quad (46a)$$
$$g^H_1 \leq e^H - b^H, \quad (46b)$$
$$b^H \geq \tilde{b}, \quad (46c)$$
$$x^H \leq R\tilde{\theta} \left( \alpha^H z^H + (1 - \alpha^F) z^F \right). \quad (46d)$$
A.4 Proofs

A.4.1 Proof of Proposition 1

The supranational authority sets $\tau \geq 0$ and $\bar{x} \geq 0$ in order to maximize (14) given (15) and (16). Constraint (15) expands to

$$(1 - \gamma^H) v(r^H) + \gamma^H U^H(x^H, x^F, g^H, g^H_1) \geq (1 - \gamma^H) v(r^{H0})$$

$$+ \gamma^H U^H(x^{H0}, x^{F0}, g^{H0}, g^{H0}_1)$$

where $x^F = x^{F0}$ are the policies chosen by the Home government given $(\tau, \bar{x})$ and $(r^{H0}, x^{H0}, g^{H0}, g^{H0}_1)$ are the policies chosen by the Home government without a banking union.

The minimum intervention level requires $r^H + x^H \geq \bar{x}$; if $\bar{x}$ is binding, this implies that $r^H + x^H > r^{H0} + x^{H0}$; otherwise $r^H + x^H = r^{H0} + x^{H0}$. Since the politician’s utility from rents $r^H$ and recapitalizations $x^H$ is concave and additive, a binding intervention level $\bar{x}$ induces the politician to increase both rents and recapitalizations, so $r^H > r^{H0}$ and $x^H > x^{H0}$. This implies $v(r^H) > v(r^{H0})$. Therefore, (47) is satisfied as long as

$$U^H(x^{H0}, x^{F0}, g^{H0}, g^{H0}_1) - U^H(x^H, x^F, g^H, g^H_1) \leq \frac{(1 - \gamma^H)}{\gamma^H} [v(r^H) - v(r^{H0})].$$

Therefore, (47) is satisfied if $U^H(x^{H0}, x^{F0}, g^{H0}, g^{H0}_1) - U^H(x^H, x^F, g^H, g^H_1) < 0$ or

$$0 \leq U^H(x^{H0}, x^{F0}, g^{H0}, g^{H0}_1) - U^H(x^H, x^F, g^H, g^H_1) \leq \frac{(1 - \gamma^H)}{\gamma^H} [v(r^H) - v(r^{H0})].$$

We first show that $U^H(x^H, x^F, g^H, g^H_1)$ is an increasing function of $\eta$. With $\bar{x}$ binding, the Home government’s budget constraint is

$$g^H + \beta g^H_1 \leq e^H (1 + \beta) + \tau - \bar{x}.$$ 

The Home government utility is therefore increasing in $\tau - \bar{x}$. For a binding $\bar{x}$,
it is decreasing in $x$. For the supranational authority, the first-order conditions to the maximization problem when the participation constraints do not bind are

$$
\left[u_{xH}^H(x^H, x^F, \theta) + \frac{(1 - \eta)}{\eta} u_{xH}^F(x^F, x^X, \theta)\right] \frac{\partial x^H}{\partial x} = w_g(g^H) \quad (49)
$$

$$
w_g(g^H) = \frac{(1 - \eta)}{\eta} w_g(g^F) \frac{\partial g^F}{\partial \tau} \quad (50)
$$

From the Home government’s maximization problem, $\frac{\partial x^H}{\partial x}$ is increasing in $x$ and $\frac{\partial g^H}{\partial \tau} = -1$. Then, (49) implies that the equilibrium $x$ decreases in $\eta$. From the Foreign government’s maximization problem, $-1 \leq \frac{\partial g^F}{\partial \tau} < 0$ and from the Home government’s maximization problem when $x$ binds, $\frac{\partial g^H}{\partial \tau} = -1$. Condition (50) implies that the equilibrium $\tau$ is increasing in $\eta$. Therefore, $\tau - x$ is increasing in $\eta$, and consequently $U^H(x^H, x^F, g^H, g_1^H)$ is increasing in $\eta$.

At $\eta \to 0$, the supranational authority maximizes the utility of the Foreign households only, so the optimal choice for $x$ will be the maximum value at which the participation constraint for the Home government binds. In this case, from (48),

$$
U^H(x^{H0}, x^{F0}, g^{H0}, g_1^{H0}) - U^H(x^H, x^F, g^H, g_1^H) = \frac{(1 - \gamma^H)}{\gamma^H} \left[v(r^H) - v(r^{H0})\right],
$$

so

$$
U^H(x^H, x^F, g^H, g_1^H) < U^H(x^{H0}, x^{F0}, g^{H0}, g_1^{H0}).
$$

At $\eta \to 1$, the supranational authority maximizes the utility of the Home households only, so we have a corner solution with non-binding $x$ and transfers at the maximum level at which the participation constraint for the Foreign government still holds. It then follows that $U^H(x^H, x^F, g^H, g_1^H) \geq U^H(x^{H0}, x^{F0}, g^{H0}, g_1^{H0})$. Given the monotonicity of $U^H(x^H, x^F, g^H, g_1^H)$ with respect to $\eta$, there exists $\eta^* \in (0, 1)$ such that

$$
U^H(x^{H0}, x^{F0}, g^{H0}, g_1^{H0}) - U^H(x^H(\eta^*), x^F(\eta^*), g^H(\eta^*), g_1^H(\eta^*)) = 0. \quad (51)
$$
A.4.2 Proof of Proposition 2

The value of \( \eta^* \) satisfies

\[
\begin{align*}
u^H(x^H(\eta^*), x^{F,MAX}, \theta) + (1 + \beta)w(g^H(\eta^*)) & = u^H(x^{H0}, x^{F,MAX}, \theta) \nonumber \\
+ (1 + \beta)w(g^{H0}). \quad (52)
\end{align*}
\]

Also, \( \frac{\partial^2 u^H(x^H, x^{F,MAX}, \beta)}{\partial (x^H)^2} < 0 \), so \( \frac{\partial u^H(x^H(\eta), x^{F,MAX}, \theta)}{\partial x^H} < \frac{\partial u^H(x^{H0}, x^{F,MAX}, \beta)}{\partial x^{H0}} \) and

\[
\begin{align*}
u^H(x^H(\eta), x^{F,MAX}, \theta) - u^H(x^{H0}, x^{F,MAX}, \theta) & = \int_{x^{H0}}^{x^H(\eta)} \frac{\partial u^H(x^H, x^{F,MAX}, \theta)}{\partial x^H} dx^H \\
< & \int_{x^{H0}}^{x^H(\eta)} \frac{\partial u^H(x^H, x^{F,MAX}, \theta)}{\partial x^H} \bigg|_{x^H = x^{H0}} dx^H \\
= & \frac{\partial u^H(x^H, x^{F,MAX}, \theta)}{\partial x^H} \bigg|_{x^H = x^{H0}} (x^H(\eta) - x^{H0}). \quad (53)
\end{align*}
\]

Also,

\[
\begin{align*}
w(g^{H0}) - w(g^H(\eta)) & = \int_{g^H(\eta)}^{g^{H0}} w'(g^H) dg^H \\
& > \int_{g^H(\eta)}^{g^{H0}} w'(g^H) \bigg|_{g^H = g^{H0}} dg^H \\
& = w'(g^{H0})(g^{H0} - g^H(\eta)). \quad (54)
\end{align*}
\]

Then, (52) holds if

\[
\frac{\partial u^H(x^{H0}, x^{F,MAX}, \beta)}{\partial x^{H0}} (x^H(\eta^*) - x^{H0}) > (1 + \beta)w'(g^{H0})(g^{H0} - g^H(\eta^*)).
\]

From the Home government’s first-order conditions, \( \frac{\partial w^H(g^{H0})}{\partial g^{H0}} = \frac{\partial w^H(g^{H0})}{\partial g^{H0}} \), so (52) holds if

\[
(x^H(\eta^*) - x^{H0}) > (1 + \beta)(g^{H0} - g^H(\eta^*)). \quad (55)
\]
Let $\Delta x(\eta) \equiv x^H(\eta) - x^{H0}$, $\Delta g(\eta) \equiv (1 + \beta) \left( g^{H0} - g^H(\eta) \right)$ and $\Delta x(\eta) = x^H(\eta) + r^H(\eta) - x^{H0} - r^{H0}$. Then, Also, notice that $x^H(\eta) + r^H(\eta) - x^{H0} - r^{H0} = (1 + \beta) \left( g^{H0} - g^H(\eta) \right) + \tau(\eta)$, so

$$\frac{\Delta x(\eta^*)}{\Delta x(\eta^*)} > \frac{\Delta g(\eta^*)}{\Delta g(\eta^*) + \tau(\eta^*)}.$$ 

Then, let $\delta^H \equiv \frac{\Delta x(\eta^*)}{\Delta x(\eta^*)}$, and re-write the above as

$$\frac{\tau(\eta^*)}{\Delta g(\eta^*)} > \frac{1 - \delta^H}{\delta^H}.$$ 

Then, $\tau(\eta^*) > \frac{1 - \delta^H}{\delta^H} \Delta g(\eta^*)$, which implies

$$\frac{w'(g^{H0} - \Delta g(\eta^*))}{w'(g^{F0} - \frac{1 - \delta^H}{\delta^H} \Delta g(\eta^*) \frac{\Delta g(\eta^*)}{\tau(\eta^*)})} > \frac{w'(g^{H0} - \Delta g(\eta^*))}{w'(g^{F0} - \tau(\eta^*) \frac{\Delta g(\eta^*)}{\tau(\eta^*)})}.$$ 

If $w'(\cdot)$ satisfies the property of decreasing absolute risk aversion, $\frac{w''(g)}{w'(g)}$ is an increasing function of $g$. So

$$\partial \left( \frac{w'(g^{H0} - \Delta \tilde{g})}{w'(g^{F0} - \frac{1 - \delta^H}{\delta^H} \Delta \tilde{g} \frac{\Delta g(\eta^*)}{\tau(\eta^*)})} \right) / \partial \Delta \tilde{g} > 0.$$ 

Let $\Delta \tilde{g} = \delta^H \cdot \theta (\alpha^H z^H + (1 - \alpha^F) z^F)$. Then, $\Delta \tilde{g} > \Delta g(\eta^*)$, and

$$\frac{w'(g^{H0} - \Delta \tilde{g})}{w'(g^{F0} - \frac{1 - \delta^H}{\delta^H} \Delta \tilde{g} \frac{\Delta g(\eta^*)}{\tau(\eta^*)})} > \frac{w'(g^{H0} - \Delta g(\eta^*))}{w'(g^{F0} - \frac{1 - \delta^H}{\delta^H} \Delta g(\eta^*) \frac{\Delta g(\eta^*)}{\tau(\eta^*)})}. \quad (56)$$

Finally, let

$$\delta^H = \max_{x^H \in [x^{H0}, x^{H, MAX}]} \left\{ \frac{1}{1 + \frac{\gamma^H u^{Hn}(x^H, x^{F, MAX}, \theta)}{(1 - \gamma^H) u' (r^H)}} \right\}$$

subject to

$$(1 - \gamma^H) u' (r^H) = \gamma^H u'H(x^H, x^{F, MAX}, \theta).$$
and
\[
\delta^F = \min_{g^F \in [0,x^H,\text{MAX} - x^H,\text{MAX} - x^H_0 - r, x^H_0]} \left\{ \frac{1}{1 + \frac{\gamma^F w'(g^F)}{(1-\gamma^F)w'(\tau - g^F)}} \right\}
\]

subject to
\[
\gamma^F w'(g^F) = (1 - \gamma^F)w'(\tau - g^F).
\]

Then,
\[
\frac{w'(g^{H_0} - \delta^H \cdot \theta(\alpha^H z^H + (1 - \alpha^F)z^F))}{w'\left(g^{F_0} - \left(1 - \delta^H\right)\delta^F \theta(\alpha^H z^H + (1 - \alpha^F)z^F)\right)} \geq \frac{w'(g^{H_0} - \Delta g)}{w'\left(g^{F_0} - \frac{1 - \delta^H + \Delta g}{\delta^F} \cdot \Delta g^F(\tau(\eta))\right)}
\]

From the supranational authority’s maximization problem, first-order condition (41):
\[
\eta w'(g^H) = (1 - \eta)w'(g^F) \frac{\partial g^F}{\partial \tau},
\]
so
\[
\eta = \frac{1}{1 + \frac{w'(g^H)}{w'(g^F)\frac{\partial g^F}{\partial \tau}}}
\]

For \( \eta = \frac{z^H}{z^H + z^F} \), this means
\[
\frac{z^F}{z^H} = \frac{w'(g^H)}{w'(g^F)\frac{\partial g^F}{\partial \tau}},
\]
\[
\frac{z^F}{z^H} = \frac{w'(g^{H_0} - \Delta g(\eta))}{w'(g^{F_0} - \Delta g^F(\eta))\frac{\partial g^F}{\partial \tau}},
\]
where \( \Delta g^F(\eta) = (1 + \beta) \left[ g^{F_0} - g^F(\tau(\eta)) \right] \).

The relationship \( \eta < \eta^* \) implies \( \frac{z^F}{z^H} > \frac{w'(g^{H_0} - \Delta g(\eta^*))}{w'(g^{F_0} - \frac{1 - \delta^H + \Delta g(\eta^*)}{\delta^F} \Delta g^F(\tau(\eta^*)))} \), so a sufficient condition for this to hold is that
\[
\frac{z^F}{z^H} \geq \frac{w'(g^{H_0} - \delta^H \cdot \theta(\alpha^H z^H + (1 - \alpha^F)z^F))}{w'\left(g^{F_0} - \left(1 - \delta^H\right)\delta^F \theta(\alpha^H z^H + (1 - \alpha^F)z^F)\right)}.
\]
This condition implies (57), which implies (56).

A.4.3 Proof of Result 1

At \( \eta^* \) the Home household utility is the same as without a banking union. Since \( r^H > r^{H0} \), this implies that \( V^H(r^H, x^H, x^F, g^H, g^H_1) \geq V^H(r^{H0}, x^{H0}, x^{F0}, g^{H0}, g^{H0}_1) \), and the participation constraint for the Home government in the supranational authority’s problem is not binding.

Consider first the case in which the participation constraint for the Foreign government is also not binding. From the first-order condition to the supranational authority’s maximization problem, \( \eta^* \) satisfies the first-order condition (41), so

\[
\eta^* = \frac{1}{1 + \frac{w_g(g^H)}{w_g(g^F) \frac{\partial g^F}{\partial \theta}}} > \frac{1}{1 + \frac{w^t(g^{H0} - \theta(\alpha H z^H + (1 - \alpha F) z^F))}{w^t(g^{F0} - (1 - \delta^H) \delta \theta(\alpha H z^H + (1 - \alpha F) z^F)}}.
\]

The inequality follows from the proof to Proposition 2.

A.4.4 Proof of Result 2

Let \( \tau(\eta^*) \) and \( \underline{x}(\eta^*) \) be the solution to the supranational authority’s problem, derived from (41) and (42).

At \( \eta^* \), the Home household utility is the same as without the banking union:

\[
\begin{align*}
&u^H(x^H(\tau(\eta^*), \underline{x}(\eta^*), \theta), x^{F, MAX}, \theta) + w(g^H(\tau(\eta^*), \underline{x}(\eta^*), \theta)) \\
&+ \beta w(g^H_1(\tau(\eta^*), \underline{x}(\eta^*), \theta)) = u^H(x^{H0}, x^{F, MAX}, \theta) + w(g^{H0}) + \beta w(g^{H0}_1).
\end{align*}
\]
Consider an increase in $e^H$ holding $e^F$ constant. By the Envelope Theorem,

$$
u_{x}^{H}(x^{H}(\tau(\eta^{*}), x(\eta^{*}), \theta), x_{F, M A X}, \theta) \frac{\partial e^{H}}{\partial x} \left( \frac{\partial \tau}{\partial \eta} \frac{\partial \eta^{*}}{\partial e^{H}} + \frac{\partial x}{\partial e^{H}} \right) + (1+\beta)w_{g}(g^{H}(\tau(\eta^{*}), x(\eta^{*}), \theta)) \left( 1 + \frac{1}{1+\beta} \left( \frac{\partial \tau}{\partial \eta} \frac{\partial \eta^{*}}{\partial e^{H}} - \frac{\partial x}{\partial e^{H}} - \frac{\partial x}{\partial e^{H}} - \frac{\partial \tau}{\partial e^{H}} \right) \right) = 0.$$  

(58)

$$\frac{\partial \eta^{*}}{\partial e^{H}} = - \frac{u_{x}^{H}(\cdot) \frac{\partial x}{\partial x} \frac{\partial \eta^{*}}{\partial e^{H}} + w_{g}(\cdot) \left( 1 + \beta + \frac{\partial \tau}{\partial \eta} - \frac{\partial x}{\partial x} \right)}{u_{x}^{H}(\cdot) \frac{\partial x}{\partial x} \frac{\partial \eta^{*}}{\partial e^{H}} + w_{g}(\cdot) \left( \frac{\partial \tau}{\partial \eta} - \frac{\partial x}{\partial x} \right)}.$$  

From (42), $\frac{\partial \tau}{\partial \eta} > 0$ since $\frac{\partial x}{\partial x} > 0$, and $\frac{\partial x}{\partial \eta} < 0$. From (41), $\frac{\partial \tau}{\partial \eta} < 0$ and $\frac{\partial x}{\partial \eta} > 0$.

The Home household utility under the banking union is the same as without the banking union. Since without the banking union $u_{x}^{H}(x^{H_{0}}, x^{F_{0}}, \theta) = w_{g}(g^{H_{0}})$, this implies $u_{x}^{H}(\cdot) \frac{\partial x}{\partial x} < w_{g}(\cdot)$. Then, sign of the denominator in expression (58) is positive. For the numerator, notice that, applying the Envelope Theorem in (41):

$$\eta w_{g}(g^{H}) + \eta w_{g}(g^{H}) \frac{\partial g^{H}}{\partial \tau} \frac{\partial \tau}{\partial e^{H}} = (1-\eta)w_{g}(g^{F}) \left( \frac{\partial g^{F}}{\partial \tau} \right)^{2} \frac{\partial \tau}{\partial e^{H}} + (1-\eta)w_{g}(g^{F}) \frac{\partial^{2} g^{F}}{\partial \tau^{2}} \frac{\partial \tau}{\partial e^{H}},$$

so

$$\frac{\partial \tau}{\partial e^{H}} = \frac{\eta w_{g}(g^{H})}{-\eta w_{g}(g^{H}) \frac{\partial g^{H}}{\partial \tau} + (1-\eta)w_{g}(g^{F}) \left( \frac{\partial g^{F}}{\partial \tau} \right)^{2} + (1-\eta)w_{g}(g^{F}) \frac{\partial^{2} g^{F}}{\partial \tau^{2}} \frac{\partial \tau}{\partial e^{H}},}$$

$$\frac{\partial \tau}{\partial e^{H}} = \frac{\eta w_{g}(g^{H})}{-\eta w_{g}(g^{H}) \frac{1}{1+\beta} + (1-\eta)w_{g}(g^{F}) \left( \frac{\partial g^{F}}{\partial \tau} \right)^{2} + (1-\eta)w_{g}(g^{F}) \frac{\partial^{2} g^{F}}{\partial \tau^{2}} \frac{\partial \tau}{\partial e^{H}},}$$

$$\frac{\partial \tau}{\partial e^{H}} = - \frac{1 + \beta}{1 - (1+\beta) \frac{1-\eta}{\eta} \frac{w_{g}(g^{F})}{w_{g}(g^{F})} \left( \frac{\partial g^{F}}{\partial \tau} \right)^{2} - (1+\beta) \frac{1-\eta}{\eta} \frac{w_{g}(g^{F})}{w_{g}(g^{F})} \frac{\partial^{2} g^{F}}{\partial \tau^{2}} \frac{\partial \tau}{\partial e^{H}},}$$

$$< - (1+\beta).$$

Then, $w_{g}(\cdot) (1 + \beta + \frac{\partial x}{\partial \eta}) < 0$ and $\frac{\partial x}{\partial \eta} \left( u_{x}^{H}(\cdot) \frac{\partial x}{\partial x} - w_{g}(\cdot) \right) < 0$, so the
denominator of (58) is positive. Then $\frac{\partial \eta^*}{\partial e_F} > 0$.

For a change in $e_F$, the analysis is analogous, with a negative numerator in the expression for $\frac{\partial \eta^*}{\partial e_F}$ analogous to (58).

At $\eta^*$, the Home household utility is the same as without the banking union:

$$u^H(x^H(\tau(\eta^*), \bar{x}(\eta^*), \theta), x^{F,\text{MAX}}, \theta) + w(g^H(\tau(\eta^*), \bar{x}(\eta^*), \theta))$$

$$+ \beta w(g_1^H(\tau(\eta^*), \bar{x}(\eta^*), \theta)) = u^H(x^{H0}, x^{F,\text{MAX}}, \theta) + w(H0) + \beta w(g_1^{H0}).$$

### A.4.5 Proof of Result 3

Consider now a decrease in $\gamma^H$ holding everything else constant. In this case, the implicit change in $\eta^*$ is given by:

$$
\begin{align*}
&u^H_{\gamma^H}(x^H(\tau(\eta^*), \bar{x}(\eta^*), \theta), x^{F,\text{MAX}}, \theta) \left[ \frac{\partial x^H}{\partial \gamma^H} + \frac{\partial x^H}{\partial \eta^*} \left( \frac{\partial \eta^*}{\partial \gamma^H} + \frac{\partial \tau}{\partial \gamma^H} \right) \right] \\
&+ (1 + \beta) w'(g^H(\tau(\eta^*), \bar{x}(\eta^*), \theta)) \left( \frac{\partial g^H}{\partial \gamma^H} \right) + \left( \frac{1}{1 + \beta} \left( \frac{\partial \eta^*}{\partial \gamma^H} - \frac{\partial \tau}{\partial \gamma^H} + \frac{\partial \tau}{\partial \gamma^H} \right) \right)
\end{align*}
$$

So

$$\frac{\partial \eta^*}{\partial \gamma^H} = \frac{\Psi}{u^H_{\gamma^H}(\cdot)} \left( \frac{\partial x^H}{\partial \gamma^H} + \frac{\partial x^H}{\partial \eta^*} \frac{\partial \tau}{\partial \gamma^H} \right) + (1 + \beta) w'(g^H) \frac{1}{1 + \beta} \left( \frac{\partial \eta^*}{\partial \gamma^H} - \frac{\partial \tau}{\partial \gamma^H} \right),$$

where

$$\Psi = w(g^{H0}) \left( \frac{\partial x^{H0}}{\partial \gamma^H} + (1 + \beta) \frac{\partial g^{H0}}{\partial \gamma^H} \right) - u^H_{\gamma^H}(\cdot) \left( \frac{\partial x^H}{\partial \gamma^H} + \frac{\partial x^H}{\partial \eta^*} \frac{\partial \tau}{\partial \gamma^H} \right)$$

$$- w(g^H) \left( \frac{\partial g^H}{\partial \gamma^H} \right) + \left( \frac{\partial \eta^*}{\partial \gamma^H} - \frac{\partial \tau}{\partial \gamma^H} \right).$$
Since $\frac{\partial f^H}{\partial \gamma^H} = 0$,

$$
\frac{\partial \eta^*}{\partial \gamma^H} = \frac{w_g(g^{H0})}{u^H_x(\eta^*, \varphi^*(\eta^*), \theta, x^{F,F,MAX}, \theta)} \frac{\partial x^H}{\partial \eta^*} \left( \frac{\partial \tau}{\partial \gamma^F} + \frac{\partial x^L}{\partial \eta^*} \right) + \left( \frac{\partial \tau}{\partial \gamma^F} \right) \left( \frac{\partial \eta^*}{\partial \gamma^F} + \frac{\partial x^L}{\partial \eta^*} \right) = 0.
$$

Consider an increase in $\gamma^F$ holding all else equal. Then,

$$
\frac{\partial \eta^*}{\partial \gamma^F} = -\frac{u^H_x(\eta^*, \varphi^*(\eta^*), \theta, x^{F,F,MAX}, \theta)}{\left( \frac{\partial \tau}{\partial \gamma^F} \right)^2} \frac{\partial \tau}{\partial \gamma^F} \frac{\partial x^L}{\partial \eta^*} - \frac{w_g(g^H)}{\left( \frac{\partial \tau}{\partial \gamma^F} \right)^2} \frac{\partial \tau}{\partial \gamma^F} \left( \frac{\partial x^L}{\partial \eta^*} \right).
$$

From the Foreign government’s maximization problem, it follows that an increase in $\gamma^F$ implies $\frac{\partial \gamma^F}{\partial \gamma^F} > 0$ and $\frac{\partial \gamma^F}{\partial \gamma^F} < 0$, while $x^{F,F,MAX}$. Then, from (41) and (42) and $\frac{\partial \gamma^F}{\partial \gamma^F} > 0$ we obtain $\frac{\partial \gamma^F}{\partial \gamma^F} > 0$ and $\frac{\partial x^L}{\partial \eta^*} > 0$. Moreover, $\frac{\partial \gamma^F}{\partial \gamma^F} > 0$ and (41) imply $\frac{\partial \gamma^F}{\partial \gamma^F} > 0$, so $\frac{\partial \gamma^F}{\partial \gamma^F} > 0$. Then, the denominator of (60) is positive, as discussed above, and the numerator is negative. So $\frac{\partial \gamma^F}{\partial \gamma^F} < 0$.

Then, $\frac{\partial \gamma^F}{\partial \gamma^F} < 0$. 

48
A.4.6 Proof of Lemma 1

Denote by $U_{HB}^{\theta}$ the value of the Home household utility given the solution \{r^H, x^H, g^H_0, g^H_1\} to the Home government's maximization problem (1) given constraints (4a)-(4f) with $x = 0$ and $\tau = 0$. Also, denote by $U_{HL}^{\theta, \tilde{b}}$ the value of the Home household utility given the solution \{r^H, x^H, g^H_0\} to the Home government's maximization problem (1) given constraints (4a)-(4f) with $x = 0$ and $\tau = 0$ and constraint $b^H(\theta) \leq \tilde{b}$. Finally, let $e(\tilde{b})$ denote the value of $\theta$ at which the solution to the Home government maximization problem (1) subject to (4a)-(4f) with $x = 0$ and $\tau = 0$ is $g^H_1 = e^H - \tilde{b}$ (i.e., $\tilde{b}$ is the debt level chosen by the Home government when $\theta = \tilde{\theta}$). Given $f(\theta)$ the p.d.f. for $\theta$ over $\Theta = [\Theta, \overline{\Theta}]$, the function maximized by problem (28) is

$$EU(\tilde{b}) = \int_\Theta \tilde{\theta}(\tilde{b}) U_{HB}^{\theta}(\theta)f(\theta)d\theta + \int_\Theta \tilde{\theta}(\tilde{b}) U_{HL}^{\theta, \tilde{b}}(\theta)f(\theta)d\theta.$$ 

Taking the first-derivative with respect to $\tilde{b}$, we obtain

$$\frac{\partial EU(\tilde{b})}{\partial \tilde{b}} = \int_\Theta \tilde{\theta}(\tilde{b}) \frac{\partial U_{HB}^{\theta}(\theta)f(\theta)}{\partial \tilde{b}}d\theta + U_{HB}^{\theta}(\tilde{\theta}(\tilde{b}))f(\tilde{\theta}) \frac{\partial \tilde{\theta}(\tilde{b})}{\partial \tilde{b}} - 0$$

$$+ \int_\Theta \tilde{\theta}(\tilde{b}) \frac{\partial U_{HL}^{\theta, \tilde{b}}(\theta)f(\theta)}{\partial \tilde{b}}d\theta + 0 - U_{HL}^{\theta, \tilde{b}}(\tilde{\theta}(\tilde{b}), \tilde{b})f(\tilde{\theta}) \frac{\partial \tilde{\theta}(\tilde{b})}{\partial \tilde{b}}.$$ 

Then,

$$\frac{\partial EU(\tilde{b})}{\partial \tilde{b}} = U_{HB}^{\theta}(\tilde{\theta}(\tilde{b}))f(\tilde{\theta}) \frac{\partial \tilde{\theta}(\tilde{b})}{\partial \tilde{b}} + \int_\Theta \tilde{\theta}(\tilde{b}) \frac{\partial U_{HL}^{\theta, \tilde{b}}(\theta)f(\theta)}{\partial \tilde{b}}d\theta - U_{HL}^{\theta, \tilde{b}}(\tilde{\theta}(\tilde{b}), \tilde{b})f(\tilde{\theta}) \frac{\partial \tilde{\theta}(\tilde{b})}{\partial \tilde{b}}.$$ 

Notice that for $\theta = \tilde{\theta}$, we have $U_{HB}(\tilde{\theta}) = U_{HL}(\tilde{\theta}, \tilde{b})$, so

$$\frac{\partial EU(\tilde{b})}{\partial \tilde{b}} = \int_{\tilde{\theta}(\tilde{b})} \frac{\partial U_{HL}^{\theta, \tilde{b}}(\theta)\tilde{\theta}(\tilde{b})}{\partial \tilde{b}}d\theta.$$ 

(61)
The second derivative of $EU(\bar{b})$ is

\[
\frac{\partial^2 EU(\bar{b})}{\partial \bar{b}^2} = \int_{\bar{\theta}(\bar{b})}^{\bar{\theta}} \frac{\partial^2 U^{HL}(\theta, \bar{b}) f(\theta)}{\partial \bar{b}^2} d\theta - \frac{\partial U^{HL}(\bar{\theta}, \bar{b}) f(\bar{\theta})}{\partial \bar{b}} \frac{\partial \bar{\theta}(\bar{b})}{\partial \bar{b}}.
\]

But $\frac{\partial U^{HL}(\bar{\theta}, \bar{b}) f(\bar{\theta})}{\partial \bar{b}} = 0$ since any increase in $\bar{b}$ would make the debt constraint $\bar{b}^H(\bar{\theta}) \leq \bar{b}$ slack. Therefore

\[
\frac{\partial^2 EU(\bar{b})}{\partial \bar{b}^2} = \int_{\bar{\theta}(\bar{b})}^{\bar{\theta}} \frac{\partial^2 U^{HL}(\theta, \bar{b}) f(\theta)}{\partial \bar{b}^2} d\theta.
\]

Then

\[
\frac{\partial^2 U^{HL}(\theta, \bar{b}) f(\theta)}{\partial \bar{b}^2} < 0 \implies \frac{\partial^2 V(\bar{\theta})}{\partial \bar{b}^2} < 0.
\]

The fact that $U^{HL}(\theta, \bar{b})$ is a concave function of $\bar{b}$, $\forall \theta \geq \bar{\theta}(\bar{b})$, notice that the Home government’s indirect utility function is given by

\[
V^{HL}(\theta, \bar{b}) = (1 - \gamma)v(r^H) + \gamma U^{HL}(\theta, \bar{b}),
\]

and

\[
V^{HL}(\theta, \bar{b}) = \max_{\{r^H, x^H, g^H\}} (1 - \gamma)v(r^H) + \gamma u(e^H(x^H, x^{F,\text{MAX}}, \theta)) + \gamma w(g^H) + \beta \gamma w(e^H - b^H(\bar{\theta}))
\]

subject to

\[
r^H + x^H + g^H \leq e^H + \beta b^H(\bar{\theta}).
\]

Given the policy choices made by the Home government, the change in household utility due to the change in the debt limit $\bar{b}$ is given by

\[
\frac{\partial U^{HL}(\theta, \bar{b})}{\partial \bar{b}} = u^H(x^H, x^{F,\text{MAX}}, \theta) \frac{\partial x^H}{\partial \bar{b}} + w'(g^H) \frac{\partial g^H}{\partial \bar{b}} - \beta w'(e^H - \bar{b}).
\]
Then,

\[
\frac{\partial^2 U_{HL}(\theta, b)}{\partial b^2} = \left[ u^{H}(x^H, x^{F,\text{MAX}}, \theta) \left( \frac{\partial x^H}{\partial b} \right)^2 + w'(g^H) \left( \frac{\partial g^H}{\partial b} \right)^2 + \beta w''(e^H - \bar{b}) \right] \\
+ \left[ u^{H}(x^H, x^{F,\text{MAX}}, \theta) \frac{\partial^2 x^H}{\partial b^2} + w'(g^H) \frac{\partial^2 g^H}{\partial b^2} \right] \\
+w'(g^H) \left( -\frac{\partial^2 r^H}{\partial b^2} \right)
\]

Under the conditions outlined in Assumption (1), the rents taken on by the Home government are an increasing and convex function of \(b\), so \(-\frac{\partial^2 r^H}{\partial b^2} < 0\). This, together with the concave increasing functions \(u^H(\cdot)\), \(w(\cdot)\) implies

\[
\frac{\partial^2 U_{HL}(\theta, \bar{b})}{\partial b^2} < 0,
\]

and

\[
\frac{\partial^2 V(\bar{\theta})}{\partial b^2} < 0.
\]

A.4.7 Proof of Proposition 3

From the proof to Lemma 1, the first-order condition for the household expected utility maximization problem is given by

\[
\int_{\bar{\theta}}^{\bar{\theta}} \frac{\partial U_{HL}(\theta, \bar{b})}{\partial b} f(\theta) d\theta = 0. \tag{62}
\]

Consider the case in which \(\bar{b} = b^{H*}(\bar{\theta})\), the level of debt at which the utility of the Home households is maximized when \(\theta = \bar{\theta} \equiv \max_{\theta} \Theta\). Then, \(\forall \theta < \bar{\theta}, \frac{\partial U_{HL}(\theta, \bar{b})}{\partial b} < 0\). Since \(\bar{b}\) is lower than the level of debt that maximizes Home
government’s utility when $\theta = \bar{\theta}$, it follows that $\tilde{\theta}(\bar{b}) < \bar{\theta}$. So, for all nondegenerate probability distribution functions $f(\theta)$, we have $\int_{\tilde{\theta}(\bar{b})}^{\bar{\theta}} \frac{\partial U^{HL}(\theta, \bar{b})}{\partial \bar{b}} f(\theta) d\theta < 0$. Then, $\bar{b} = b^{H*}(\bar{\theta})$ cannot be the solution to (62), and the $\bar{b}$ that satisfies (62) must have the property that $\bar{b} < b^{H*}(\bar{\theta})$. Thus, $\exists \theta^{HG} \in \Theta$ such that $\forall \theta \geq \theta^{HG}$, $V^{HL}(\theta, \bar{b}) < V(\theta)$.

To derive the conditions under which the welfare of Home households decreases, consider the value of debt $b^{H}(\bar{\theta})$ chosen by the Home government in the absence of fiscal rules, when the value of the shock is $\bar{\theta}$. Then, let $\hat{b}(\bar{\theta}) < b^{H}(\bar{\theta})$ denote the value of $b^{H}$ at which $U^{HL}(\bar{\theta}, \hat{b}(\bar{\theta})) = U^{HL}(\bar{\theta}, b^{H}(\bar{\theta}))$, that is, the value of the binding debt limit that is lower than the debt level that maximizes Home household utility. $\hat{b}(\bar{\theta})$ is feasible if $\hat{b}(\bar{\theta}) > -e^{H}/\beta$, so if the household utility at $b^{H}(\bar{\theta})$ satisfies:

$$U^{HL}(\bar{\theta}, b^{H}(\bar{\theta})) > u^{H}(0, x^{F,MAX}, \bar{\theta}) + w(0) + \beta w(e^{H}(1 + \frac{1}{\beta})),$$  \hfill (63)

If condition (63) is satisfied, then the utility loss to the Home government from rent-seeking is not as high as to make any feasible debt limit preferable to no debt limit at all.

Let $b^{H}(\bar{\theta})$ chosen by the Home government in the absence of fiscal rules, when the value of the shock is $\bar{\theta}$. Then, $\hat{b}(\bar{\theta}) > b^{H}(\bar{\theta})$ if

$$U^{HL}(\bar{\theta}, b^{H}(\bar{\theta})) < U^{HL}(\bar{\theta}, \hat{b}(\bar{\theta})).$$  \hfill (64)

Conditions (63) and (64) are satisfied if

$$U^{HL}(\bar{\theta}, b^{H}(\bar{\theta})) > \max\{U^{HL}(\bar{\theta}, b^{H}(\bar{\theta}));\,$$

$$u^{H}(0, x^{F,MAX}, \bar{\theta}) + w(0) + \beta w(e^{H}(1 + \frac{1}{\beta}))\}\}.$$  \hfill (65)

Using the first-order conditions to the Home government’s problem, condition (65) can be reduced to an upper bound on $\frac{\partial r}{\partial \bar{b}}$, so an upper bound on $(1 - \gamma) v''(\cdot)$. Therefore, there exists $v(\bar{\theta}, \bar{\theta})$ such that condition (65) holds for all functions $(1 - \gamma) v(\cdot)$ for which $(1 - \gamma) v''(\cdot) < v(\bar{\theta}, \bar{\theta})$. 52
Let $\tilde{\theta}(\tilde{b})$ denote the value of $\theta$ for which $\tilde{b}(\tilde{\theta})$ is the debt level chosen by the Home government. Given the above, $\tilde{\theta} \geq \theta$ whenever condition (65) holds. Let $\theta^{FB}(\tilde{b})$ denote the value of $\theta$ at which the debt level $\tilde{b}$ is the value of debt that maximizes Home household utility. If

$$- \int_{\theta^{FB}(\tilde{b})}^{\tilde{\theta}(\tilde{b})} \frac{\partial U^{HL}(\theta, \tilde{b})}{\partial \tilde{b}} f(\theta) d\theta \geq \int_{\theta^{FB}(\tilde{b})}^{\tilde{\theta}(\tilde{b})} \frac{\partial U^{HL}(\theta, \tilde{b})}{\partial \tilde{b}} f(\theta) d\theta,$$

then the optimal choice for the fiscal rule must be $\tilde{b} \leq \tilde{b}$. Thus, $U^{HL}(\tilde{\theta}, \tilde{b}) \leq U^{HL}(\tilde{\theta}, b^{H}(\tilde{\theta}))$.

Condition (66) requires that the probability distribution function $f(\theta)$ has sufficiently low mass over $[\theta^{FB}(\tilde{b}), \tilde{\theta}]$. This requires a positive skewness of the $f(\theta)$ distribution. Let $\rho_{1}^{MIN}(\theta)$ denote the skewness of the distribution of $\theta$ for which (66) holds with equality. Then, for $\rho(\theta) > \rho_{1}^{MIN}(\theta)$, $\exists \theta^{HH} \in \Theta$ such that

$$- \int_{\theta^{HH}}^{\theta^{FB}(\tilde{b}(\theta^{HH}))} \frac{\partial U^{HL}(\theta, \tilde{b}(\theta^{HH}))}{\partial \tilde{b}} f(\theta) d\theta = \int_{\theta^{FB}(\tilde{b}(\theta^{HH}))}^{\tilde{\theta}(\theta^{HH})} \frac{\partial U^{HL}(\theta, \tilde{b}(\theta^{HH}))}{\partial \tilde{b}} f(\theta) d\theta,$$

which implies that $U^{HL}(\theta, \tilde{b}) \leq U^{HL}(\theta, b^{H}(\theta)) \forall \theta \geq \theta^{HH}$. From the Home government’s first-order conditions, $\frac{\partial v(\tilde{b})}{\partial \tilde{b}} < 0$, so $v(\tilde{r}(\theta, \tilde{b})) < v(\tilde{r}(\theta, b^{H}(\theta))) \Rightarrow V^{HL}(\theta, \tilde{b}) < V^{HL}(\theta, b^{H}(\theta)) \Rightarrow \theta^{HH} > \theta^{HG}$.

A.4.8 Proof of Proposition 4 (sketch)

In the presence of fiscal rules, the supranational authority faces the same first-order conditions (41) and (42). If the debt limit $\tilde{b}$ binds, then from (41), the transfer under fiscal rules, $\tau^{FR}$ satisfies $\tau^{FR} > \tau$. Then, at $\eta^{*}$, Home household utility must be larger than under no fiscal rules. This follows from $\tau^{FR} > \tau$, which ensures that the decrease in $x$ compared to the no fiscal rules case is smaller than the increase in $g^{H}$ compared to the no fiscal rules case. Given the monotonicity of household utility as a function of $\eta$, it follows that $\eta^{**} \leq \eta^{*}$.
A.4.9 Proof of Proposition 5 (sketch)

From the first-order conditions to the Home government’s maximization problem with a binding fiscal rule \( b \), \( \frac{\partial g^H}{\partial \gamma^H} > 0 \) and \( \frac{\partial x^H}{\partial \gamma^H} > 0 \). Then, from the first-order conditions to the supranational authority’s problem, (41) and (42), \( \frac{\partial r}{\partial \gamma^H} < 0 \) and \( \frac{\partial (x - \tau)}{\partial \gamma^H} < 0 \). From the Home government’s first-order conditions, this implies
\[
\frac{\partial \left[ (U^H(\tau, \bar{x}, \theta) - U^H(0, 0, \theta)) \right]}{\partial \gamma^H} \leq 0,
\]
so
\[
\frac{\partial \eta^{**}}{\partial \gamma^H} \leq 0.
\]

A.4.10 Proof of Corollary 1 (sketch)

Comparing the first-order conditions (41) and (42) in the case in which the debt limit binds and the case without a debt limit, then
\[
\frac{\partial \left[ (x - \tau) - (\bar{x}^L - \tau^L) \right]}{\partial \gamma^H} < 0,
\]
where \( (\bar{x}^L - \tau^L) \) denotes the solution when the debt limit is binding. Also,
\[
\frac{\partial (\tau^L - \tau)}{\partial \gamma^H} < 0,
\]
(68)

Let \( U^{HL}(\tau, \bar{x}, \theta) \) denote the value of household utility under policy \( (\tau, \bar{x}) \) when the debt limit is \( \bar{b} \) and let \( U^H(\tau, \bar{x}, \theta) \) denote the value of household utility under policy \( (\tau, \bar{x}) \) and no fiscal rules. Then, (67) and (68) imply
\[
\frac{\partial \left[ (U^{HL}(\tau, \bar{x}, \theta) - U^{HL}(0, 0, \theta)) - (U^H(\tau, \bar{x}, \theta) - U^H(0, 0, \theta)) \right]}{\partial \gamma^H} \leq 0,
\]
so
\[
\frac{\partial [\eta^{**} - \eta^*]}{\partial \gamma^H} \leq 0.
\]
A.4.11 Proof of Proposition 6

Consider a binding fiscal rule $\bar{b}$ along with a binding retrospective rule $\bar{u}$. The participation constraint for the Home government is given by

\[
(1 - \gamma)v(r^H) + \gamma u^H(x^H, x^{F,MAX}, \theta) + \gamma w(g^H) + \beta \gamma w(e^H - \bar{b}) \geq (1 - \gamma)v(r^{H0}) + \gamma u^H(x^{H0}, x^{F,MAX}, \theta) + \gamma w(g^{H0}) + \beta \gamma w(e^H - \bar{b}),
\]

subject to

\[
u^H(x^H, x^{F,MAX}, \theta) + w(g^H) \geq u.
\]

The above conditions make it clear that Home household utility will be fixed at $u + \beta \gamma w(e^H - \bar{b})$ both with and without the partial banking union. Therefore, the supranational authority maximizing objective (38) will set $\tau$ and $\bar{z}$ such that $r^H = r^{H0}$. Then, Home household utility is unchanged, rents are unchanged, but $x^H > x^{H0}$ and $g^H < g^{H0}$, so that Foreign household utility is increased. Therefore, a Pareto improvement is achieved without the increase in Home government rents.