Debt Crises: For Whom the Bell Tolls*

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Abstract

What a country has done in the past, and what other countries are doing in the present can feedback for good or for ill. We develop a simple model that can address hysteresis and contagion in sovereign debt markets. When a country’s fundamentals change, those changes affect information acquisition about that country but also affect the allocation of investment funds worldwide, inducing changes in the dynamics of sovereign spreads in seemingly unrelated economies.

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1 Introduction

Several features of sovereign debt markets are difficult to explain. First, contagion. Sovereign debt crises tend to be highly correlated across countries and sovereign spreads (the sovereign’s cost of external funding), tend to co-move strongly. The most recent example is the 2010 debt crisis in Europe. Beirne and Fratzscher (2013), using information for 31 advanced and emerging economies during the crisis, find that there was a sharp and simultaneous increase in sovereign spreads in both European and non-European countries. Similar forces were at play in the debt crises initiated by Poland in 1981, Mexico in 1994, Thailand in 1997, Russia in 1998, and Argentina 2001.

Previous work has attempted to explain contagion by appealing to different types of linkages between countries. One branch of the literature focuses on real linkages. For example, trade in goods or financial assets between countries may transmit negative shocks from one country to the next and lead to co-movements in sovereign spreads (e.g., Alter and Beyer (2014) and Gross and Kok Sorensen (2013)). A second branch focuses on belief linkages through learning and herding. In this view (e.g., De Santis (2012)), contagion is driven by the correlation of beliefs about fundamentals in different countries, so that bad news about one country make investors pessimistic about other countries. Of course, a prerequisite for belief correlation to cause contagion is that observations about one country hold information about other countries. This requires correlation in fundamentals across countries, or the existence of a common unobservable variable linking all countries. Theories of contagion based on belief linkages therefore also require real linkages between countries. Finally, a third set of explanations relies on the rationalization of crises as self-fulfilling roll-over problems a la Cole and Kehoe (1996). To explain contagion, however, this literature requires a correlated structure of sunspots to induce simultaneous roll-over crisis episodes in many countries at the same time.

Because many extant theories of contagion rely on the existence of structural links across countries, finding evidence for such linkages is imperative in providing support for them. Problematically, however, it is often difficult to empirically identify linkages that are plausibly powerful enough to induce the degree of contagion observed in many debt crisis episodes. Again taking the recent European experience

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¹For a survey of these cases see Reinhart and Rogoff (2009).
as an example, Beirne and Fratzscher (2013) explore empirical models with economic fundamentals and find that “the market pricing of sovereign risk may not have been fully reflecting fundamentals prior to the crisis.”

Second, sovereign risk premia seem only loosely connected to the country’s fundamentals more generally: they frequently exhibit sudden changes without obvious changes in underlying fundamentals, and sometimes fluctuate without any observable changes in fundamentals at all. Indeed, sovereign risk premia seem to react differently to a given change in fundamentals at different points in time.

Third, there seems to be history dependence in the borrowing conditions faced by different countries: the same change in fundamentals may have different effects in different countries, and these differences are persistent over time. Indeed, a given country’s past behavior seems to matter for how sovereign spreads react to changes in fundamentals. Consider, for example, the diverging experiences of Argentina and the United States. The U.S. seems to be in a “stable” environment that allows it to accumulate high debt levels without triggering increases in spreads, while Argentina, in contrast, seems to be in an “unstable” environment in which slight changes in fundamentals cause large and sudden changes in spreads.

To jointly accommodate all of these features within a single framework, we construct a model of sovereign bond markets with many countries and two key elements. First, there is a global pool of risk-averse investors who freely allocate funds across sovereign bond markets. Second, these investors can choose to produce information about a country’s fundamentals at a cost. This information is valuable because informed investors are able to exploit their superior knowledge of a country’s fundamentals to outbid uninformed investors in particularly attractive states of the world. In equilibrium, this benefit is exactly offset by the cost of becoming informed.

Our first result is that the free flow of capital across countries can generate contagion across countries, even in the absence of any real linkages, correlation of fundamentals, or belief updating about one country due to equilibrium outcomes in another country. Specifically, when investor preferences exhibit prudence (that is, \( u''(c) > 0 \), as is the case for CRRA utility functions), an increase in the probability of default in one country increases sovereign spreads for all sovereign bonds held by the investor. This is because an increase in the default risk of a given country increases the background risk inherent in the entire portfolio of sovereign bonds, and thereby reduces the investor’s appetite to invest in sovereign debt more generally. Hence, sovereign
bond prices fall across all countries when one country becomes more likely to default. If this effect is sufficiently large and the increase in spreads is severe enough, it may no longer be feasible for countries to roll over their debt, causing a wave of debt crises.

Our contagion result relies only on investor prudence and the fact that there is a common pool of investors for all countries. Hence it does not rely on changes in investors’ wealth (as in Kyle and Xiong (2001) or Goldstein and Pauzner (2004)), borrowing constraints (as in Yuan (2005)) or short-selling constraints (as in Calvo and Mendoza (1999)). Indeed, contagion stems only from the portfolio rebalancing of prudent investors in response to an increase in the riskiness of a subset of assets at their disposal. For empirical evidence about the importance of portfolio effects on contagion see Broner, Gelos, and Reinhart (2004). For empirical evidence about the importance of risk aversion to explain sovereign spreads see Lizarazo (2013).

Our second result is that the option to produce information about countries’ fundamentals can generate multiple equilibria. In particular, an uninformed equilibrium, in which no investor acquires information about the country’s fundamentals, may co-exist with an informed equilibrium, in which some investors do acquire information about the country’s fundamentals. These information regimes have real effects: taking as given the stochastic process for fundamentals, the average level and the volatility of spreads differ across regimes. In the uninformed equilibrium, spreads are stable and low on average, because investors are relatively insensitive to variation in fundamentals. In the informed equilibrium, in contrast, spreads are volatile and high on average, because investors strongly react to variation in fundamentals and demand very high risk premia in bad states of the world. For this reason, sovereigns strictly prefer an uninformed equilibrium to an informed equilibrium. Because information acquisition is costly, and information rents come at the expense of other investors, the same is true for investors.

Why do all agents, investors and countries alike, lose in the informed equilibrium? In our setting information does not affect any real variable, so there are no benefits of information. Still information is costly and uses real resources that could be consumed otherwise, but are lost in equilibrium because of investors competing for a larger share of resources. We do not claim that information does not have benefits in terms of disciplining governments or allocating funds to productive investment opportunities, but we assume away those benefits to focus on the forces behind in-
formation acquisition. Any benefit of information will naturally go in the direction of making the uninformed equilibrium less desirable.

An important upshot from our analysis is that, because investors’ optimal portfolio choice and the information regime jointly determine the mapping from country fundamentals to sovereign bond spreads and the likelihood of debt crises, there need not exist a unique mapping from economic fundamentals to spreads in sovereign bond markets even in the absence of roll-over crises driven by coordination failures. Indeed, since investors choose their portfolio by taking the fundamentals and information regimes in all countries into account, the mapping from fundamentals to prices in a single country depends on equilibrium outcomes in all other countries. To the extent that a given pool of investors prices sovereign bonds in multiple countries, understanding contagion and default risk therefore requires a “global” view of bond markets.

Finally, to the extent that informational regimes are persistent (in the sense that there is a change in regime only if the only if the old regime can no longer be sustained), only large changes in fundamentals can force a transition across regimes. This implies that a country starting out in an uninformed equilibrium begins to attract informed investors only if its fiscal situation worsens substantially, while a country starting out in an informed equilibrium requires a substantial improvement of their fiscal situation to discourage information acquisition. In the absence of such large shocks, two given countries may therefore be in different informational regimes, and thus have to pay different spreads, even when their current fundamentals are similar. A country’s past sins or virtues may therefore be important determinants of current borrowing conditions, and may remain with the country for a long time. We call this phenomenon hysteresis. This also implies that understanding contagion and default risk therefore also requires a “historical” view of bond markets.

In the next section we present a model with a single country in which we discuss multiplicity of equilibria in terms of information acquisition and the outcome in terms of sovereign spreads. In Section 3 we extend the results for two countries, discussing contagion of sovereign spreads in its purest form, without any fundamental linkage and no information acquisition and contagion of information regimes. In Section 4 we reinterpret the dynamics of sovereign spreads in the recent european debt crisis from the point of view of our model. In Section 5 we conclude.
2 A Single Country Model

2.1 Setting

Environment: This is a two period model with a mass 1 of investors and a government. Investors have wealth $W$ in the first period and only care about consumption $c$ in the second period. Their preferences over consumption are given by a strictly concave utility function $u(c)$ that satisfies the Inada conditions. Since investors only care about consumption in period 2, their problem is deciding how to invest their wealth in period 1, choosing between a safe asset that has gross return 1 (storage), and risky government debt. We describe the source of this risky debt next.

In period 1 the government has an amount of outstanding legacy debt $D$ coming due. This debt is new of the country’s period 1 income and is owed to previous, unspecified, investors. This implies that, in order to repay $D$ the government has to roll over the debt. We assume that the government rolls over this debt using pure discount bonds via an auction-type market. In this market, investors specify combinations (possibly menus) of prices $P$ and quantities $B$ they wish to purchase. The government sells debt to the highest bidder until it either exhausts the bids or sells enough to roll over its debt. If the government cannot roll over its debt then it must default on initial investors, a situation we call a debt crisis.

In period 2 the debt issued in period 1 comes due. The government then chooses whether to repay its debt using its income $Y$ generated in period 2, or to default. If the government defaults in either period the total output that remains is $(1 - \theta)Y$ where $\theta \in [0, 1]$ is the cost of default in terms of lost income. Both the government’s default cost factor $\theta$ and its income $Y$ are random. We assume the cost of default is independent of whether the government defaults in period 1 or period 2, and since the government is just seeking to roll over its debt during the first period, it will always do so if it can; reserving the decision to default for the second period.

While the realization of $Y$ is drawn in period 2 from a distribution $F(Y)$, the realization of $\theta$ is drawn in period 1 from a discrete distribution with $S$ elements $\Theta = \{\theta_1, \ldots, \theta_S\}$, such that $\theta_1 > \ldots > \theta_s > \ldots > \theta_S$. The realization of $\theta$ is unknown to investors, they can choose to become informed about it in period 1 at a utility cost $K$.

Auctions: Investors are ex-ante identical but end up being one of two types based upon their information choice: informed and uniformed. Denote by $n$ the fraction of
informed investors and by $P$ the marginal price of government debt in period 1. If there are informed investors then this marginal price will depend upon the realized $\theta$, and in this case we will denote it by $P(\theta)$. Because informed traders know $\theta$, they know the price that the marginal investor must pay for government debt and hence bid the price $P(\theta)$ along with the (conditional) quantity that they wish to purchase at that price, $B^I(\theta)$. The uninformed traders may (and will) find it advantageous to bid heterogeneous price-quantity pairs. Because they know the set of possible marginal prices, $\{P(\theta_1), \ldots, P(\theta_S)\}$ in an equilibrium with a fraction $n$ of informed investors, they will choose the quantities to bid at each one of these prices. Let $B^U(\theta)$ denote the amount that an uninformed trader bids if he chooses to bid at price $P(\theta)$.

The auction arrangement leads to the following budget constraints for the government. In period 1, and for a given $\theta$, if it can roll over its debt in period 1, then

$$nB^I(\theta)P(\theta) + (1 - n) \sum_{\hat{\theta}:P(\hat{\theta}) \geq P(\theta)} B^U(\hat{\theta})P(\hat{\theta}) = D.$$  \(\text{(1)}\)

Notice that the previous sum represents that, if the cost of default is $\theta$, uninformed investors get to buy all their bids at prices larger than $P(\theta)$.

If the government cannot roll over the debt in period 1, then

$$nB^I(\theta)P(\theta) + (1 - n) \sum_{\hat{\theta}:P(\hat{\theta}) \geq P(\theta)} B^U(\hat{\theta})P(\hat{\theta}) < D,$$

in which case it must default. We will refer to this second case as a debt crisis.

If the government hasn’t defaulted in period 1, its debt coming due in period 2 is

$$R(\theta) = nB^I(\theta) + \sum_{\hat{\theta}:P(\hat{\theta}) \geq P(\theta)} (1 - n)B^U(\hat{\theta})$$

In this case the government’s payoff if it doesn’t default in period 2 is $Y - R(\theta)$, while it is $(1 - \theta)Y$ if it does default in period 2. This leads to a simple cut-off rule in which the government defaults in period 2 if and only if $Y < \bar{Y}(\theta)$, where

$$\bar{Y}(\theta) \equiv \frac{R(\theta)}{\theta}.$$  \(\text{(2)}\)

Since we assume that the government’s income is $(1 - \theta)Y$ if it has already defaulted
in period 1; irrespective of whether it defaults in period 2, the government is always weakly better off waiting to default in period 2 if possible (there are no gains from defaulting in the first period rather than rolling over with the possibility of repaying in the second period).

Since $\bar{Y}(\theta)$ denotes the government’s cut-off rule for defaulting as a function of $\theta$, the realized return to an investor is 1 if $Y \geq \bar{Y}(\theta)$ and 0 otherwise. In other words, it is 1 with probability $\Pr \{ Y \geq \bar{Y}(\theta) \}$ and 0 with probability $1 - \Pr \{ Y \geq \bar{Y}(\theta) \}$. Then, so long as the total amount coming due, $R(\theta)$, is weakly decreasing in $\theta$ (the higher the cost of default, the higher the price of debt and the less debt comes due in period 2), it follows that the default cut-off is strictly decreasing in $\theta$ and the default probability is also weakly decreasing in $\theta$. In words, the higher the cost of default $\theta$ the less likely is that the country defaults, this decreases the repayment needs and reduces the probability of default, which is consistent with a lower repayment need.

**Short-sale Prohibition:** We will assume that our private investors cannot short the government’s bond. We make this assumption for two reasons. First, in our two-period context shorting the bond does not mean pledging to deliver a unit of the bond later. Rather it means committing to the same state-contingent payoff profile as the government. But, in order to do this, the private investor would need access to exactly the sort of commitment technology as the government. Second, in an equilibrium in which uninformed investors were seeking to short the bond the ability to trade would reveal information about the realization of $\theta$. This is because the other party to the trade will be an informed trader who is only willing to buy because the marginal price is weakly greater than the price asked by the uniformed investor. Similarly, an offer to sell the bond outside of the auction by an informed investor to an uninformed investor would also reveal information about the realization of $\theta$.

**Investors’ Problem:** An informed agent knows $\theta$ and takes as given the marginal price of debt $P(\theta)$. Therefore, their maximization problem is given by

\[
U^I(\theta) = \max_{B^I(\theta) \geq 0} u \left( W + [1 - P(\theta)] B^I(\theta) \right) \Pr \{ Y \geq \bar{Y}(\theta) \} \\
+ u \left( W - P(\theta) B^I(\theta) \right) \left[ 1 - \Pr \{ Y \geq \bar{Y}(\theta) \} \right] - K,
\]
which implies that their first-order condition is,

\[ u'(W + [1 - P(\theta)] B^I(\theta)) [1 - P(\theta)] \Pr \{ Y \geq \hat{Y}(\theta) \} + u'(W - P(\theta) B^I(\theta)) [-P(\theta)] [1 - \Pr \{ Y \geq \hat{Y}(\theta) \}] \leq 0, \]  \hspace{1cm} (4)

and with strict equality if \( B^I(\theta) > 0 \).

An uninformed agent must choose how much to bid at each one of the possible marginal prices \( P(\theta) \). The maximization problem of an uninformed agent is then

\[ U_U = \max_{\{B^U(\hat{\theta})_1, \ldots, B^U(\hat{\theta})_s\}} \sum_{\theta \in \Theta} \Pr(\theta) \left\{ \begin{array}{c} u \left( W + \sum_{\theta' \geq \theta} \left[ 1 - P(\hat{\theta}) \right] B^U(\hat{\theta}) \right) \Pr \{ Y \geq \hat{Y}(\theta) \} \\ u \left( W - \sum_{\theta' \geq \theta} P(\hat{\theta}) B^U(\hat{\theta}) \right) [1 - \Pr \{ Y \geq \hat{Y}(\theta) \}] \end{array} \right\}. \]  \hspace{1cm} (5)

which implies that his first-order condition for \( B^U(\hat{\theta}) \) is,

\[ \sum_{\theta' \leq \hat{\theta}} \Pr \{ \theta' \} \left\{ \begin{array}{c} u' \left( W + \sum_{\theta' \leq \theta} \left[ 1 - P(\theta') \right] B^U(\theta') \right) \left[ 1 - P(\hat{\theta}) \right] \Pr \{ Y \geq \hat{Y}(\theta) \} \\ u' \left( W - \sum_{\theta' \leq \theta} P(\theta') B^U(\theta') \right) [-P(\hat{\theta})] [1 - \Pr \{ Y \geq \hat{Y}(\theta) \}] \end{array} \right\} \leq 0, \]  \hspace{1cm} (6)

where this condition holds as an equality if \( B^U(\hat{\theta}) > 0 \). As the decision of the quantities to bid at different prices are linked through first order conditions, the bids in equilibrium are the solution to the system of equations (6) for all \( \theta \).

Finally, if an interior fraction of investors choose to become informed, \( n \in (0, 1) \), then the investors must be indifferent between being informed or staying uninformed. If none of the investors become informed then this condition becomes an inequality with \( U^U \) being weakly preferred. Alternatively, if \( n = 1 \) then the inequality is reversed with the payoff from being informed. Hence

\[ \sum_{\theta} \Pr \{ \theta \} U^I(\theta) - K = U^U \]  \hspace{1cm} if \( n = 0 \),

\[ \geq U^U \]  \hspace{1cm} if \( n \in (0, 1) \),

\[ = U^U \]  \hspace{1cm} if \( n = 1 \). \hspace{1cm} (7)

**Equilibrium:** The previous discussion summarizes the main elements of the problem of a single country, which is completely indexed by \( n \) from equation (7). Next we define the equilibrium.

**Definition 1** An equilibrium will consist of a set of cut-offs \( \hat{Y}(\theta) \), prices \( P(\theta) \), quantities for
the informed and uninformed (\( B_I(\theta) \) and \( B_U(\theta) \) respectively), a fraction of informed investors (\( n \)) such that the following conditions are satisfied.

1. The period 1 bond market from equation (1) clears in each country for each state \( \theta \), or 
   \[ P(\theta) = 0 \] 
   then \( \bar{Y}(\theta) = \infty \) and there is a debt crisis in state \( \theta \).

2. The set of cut-offs, \( \bar{Y}(\theta) \), satisfy the threshold condition (2).

3. The choices of \( B_I(\theta) \) and \( B_U(\theta) \) are solutions to the informed and uniformed investors’ problems (first order conditions (4) and (6) respectively).

4. The fraction of informed investors \( n \) must satisfy the indifference condition \( (7) \). The country is an informed equilibrium when \( n > 0 \) and in an uninformed equilibrium when \( n = 0 \).

There are a variety of equilibria. This is in part because the price of government debt affects the likelihood of repayment, and this in turn can rationalize different prices of the debt. For example, no-lending with \( P(\theta) = 0 \) and \( \bar{Y}(\theta) = \infty \) for all \( \theta \) is always an equilibrium. At a zero price the government will not be able to rollover its debt and therefore must default, this in turn rationalizes the zero price. This multiplicity is well-known since the work of Calvo (1988) and Cole and Kehoe (1996). Next we present a simplified special case to characterize the other (potentially multiple because of information acquisition) equilibria in a tractable and intuitive way.

**Simplifications:** First, we assume just two possible costs of default (that is, \( S = 2 \)), such that \( 0 < \theta_L < \theta_H < 1 \), where \( \theta_H \) is realized in period 1 with probability \( a \) (situation that we denote as good state) and \( \theta_L \) is realized in period 1 with probability \( 1 - a \) (situation that we denote as bad state). Second, we assume just three possible income realizations in period 2, \( Y_L < Y_M < Y_H \), where \( Y_L \) happens with probability \( x \) and \( Y_M \) with probability \( z \). Finally, we assume \( \theta s \) and \( Y s \) are such that default cutoffs are exogenous. Formally,

**Assumption 1**

\[ Y_L < \bar{Y}(\theta_H) < Y_M < \bar{Y}(\theta_L) < Y_H \]

This assumption guarantees that when the cost of default is high (good state), the country only defaults when the income is low, which implies a default probability of
When the cost of default is low (bad state), the country only repays when the income is high, which implies a default probability of $\kappa_L \equiv x + z$.

The previous assumption relies on endogenous variables (the prices of debt $P(\theta_L)$ and $P(\theta_H)$). For example, for any $D$ and $\theta$, if $P = 0$, then $\bar{Y} = \infty$, which implies that the country always defaults and $P = 0$ is indeed an equilibrium. There may be other equilibria under which the income level is equal to the cutoff and the government randomizes between repaying or not in case such income is realized. We do not consider these equilibria as we are interested in the impact of changes in the probability of default of a country on prices and not on the impact of changes in prices on the probability of default. Notice that both effects are intertwined in the general setting in which the probability of default and prices are jointly determined. We relax this restriction later.

### 2.2 Characterization of Equilibria

First we study an equilibrium in which no investor is informed about the state of the country in terms of its cost of default. Then we study equilibria in which some investors may decide to become informed about the state of the country. Finally we describe the possibility of multiplicity, in which these equilibria coexist, and discuss its robustness to changes in parameters.

#### 2.2.1 Uninformed Equilibrium

Define the expected probability of default as

$$\hat{\kappa} \equiv ax + (1 - a)(x + z)$$

Since there is no information about the country’s state there is a single marginal price $P$. Given this price, we can rewrite the first order condition (4) as

$$\frac{u'(W + [1 - P]B)}{u'(W - PB)} = \frac{P\hat{\kappa}}{(1 - P)(1 - \hat{\kappa})}$$

The next proposition displays properties of this first-order condition in terms of how bid quantities depend on parameters.
**Proposition 1** The investors’ demand of sovereign bonds is decreasing on the price of the bond and on its default probability.

**Proof** Rewriting the first order condition (8) as

\[ F(B|P, \hat{\kappa}) \equiv \frac{u'(W + [1 - P]B)}{u'(W - PB)} - \frac{P\hat{\kappa}}{(1 - P)(1 - \kappa)} = 0 \]

define \( u'(+) \equiv u'(W + [1 - P]B) \) and \( u'(-) \equiv u'(W - PB) \). Differentiating with respect to \( \hat{\kappa} \), \( \frac{dB}{d\hat{\kappa}} \) is negative as

\[ \frac{\partial F}{\partial B} = \frac{(1 - P)u''(+)u'(-) + Pu''(-)u'(+) - u'(-)u''(+)u'}{u'^2(-)} < 0 \]

and

\[ \frac{\partial F}{\partial \hat{\kappa}} = -\frac{P}{(1 - P)(1 - \hat{\kappa})^2} < 0 \]

Similarly, differentiating with respect to \( P \), \( \frac{dB}{dP} \) is negative if

\[ \frac{\partial F}{\partial P} = \frac{B}{u'^2(-)}[u''(-)u'(+) - u''(+)u'(-)] - \hat{\kappa} \frac{u'(-)}{u'(+)} \frac{(1 - P)^2(1 - \kappa)}{(1 - P)(1 - \hat{\kappa})} < 0 \]

A sufficient condition for this to be the case is that \( \frac{u''(-)}{u'(-)} \leq \frac{u''(+)}{u'(+)}, \) which is always the case for CRRA and CARA preferences. Q.E.D.

The first-order condition together with the resource constraint pins down the price in equilibrium. Substituting the resource constraint \( PB = D \) into the first-order condition,

\[ \frac{u'(W - D + \frac{P}{P^*})}{u'(W - D)} = \frac{P\hat{\kappa}}{(1 - P)(1 - \kappa)} \]  

**Proposition 2** There is always an equilibrium with rollover failure and a debt crisis at \( P = 0 \). If there exist other equilibria with \( P > 0 \), the highest price equilibrium price decreases with the probability of default \( \hat{\kappa} \) and the country’s debt \( D \).

**Proof** Define

\[ F(P|\hat{\kappa}) = \frac{u'(W - D + \frac{P}{P^*})}{u'(W - D)} - \frac{P\hat{\kappa}}{(1 - P)(1 - \kappa)} \]

A price \( P^* \) in equilibrium is given by \( F(P^*|\hat{\kappa}) = 0 \). At one extreme, the zeros of the function \( F \) include \( P = 0 \). To see this note that under the Inada conditions (this is
\(c \to \infty\) the first term is zero and trivially the second term is 0 too. Hence, \(F(P = 0|\tilde{\kappa}) = 0\). However, assuming \(Y\) has finite support, then a rollover failure occurs at \(P^* = 0\) and hence this price can be an equilibrium. At the other extreme, for \(P = 1 - \tilde{\kappa}, F(P = 1 - \tilde{\kappa}|\tilde{\kappa}) < 0\) (the first term on \(F(P|\tilde{\kappa})\) is less than one and the second term is equal to one), then \(P = 1 - \tilde{\kappa}\) is never an equilibrium with risk aversion. Indeed, under risk-aversion, \(P < 1 - \tilde{\kappa}\) as the first term of \(F(P|\tilde{\kappa})\) is less than 1 and then the second term should also be less that 1.

If parameters are such that \(F(P|\tilde{\kappa}) < 0\) for all \(P \in (0, 1 - \tilde{\kappa})\), then the only equilibrium is given by \(P^* = 0\). If \(F(P|\tilde{\kappa}) > 0\) for some \(P \in (0, 1 - \tilde{\kappa})\), then there are other equilibria besides \(P^* = 0\). Among those, the maximum \(P^*\) sustainable in equilibrium is such that \(\frac{\partial F}{\partial P < 0}\) (recall \(F(P^*|\tilde{\kappa}) = 0\) and \(F(P = 1 - \tilde{\kappa}|\tilde{\kappa}) < 0\)).

The maximum sustainable price in equilibrium is decreasing in \(\tilde{\kappa}\) and \(D/W\) as, on the one hand, \(\frac{\partial P}{\partial \tilde{\kappa}} = -\frac{\partial F}{\partial \tilde{\kappa}}\) and \(\frac{\partial F}{\partial P} = -\frac{P}{(1-P)(1-\tilde{\kappa})^2} < 0\) whereas on the other hand, \(\frac{\partial P}{\partial D} = -\frac{\partial F}{\partial D}\)

and \(\frac{\partial F}{\partial D} = -\frac{\frac{1-P}{u''(+) + \frac{u''(+)}{u''(-)}}}{u''(-)} < 0\) Q.E.D.

To provide intuition, the next figure plots the left-hand side of equation (9), in black and the right-hand side in different colors for three different levels of \(\tilde{\kappa}\). The equilibrium price is determined by the intersection of the two curves. The higher is the expected probability of default, the higher is the right-hand side and the smaller is the price \(P\) in equilibrium. When \(\tilde{\kappa}\) is large enough, the only feasible equilibrium is \(P^* = 0\) and there is a debt crisis.

The next figure shows the right hand side of equation (9) in black and the right hand side in different colors for three different levels of \(D/W\). As before, the equilibrium price is determined by the intersection of the two curves. The higher is the relative indebtedness of the country, the higher is the left hand side and the smallest the price \(P\) in equilibrium. When \(D/W\) is large enough, the only feasible equilibrium is a \(P^* = 0\) and there is a debt crisis.

When is an uninformed equilibrium sustainable? To answer this question, we have to determine the incentives for a single uninformed investor to deviate and acquire information, paying a utility cost \(K\). Because a single investor’s bidding behavior does not impact equilibrium prices, the benefits of acquiring information come from the possibility of re-optimizing the quantities the investor bids at the marginal price \(P\) in equilibrium, given that there is a single price.
If the investor learns the state is good, he would like to bid more than uninformed individuals. This is immediate from the first order condition (8) evaluated at $P$ and $\kappa_H$, as the bid is decreasing in the probability of default and $\kappa_H < \hat{\kappa}$. Similarly, if the investor learns the state is bad, he would like to bid less than if he were uninformed.

Defining the expected benefits of acquiring information as

$$\chi^U \equiv a [U(B(\kappa_H, P)) - U(B(\hat{\kappa}, P))] + (1 - a) [U(B(\kappa_L, P)) - U(B(\hat{\kappa}, P))]$$

As $U(B(\kappa, P))$ is obtained by re-optimizing the quantities bid, it is clear that $\chi^U$ cannot be negative (as the investor can always replicate his uninformed bid). Then, the uninformed equilibrium is feasible as long as

$$K > \chi^U \geq 0$$

Notice that the difference between the optimal bid in each state and the bid without information (this is, $B(\kappa_s, P) - B(\hat{\kappa}, P)$) is increasing in the absolute difference $\kappa_s - \hat{\kappa}$. Since $\hat{\kappa} - \kappa_H = (1 - a)z$ and $\kappa_L - \hat{\kappa} = az$, the gap increases with $z$ and it is maximized at intermediate levels of $a$. In the extremes, when $a = 0$, $U(B(\kappa_L, P)) = U(B(\hat{\kappa}, P))$ and $\chi^U = 0$. This is also the case for $a = 1$.

The incentives to acquire information is also increasing in $D/W$ as more exposure
to the risky asset increases the differences in utility from knowing the probability of default in each state.

2.2.2 Informed Equilibrium

The critical difference between the informed and uninformed equilibrium is that in the informed equilibrium there will be as many prices as states, as informed investors will bid different quantitates in those different states. In a sense, the existence of informed investors changes the structure of equilibrium as they will generate prices that are conditional on underlying fundamentals that are unknown unless information is produced, the cost of default $\theta$ in our setting.

Denoting the two prices $P_L \equiv P(\theta_L)$ and $P_H \equiv P(\theta_H)$ we can rewrite the first order condition (4) as

$$
\frac{u'(W + [1 - P_s]B^s)}{u'(W - P_sB^s)} = \frac{P_s\kappa_s}{(1 - P_s)(1 - \kappa_s)}
$$

(10)

where $\kappa_s \in \{\kappa_L, \kappa_H\}$ are the expected probabilities of default in each state $s$ and $P_s \in \{P_L, P_H\}$ are the prices in each state $s$.

As in the uninformed equilibrium, the next proposition describes the features of these first order conditions, which are identical to those in Proposition 1, as is the proof.
Proposition 3  Informed investors’ demand of sovereign bonds is decreasing on the price of the bonds and on their default probability.

For uninformed investors bidding in the informed equilibrium, we can rewrite the first-order condition (6) for the bid at the marginal price in the low state, $B_U^{l}$, as

$$P_L \kappa_L u'(W - P_H B_H^U - P_L B_L^U) = (1 - P_L)(1 - \kappa_L)u'(W + (1 - P_H)B_H^U + (1 - P_L)B_L^U) \tag{11}$$

and for the bid at the marginal price in the high state, $B_H^U$, as

$$a \left[ P_H \kappa_H u'(W - P_H B_H^U) \right] + (1 - a) \left[ P_H \kappa_L u'(W - P_H B_H^U - P_L B_L^U) \right] = a \left[ (1 - P_H)(1 - \kappa_H)u'(W(1 - P_H)B_H^U) \right] + (1 - a) \left[ (1 - P_H)(1 - \kappa_L)u'(W + (1 - P_H)B_H^U + (1 - P_L)B_L^U) \right] \tag{12}$$

Critically, auctions with discriminatory pricing implies that, as $P_H > P_L$, the sovereign will sell $B_H^U$ to the uninformed at $P_H$ in the good state (as the price is $P_H$) but also in the bad state, in which the marginal price is $P_L$. This implies that uninformed understand that they will always buy whatever they decide to bid at $P_H$, but in a bad state they are buying at an overprice.

By comparing these first order conditions, the next proposition describes general properties of the total expenditures on sovereign debt by uninformed investors.

Proposition 4  Uninformed investors spend more than informed investors in the bad state and less than informed investors in the good state.

Proof  First, we prove that uninformed investors spend less than informed investors in the bad state, that is $P_L B_L^I < P_H B_H^U + P_L B_L^U$.

Suppose not, so that $P_L B_L^I \geq P_H B_H^U + P_L B_L^U$. Then

$$P_L \kappa_L u'(W - P_L B_L^I) \geq P_L \kappa_L u'(W - P_H B_H^U - P_L B_L^U)$$

From the first-order conditions for informed investors in the bad state (10) and the first-order condition for uninformed investors at the marginal price for the bad state (11), this implies

$$u'(W + (1 - P_L)B_L^I) \geq u'(W + (1 - P_H)B_H^U + (1 - P_L)B_L^U)$$
or
\[
B^I_L - (B^U_H + B^U_L) \leq P_L B^I_L - (P_H B^U_H + P_L B^U_L) < B^I_L - (P_H B^U_H + B^U_L)
\]

where the second strict inequality is the result of \( P_L < 1 \). This is a contradiction for all \( P_H > P_L \).

Second, we prove that uninformed investors spend more than informed investors in the good state, this is, \( P_H B^I_H > P_H B^U_H \). Notice the first-order condition for uninformed investors for bidding at the marginal price for the good state (12) can be rewritten as

\[
(1-a) \left[ P_H \kappa_L u'(W - P_H B^U_H - P_L B^U_L) - (1-P_H)(1-\kappa_L)u'(W + (1 - P_H)B^U_H + (1 - P_L)B^U_L) \right] = \]

\[
a \left[ (1-P_H)(1-\kappa_H)u'(W(1 - P_H)B^U_H) - P_H \kappa_H u'(W - P_H B^U_H) \right]
\]

From equation (11) and \( P_H > P_L \) the left hand side is positive. This implies

\[
\frac{u'(W + [1 - P_H]B^U_H)}{u'(W - P_H B^U_H)} > \frac{P_H \kappa_H}{(1-P_H)(1-\kappa_H)}
\]

Comparing with the first order conditions for informed investors in the good state (10), then \( B^U_H < B^I_H \). Q.E.D.

The intuition for this result is as follows. On the one hand, Uninformed investors pay an overprice for a fraction \( \frac{B^U_H}{B^I_H + B^U_H} \) of the debt that they purchase in the bad state. This implies that, if uninformed investors spend the same amount as informed investors in the bad state, they incur the same losses as the informed in case of default, but receive smaller gains in case of repayment as \( B^U_H + B^U_L < B^I_L \). The marginal benefits of spending more in the bad state are thus larger than the marginal costs, which induces the uninformed to spend more than informed in the bad state. On the other hand, whatever uninformed spend in the good state, they also spend in the bad state. As they are overexposed to sovereign debt in the bad state they would rather reduce their exposure in the good state when compared to informed investors.

We refer to the set of parameters under which \( B^U_H = 0 \) (that is, parameters under which short selling constraints bind and uninformed investors bid nothing at \( P_H \), not purchasing any bond in the high state), as the partial participation region (partial because only informed investors participate in the good state and purchase debt).
For completeness, we refer to the set of parameters under which $B_{ih}^U > 0$, so that uninformed investors also purchase some debt in the good state on the good state, as the full participation region.

Notice that, in the partial participation region, uninformed investors know the default probability conditional on being able to purchase debt in equilibrium, because they know they are only able to purchase debt in the bad state. Hence, the informed and the uninformed behave symmetrically in the bad state, bidding the same amount at the same price, $P_L$. This is straightforward from replacing $B_{ih}^U = 0$ in the first order condition (11) and comparing it with the first-order condition (10). This implies that all information rents in the partial participation region stem from informed investor’s ability to purchase bonds in both states of the world.

Now that we have characterized how informed and uninformed investors bid at different prices, we continue solving the informed equilibria as follows. Using the demand functions for bonds in each state along with market clearing in each state, we can characterize properties of the prices as a function of the fraction of investors that are informed, which we denote by $n$. Then we will endogenize the fraction of investors in equilibrium, $n^*$, by exploiting a free-entry condition under which investors are indifferent between being informed or uninformed.

**Proposition 5** Consider the equilibrium with the highest sustainable prices. The good state price, $P_H$, increases with the fraction of informed investors, $n$.

**Proof** If the economy is in a partial participation region, market clearing for the good state is just

$$nP_H B_{ih}^I = D$$

Increasing $n$ is isomorphic to decreasing $D$, and as we showed in Proposition 2 this implies $\frac{dP_H}{dn} > 0$.

In contrast, if the economy is in a full participation region, market clearing for the good state is

$$nP_H B_{ih}^I + (1 - n)P_H B_{ih}^U = D,$$

which we can rewrite it in terms of excess demand as

$$ED(P_H) = B_{ih}^U + n(B_{ih}^I - B_{ih}^U) - \frac{D}{P_H} = 0.$$
Then
\[ \frac{dP_H}{dn} = -\frac{B_H^I - B_H^U}{n \frac{\partial B_H^I}{\partial P_H} + (1 - n) \frac{\partial B_H^U}{\partial P_H} - \left(- \frac{D}{P_H}\right)} > 0. \]

To see that this fraction is positive for the highest equilibrium price, note first that the numerator is positive, as we have shown that $B_H^I > B_H^U$. With respect to the denominator, however, as the slope of the demand (given by $n \frac{\partial B_H^I}{\partial P_H} + (1 - n) \frac{\partial B_H^U}{\partial P_H}$) and of the supply (given by $-\frac{D}{P_H}$) are both negative, in principle the denominator could be positive or negative. For the highest price in equilibrium, however, the denominator is negative: when evaluated at $P_H = 1 - \kappa$ there is an excess of supply, as $B_H^L = 0$ and $B_H^L = 0$ (then there is no demand), while the supply is given by $\frac{D}{1 - \kappa}$. The highest price in equilibrium is computed at the highest price at which demand and supply equalize, which implies that $n \frac{\partial B_H^I}{\partial P_H} + (1 - n) \frac{\partial B_H^U}{\partial P_H} < \left(- \frac{D}{P_H}\right) < 0$. Q.E.D.

In Figure 3 we illustrate how prices $P_H$ and $P_L$ depend on the fraction of informed investors $n$ in the economy. As reference we also include in the figure the price for the uninformed equilibrium, which we denote by $P_U$. There are two distinct regions in the graph. When $n$ is low, the economy is in a full participation region and when $n$ is high (in the figure to right of the arrows, for $n$ above 0.55), the economy is in a partial participation region.

In the partial participation region, $P_L$ does not change with $n$ as $B_L^I = B_L^U$ and the resource constraint in the bad state is just $P_L B_L^I = D$, which is independent of $n$. Even though in the figure it looks as if $P_L$ always declines with $n$ in the full participation region, this is not necessarily the case, as $P_H$ also enters in the market clearing for the bad state and the the evolution of $P_L$ is jointly determined by an increase in $n$ and by an increase in $P_H$, which act as forces in opposite direction.

In contrast, $P_H$ increases with the fraction of investors that are informed in the market, $n$, in both regions. In the full participation region the sensitivity of $P_H$ to $n$ is moderated by the participation of the uninformed investors, but in the partial participation region the sensitivity is larger (the rate of increase of $P_H$ with $n$ is larger) as there is a pure Cannibalization effect among informed investors, as the market in the good state is populated by a larger mass of informed investors, driving up demand and, thus, prices.

In Figure 4 we show how the utility of both informed and uninformed investors depend on the fraction of informed investors in the market. These utilities depend on
the evolution of prices, which we have shown depend on the fraction of informed investors. We also show the utility of investors in the uninformed equilibrium for reference. While the utility of uninformed investors decline with $n$ in the full participation region, it is independent of $n$ in the partial participation region as $P_L$ is independent on $n$ in this region, and this is the only price at which uninformed investors participate. For informed investors, however, utility always declines in the partial participation region (because of the cannibalization effect), while the utility in the full participation region may increase and then decline. Even though in the figure the utility of informed investors always decline with $n$, the reason is that in this specific numerical example $P_L$ always declines with $n$ as well.

The utility of informed investors in the informed equilibrium is always above the utility of investors in the uninformed equilibrium and the utility of uninformed investors in the informed equilibrium is always below their utility in the uninformed equilibrium. This does not imply, however, that informed investors are better-off in the informed equilibrium, as they have to spend utility costs to become informed in the first place. In Figure 5 we show that the informed equilibrium is characterized by the fraction of investors $n^*$ that make investors indifferent between being informed or uninformed, this is $U^I(n^*) - u(K) = U^U(n^*)$, which implies that all investors are always worse-off in the informed equilibrium.

It is important to highlight at this point that the utility of investors in the informed
equilibrium is lower as they end up spending resources to acquire information that is only useful to compete for whom gets the larger fraction of resources. There are no real benefits from information acquisition, and then information just implies a costly redistribution of resources across investors.

2.2.3 Multiplicity

Here we show that both the uninformed and informed equilibrium can coexist. Figure 5 shows a situation of multiple equilibria. The informed equilibrium, as discussed above, is the point at which the utility gap between informed and uninformed investors is equal to the utility cost of producing information $K$. In this specific case, a situation where all investors are uninformed is also an equilibrium since $\chi^U < K$.

The complementarity among the informed that generate this multiplicity is somewhat complicated. There is no complementarity among informed investors in the informed equilibrium (the utility of informed investors decrease as there are more informed investors, $n$), which is the reason there is an equilibrium $n^*$. However, there is an initial complementarity that arises from moving from a regime with a single price in equilibrium (uninformed equilibrium) to multiple prices (informed equilibrium). A deviation of becoming informed when there is a single price is lower than a deviation of becoming uninformed when there are many prices in equilibrium.
In other words, multiple equilibria arises because of the discontinuous increase in the incentives to become informed at $n = 0$ when comparing the situation under which information allows to reoptimize quantities bid at a single price (uninformed equilibrium) and the situation under which information allows paying the right price in each state. The larger gains are intuitively higher as they imply reoptimizing bidding at prices that more closely aligned with correct default probabilities.

Figure 5: Equilibrium Multiplicity

![Equilibrium Multiplicity](image)

Now we show how equilibria changes when changing fundamentals, in particular the average probability of default, $\hat{\kappa}$. There are in principle different ways in which $\hat{\kappa}$ can increase. For example, there can be an increase in the probability of a mediocre output $z$, an increase in the probability of a bad output $x$, and an increase in the probability of a bad state, $a$. As we discuss later all these changes will have different implications for equilibria, which implies that it is not sufficient to know how the average probability of default changes to predict changes in bond prices. Instead, one needs to know where the change comes from (a reduction in the probability of a very good output, an increase in the probability of a very bad output, a change in the cost of default) to predict how information and bond prices change in our setting.

We start by analyzing how our equilibria change when there is an increase in $z$ (a reduction in the probability of a high income realization and an increase in the probability of a mediocre realization). This change induces an increase in the gap between $\kappa_L$ and $\kappa_H$. Figure 6 shows how the set of possible equilibria change in response to
an increase in \( z \). An increase in the probability of default, which is generated by an increase in the gap between the two states, induces more information acquisition.

Figure 6: Effect of \( z \) on Equilibrium Multiplicity

The solid lines represent a low \( z \) and the dotted lines a higher \( z \). On the one hand, an increase in \( z \) increases the individual incentives to deviate and become informed in the uninformed equilibrium (increasing \( \chi^U \)). In the case of the numerical simulation this effect is large enough for the uninformed equilibrium to become unsustainable. On the other hand, it increases the gap between informed and uninformed investors in the informed equilibrium, thus increasing \( n^\ast \) in the informed equilibrium (the point at which the red solid line and the dotted black curve cross). A similar figure arises if we compare two levels of indebtedness, with a higher debt \( D/W \) also increases the incentives to become informed in both equilibria.

Figure 7 shows the equilibrium fraction of informed investors, \( n^\ast \), in the informed equilibrium, as we change the gap between the states in terms of default probabilities, \( z \), and also as we increase \( D/W \), the indebtedness of the country.

Now that we have characterized both the conditions for the uninformed equilibrium and the equilibrium fraction of informed investors in the informed equilibrium, we can compute the price \( P_U \) in the uninformed equilibrium and the prices \( P_H \) and \( P_L \) in the informed equilibrium for different levels of \( z \) (for the optimal \( n^\ast \) at each fundamental \( z \)). We displayed these prices in Figure 8.
First, there are clearly three regions of equilibria as a function of $z$. For low levels of $z$ there are low incentives to acquire information and only the uninformed equilibrium is sustainable. In contrast, for high levels of $z$ there are high incentives to acquire information and only the informed equilibrium is sustainable. For intermediate region of $z$ both equilibria coexist. Interestingly, once we compute the weighted average of prices $E(P) = a P_H + (1-a) [\omega P_H + (1-\omega) P_L]$, where $\omega = \frac{(1-n)B_H^U}{(1-n)(B_H^U + B_L^I) + n B_L^I}$, the informed equilibrium is not only characterized by higher volatility of prices (which can fluctuate between $P_L$ and $P_H$), but also by a lower average price, $E(P)$.

This result is important because not only investors are worse off in the informed equilibrium, as his cussed before, but also countries are worse off, both because debt can be roll over at lower prices and because they face higher volatility on those prices. This result on prices translate into the debt burden of countries: as the expected prices at which a country raises funds in the informed equilibrium are lower than in the uninformed equilibrium, the expected debt burden is higher in the informed equilibrium, as shown in Figure 9.

In other words, the informed equilibrium is inferior from both the country’s and the investors’ point of view. As we explained above, in this model information does not affect allocations, and then its costly acquisition motivated by obtaining a larger share of resources is only detrimental. even though the cost of information acquisition lies on investors, there is a pass through to the country in the form of lower sovereign
bond prices.

This characterization of equilibria and potential multiplicity has implications for interpreting how shocks to fundamentals affect a country’s debt burden, as well as the volatility that countries experience in their sovereign spreads. Assume for example a simple and plausible equilibrium selection under which a country remains in a given equilibrium as long as sustainable. This “conservative” equilibrium selection
introduces history dependence, or hysteresis, such that small shocks to fundamentals may generate large changes in the behavior of sovereign prices. In different words, the past matters and two countries with identical fundamentals can have different average price of their debt, different debt burdens and different price volatility just because their past was different.

These results are relevant in interpreting the mapping from fundamentals to sovereign debt prices. Periods of calm sovereign experiences do not necessarily imply that fundamentals are calm, as it may be that the country raises funds in an uninformed equilibrium, in which prices are simply not sensitive to movements in fundamentals. In contrast, periods of turbulent sovereign experiences do not necessarily imply that fundamentals have become much more turbulent than normal, as it may be that the country transitioned to an informed equilibrium in which prices are more sensitive to movements in fundamentals.

3 Two-Country Model

So far we have studied the different informational equilibria under which a single country may raise funds. Now we study a setting in which the same mass 1 of investors bid in two different countries, with the same characterization of income $Y$ and default costs $\theta$, but possibly different parameters. First, we focus on a situation in which information about $\theta$ cannot be produced and discuss contagion on sovereign debt prices and debt crises in its purest form, without any fundamental linkage across countries other than this common pool of investors. We show in this case that the condition for contagion just relies on the utility functions displaying prudence, that is $u''(c) > 0$.

Second, we introduce again the possibility of information acquisition and we show that there are complementarities across countries in the incentives to acquire information. This leads to a second form of contagion based on information regimes: A country transitioning to an informed equilibrium increases the likelihood the other country moves to an informed equilibrium as well.

To extend our analysis to a two-country environment and maintain tractability we make the following assumptions. We assume that the investor household is com-

\footnote{A history-dependent selection criterion is formally proposed and solved by Cooper (1994).}
posed of two members who each attend one of the two simultaneous auctions in the two countries. Before each auction takes place the household splits its wealth up between its two members. Each household member can become informed about the country whose auction they are attending but cannot communicate this information to the other member at the other country’s auction. They then place their bids without knowing the bids of the other household member. These assumptions allow us to restrict the number of equilibrium prices, and the conditional bids that a household might undertake. Thus, reducing the dimensionality of the problem to draw crisp conclusions.

Without these assumptions the number of prices can become quite large as the number of countries increases to 2. If there were $J$ values of $\theta$ in each country the number of prices in each country would be $J^2$ if some investors were becoming informed in each country. This in turn would imply that a household that was not informed about a particular country would be choosing $J^2$ possible marginal bid levels in each country for a potential total of $2J^2$ bids across the two countries. Our assumption reduces this number to a maximum of $2J$ prices in equilibrium.

### 3.1 Pure Contagion on Sovereign Debt Prices

We start by analyzing the simpler case in which both countries are in the uninformed equilibrium, and hence no investor is informed about the state in either country. This case turns out to be a fairly straightforward extension of the one-country uniformed case. Since there are now three possible assets (a safe asset, or no investment, country 1’s bonds and country 2’s bonds) the maximization problem can be written simply as

$$\max_{B_1, B_2} U = \tilde{\kappa}_1 [\tilde{\kappa}_2 u(W - P_1 B_1 - P_2 B_2) + (1 - \tilde{\kappa}_2) u(W - P_1 B_1 + (1 - P_2) B_2)]$$
$$+ (1 - \tilde{\kappa}_1) [\tilde{\kappa}_2 u(W + (1 - P_1) B_1 - P_2 B_2) + (1 - \tilde{\kappa}_2) u(W + (1 - P_1) B_1 + (1 - P_2) B_2)]$$

The first-order condition for the quantities bid in country $j$ is

$$\frac{E_j(u'(+))}{E_j(u'(-))} = \frac{P_j \tilde{\kappa}_j}{(1 - P_j)(1 - \tilde{\kappa}_j)}$$
where

\[ E_j(u'(-)) = \tilde{\kappa}_{-j}u'(W - P_jB_j - P_{-j}B_{-j}) + (1 - \tilde{\kappa}_{-j})u'(W - P_jB_j + (1 - P_{-j})B_{-j}) \]

and

\[ E_j(u'(+)) = \tilde{\kappa}_{-j}u'(W + (1 - P_j)B_j - P_{-j}B_{-j}) + (1 - \tilde{\kappa}_{-j})u'(W + (1 - P_j)B_j + (1 - P_{-j})B_{-j}) \]

The next proposition shows that, when utilities follow CRRA utility functions and display prudence (that is \( u'''(c) > 0 \)), an increase in the expected default probability in one country reduces the sovereign price in the other country. Notice we have constructed a simple portfolio problem where the returns on the two risky assets are i.i.d. and there is no feedback other than the one imposed by investors rebalancing their portfolio. Furthermore, there is no feedback from information acquisition, which we explore in the next subsection.

**Proposition 6** There is contagion (i.e. \( \frac{\partial P_j}{\partial \kappa_{-j}} < 0 \)) when preferences are CRRA.

**Proof** Impose resource constraints \( P_1B_1 = D_1 \) and \( P_2B_2 = D_2 \) for each country in the first order conditions. Denoting \( R = P_1B_1 + P_2B_2 = D_1 + D_2 \), write first-order conditions as

\[ \frac{\tilde{\kappa}_{-j}u'(W - R + \frac{D_j}{P_j}) + (1 - \tilde{\kappa}_{-j})u'(W - R + \frac{D_j}{P_j} + \frac{D_{-j}}{P_{-j}})}{\tilde{\kappa}_{-j}u'(W - R) + (1 - \tilde{\kappa}_{-j})u'(W - R + \frac{D_{-j}}{P_{-j}})} - \frac{P_j\tilde{\kappa}_j}{(1 - P_j)(1 - \tilde{\kappa}_j)} = 0 \]

For simplicity

\[ \frac{\tilde{\kappa}_{-j}u'(+ -) + (1 - \tilde{\kappa}_{-j})u'(+ +)}{\tilde{\kappa}_{-j}u'(- -) + (1 - \tilde{\kappa}_{-j})u'(- +)} - \frac{p_j\tilde{\kappa}_j}{(1 - p_j)(1 - \tilde{\kappa}_j)} = 0 \]

where the first argument of \( u' \) corresponds to the repayment or not of country \( j \) and the second argument to the repayment or not of country \( -j \).

\[
\frac{dP_j}{d\tilde{\kappa}_{-j}} = -\left[ \frac{u'(-) - u'(+ -) - (1 - \tilde{\kappa}_{-j})D_j}{P_j E_j(u'(-))} \right] + \left[ \frac{D_j}{P_j} \frac{\partial u''}{\partial \kappa_{-j}} u''(+ -) \right] - \left[ \frac{d}{d\tilde{\kappa}_{-j}} \left( \frac{u'(-) - u'(+ -) - (1 - \tilde{\kappa}_{-j})D_j}{P_j E_j(u'(-))} \right) \right] + \left[ \frac{D_j}{P_j} \frac{\partial u''}{\partial \kappa_{-j}} u''(+ -) \right] - \frac{\tilde{\kappa}_j}{(1 - P_j)^2(1 - \tilde{\kappa}_j)}
\]
There is contagion, by which we mean \( \frac{dP_j}{d\kappa_{-j}} < 0 \), when the denominator is negative which is the case (as was discussed in the one country case) for the highest \( P_j^* \) in equilibrium, and the numerator is also negative. The numerator is negative when,

\[
\frac{u'(+-) - u'(++)}{1-\kappa_{-j}} - \frac{D_{-j}}{P^2_{-j}} \frac{\partial P_{-j}}{\partial \kappa_{-j}} u''(++) < \frac{u'(-+)-u'(+)}{1-\kappa_{-j}} - \frac{D_{-j}}{P^2_{-j}} \frac{\partial P_{-j}}{\partial \kappa_{-j}} u''(-+)
\]

In words, the relative change in the gains from bidding in country \( j \) are smaller than the relative change in the losses. This implies a reduction in bidding in country \( j \), a decline in the demand and then a decline in sovereign prices. Q.E.D.

Figure 10 is similar to Figure 2, but for different levels of risk aversion (which, for CRRA utility functions, also implies different levels of prudence) and with the left hand side computed by the ratio of marginal utilities in expectation (which depends on the probabilities of default in the country that suffers a shock). We can draw several conclusions from the figure. First, as we already discussed, the larger the level of risk aversion the smaller the sovereign price in equilibrium.

Second, we show in blue a situation in which the other country has a low expected probability of default and in red when the expected probability of default in the other country is higher. As is clear from the figure, given a shock in the probability of default in the other country, contagion is stronger the larger the risk aversion (and then the larger the prudence). This result arises for two reasons. On the one hand, the higher level of prudence the larger is the reaction of investors, moving investment away from risky sovereign bonds. On the other hand, the higher the level of risk aversion the lower the price in equilibrium and more sensitive it is to movements in the left hand side (this is, the left and right hand sides coincide in flatter regions).

### 3.2 Contagion on Information Regime

With informed investors in at least one of our countries, the model becomes more complicated. To handle that and impose the restrictions implied by our assumptions about time and lack of information sharing within investor households, we define the state vector as \( s = (\theta_1, \theta_2, Y_1, Y_2) \) where \( \theta_i \) is the realized default cost and \( Y_i \) is the realized output level in country \( i \) and we denote by \( B_{ij}(\theta_i) \) the investor’s investment in country \( i \) at marginal price \( j \), \( P_{ij} \). Given this notation the payoff to the investor’s
Figure 10: Contagion Depends on Risk Aversion

portfolio is given by

\[ W + \sum_{i=1}^{2} \sum_{j: \theta_j \geq s_{\theta_i}} \left[ \mathbb{I}(Y_i > \bar{Y}(\theta_i)) - P_{ij} \right] B_{ij}(\theta_i), \]

where \( s_{\theta_i} \) is the realized value of \( \theta_i \). For the uninformed investor in country \( i \), there is an additional restriction which takes the form of a simple measurability condition, or

\[ B_{ij}(\theta_j) = B_{i,j'}(\theta_{j'}) \text{ for all } j \text{ and } j'. \]  

The portfolio payoff to the investor is given by

\[ \sum_{s} U \left( W + \sum_{i=1}^{2} \sum_{j: \theta_j \geq s_{\theta_i}} \left[ \mathbb{I}(Y_i > \bar{Y}(\theta_i)) - P_{ij} \right] B_{ij}(\theta_i) \right) \Pr\{s\}, \]

where \( \Pr\{s\} \) denotes the probability of state \( s \). The maximization problem of an uninformed investor household is to choose \( \{B_{ij}(\theta_i)\} \) for \( i = 1, 2 \) and \( j = 1, 2 \) subject to our measurability condition (13) for both \( i = 1, 2 \). The problem of an investor who is informed in country 1 is to choose \( \{B_{ij}(\theta_i)\} \) subject to our measurability condition (13) for both \( i = 1, 2 \). Note that the payoff is (14) minus the cost of information, \( K \).
The problem of an investor who is informed in both countries is to choose \( \{B_{ij}(\theta_i)\} \) to maximize (14) where we need to subtract \( 2K \) to get the final payoff.

Figure 11 shows a situation in which both countries are symmetric with respect to the fundamentals and plots the incentives to acquire information vs. the cost in three different equilibrium configurations with respect to information acquisition. Two of the configurations are symmetric - both countries in the uniformed equilibrium and both countries in the informed equilibrium. In these two cases we focus on symmetric equilibria. This means that the price in each country will be the same in the uniformed equilibria. It also means that prices \( P_H \) and \( P_L \) are the same in both countries in the informed equilibrium.

The third case we consider in figure 11 is an equilibrium in which the home country is informed and the other country is uniformed. In this case, the incentives \( \chi^I \) to acquire information in a country are computed imposing the no information equilibrium in the other country (the solid black curve). In this case there is a single price in the other country and the the information incentives depend on the marginal prices in the country where information is a possibility.

The green functions in the figure show the incentives to acquire information under
a symmetric informed equilibrium in which both countries have the same fraction of informed investors. While $\chi_1^I$ shows the incentives to acquire information in one country (solid green function), $\chi_2^I$ shows the incentives to acquire information in a second country (this is, the additional gains from acquiring information on a second country, the dashed green function). As we focus on a symmetric equilibrium, when $n < 0.5$ there is a mass $2n$ of investors who are informed in one of the countries, and $1 - 2n$ uninformed investors. When $n > 0.5$, then all investors are informed, with mass $2(n - 0.5)$ informed in both countries and mass $2(1 - n)$ informed in only one of the countries.

The Figure shows a situation in which, conditional on no one being informed in the other country, only the uninformed equilibrium is sustainable (this is, $\chi^U < K$ and $\chi^I < K$ for all $n$). In contrast, conditional on a symmetric information equilibrium, there are two equilibria that are sustainable, an uninformed equilibrium where no investor is informed about any country (as $\chi^U < K$) and an informed equilibrium where all investors are informed about at least one country (as $\chi_1^I > K$ and $\chi_2^I = K$ for $N^* > 0.5$). This shows the strength of complementarity across countries in the incentives to acquire information.

This result has important implications for the contagion of information regimes, with their implications on expected prices, volatility of prices and debt burden. As long as a country remains in the uninformed equilibrium it is less likely that investors decide to acquire information about other countries. As soon as a country changes to an informed equilibrium, then there are more incentives to acquire information in other countries.

The intuition for this result can be explained as follows: If one country is in an uninformed equilibrium while the other is in the informed equilibrium, investors that are uninformed in both countries do not have that much of an incentive to become informed as they can always participate more in the country with a single price. In contrast, when both countries have informed investors, investors that are uninformed in both countries cannot avoid paying excessive prices in some states of the world, and this increases their incentives to acquire information.

This intuition can be corroborated in Figure 12, which shows the bidding in country 1 of an investor who is uninformed about country 1. The bids are plotted as function of $z_1 = z_2$ shows how these bids react to a symmetric increase in both $z$’s. $B_1^U$ shows the bidding in country 1 when both countries are in the uniformed equilibrium. In
contrast, $B_{1,H}^I$ and $B_{1,L}^I$ shows the bidding in country 1 when both counties are in the informed equilibrium and the investor is informed in country 2 (but not about country 1). In the informed equilibrium case there are two marginal prices in each country and the graph shows his bids for each of these prices. $H$ and $L$ respectively. This shows the segmentation that information generates. Informed investors tend to invest more in the country in which they are informed, and at some point exclusively in such a country.

![Figure 12: Segmentation](image)

4 An illustration Based on the European Debt Crisis

Sovereign bond spread in Europe have displayed an interesting pattern since the Euro was introduced in 1999. After a long period in which government bond yields were relatively stable and quite similar across countries, they showed the first signs of divergence on September 2008 (right after the banking crisis in Ireland that followed the collapse of Lehman). This divergence then became magnified during 2010 and 2011 (during the so-called “Greek sovereign crisis”). As can be seen in Figure 13, after 2009
government bond yields increase significantly in comparison to the mean during the five years preceding the crisis for some countries (notably Greece, Ireland and Portugal), while for some other countries declined (notably Germany, France and Netherlands). The “fanning out” of spreads across European countries stopped right after Mario Draghi’s influential statement during a panel discussion in July 2012, where he claimed that the ECB “...is ready to do whatever it takes to preserve the Euro. And believe me, it will be enough.” Since then, the spreads of most countries started a process of convergence.

One explanation of this pattern is that government yields closely reflect fundamentals and that these fundamentals diverged considerably following the 2008 global crisis, substantially deteriorating in countries like Greece and Ireland and improving in countries like Germany and France. Another explanation is that, even though fundamentals did not change dramatically, the sensitivity of yields to fundamentals increased during the crisis.
To capture these explanations we run the following simple OLS regression

\[
Yields_{it} = (\beta_1 + \beta_2 I_c) \Delta GDP_{it} + (\beta_3 + \beta_4 I_c) \left( \frac{Debt}{GDP} \right)_{it} + \eta_i + \eta_t + \epsilon_{it}
\]

with yearly data from Eurostat for 28 European countries since 2000.\(^3\) \(Yields_{it}\) correspond to 10-year government bond yields for country \(i\) in year \(t\). The observed fundamentals we include are the yearly change of real GDP per capita, \(\Delta GDP_{it}\) and the outstanding level of public debt over GDP, \(\left( \frac{Debt}{GDP} \right)_{it}\). We allow for country and year fixed effects and also for the possibility that the sensitivity of yields to fundamentals changes during crises, captured by the indicator \(I_c\), which is equal to 1 for the crisis years, 2009-2013.

This regression controls for the first explanation, as GDP growth and debt over GDP seem to be significant variables explaining the evolution of yields, and shows that the second explanation is partly correct, as the sensitivity of yields to GDP growth and debt over GDP increases significantly during the crisis. Still these explanations are not enough to explain the evolution of sovereign yields during the recent European debt crisis, as shown in the evolution of the regression errors, \(\epsilon_{it}\) for 2009-2013. Figure 14 shows that the regression errors increased significantly between 2009 and 2013, which is the time frame in which yields diverged significantly.\(^4\)

Consistent with these results, Bocola and Dovis (2015) find that standard empirical models that tend to capture the evolution of yields in normal times, are not able to ac-

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\(^3\)Countries are Austria, Belgium, Bulgaria, Croatia, Cyprus, Czech Republic, Denmark, Estonia, Finland, France, Germany, Greece, Hungary, Ireland, Italy, Latvia, Lithuania, Luxembourg, Malta, Netherlands, Poland, Portugal, Romania, Slovakia, Slovenia, Spain, Sweden and United Kingdom.

\(^4\)For more involved empirical analyses, but similar results, see Borgy et al. (2012), von Hagen, Schuknecht, and Wolswijk (2011) and Baldacci and Kumar (2010).
commodate their dynamics during the recent European sovereign crisis. This implies the divergence cannot be explained by the observed behavior of the usual fundamentals, such as GDP growth or the level of indebtedness of the country. More specifically, the standard deviation of the regression errors increased by a factor of three during the crisis when compared to normal times.

Our paper provides an alternative interpretation of this residual, which cannot be accommodated by the more standard explanations. The divergence of the errors and the higher sensitivity to publicly observable fundamentals during crises may be the reflection of a different information regimen. This explanation can account not only for variables that are publicly observable but also for variables that are not public and costly to obtain by investors. In the recent European debt crisis these variables may include the political cost of default, the health of the domestic financial institutions, the exposure of domestic banks to certain assets, etc.

Indeed both the larger sensitivity to observed fundamentals and the larger errors from a regression based on those fundamentals can be explained by our model when one country suffers a shock that pushes it into an informed equilibrium. To see this, imagine a situation with seven countries with different $z$ levels, which can be interpreted as the inverse of the GDP growth (the larger the GDP growth, the lower the
expected probability of default for the country). Imagine also that during normal times these countries are all in an uninformed equilibrium. In Figure 15 this is captured by the seven green dots having a sovereign price according to $p^U$. As can be seen, prices are not very sensitive to fundamentals and can be perfectly explained by the observed fundamental $z$ (in this extreme there would be no errors if running a regression as the one above).

Imagine now that the country with the largest $z$ (or the smaller GDP growth) has a negative, relatively small, shock that reduces its GDP growth even more. For such a country the uninformed equilibrium would become unsustainable, then attracting information about its economy. As now there is information in some other country there are more incentives to acquire information about the countries that have not being hit by the GDP growth shock, and some of them would also move to an informed equilibrium. After this reinforcing effect on information acquisition across countries, the Figure 15 may change to Figure 16.

In Figure 16, the five countries with the lowest GDP growth (highest $z$) have moved to an information equilibrium. Sovereign bond prices reflect now information not only about $z$ but also about $\theta$, which is not publicly observable and is not included in the regression. This effect has two implications. First, for some countries information about $\theta$ is “positive” ($\theta_H$ or high cost of default) and their price will be $p_H$, which is above what $p^U$ would imply. For some countries information about $\theta$ is “negative”
(θ_L or low cost of default) and their price will be p_L, which is below what p_U would imply. This immediately implies that during a crisis any regression model that uses standard publicly observable data to explain yields will have more errors that in normal times, as there are variables not observed by the econometrician that enter into the pricing of debt. Second, since in the informed equilibrium the average price is lower than in the uninformed equilibrium, having more countries in the informed equilibrium makes the sensitivity of prices to fundamentals z larger (higher slope in the regression, which is now an average between the blue p_U that uninformed countries follow and the purple E(p) that country sin the informed equilibrium follow in expectation).

5 Conclusions

We constructed a simple model of portfolio choice with information acquisition, where the portfolio is composed by sovereign debt of different countries and information about some determinants of default are not easily observable and costly to acquire.

For a single country we have shown that the participation of informed investors (informed equilibrium) is more likely when the country is highly indebted and when there is more certainty about its fundamentals. An equilibrium in which a country
raises funds from informed investors is inferior, as investors obtain less utility and the country faces higher and more volatile prices, then higher debt burden.

Given that an informed and a uninformed equilibrium may coexist, small changes in fundamentals can generate large changes in the sovereign debt experience. If the selection of equilibrium is hysteresis (the country remains in a given equilibrium as long as it is sustainable) then the sovereign price of two countries with the same fundamentals but different past can have very different experiences.

Once we allow for many countries, there are two important sources of contagion. On the one hand, contagion of sovereign debt prices does not require fundamental linkages or common factors, just a common pool of investors that react to changes in fundamentals of each country and rebalance the portfolio. On the other hand, the information regime is also contagious, as one country moving to an informed equilibrium increases the incentives to acquire information about other countries, even in the absence of economies of scale to acquire information.

Our results show why it is not straightforward to interpret changes in sovereign debt prices as informative about the country’s fundamentals, as they depend not only on publicly observable fundamentals but sometimes also on fundamentals that are not easily observable, as they depend not only on the country’s own fundamentals but also on other countries’ fundamentals, as they depend not only on the country’s informational regime (and thus, potentially on past fundamentals) but also on other countries’ informational regime.

We have highlighted the main forces behind information acquisition and then behind the mapping between observable and non-observable fundamentals to sovereign debt spreads. There are many reasons why we may expect these forces to be also quantitatively relevant. Just to mention a few magnifying forces. First, the probability of default is endogenous and depends on sovereign prices. There is a feedback effect across countries: an exogenous increase in default probability in one country induces a reduction of prices in several other countries, increasing the probabilities of default in all those countries, further reduction of prices, and so on. Second, fundamental linkages across countries naturally magnify contagion. Third, if there is time varying prudence, for example because of time varying risk-aversion or time varying wealth. Fourth, market segmentation can concentrate contagion in certain regions, buffering others. Finally, how a shock in a country changes the informational equilibrium in
other countries depend on the structure of the costs to acquire information: if a coun-
try attracts informed investors and then makes easier for them to acquire information
about other similar countries, then it is more likely that those other countries also
attract informed investors.
References


