When transparency improves, must prices reflect fundamentals better?*

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Abstract

No. Regulation often mandates increased transparency to improve how informative prices are about fundamentals. We show that such policy can be counterproductive. We study the optimal decision of an investor who can choose to acquire costly information not only about asset fundamentals but also about the behavior of liquidity traders. We characterize how changing the cost of information acquisition affects the extent to which prices reflect fundamentals. When liquidity trading is price-dependent (e.g., due to forced deleveraging), surprising results emerge: higher transparency, even if exclusively targeting fundamentals, can decrease price informativeness, while cheaper access to non-fundamental information can improve efficiency.

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The sharp decline in ABS prices during the financial crisis, highlighted in Figure 1, precipitated a period of extraordinary uncertainty that was characterized by two salient features. First, investors were unable to discern whether these price drops were driven by fundamentals or trading by other investors (e.g., due to deleveraging or hedging). In fact, Stanton and Wallace (2011) argue that market prices for AAA ABX indices in June 2009 were “inconsistent with any reasonable assumptions for future default rates.”

Second, much of the non-fundamental trading demand appeared to be price-dependent, i.e., was driven by feedback trading. As Acharya, Philippon, Richardson, and Roubini (2009) discuss, the price collapse led to a “cascading vicious circle of falling asset prices, margin calls, fire sales, deleveraging, and further asset price deflation.”

Figure 1: Levels for the Markit ABX AAA and AA series from 2006

The figure plots the level of the Markit ABX index tracking bonds with a rating of AAA and AA issued in the first half and second half of 2006.

The above episode highlights that while investors, regulators and academics turn to security prices to infer the “market’s expectation” of fundamentals, frictions can drive a wedge between the two and amplify investor uncertainty, especially during crises. A natural regulatory response to reduce this uncertainty is to improve transparency. For instance, the events of the subprime crisis led to the introduction of higher requirements for disclosure of loan-level data as part of the Dodd-Frank Act (2010), disclosure of bank stress test results (see Goldstein and Sapra (2012)), and the provision of forward guidance on monetary policy (see Bernanke (2013)).

\[1\] For example, they show that even if the realized recovery rate fell below any ever observed before in the United States, the AAA 06-2 price implied default rates in excess of 100%.

\[2\] Prior to the financial crisis, issuers of MBS were only required to provide aggregate data, such as the
such situations, a question naturally arises: when investors face uncertainty about fundamentals and other traders, and can choose what information to acquire, do these policies necessarily increase the extent to which prices reflect fundamentals?

Our analysis suggests that the answer to this question is \textit{no}. In the presence of frictions, investors often choose to learn not only about asset fundamentals (e.g., cash-flows, systematic risk), but also liquidity trading by other traders, and prices reflect both types of information. We find that when liquidity trading is price-dependent (e.g., due to deleveraging), learning exhibits \textit{complementarity} — learning about fundamentals increases the value of learning about liquidity trading, and vice versa. As we show, this complementarity implies that better access to information can \textit{decrease} price informativeness about fundamentals. Perhaps surprisingly, this is true even when the increase in transparency targets fundamental information; similarly, making it easier to learn about other traders exclusively can lead to an \textit{increase} in price efficiency.

We consider a three-date (two-period) model. At date 3, the risky asset pays a terminal dividend, which reflects the asset’s fundamental value. We study the costly information acquisition and trading decisions of a risk-neutral investor, who maximizes terminal wealth, faces quadratic transaction costs, and anticipates trading the risky security with liquidity (noise) traders at dates 1 and 2. She is also subject to a shock that can force her to liquidate and exit the market at date 2 — this generates an incentive to learn about short-term prices, and consequently, about both fundamentals and liquidity trading. The key feature in our model is that the liquidity trading can be price-dependent: a component of the aggregate noise trader demand is generated by feedback trading.\footnote{While the assumption of price-independent noise is often made for modeling convenience, \textit{price-dependent} noise trading is likely to be quite relevant. As we discuss in Section 2.4, such trading behavior may arise not only due to behavioral biases (e.g., extrapolative expectations), but also as a consequence of institutional and market frictions such as forced deleveraging during financial crises and performance-flow sensitivity in delegated asset management. A number of recent papers consider alternative specifications in which noise trading depends on the contemporaneous price (e.g., Goldstein, Ozdenoren, and Yuan (2013b), Goldstein and Yang (2014a)). In contrast, feedback trading in our model depends on lagged price changes (as in DeLong, Shleifer, Summers, and Waldmann (1990) and others).} The investor derives value from providing liquidity, and the extent to which she can take advantage of such opportunities depends on the precision of the information she chooses to learn about fundamentals and noise trading.

A key mechanism in generating our results is that when liquidity demand is price-dependent, learning about fundamentals and liquidity is complementary. This is because liquidity demand...
responds endogenously to price changes: the feedback demand tomorrow depends upon the price today, which in turn, depends on the investor’s expectation of feedback demand tomorrow. Therefore, more precise information about fundamentals leads to a bigger change in today’s price, which leads to more feedback trading tomorrow — this increases the value of learning more about liquidity trading. This complementarity gives rise to predictions that differ from those in linear, noisy RE models (e.g., Grossman and Stiglitz (1980), Kyle (1985)), where noise trading is price-independent. In such models, improving information about fundamentals usually leads to higher price efficiency since the price is a linear combination of information about fundamentals and noise. In our model, the price depends non-linearly on the investor’s information, and as we show, this implies that increasing information about fundamentals, when accompanied by sufficient learning about liquidity traders, can make the price less informative about fundamentals.

We consider two measures of price informativeness. The first, which we denote as accuracy, captures how closely, on average, the level of prices reflects fundamentals. The second, which we call efficiency, measures the error in the conditional expectation of fundamentals, given the information in the price. In the absence of feedback trading, the two measures are closely related — more learning about fundamentals (liquidity trading) leads to an increase (decrease) in both accuracy and efficiency. In the presence of price-dependent liquidity trading, the two measures can move in opposite directions. In particular, we show that while more learning about feedback trading always decreases accuracy, it can actually increase efficiency. Similarly, while learning more about fundamentals tends to increase efficiency, it can also lead to a decrease in accuracy. From an empirical or regulatory perspective, it is not apparent which measure is the right one, especially when investors, academics and regulators are uncertain about the structure of the economy. While efficiency is the appropriate theoretical measure for agents within the model, it is difficult to measure in practice since it requires knowledge of the joint distribution of fundamentals, liquidity demand, and prices. In contrast, accuracy is easier to estimate empirically, may be more robust to mis-specification, and appears to match “real-world” measures commonly used by empirical studies and regulators.

We provide an analytical characterization of how transparency affects both accuracy and efficiency, and then explore its implications using a series of examples. The complementarity between learning about fundamentals and feedback trading implies that an increase in overall transparency can decrease both efficiency and accuracy, and that the effects may be disproportionately large. The complementarity also generates counterintuitive predictions when

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4 More precisely, given fundamentals $\phi$ and price $P$, accuracy captures $-\mathbb{E} \left[ (\phi - P)^2 \right]$ and efficiency captures $-\mathbb{E} \left[ (\phi - \mathbb{E}[\phi|P])^2 \right]$. 

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transparency is targeted along a specific dimension. When the increased transparency targets fundamental information, the investor learns more about fundamentals. But this makes learning about feedback trading more valuable, and the resulting increase in learning about noise trading can actually decrease accuracy and efficiency. Similarly, a decrease in transparency about noise trading leads to less learning about feedback trading, which indirectly reduces the value of learning about fundamentals, and as a result, can lead to lower efficiency and accuracy.

Regulation that increases transparency about fundamentals (e.g., forward guidance by central banks, disclosure of stress tests outcomes) is generally introduced in response to increased uncertainty during economic crises, and often with the intent of reducing such uncertainty in the future. And while it is of interest throughout the business cycle, price informativeness is of particular importance during episodes of large price declines and run-ups, which are accompanied by price-dependent trading. As such, the mechanism we describe is important for studying the impact of regulatory changes to transparency, and our analysis highlights one channel through which such regulation can actually exacerbate the problem it is intended to address.\textsuperscript{5} Our analysis also cautions against policies that limit the availability or timeliness of information about other participants, as have been recently proposed to mitigate the adverse effects of high-frequency traders (e.g., Harris (2013)). Finally, we show that allowing feedback trading to respond to an increase in transparency can amplify the effects on price uninformativeness, instead of dampening them. When learning becomes easier for the investor, a natural response for liquidity traders is to reduce the intensity of their feedback trading. But such a response decreases the investor’s opportunities to speculate, and therefore reduces her incentive to acquire information. As a result, when the response by liquidity traders is large enough, we show that an increase in transparency can actually lead to less learning about fundamentals, which can lead to lower efficiency and accuracy.

The paper proceeds as follows. The rest of the introduction provides a brief example to highlight the intuition for some of our results. The next section discusses the relevant related literature. Section 2 describes the model, characterizes the equilibrium, and solves for the investor’s optimal information acquisition. Section 3 presents the main analysis of the paper. It describes how price accuracy and efficiency change with general and targeted transparency, and how our results are affected when feedback trading can respond to changes in transparency. Section 4 discusses some implications of our analysis and concludes. Proofs for the main results can be found in the Appendix.

\textsuperscript{5}Filardo and Hofmann (2014) empirically evaluate the impact of forward guidance by the Federal Reserve, the Bank of Japan, the ECB, and the Bank of England on the level and volatility of interest rate expectations, and discuss its role in potentially encouraging excessive risk-taking by investors.
An example

To highlight the intuition for how higher transparency can reduce price informativeness, consider the following example. Suppose the asset’s fundamental value is normally distributed around $100, and the price-independent component of noise trading is zero. Moreover, suppose feedback trading intensity is equally likely to be positive (high) or negative (low) and is independent of the fundamental value. Suppose the realization of the fundamental is $65, and feedback intensity is high: for instance, there could be investors or intermediaries who, faced with financing constraints, are forced to delever as prices fall.

Without any learning about fundamentals, the initial price of the risky asset is its unconditional expected value, $100. This is true even if the investor learns about feedback trading — if the investor’s fundamental valuation is unchanged, there is no change in the date 1 price to trigger demand from liquidity traders. However, if the investor learns the asset’s fundamental value is $65 before trading at date 1, but learns nothing about liquidity demand, then the date 1 price is $65. In this case, the price is efficient and accurate, since it reflects the investor’s expectation of the asset’s fundamentals.

When the investor learns that feedback intensity is high and the fundamentals are low, the date 1 price is lower than $65. Given the negative signal about fundamentals, she anticipates liquidity traders will sell the asset, depressing tomorrow’s price. Consequently, the current price is below her conditional expectation of $65 — learning about feedback traders decreases price accuracy. Moreover, if the investor anticipates liquidity selling to be sufficiently high, the price can fall to a level that is further from the fundamental value of $65 than if the investor had not learn about fundamentals. As a result, when the investor expects the feedback intensity to be sufficiently high, learning more about fundamentals can make the price less accurate!

Moreover, learning about liquidity trading makes the price a noisier signal of fundamentals. Consider the price paths in Figure 2. If the investor does not learn about liquidity traders, a date 1 price of $65 corresponds to a fundamental valuation of $65. However, with noisy learning about feedback, a date 1 price of $65 to an outside observer could correspond to fundamentals of $70 and a (noisy) signal that the feedback effect intensity will be weak (solid line) or fundamentals of $85 and and a signal that the feedback effect will be strong (dotted line). In this case, conditional on the price, the outside observer is more uncertain about fundamentals when the investor learns about liquidity demand i.e., efficiency is lower.

While more learning about liquidity traders always decreases accuracy, it can sometimes increase efficiency. For instance, suppose the investor learns about feedback trading intensity perfectly and the date 1 price is $30 (dashed and dot-dashed lines in Figure 2). This can arise either because feedback intensity is high and fundamentals are $70, or feedback intensity is low and fundamentals are $35. However, given that the prior distribution of fundamentals is normal
Figure 2: Learning about price-dependent liquidity demand

The figure plots the price as a function of the date \((t = 0, 1, 2, 3)\) for the following cases. The solid and dotted lines correspond to noisy learning about feedback: either fundamentals are $70 and expected feedback is low (solid line), or fundamentals are $85 but feedback is expected to be high (dotted line). In either case, realized feedback intensity is high. The dashed and dot-dashed correspond to perfect learning about intensity: either fundamentals and feedback intensity are both high (dashed), or fundamentals and feedback intensity are both low (dot-dashed).

around $100, an outside observer is more certain that fundamentals are high because fundamentals of $35 are not very likely. Intuitively, when the investor has sufficiently precise information about feedback intensity, small differences in fundamentals become amplified, which can make some price realizations very informative about both feedback intensity and fundamentals. As a result, more learning about liquidity trading can make the price more efficient. Moreover, as discussed in Section 3.1, more extreme price realizations can be more informative about fundamentals in our model. This result distinguishes our predictions from those in standard, linear models with normally-distributed shocks, in which the posterior variance about fundamentals, conditional on the price, is constant.\(^6\)

Our analysis builds on the intuition above. Instead of taking the choice of information as given, we solve for how the investor’s optimal choice of information responds to changes in transparency, and how this, in turn, affects efficiency and accuracy. As we show, the mechanism described above together with the complementarity in learning between fundamentals and noise trading, implies that increasing transparency (even if it targets fundamentals) can decrease efficiency and accuracy.

\(^6\text{In such linear-normal models, the price is a conditionally normal signal of the fundamental, and so its realization does not affect the conditional variance of fundamentals.}\)
1 Related literature

A number of papers, including DeLong et al. (1990), Cutler, Poterba, and Summers (1991), Hong and Stein (1999), and Barberis, Greenwood, Jin, and Shleifer (2014), explore the impact of feedback trading on asset prices. In our paper, we extend the analysis of DeLong et al. (1990) by endogenizing the information received by the rational investor. Empirically, Greenwood and Shleifer (2014) use survey data to document the existence of investors with extrapolative expectations, which can induce feedback demand. We show that the anticipation of this demand (not simply the demand shock itself) can reduce the extent to which prices reflect fundamental information.

Our paper is related to the large literature on endogenous information acquisition in financial markets. Counter to the standard intuition of Grossman and Stiglitz (1980), a number of papers have identified different channels through which learning about fundamentals can be complementary across investors. Most closely related are papers in which this complementarity results from the fact that, in the presence of persistent (but price-independent) noise trading, learning more about fundamentals makes the price a more informative signal about noise trading (as in Avdis (2012)) — hence, acquiring fundamental information can become more valuable as the number of informed investors increases. The nature of complementarity and the underlying mechanism are very different in our model. Our analysis focuses on complementarity in learning about different payoff components (fundamentals vs. liquidity trading) for a single investor. As such, the decrease in price efficiency in our model is not driven by higher order beliefs or “beauty contest” effects (as in Cespa and Vives (2014)). Our results also highlight an important distinction between persistent, but price-independent, noise trading, and price-dependent noise trading: the complementarity we focus on, arises even in the absence of persistence in price-independent liquidity trading, but requires the possibility of price-dependent liquidity demand. Finally, in standard, noisy RE models with heterogeneous information, the equilibrium price serves the additional role of imperfectly “aggregating” private information about fundamentals. Since there is a single informed investor in our model, the price “reflects” her information perfectly. However, the price may still be inefficient in that an outside observer’s ability to form expectations about fundamentals may be limited.

Our results are more closely related to that of Goldstein and Yang (2014b) who show that acquiring information about different components of fundamentals can be complementary. These papers include Froot, Scharfstein, and Stein (1992), Barlevy and Veronesi (2000), Veldkamp (2006), Ganguli and Yang (2009), Garcia and Strobl (2011), Breon-Drish (2011), Avdis (2012), Cespa and Vives (2014), and Goldstein, Li, and Yang (2013a). In a more general setting, Hellwig and Veldkamp (2009) show that when agents’ actions exhibit complementarity, so do their information choices. In their noisy RE model, learning about the first component of fundamentals reduces uncertainty about trading on, and encourages learning about, the second component.

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8In their noisy RE model, learning about the first component of fundamentals reduces uncertainty about trading on, and encourages learning about, the second component.
Moreover, we consider a setting in which an investor can obtain a direct signal about aggregate noise trader demand, similar to Ganguli and Yang (2009). The feature that distinguishes our model from this class of linear RE models, however, is that the noise trading in our setting is price-dependent: more precise learning about fundamentals leads to larger price changes today, which increases future demand from feedback traders, thereby increasing the value of information about others. The endogenous nature of noise trading generates predictions that do not naturally arise in linear RE models — for instance, an increase in transparency about fundamentals can lead to a decrease in price informativeness, even when the investor learns more about fundamentals.

Finally, our paper relates to the broad literature that studies the costs and benefits of higher transparency and disclosure. Our model is stylized to highlight a novel tradeoff associated with increased transparency, and as such, abstracts from other tradeoffs already analyzed in the literature (see Goldstein and Sapra (2012) and Bond, Edmans, and Goldstein (2012) for recent surveys). On the one hand, improved transparency and disclosure can decrease adverse selection across market participants, reveal valuable information to real decision makers (and hence induce better allocative efficiency), and provide better market and supervisory discipline for firms. On the other hand, such changes can also reduce risk-sharing (i.e., the Hirshleifer (1971) effect), lead to over-investment in disclosure (e.g., Fishman and Hagerty (1989)), induce risk-shifting and short-termism in managerial decisions (e.g., Sapra (2002)), generate inefficient coordination on public information (e.g., Morris and Shin (2002)), crowd out the ability of managers or regulators to learn from the market (e.g., Bond and Goldstein (2013), Goldstein and Yang (2014c)) and reduce expected returns for investors (e.g., Kurlat and Veldkamp (2013)).

2 The model

This section introduces the model and presents some preliminary results. The first subsection describes the setup of the model. Section 2.2 characterizes the financial market equilibrium (prices and quantities). Section 2.3 formally characterizes the investor’s optimal information acquisition problem, and discusses the incentives for the investor to learn along various dimensions. Section 2.4 provides a discussion of our assumptions and results.

2.1 Model setup

Assets and payoffs. There are three dates (i.e., $t \in \{1, 2, 3\}$) and two assets. The risk-free security is in perfectly elastic supply, and the net risk-free rate is normalized to zero. The risky asset is in zero net supply, is traded at dates $t \in \{1, 2\}$ at price $P_t$, and pays a liquidating
dividend $\phi + \theta$ at date 3, where $\phi$ and $\theta$ are independent and have finite first and second moments.\(^9\) The predictable component of the asset’s payoff is captured by $\phi$ — the realization of $\phi$ is revealed at date 2, and the rational investor can acquire costly information about $\phi$ prior to trading at date 1. In contrast, $\theta$ reflects the residual / unpredictable component of the asset’s payoff since no information about $\theta$ is revealed before the payoff is realized at $t = 3$. Without loss of generality, we set $\mathbb{E}_0[\theta] = 0$ and $\mathbb{E}_0[\phi] = 0$. We make additional distributional assumptions about $\phi$ when we solve for the optimal information acquisition decisions in Section 2.3.

**Market participants.** There is a risk-neutral investor who faces demand from liquidity (noise) traders. The liquidity demand for the risky asset, denoted by $Z_t$, is given by:

$$Z_t = Z_{t-1} + \beta(P_{t-1} - P_{t-2}) + u_t,$$

(1)

where $u_t$ and $\beta$ are independent of each other and of $\phi$ and $\theta$, and for completeness, we set $P_0 = P_{-1} = \mathbb{E}_0[\phi + \theta] = 0$.\(^{10}\) There are two components to noise trading in our model. The first component, $u_t$, captures price-independent liquidity demand, and corresponds to the standard specification of aggregate liquidity shocks in most rational expectations models.\(^{11}\) The second component, $\beta(P_{t-1} - P_{t-2})$, captures price-dependent feedback trading. As in the noisy RE literature, we do not explicitly model the preferences of noise traders in order to maintain tractability, but our reduced form specification is consistent with empirical evidence (see Section 2.4).

The risk-neutral investor maximizes terminal wealth, subject to quadratic transaction costs. With probability $\rho \equiv \Pr(\xi = 1)$ she is subject to forced liquidation before trading at date 2, where $\xi \in \{0, 1\}$ is an indicator variable for the liquidity shock which is independent of both fundamental and noise shocks. As such, when $\rho \neq 0$, she has incentive to learn about factors which affect the short-term value of the asset. Denote her optimal demand at date $t$ by $x_t$. Conditional on being able to trade at date 2 (i.e., if $\xi = 0$), the investor chooses a limit order

\(^9\)The assumption of zero net supply is without loss of generality. An alternative formulation, which generates identical results, specifies that the risk-neutral investor holds the entire supply of the risky asset $Q \neq 0$ at date zero.

\(^{10}\)If we allowed for trading at date 0, before the investor updates her beliefs based on her acquired signals, the equilibrium price would be $\mathbb{E}_0[\phi + \theta]$. For expositional clarity, we do not show the equilibrium derivation here.

\(^{11}\)In an infinite-horizon setting, allowing price-independent demand to follow a random walk would generate concerns about stationarity; since we consider a finite horizon model, such concerns are moot. The assumption price-independent component of noise trader demand follows a random walk is not necessary for our results to hold. Qualitatively similar results arise, for instance, if this component follows an AR(1) process i.e., if $Z_2 = \beta P_1 + u_2 + a_u Z_1$. 

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\(x_2\) to maximize
\[
V_2(\phi, P_2, x_1) = \max_x \mathbb{E}_2 \left[ x (\phi + \theta - P_2) - \frac{c}{2} (x - x_1)^2 \right],
\]  
(2)
where \(c > 0\) is the marginal cost of the transaction \(x - x_1\). If forced to liquidate (i.e., \(\xi = 1\)), she is replaced by an investor who lives for one period and submits a limit order \(x^\rho\) to maximize
\[
V_2^\rho = \max_x \mathbb{E}_2^\rho \left[ x (\phi + \theta - P_2) - \frac{c}{2} x^2 \right].
\]  
(3)
This is to ensure that there is an investor to supply liquidity to the noise traders at date 2.\(^{12}\)

At date 1, the risk-neutral investor chooses a limit order \(x_1\) to maximize her expected utility, conditional on any information about fundamentals and liquidity trading she has chosen to acquire, i.e., \(x_1\) is chosen to maximize
\[
V_1 = \max_x \mathbb{E}_1 \left[ (1 - \rho) V_2(\phi, P_2, x) + x (P_2 - P_1) - \frac{c}{2} x^2 \right],
\]  
(4)
conditional on her information set at date 1. At date 0, the investor chooses the precision of her date 1 information about fundamentals (i.e., \(\phi\)) and liquidity trading (i.e., \(\beta\) and \(u_t\)), which we shall describe in the next subsection.

Finally, the date 2 and date 1 prices are determined by market clearing conditions, which are given by
\[
\xi x_2^\rho + (1 - \xi) x_2 + Z_2 = 0 \text{ and } x_1 + Z_1 = 0,
\]  
(5)
respectively.

### 2.2 Financial market equilibrium

We solve for the equilibrium by working backwards. At date 3, all uncertainty is resolved, and investors are paid the realized dividend, \(\phi + \theta\).

**Date 2:** Before trade occurs, \(\phi\) is publicly revealed. If the risk-neutral investor is not forced to liquidate before trading at date 2, her optimal demand maximizes the objective in (2) and is given by
\[
x_2 = x_1 + \frac{1}{\xi} (\phi - P_2),
\]  
(6)
\(^{12}\)An alternate specification would be to assume a continuum of investors, with a fraction \(\rho\) to be replaced at date 2. While this does not affect the equilibrium at date 2, it may affect the equilibrium price at date 1, if investors can update their beliefs using the price. See Section 2.4 for a discussion of asymmetric information in this setting.
and her value function simplifies to

$$V_2 = \frac{1}{2c} (\phi - P_2)^2 + x_1 (\phi - P_1) - \frac{c}{2} x_1^2.$$  \tag{7}

If she is forced to liquidate, the optimal demand of the investor who replaces her solves (3), and is given by

$$x^d_2 = \frac{1}{c} (\phi - P_2).$$  \tag{8}

The market clearing condition at date 2 implies that, in either case, the date 2 price is given by

$$P_2 = \phi + c (Z_2 + (1 - \xi) x_1).$$  \tag{9}

**Date 1:** Before trading, the investor observes her private information about fundamentals and noise trading. She maximizes the objective function in (4), which after substituting in (7), simplifies to

$$V_1 = \max_x \mathbb{E}_1 \left[ (1 - \rho) \frac{1}{2c} (\phi - P_2)^2 + x ((1 - \rho) \phi + \rho P_2 - P_1) - \frac{c}{2} x^2 \right].$$  \tag{10}

This implies that the optimal date 1 demand is given by

$$x_1 = \frac{1}{c} ((1 - \rho) \mathbb{E}_1 [\phi] + \rho \mathbb{E}_1 [P_2] - P_1),$$  \tag{11}

and market clearing at date 1 implies that the price is given by

$$P_1 = (1 - \rho) \mathbb{E}_1 [\phi] + \rho \mathbb{E}_1 [P_2] + c Z_1.$$  \tag{12}

Ignoring noise trader demand, the date 1 price is a weighted-average of the investor’s expectation of the asset’s short- and long-term value, which is a result of the investor’s partial myopia. Finally, note that since $P_0 = P_{-1} = 0$, $Z_1 = u_1$ and $Z_2 = \beta P_1 + u_2 + u_1$. In particular, there is no price-dependent demand at date 1, and this ensures that the investor does not learn any additional information about feedback intensity (i.e., $\beta$) from $P_1$. Combining this and the expression for the date 2 price in (9) implies that the date 1 price is

$$P_1 = \frac{\mathbb{E}_1 [\phi] + \rho \mathbb{E}_1 [u_2] + c (1 + \rho^2) u_1}{1 - \rho c \mathbb{E}_1 [\beta]}.$$  \tag{13}

It is important to note that for this to be an equilibrium price, one needs $\mathbb{E}_1(\beta) \leq 1/c$ — otherwise, the equilibrium does not exist (see DeLong et al. (1990)). We impose a sufficient condition (i.e., an upper bound on $\beta$) to ensure that an equilibrium exists in the following
Proposition 1. Suppose $\beta < 1/c$. Then equilibrium prices are given by:

$$P_1 = \frac{E_1[\phi + \rho c E_1[u_2] + c(1+\rho^2)u_1]}{1 - \rho c E_1[\beta]}, \quad \text{and} \quad P_2 = \phi + c(\beta P_1 + u_2 + \xi u_1).$$

(14)

By conditioning on the price at date 1, the investor can infer $Z_1 = u_1$ perfectly. We can now simplify the objective function in (4):

$$V_1 = (1 - \rho) \frac{c}{2} E_1[(\beta P_1 + u_2 + \xi u_1)^2] + \frac{c}{2} u_1^2.$$  

(15)

We use this expression to characterize the optimal information acquisition decision in the next subsection.

2.3 Optimal information acquisition

At date 0, the investor optimally chooses the precision of information she will observe before trading at date 1, given a cost of information acquisition. We allow her to learn about fundamentals $\phi$, the price-independent component of liquidity trading $u_2$ and the intensity of price-dependent liquidity trading $\beta$. In order to completely and tractably characterize the optimal information acquisition decision, we specify distributional assumptions. We assume that $\phi \sim N(0, \sigma_f^2)$, $\theta \sim N(0, \sigma_\theta^2)$, $u_t \sim N(0, \sigma_{u,t}^2)$ and $\beta \sim \{-b, b\}$ with equal probability and $0 < b < 1/c$. The mean-zero assumption for these variables is a normalization. Empirical evidence (see 2.4 below) suggests that positive feedback is more likely (i.e., $E[\beta] > 0$), and that learning about the strength of the feedback effect (rather than its sign) is more natural. However, setting $E[\beta] = 0$ ensures that with no learning about $\beta$, the price corresponds to linear combination of expectations of $\phi$ and $u_2$ (as in a linear, noisy RE model) and, importantly, biases us against finding any negative impact of learning on price informativeness. As such, the possible realizations of $\beta$ should be interpreted, not literally as positive vs. negative feedback, but as higher vs. lower intensity of feedback.

We also assume that the information acquisition technology allows the investor to acquire the following signals:

$$S_f = \phi + e_f, \quad \text{where} \quad e_f \sim N(0, \sigma_{f,e}^2)$$

(16)

$$S_u = u_2 + e_u, \quad \text{where} \quad e_u \sim N(0, \sigma_{u,e}^2)$$

(17)

$$S_b = \begin{cases} 
\beta & \text{with probability } q_b \\
-\beta & \text{with probability } 1-q_b
\end{cases}.$$  

(18)
For convenience, we define \( \kappa_f \equiv \frac{\sigma_f^2}{\sigma_f^2 + \sigma_{f,e}^2} \), \( \kappa_u \equiv \frac{\sigma_u^2}{\sigma_u^2 + \sigma_{u,e}^2} \) and \( \kappa_b \equiv (2q_b - 1) \), and note that

\[
E[\phi | S_f] = \kappa_f S_f, \quad E[u_t | S_{ut}] = \kappa_u S_u, \quad E[\beta | S_b] = \kappa_b S_b.
\] (19)

Note that for \( i \in \{f, u, b\} \), \( \kappa_i \in [0, 1] \) is a normalized measure of the precision of signal \( S_i \). When \( \kappa_i = 1 \), the signal \( S_i \) is perfectly informative; when \( \kappa_i = 0 \), it is perfectly uninformative. Although the model is extremely stylized, \( S_f \) represents fundamental information (e.g., earnings and balance sheet information, analyst reports for firms, macroeconomic analysis and forecasts for the aggregate economy), while \( S_u \) and \( S_b \) characterize information about the trading behavior of other investors (e.g., portfolio and position information based on 13-F filings, information from limit order books, observation of order flow, counterparty exposure, fund flow data, and fund performance history).

For a given choice of precisions \{\( \kappa_f, \kappa_u, \kappa_b \)\}, the investor pays a cost \( C(\kappa_f, \kappa_u, \kappa_b, h) \), where \( h \) parameterizes transparency. In particular, we assume

\[
C_h \equiv \frac{\partial C}{\partial h} < 0, \quad C_{hf} \equiv \frac{\partial^2 C}{\partial h \partial \kappa_f} \leq 0, \quad C_{hu} \equiv \frac{\partial^2 C}{\partial h \partial \kappa_u} \leq 0, \quad C_{hb} \equiv \frac{\partial^2 C}{\partial h \partial \kappa_b} \leq 0.
\] (20)

As such, increasing transparency, \( h \), decreases the (marginal) cost of learning about \( \phi, u_2 \) and \( \beta \). We also assume that the cost function is increasing and convex in the precisions i.e.,

\[
C_i \equiv \frac{\partial C}{\partial \kappa_i} > 0, \quad C_{ii} \equiv \frac{\partial^2 C}{\partial \kappa_i^2} > 0,
\] (21)

and is separable along the three dimensions i.e., for \( i \in \{f, u, b\} \) and \( j \in \{f, u, b\} \neq i \), \( C_{ij} = 0 \).\(^{13}\)

At date 0, the investor optimally chooses \{\( \kappa_f, \kappa_u, \kappa_b \)\} to maximize her expected utility subject to the cost function \( C(\kappa_f, \kappa_u, \kappa_b, h) \). Given the characterization of the financial market equilibrium, the investor’s optimal choice of signals can be represented as:

\(^{13}\)The assumption that \( C_{ij} = 0 \) ensures that there is no complementarity / substitutability in learning driven by the cost function. This allows us to focus on the complementarity in learning that arises endogenously due to speculative incentives, without potentially confounding effects that depend on the specific cost function. Since in our setting, learning about fundamentals and noise trading is complementarity, it seems reasonable to conjecture that an information producer may choose to bundle these types of information (and the resulting cost is no longer separable). However, given the lack of empirical evidence for such bundling of fundamental and non-fundamental financial information, we defer this analysis to future work.
\[
\{\kappa_f, \kappa_u, \kappa_b\} = \arg\ max_{\kappa_f, \kappa_u, \kappa_b} \mathbb{E}_0 [V_1] - C(\kappa_f, \kappa_u, \kappa_b, h) 
\]
(22)

\[
= \arg\ max_{\kappa_f, \kappa_u, \kappa_b} \mathbb{E}_0 \left[ (1 - \rho) \frac{c}{2} \mathbb{E}_1 \left[ (\beta P_1 + u_2 + \xi u_1)^2 \right] \right] - C(\kappa_f, \kappa_u, \kappa_b, h) 
\]
(23)

\[
= \arg\ max_{\kappa_f, \kappa_u, \kappa_b} \left( 1 - \rho \right) \frac{c}{2} \mathbb{E}_0 \left[ \beta \left( \frac{\mathbb{E}_1[\phi] + \rho \mathbb{E}_1[u_2] + c(1 + \rho^2)u_1}{1 - \rho \mathbb{E}_1[\beta]} \right) + u_2 + \xi u_1 \right]^2 
\]
(24)

\[
\equiv \arg\ max_{\kappa_f, \kappa_u, \kappa_b} V_0 - C(\kappa_f, \kappa_u, \kappa_b, h). 
\]
(25)

The above characterization highlights the mechanism underlying our central result: the complementarity between learning about fundamentals and liquidity trading. First, note that learning is valuable to the investor only if she can provide liquidity i.e., in the absence of feedback trading (i.e., \(\beta = 0\)), the investor has no incentive to acquire information. Second, conditional upon \(\beta \neq 0\), equation (24) implies that learning about \(\phi\) and \(u_2\) is always valuable. Specifically, the objective function \(U\) is convex in \(\mathbb{E}_1[\phi]\) and \(\mathbb{E}_1[u_2]\) — more learning about \(\phi\) and \(u_2\) at date 1 increases the variance of these conditional expectations, and therefore increases expected utility \(V_0\). Finally, the value of learning about \(\phi\) (and \(u_2\)) at date 1 increases in the precision of the investor’s information about \(\beta\) — the second derivative of \(U\) with respect to \(\mathbb{E}_1[\phi]\) (and \(\mathbb{E}_1[u_2]\), respectively) is convex in \(\mathbb{E}_1[\beta]\).

If ease of learning corresponds to sophistication, the above characterization suggests that more sophisticated investors choose to learn more about the behavior of other traders. Although at first glance this may appear inconsistent with the standard notion of financial sophistication, it provides a natural interpretation for the behavior of extremely sophisticated institutional investors (e.g., statistical arbitrageurs and high-frequency traders) who focus much of their attention on learning about other market participants. Given the increase in transparency and technological sophistication over the last few decades, our results can help explain the recent increase in popularity of strategies that exploit predictability in order-flow, and why investors are willing to pay for such information.

Under sufficient regularity assumptions on the cost of acquiring information, the following proposition characterizes the investor’s optimal information acquisition decision.

**Proposition 2.** Suppose for every \(h\), the cost function \(C\) is separable, increasing, and convex in the choice of precisions \(\{\kappa_f, \kappa_u, \kappa_b\}\). Then, the optimal choice of precisions \(\kappa_f(h), \kappa_u(h)\)
and \( \kappa_b(h) \) is characterized by \( V - C \geq 0 \) and the following first order conditions:

\[
V_f - C_f \leq 0, \quad V_u - C_u \leq 0, \quad V_b - C_b \leq 0,
\]

where \( V_i = \frac{\partial V_0}{\partial \kappa_i} \) is the marginal value of learning along dimension \( i \), and the equalities are strict when the corresponding choice of precisions is strictly greater than zero (i.e., \( V_i - C_i = 0 \) when \( \kappa_i(h) > 0 \)).

### 2.4 Discussion

Transactions costs play a critical role in generating the feedback bubble. The investor’s expected utility is increasing in the cost parameter \( c \) — in fact, \( V_0 \) is zero, its lowest value, when there is no cost to trade. Intuitively, when the investor is unconstrained, the price of the risky asset is unaffected by the actions of the noise traders and simply reflects the fundamental value (i.e., \( P_2 = \phi \) and \( P_1 = \mathbb{E}_1[\phi] \)). In order for feedback traders to have an effect on the price \( P_2 \), and consequently for valuable speculative opportunities to exist, the investor must be constrained. While transactions costs provide a transparent way to capture such a constraint, alternative assumptions (e.g., risk-aversion) suffice as well.

Similarly, the liquidity shock \( \xi \) plays an important role: in the absence of a liquidity shock (i.e., if \( \rho = 0 \)), the investor is long-lived and her demand at date 1 only depends on her beliefs about fundamentals. In turn, this implies she has no motive to learn about noise trading. Other motivations (e.g., short-term, performance-based compensation for institutional investors) that generate similar incentives for the investor to learn about intermediate prices would yield similar implications. At the other extreme, if the investor is short-lived and can never trade at date 2 (i.e., when \( \rho = 1 \)), she is unable to provide liquidity to the noise trader in that period, and so has no incentives to learn about \( \beta, \phi \) or \( u_2 \).

We focus on the case of a single investor purely for tractability. A setting with heterogeneity in information acquisition would allow us to study the impact of transparency on the information asymmetry, and whether specialization in information acquisition across investors could arise endogenously. However, such an extension with asymmetric information and learning from prices does not seem analytically tractable in our current framework, since the price depends nonlinearly on the investor’s conditional expectations. We hope to explore this in future work.

In the absence of learning from prices, one can show that our results are qualitatively similar in settings with multiple, symmetrically-informed investors. In particular, even though we have a single investor, she takes prices as given and does not manipulate today’s price strategically (e.g., as discussed in Kyle and Viswanathan (2008)) — we leave an analysis of such behavior for future work.
Our analysis highlights that the presence of price-dependent liquidity trading can have important consequences for the effectiveness of regulation that increases transparency. Such policy is often introduced in response to, and in an effort to decrease, the uncertainty that is generated during economic crises. Since crises are also accompanied by deleveraging cycles and forced liquidations (and other examples of price-dependent trading), this suggests that the tradeoff we describe may be of first-order importance in assessing the effect of such regulation. For instance, the 2007 subprime crisis highlighted the impact of leverage constraints, especially on financial institutions like hedge funds. As security prices fell, lenders demanded higher margins and more collateral, which forced hedge funds to delever by selling the underlying assets, further lowering prices (see Acharya et al. (2009) for a narrative of the financial crisis). Exacerbating the issue, hedge funds were also hit with large increases in redemptions over this period; in fact, their redemptions exceeded those suffered by mutual funds. The impact of their delevering was economically significant. For instance, Ben-David, Franzoni, and Moussawi (2012) show that hedge funds reduced their U.S. equity holdings by about 6% in each of the third and fourth quarters of 2007 and by about 15% in each of the third and fourth quarters of 2008, on average. Furthermore, about 80% of this decrease can be explained by redemptions and reduced leverage. Finally, Ang, Gorovyy, and Van Inwegen (2011) and Aragon and Strahan (2012) document that this deleveraging is predictable, and has price impact on the underlying assets. Taken together, this evidence suggests that deleveraging episodes can generate feedback trading by financing-constrained, sophisticated investors.

More generally, a number of papers, including Lakonishok, Shleifer, and Vishny (1992), Grinblatt, Titman, and Wermers (1995), Wermers (1999), and Grinblatt and Keloharju (2000), document trading behavior by both individuals and institutions which is consistent with feedback trading. Cohen and Shin (2003) find evidence of positive feedback trading in the U.S. Treasury market during a period of high market stress. A standard explanation for feedback trading is that investors exhibit extrapolative expectations, and a number of papers provide survey evidence consistent with such behavior (e.g., Frankel and Froot (1987); Malmendier and Nagel (2011); and Greenwood and Shleifer (2014)). While feedback trading can naturally arise through such behavioral channels (see Shleifer and Summers (1990) and Hirshleifer (2001) for comprehensive surveys), there may be other reasons for such predictability in trading. We focus our discussion on two alternatives: deleveraging episodes and fund-flows to delegated portfolio managers.

Delegated asset management, and the nature of investor behavior therein, provides another natural channel through which predictable feedback trading may arise, even in the absence of leverage crises. A number of papers, including Chevalier and Ellison (1997) and Sirri and Tufano (1998), document a strong relation between past performance and mutual fund flows. Coval
and Stafford (2007) and Lou (2012) show that such flows generate predictable price pressure in
the underlying stocks. This predictability in trading behavior by mutual funds corresponds
closely to the feedback trading we consider. Moreover, there is evidence to suggest that, as
in our model, some investors learn about, and could profitably trade on, this predictability in
mutual fund demand. For instance, Chen, Hanson, Hong, and Stein (2008) document that,
for an individual stock, short interest tends to increase prior to its sale by mutual funds which
experience large outflows. Similarly, Dyakov and Verbeek (2013) present evidence that a trading
strategy which uses public information to predict price pressure (and trades accordingly) can
generate excess returns.

3 The extent to which prices reflect fundamentals

This section presents the main analysis of the paper. In Section 3.1, we define two measures of
informativeness — accuracy and efficiency — and describe conditions under which an increase
in one measure may be accompanied by a decrease in the other. In Section 3.2, we characterize
necessary and sufficient conditions under which, given the endogenous choice of precisions
described above, an increase in transparency (i.e., an increase in \( h \)) can lead to decreases in
accuracy and efficiency. Using specific examples of cost functions, we illustrate the implications
of these results by analyzing the impact of changes in general transparency (in Section 3.3)
and targeted transparency (in Section 3.4). Finally, in Section 3.5 we allow feedback trading
to respond endogenously to an increase in transparency, and show how this can exacerbate the
decrease in informativeness.

3.1 Price accuracy and price efficiency

While analysis of regulatory policy often focuses on a measure of welfare, this is difficult to
characterize in our model: the risk neutral investor trades against liquidity traders who do
not have a well-defined utility function, and the investor’s trading gains are exactly offset by
the losses of the liquidity traders. Instead, our focus is on the impact of transparency on
price informativeness. Not only is price informativeness itself of general interest to academics,
practitioners and regulators, it is also closely related to real (allocative, or Pareto) efficiency in more general settings.\textsuperscript{16}

We define two measures of the extent to which prices reflect fundamentals. The first measure captures how close the date 1 price is to fundamentals in expectation.\textsuperscript{17}

**Definition 1.** Price *accuracy* of the date 1 price is given by \( A \equiv -E \left[ (\phi - P_1)^2 \right]. \)

The second measure captures how close the conditional expectation of fundamentals, given the date 1 price, is to fundamentals in expectation.

**Definition 2.** Price *efficiency* of the date 1 price is given by \( E = -E \left[ (\phi - E[\phi|P_1])^2 \right]. \)

Note that estimating efficiency empirically not only requires observations of \( \phi \) and \( P_1 \), but also requires that the observer knows the structure of the economy, and in particular, the joint distribution of fundamentals and noise trading. In contrast, price accuracy can be interpreted as a more robust measure since it can be estimated using observations of \( \phi \) and \( P_1 \) alone. Furthermore, our notion of accuracy captures how informative the date 1 price is about the predictable component of fundamentals, relative to its *frictionless* benchmark, since in the absence of transaction costs (i.e., \( c = 0 \)) and with perfect transparency, \( P_1 = \phi. \textsuperscript{18} \) Finally, while efficiency captures the notion of price informativeness for agents within the model, accuracy seems to more closely match the concept of price informativeness used in the empirical literature and by market participants and regulators in practice.

\textsuperscript{16}See Goldstein et al. (2013b) for a recent theoretical example and Chen, Goldstein, and Jiang (2007) for empirical evidence consistent with this hypothesis. As a simple example, suppose at date 1, a manager (or capital provider) must decide how much to invest in a risky technology, subject to a quadratic investment cost function. The average productivity of the risky technology is given by \( \phi \), but at date 1, \( \phi \) is not known to the manager. However, suppose the manager can update her beliefs conditional on the date 1 price. Her objective is to maximize the payoff to investing amount \( I \), net of costs, which is given by:

\[
I^* = \arg \max_I U(I) = \arg \max_I E[\phi|P_1]I - \frac{1}{2}I^2 = E_1[\phi|P_1].
\] (27)

*Real* (allocative) efficiency can be characterized as the expected difference in total output relative to the full-information, efficient choice which sets \( I = \phi \), i.e.,

\[
RE = E_0 [U(I^*) - U(\phi)] = -\frac{1}{2}E_0 \left[ (\phi - E[\phi|P_1])^2 \right],
\] (28)

which is a multiple of our *price* efficiency measure \( E \) defined below.

\textsuperscript{17}While the focus of our analysis is on price informativeness at date 1, one could also consider how well the date 2 price reflects fundamentals. In our model, this is not very interesting: the investor’s value function increases as price accuracy at date 2 falls, and so as we make it easier to learn, this measure must fall.

\textsuperscript{18}The difference between \( P_1 \) and \( \phi \) is dependent upon the realization of the signals selected, which is random. We take the date 0 expectation over the possible signal realizations to simplify our characterization of the results. This also ensures our measure of efficiency can be empirically implemented — an econometrician who is able to observe prices and fundamentals (i.e., \( \phi \)), but not the information of investors, would still be able to construct our measure. Finally, our measure is common in the theoretical literature (e.g., Gao (2008)).
To gain some intuition for these measures, it is useful to rewrite the date 1 price in terms of the investor’s signals. First, denote

\[ y \equiv \kappa_f S_f + \rho c \kappa_u S_u + c \left(1 + \rho^2\right) u_1. \]  

(29)

In the absence of feedback trading (i.e., \( \beta = 0 \)), this linear combination of \( S_f, S_u \) and \( u_1 \) is the date 1 price (i.e., \( P_1 = y \)); more generally, the date 1 price is given by \( P_1 = \frac{y}{1 - \rho \kappa_s S_b} \). As the investor learns more about both fundamentals and noise trading (i.e., \( u_2 \)), the variance of \( y \) increases, which in turn affects both the investor’s expected utility as well as an uninformed observer’s ability to extract information. We denote

\[ \alpha \equiv \frac{\rho^2 c^2 \sigma_{u,2}^2}{\sigma_f^2} \quad \text{and} \quad \omega \equiv \frac{c^2(1 + \rho^2)^2 \sigma_{u,1}^2}{\sigma_f^2}, \]  

(30)

so that the variance of \( y \) is given by:

\[ \sigma_y^2 = \kappa_f \sigma_f^2 + \kappa_u \rho^2 c^2 \sigma_{u,2}^2 + c^2 \left(1 + \rho^2\right)^2 \sigma_{u,1}^2 = \sigma_f^2 \left(\kappa_f + \alpha \kappa_u + \omega\right). \]  

(31)

Given this parameterization, \( \alpha \) represents the relative variation in the date 1 price generated by uncertainty about \( u_2 \) versus prior uncertainty about fundamentals (i.e., \( \phi \)). Similarly, \( \omega \) represents the relative variation in the price due to uncertainty about noise trading at date 1 versus prior fundamental uncertainty.

In the absence of feedback trading (i.e., when \( \beta = 0 \)), the two measures are intimately related.\(^{19}\) In this case, since \( P_1 = y \) (the linear component of the price), accuracy is given by

\[ A = -\mathbb{E} \left[ (\phi - y)^2 \right] = \sigma_f^2 \left(\kappa_f - \alpha \kappa_u - \omega - 1\right), \]  

(32)

and efficiency is given by

\[ E = -\mathbb{E} \left[ \text{var} (\phi | y) \right] = \sigma_f^2 \left(\frac{\kappa_f}{\kappa_f + \alpha \kappa_u + \omega} - 1\right). \]  

(33)

Note that in this case both accuracy and efficiency increase in the precision of fundamental information (i.e., \( \kappa_f \)) and decrease in the quality of information about noise trading (i.e., \( \kappa_u \)). More generally, however, since the price depends nonlinearly on the investor’s beliefs about feedback traders, the two measures are generically different. This is summarized in the following result.

\(^{19}\)This is also true in standard noisy RE models, where the price is a linear function of normally distributed shocks.
Theorem 1. Efficiency and accuracy can be expressed as

\[ E = \sigma_f^2 \left( \frac{\kappa_f^2}{\kappa_f + \alpha \kappa_u + \omega} (1 - G(\lambda)) - 1 \right), \] 
and

\[ A = \sigma_f^2 \left( \frac{1 - 3\lambda^2}{(1 - \lambda^2)^2} \kappa_f - \frac{1 + \lambda^2}{(1 - \lambda^2)^2} (\alpha \kappa_u + \omega) - 1 \right) \] 

where \( \lambda \equiv \rho_c \kappa_b \), \( G(\lambda) \equiv 4\lambda^2 \mathbb{E} \left[ \pi (1 - \pi) P_1^2 \right] \), and \( \pi \equiv \Pr (S_b = b | P_1) \).

The above expressions for \( E \) and \( A \) immediately imply the following results.

Proposition 3. Efficiency always increases in \( \kappa_f \), always decreases in \( \kappa_u \), and decreases in \( \kappa_b \) when \( \frac{d}{d \kappa_b} G > 0 \). Accuracy always decreases in \( \kappa_u \), always decreases in \( \kappa_b \), and decreases in \( \kappa_f \) when \( 3(\rho_c \kappa_b)^2 > 1 \).

Recall that under our distributional assumptions, the price at date 1 is given by

\[ P_1 = \frac{\kappa_f S_f + \rho_c \kappa_u S_u + c(1 + \rho^2) u_1}{1 - \rho_c \kappa_b S_b} = \frac{\kappa_f}{1 - \rho_c \kappa_b S_b} \phi + \frac{\kappa_f e_f + \rho_c \kappa_u S_u + c(1 + \rho^2) u_1}{1 - \rho_c \kappa_b S_b}. \] 

The result that efficiency and accuracy fall when the investor learns more about \( u_2 \) (i.e., \( \kappa_u \) increases) is intuitive — in this case, the price becomes a noisier signal about fundamentals. Similar results obtain with increases in \( \sigma_u^2 \) and \( \sigma_{u,1}^2 \) — increasing the variance of \( u_1 \) and \( u_2 \) also make the price a noisier signal of \( \phi \) on average.

When the investor learns more about fundamentals (i.e., \( \kappa_f \) increases), the price becomes a more informative signal about \( \phi \), which leads to greater efficiency. However, the multiplicative term in the price, \( \frac{1}{1 - \rho_c \kappa_b S_b} \), changes the extent to which the level of \( P_1 \) responds to better information about fundamentals. Note that on average, the multiplier term is increasing in \( b \kappa_b \) since

\[ \mathbb{E} \left[ \frac{1}{1 - \rho_c \kappa_b S_b} \right] = \frac{1}{1 - (\rho_c \kappa_b)}. \] 

Intuitively, when \( b \kappa_b \) is low, either because the feedback effect (i.e., \( b \)) is small or the investor knows little about it (i.e., \( \kappa_b \) is low), the multiplier term is close to one, on average, and so learning more about fundamentals (i.e., higher \( \kappa_f \)) pushes the level of the price closer to \( \phi \). However, when the feedback effect is large and the investor learns about it precisely (i.e., \( b \kappa_b \) is large enough), the multiplier term is large and learning more about \( \phi \) pushes the price away from the fundamental value — a decrease in accuracy.

This multiplier effect also implies that accuracy always decreases as the investor learns more about \( \beta \) — the multiplier is equal to one when the investor chooses to learn nothing about \( \beta \) (i.e., when \( \kappa_b = 0 \)). However, the effect of an increase in \( \kappa_b \) on efficiency is more nuanced. To
see this, note that conditional on a specific value of \( S_b \), the price is a linear signal about \( \phi \), since

\[
P_1 = \begin{cases} \frac{\kappa_f}{1-\rho \kappa_b} e^{\phi} + \frac{\kappa_f e_f + \rho \kappa_b S_u + c(1+\rho^2)u_1}{1-\rho \kappa_b} & \text{when } S_b = b \\ \frac{\kappa_f}{1+\rho \kappa_b} e^{\phi} + \frac{\kappa_f e_f + \rho \kappa_b S_u + c(1+\rho^2)u_1}{1+\rho \kappa_b} & \text{when } S_b = -b \end{cases}
\] (38)

However, for an investor who does not observe the realization of \( S_b \), but instead must form an expectation of \( \phi \) conditional on \( P_1 \) only, the uncertainty about \( S_b \) generates additional uncertainty about fundamentals, as it is not clear which of the two signals in (38) is being observed. Specifically, note that the law of total variance implies that

\[
\text{var} \left( \phi | P_1 \right) = \mathbb{E} \left[ \text{var} \left( \phi | P_1, S_b \right) | P_1 \right] + \mathbb{E} \left[ \text{var} \left( \phi | P_1, S_b \right) | P_1 \right].
\] (39)

The second term in this decomposition, \( \mathbb{E} \left[ \text{var} \left( \phi | P_1, S_b \right) | P_1 \right] \), captures the additional uncertainty about \( \phi \) generated by the fact that \( S_b \) is uncertain. As we show in the proof of Proposition 1, this term depends on the probability that the observer assigns to \( S_b = b \), conditional on the realized price \( P_1 \), i.e., on \( \pi \equiv \Pr (S_b = b|P_1) \). Moreover, we show that the unconditional expectation of this term is non-monotonic in \( \kappa_b \). At \( \kappa_b = 0 \), there is no additional uncertainty since there is no effect of \( S_b \) on the price. Increasing \( \kappa_b \) when it is low increases the additional uncertainty, since the price is sensitive to the realization of \( S_b \), but \( S_b \) is difficult to detect from the price (i.e., \( \pi (P_1) \) is close to its prior value of \( \frac{1}{2} \)). However, when \( \kappa_b \) is sufficiently large, the price is sufficiently sensitive to the realization of \( S_b \), so that it becomes increasingly easy to distinguish whether \( S_b = b \) or \( S_b = -b \) from the price itself i.e., \( \pi (P_1) \) becomes closer to zero or one.\(^{20}\) In this region, increasing \( \kappa_b \) further decreases the variance about \( S_b \), conditional on the price, which consequently decreases the additional variance in the inference of fundamentals. To summarize, an increase in \( \kappa_b \) leads to an increase in efficiency when it decreases the additional variance due to \( S_b \) being unknown i.e., when it decreases \( \mathbb{E} \left[ \text{var} \left( \phi | P_1, S_b \right) | P_1 \right] \), or equivalently, when \( \frac{d}{d\kappa_b} G < 0 \). While \( G (\lambda) \) cannot be computed in closed form, because \( \lambda \equiv \rho \kappa_b \in [0, 1] \), \( G (\lambda) \) can be completely characterized by its plot over the region \( \lambda \in [0, 1] \).

As is apparent from Figure 3, \( G (\lambda) \) is always non-negative and single peaked at \( \lambda^* \approx 0.7 \).

Finally, the above variance decomposition highlights that in the presence of price-dependent liquidity, more extreme price realizations can be more informative about fundamentals. This distinguishes our model’s predictions from those of standard, linear noisy RE models with normally distributed shocks, where the posterior variance in fundamentals, conditional on the price, is constant (i.e., \( \text{var}(\phi|P_1) \) does not depend on \( P_1 \)). In contrast, more extreme price realizations are more likely to occur when the investor anticipates a large feedback effect in our

\(^{20}\)Note that the minimum \( \kappa_b \) for which this occurs is falling in the size of the feedback effect \( (b) \) and frictions \( (\rho, c) \) as these increase the sensitivity of \( P_1 \) to \( \kappa_b \).
setting. As a result, upon observing an extreme price realization, an outside observer is able to better discern $S_b$, and in turn, $S_f$, and so price realizations in the tails are relatively more informative about fundamentals.

3.2 When does greater transparency decrease accuracy / efficiency?

We use the above results to characterize conditions for which accuracy and efficiency can decrease with an increase in transparency. Recall that the investor optimally chooses precisions $\kappa_f$, $\kappa_u$, and $\kappa_b$, subject to a cost function $C(\kappa_f, \kappa_u, \kappa_b, h)$, which is separable, increasing, and convex in the precisions (i.e., equation (21) holds), and where $h$ parameterizes transparency (i.e., equation (20) holds). In this section, we consider a general decrease in transparency, which implies that an increase in $h$ leads to a decrease in the marginal cost of acquiring information across all dimensions. One might expect that increasing transparency should lead to an increase in the extent to which prices reflect fundamentals. The following result characterizes conditions under which this is not the case.

Theorem 2. Let $\lambda = \rho c \kappa_b$, and suppose the investor’s optimal choice of precisions, as characterized in Proposition 2, is given by $\kappa_f(h)$, $\kappa_u(h)$ and $\kappa_b(h)$. Then,

(i) Efficiency $\mathcal{E}$ decreases in transparency (i.e., $h$) if and only if

$$\frac{2}{\kappa_f} \frac{d\kappa_f}{dh} - \frac{\sigma_f^2}{\sigma_b^2} \left( \frac{d\kappa_f}{dh} + \alpha \frac{d\kappa_u}{dh} \right) - \frac{\rho cb}{1 - G(\lambda)} \frac{dG(\lambda)}{d\lambda} \frac{d\kappa_b}{dh} < 0. \quad (40)$$

(ii) Accuracy $\mathcal{A}$ decreases in transparency (i.e., $h$) if and only if

$$\frac{d\kappa_f}{dh} - \left[ \alpha \frac{d\kappa_u}{dh} + \lambda^2 \left( \frac{d\kappa_f}{dh} + \alpha \frac{d\kappa_u}{dh} \right) \right] - \frac{2\lambda \left( (1+3\lambda^2)\kappa_f + (3+\lambda^2)(\alpha \kappa_u + \omega) \right)}{(1-\lambda^2)} \rho cb \frac{d\kappa_b}{dh} < 0 \quad (41)$$

The result follows from simplifying the conditions for which $\frac{d\mathcal{A}}{dh} < 0$ and $\frac{d\mathcal{E}}{dh} < 0$, given the expressions in Theorem 1. Standard intuition suggests that both accuracy and efficiency would
fall with an increase in transparency when the resulting increase in precision about fundamentals (i.e., $\kappa_f$) is sufficiently less than the increase in precision about noise trading (i.e., $\kappa_u$ and $\kappa_b$). Interestingly, however, this intuition does not always hold. Specifically, note that while the coefficient on $\frac{d\kappa_u}{dh}$ is always negative in both inequalities, the coefficient on $\frac{d\kappa_b}{dh}$ can be positive in (40), and the coefficient on $\frac{d\kappa_f}{dh}$ can be negative in (41). As such, an increase in $\kappa_u$ due to higher transparency always increases the likelihood that accuracy and efficiency fall. However, while accuracy is always more likely to fall with with an increase in $\kappa_b$, it may be that learning about feedback trading makes it more likely that efficiency increases (because when $\frac{dG(\lambda)}{d\lambda} < 0$, it becomes easier to extract information about $\phi$ from the price, as discussed above). Similarly, while learning more about fundamentals always makes it more likely that efficiency increases, it may lead to lower accuracy when $\kappa_b$ (and hence $\lambda = \rho c \kappa_b$) is sufficiently high, due to the multiplier effect.

To gain a better understanding of the underlying mechanism, first consider condition (40). The first term — the semi-elasticity of $\kappa_f$ with respect to transparency — captures the increase in price efficiency as a result of more learning about fundamentals. Note that this effect decreases as $\kappa_f$ approaches 1 — the marginal increase in price informativeness decreases as $\kappa_f$ increases. The second term — the sensitivity of $\sigma_y^2$ with respect to transparency — captures the increase in the variance of $y$ as a result of more learning about fundamentals (captured by the $\frac{d\kappa_f}{dh}$ term) and about $u_2$ (captured by the $\frac{d\kappa_u}{dh}$ term). The increased variance of $y$ makes it more difficult to infer $\phi$ from the price. When there is relatively more prior uncertainty about $u_2$ (i.e., when $\alpha$ is higher), learning more about $u_2$ makes it more likely that efficiency will fall. The third term measures the effect of a change in $\kappa_b$. In the region when $\frac{dG(\lambda)}{d\lambda}$ is negative, learning more about feedback trading makes the price more informative about fundamentals. On the other hand, when $\frac{dG(\lambda)}{d\lambda}$ is positive, efficiency is more likely to fall with an increase in transparency, and the effect of learning about feedback trading can actually increase as $\kappa_b$ gets larger. To summarize, an increase in transparency is most likely to decrease efficiency when the investor already chooses to learn a lot about fundamentals and sufficient information about feedback trading.\(^{21}\) Paradoxically, making it easier to learn is worse for efficiency when the investor already possesses precise information about fundamentals.

Turning to condition (41), we obtain a similar decomposition. The first term captures the increase in accuracy due to more learning about fundamentals — the price reflects $\phi$ more accurately as the investor learns more about it. The second term captures the decrease in accuracy resulting from more learning about $u_2$ (the $\frac{d\kappa_u}{dh}$ term) and, because of the multiplier effect, from more learning about fundamentals (the $\lambda^2 \frac{d\kappa_f}{dh}$ term). In particular, note that when

\(^{21}\)These are not necessary conditions. As we show below, efficiency can fall when the investor is not learning anything about feedback trading.
there is no feedback effect (i.e., $b = \lambda = 0$), the coefficient on $\frac{d\kappa}{dh}$ in the second term is zero — in the absence of the feedback effect, accuracy always increases with the quality of information about fundamentals. Finally, the third term captures the decrease in accuracy due to more learning about feedback trading — learning more about feedback trading always decreases accuracy, even though at times, it may increase price efficiency.

Theorem 2 describes the necessary and sufficient conditions under which accuracy and efficiency fall with an increase in transparency. Our characterization of these conditions relies upon not just the assumed parameters of the model, but crucially upon the endogenous choice of optimal precisions made by the investor. It is essential, then, that we demonstrate under reasonable specifications of the cost function that the endogenous response of the investor can meet these conditions. The following sections take up this exercise.

3.3 General transparency

In this subsection, we illustrate the implications of Theorem 2 for changes in general transparency, which affects the marginal cost of acquiring information about both fundamentals and liquidity trading. We interpret changes in general transparency as low frequency changes in technology and the information environment that make it easier for investors to acquire and process various types of information (e.g., the introduction of computers and the internet). We consider two specifications for the cost function. The log-linear cost specification in Section 3.3.1 considers a cost function that ensures an interior choice of precision. Moreover, for some special cases of the model, it allows us to analytically characterize sufficient conditions for accuracy and efficiency to decrease with transparency. The specification in Section 3.3.2 generalizes a specification where the cost of information is proportional to the reduction in entropy, a commonly-used measure of information.

3.3.1 Log-linear cost of precision

To ensure that the optimal choice of precision is interior (i.e., $\kappa \in (0, 1)$), the marginal cost of $\kappa$ should be zero at $\kappa = 0$ and infinite at $\kappa = 1$. A parsimonious functional form which features both properties assumes a marginal cost that is proportional to $\frac{\kappa}{1-\kappa}$, which can be generated by the following cost function:

$$C(\kappa_f, \kappa_u, \kappa_b) = e^{-h} \left[ \sum_i e^{-h_i (\kappa_i - \log(1 - \kappa_i))} \right] \quad i \in \{f, u, b\},$$

where the parameters $\{h, h_f, h_u, h_b\}$ capture different dimensions of transparency. We refer to the parameter $h$ as general transparency, since it uniformly affects the marginal cost of acquiring
information along any of the three dimensions. In contrast, the targeted transparency parameter $h_i$ (for $i \in \{f, u, b\}$) affects the marginal cost of increasing $\kappa_i$, but not $\kappa_j$ for $j \neq i$. This allows us to capture the possibility that learning certain types of information may be more costly than others. Moreover, in Section 3.4, we study the effects of targeted changes in transparency by characterizing the effects of changing $h_i$, specifically.\footnote{This parameterization implies that a shift in transparency affects both the total and marginal cost of information. Given (20) and (21), this affects only whether $\kappa$ is zero or non-zero (i.e., the initial decision to learn). In the log-linear setting considered here, and in the presence of date 1 noise, as is shown below, $\kappa$ is always non-zero, and so our assumption is without loss of generality. In contrast, for the entropy-based cost function we consider in Section 3.3.2, the marginal cost of information is not zero when $\kappa = 0$, and consequently, the investor chooses not to become informed at all if transparency is too low.}

For tractability, we first consider the case when there is no noise trading at date 1 (i.e., $\sigma_{u,1}^2 = 0$). Under this assumption, we show that the investor does not initially learn about feedback trading for sufficiently low transparency. This leads us to consider two cases: when the investor optimally chooses to only learn about $\phi$ and $u_2$ (i.e., the optimal choice of $\kappa_b = 0$), and when the investor begins to learn along all three dimensions. Finally, we numerically illustrate that our conclusions are robust to $\sigma_{u,1}^2 \neq 0$.

**Corollary 1.** Let $\sigma_{u,1}^2 = 0$. There exists $\bar{h}$ such that for all values of $h < \bar{h}$, the investor chooses only to learn about $\phi$ and $u_2$. Otherwise, the investor learns along along all three dimensions.

The above result immediately follows from the expressions for the marginal value of learning along each dimension, which are given in the appendix. In particular, while the marginal values of learning about $\phi$ and $u_2$, i.e., $V_f$ and $V_u$ respectively, are non-zero when $\kappa_f = \kappa_u = \kappa_b = 0$, $V_b = 0$ in this case.

**Case 1 ($\kappa_f, \kappa_u \neq 0, \kappa_b = 0$):** Using the expressions for $V_f$ and $V_u$, and Proposition 2, it can be shown that the optimal precisions are given by

$$\kappa_f = \frac{(1-\rho)\frac{\alpha}{\alpha+1}b^2\sigma_f^2 e^{h+hf}}{1+(1-\rho)\frac{\alpha}{\alpha+1}b^2\sigma_f^2 e^{h+hf}} \quad \text{and} \quad \frac{d\kappa_f}{dh} = \frac{\kappa_f}{1+(1-\rho)\frac{\alpha}{\alpha+1}b^2\sigma_f^2 e^{h+hf}}, \quad (43)$$

$$\kappa_u = \frac{\alpha(1-\rho)\frac{\alpha}{\alpha+1}b^2\sigma_u^2 e^{h+hu}}{1+\alpha(1-\rho)\frac{\alpha}{\alpha+1}b^2\sigma_u^2 e^{h+hu}} \quad \text{and} \quad \frac{d\kappa_u}{dh} = \frac{\kappa_u}{1+\alpha(1-\rho)\frac{\alpha}{\alpha+1}b^2\sigma_u^2 e^{h+hu}}. \quad (44)$$

Taking the ratio of $\frac{d\kappa_u}{dh}$ and $\frac{d\kappa_f}{dh}$, we see that the relative rate at which the investor learns about $\phi$ and $u_2$ as transparency improves depends upon (i) how much information the investor is currently choosing to learn, (ii) the relative prior uncertainty (i.e., $\alpha$), and (iii) the relative ease of learning (i.e., $h_f$ versus $h_u$). Theorem 2 and the equilibrium solution above imply that efficiency and accuracy fall when

$$\kappa_f < \kappa_u \left( \frac{(1-\rho)\frac{\alpha}{\alpha+1}b^2\sigma_f^2 e^{h+hf}}{1+\alpha(1-\rho)\frac{\alpha}{\alpha+1}b^2\sigma_u^2 e^{h+hu}} - 2\alpha \right) \quad \text{and} \quad \kappa_f < \kappa_u \left( \frac{(1-\rho)\frac{\alpha}{\alpha+1}b^2\sigma_f^2 e^{h+hf}}{1+\alpha(1-\rho)\frac{\alpha}{\alpha+1}b^2\sigma_f^2 e^{h+hu}} \right), \quad (45)$$
respectively. In the limit, as general transparency increases (i.e., \( h \) tends to infinity), efficiency falls if \( 2\alpha < e^{(h_f-h_u)} - 1 \), while accuracy falls when \( 0 < e^{(h_f-h_u)} - 1 \). Thus, if it is relatively easier to learn about fundamentals \( (h_f > h_u) \), and general transparency is already sufficiently high, both price efficiency and accuracy will fall as general transparency increases. It is also clear that, if efficiency falls when transparency improves, accuracy must as well; however, the opposite need not be true. As discussed above, efficiency is most likely to fall when \( \kappa_f \) is high, while \( \kappa_u \) is low. If \( \alpha \) is too large, the investor will learn about both relatively equally (or will learn more about \( u_2 \)), so that this condition no longer holds.

**Case 2** \((\kappa_f, \kappa_u, \kappa_b \neq 0)\): At \( \bar{h} \), the investor begins to learn about the feedback effect, decreasing both efficiency and accuracy. Complementarity, however, implies that this increases the marginal value of learning about both \( u_2 \) and \( \phi \), the latter of which could counteract any fall in efficiency due to learning about \( \beta \). The following result shows that, even with this complementarity, for sufficiently high transparency, efficiency and accuracy fall as it becomes easier to learn.

**Proposition 4.** Let \( \sigma_{u,1}^2 = 0 \). For sufficiently low \( h_b \), and sufficiently high \( h_f \), increasing transparency leads to a fall in efficiency at \( h = \bar{h} \). Furthermore, \( \mathcal{E}(\bar{h}) > \lim_{h \to \infty} \mathcal{E}(h) \). Accuracy falls for all \( h > \hat{h} \), where \( \kappa_b(\hat{h}) = \frac{1}{\sqrt{3\rho_{cb}}} \).

Figure 4 illustrates both cases. The intuition is as follows. As transparency increases, initially the investor chooses to learn only about \( \phi \) and \( u_2 \) (Case 1). Since it is sufficiently more costly to learn about noise trading than fundamentals (i.e., \( h_f > h_u \)), the investor chooses to learn faster about \( \phi \) initially — in this region, efficiency and accuracy increase with transparency. However, given decreasing returns to scale, once the investor’s choice of \( \kappa_f \) is large enough, she chooses to learn about noise trading (i.e., \( u_2 \)) at a faster rate — in this region, efficiency and accuracy begin to decrease. Finally, when general transparency is high enough, the investor chooses to learn along all three dimensions (Case 2). She continues to learn about \( u_2 \) at a faster rate, which in combination with the information she obtains about feedback trading, increases the rate at which efficiency and accuracy fall. Moreover, at this point, the complementarity in learning kicks in. This makes learning about \( \phi \) and \( u_2 \) more valuable, which triggers even faster learning along both dimensions. However, as she has little remaining information to learn about \( \phi \), the effect of learning about noise dominates, which leads to efficiency and accuracy falling more steeply when the investor begins to learn about the feedback effect.

Figure 5 illustrates the effect of introducing date 1 noise (i.e., \( \sigma_{u,1}^2 \neq 0 \), holding the other parameters fixed. The plots suggest that our general conclusions are robust to its inclusion. The main effect of introducing date 1 noise is to increase the value of learning about feedback trading. As a result, the investor chooses to start learning about \( \beta \) for a lower level of \( h \).
This figure plots (a) optimal choice of precisions, (b) efficiency, and (c) accuracy, as a function of general transparency $h$, when the cost of precision is given by (42). The other parameters are set to the following values: $\rho = 0.6, c = 1, b = 1, \sigma_f^2 = 1, \sigma_{u_2}^2 = 0.4, h_f = 4, h_u = 0, h_b = -4$. The choice of precisions in panel (a) are given by: $\kappa_f$ (solid), $\kappa_u$ (dashed) $\kappa_b$ (dot-dashed). The dotted lines in panels (b) and (c) mark the level of maximum efficiency and accuracy, respectively, for the parameter range plotted.

(Compared to the case when $\sigma_u^2 = 0$). However, as before, when learning about fundamentals is already sufficiently easier than learning about noise trading, increasing transparency can decrease efficiency and accuracy.

Note that for accuracy to fall with an increase in transparency, it need not be the case that learning about fundamentals is easier than learning about noise trading: in fact, it may be more likely when the opposite is true. If learning about feedback trading is sufficiently easy, and the investor obtains a precise enough signal about $\beta$, further learning along any dimension (caused by the increase in transparency) will cause accuracy to fall.

### 3.3.2 Entropy-based cost of precision

To facilitate a comparison to the information acquisition literature (e.g., Sims (2003), Van Nieuwerburgh and Veldkamp (2010)), and to highlight the robustness of our results, we consider a cost function where the cost of learning along a particular dimension is proportional to the corresponding reduction in entropy. Recall that for a normal random variable $\lambda$ with variance $\sigma^2$, the entropy is given by $H(\lambda) = \frac{1}{2} \log_e (2\pi e \sigma^2)$, and for a random variable $Y$ drawn from a binomial distribution with probabilities $\{p, 1-p\}$, the entropy is given by $H(Y) = -p \log_2(p) - (1-p) \log_2(1-p)$.

This implies that

$$H(\phi) - H(\phi|S_f) = -\frac{1}{2} \log_e (1 - \kappa_f), \quad H(u_2) - H(u_2|S_u) = -\frac{1}{2} \log_e (1 - \kappa_u) \quad (46)$$

and

$$H(\beta) - H(\beta|S_b) = \frac{1}{2} ((1 + \kappa_b) \log_2 (1 + \kappa_b) + (1 - \kappa_b) \log_2 (1 - \kappa_b)) \quad (47)$$
This figure plots (a) optimal choice of precisions, (b) efficiency, and (c) accuracy, as a function of general transparency $h$, when the cost of precision is given by (42). The other parameters are set to the following values: $\rho = 0.6$, $c = 1$, $b = 1$, $\sigma_f^2 = 1$, $\sigma_{u,2}^2 = \sigma_{u,1}^2 = 0.4$, $h_f = 4$, $h_u = 0$, $h_b = -4$. The choice of precisions in panel (a) are given by: $\kappa_f$ (solid), $\kappa_u$ (dashed) $\kappa_b$ (dot-dashed). The dotted lines in panels (b) and (c) mark the level of maximum efficiency and accuracy, respectively, for the parameter range plotted.

As before, in order to allow for heterogeneity in the cost of information across dimensions, we specify the cost function as follows:

$$C(\kappa_f, \kappa_u, \kappa_b) = e^{-h} \left[ e^{-h_f} (H(\phi) - H(\phi|S_f)) + e^{-h_u} (H(u_2) - H(u_2|S_u)) + e^{-h_b} (H(\beta) - H(\beta|S_b)) \right],$$

(48)

so that $h$ parametrizes general transparency and $\{h_f, h_u, h_b\}$ reflect targeted transparency. Since $\phi$, $u_2$, and $\beta$ are independent, the special case when $h_f = h_u = h_b = 0$ implies that the cost of acquiring information is proportional to the reduction in total entropy, i.e.,

$$C(\kappa_f, \kappa_u, \kappa_b) = e^{-h} [(H(\phi) - H(\phi|S_f)) + (H(u_2) - H(u_2|S_u)) + (H(\beta) - H(\beta|S_b))]$$

(49)

$$= e^{-h} (H(\phi, u_2, \beta) - H(\phi, u_2, \beta|S_f, S_u, S_b)).$$

(50)

The cost specification in (48) differs from the log-linear specification in (42) along two important dimensions. First, the marginal cost at $\kappa_i = 0$ is no longer zero. This implies that the marginal benefit of learning along any dimension must be sufficiently high before the investor chooses to learn along this dimension. More specifically, when transparency is low enough (i.e., $h$ is low enough), the investor may optimally choose to obtain no information. Second, while the marginal cost for $\phi$ and $u_2$ is infinite at $\kappa_f = 1$ and $\kappa_u = 1$, respectively, the marginal cost at $\kappa_b = 1$ is finite. This implies that, unlike the log-linear specification, the investor can choose to learn perfectly about feedback trading (when doing so is sufficiently valuable).

Despite these differences, our main conclusions remain true. As before, if learning about fundamentals is sufficiently easier than learning about noise trading, increasing general trans-
Figure 6: Entropy based cost of precision

This figure plots (a) optimal choice of precisions, (b) efficiency, and (c) accuracy, as a function of
general transparency \( h \), when the cost of precision is given by (48). The other parameters are set to
the following values: \( \rho = 0.6, c = 1, b = 1, \sigma_f^2 = 1, \sigma_{u,2}^2 = \sigma_{u,1}^2 = 0.4, h_f = 4, h_u = 0, h_b = -4 \). The
choice of precisions in panel (a) are given by: \( \kappa_f \) (solid), \( \kappa_u \) (dashed) \( \kappa_b \) (dot-dashed). The dotted
lines in panels (b) and (c) mark the level of maximum efficiency and accuracy, respectively, for the
parameter range plotted.

3.4 Targeted transparency

In this section, we consider the impact of an increase in targeted transparency, which affects
the cost of learning along a specific dimension. Many prominent examples of financial market
regulation share a stated objective, held by regulators, market participants, and political figures alike, to make relevant financial information more readily available. These examples include the Securities Act of 1933 (requires registration of traded securities), Regulation FD (standardizes information made available to the public), and Sarbanes-Oxley (targets increased accuracy of financial statements). Targeted transparency can also increase when policymakers directly provide public information to investors (e.g., forward guidance, disclosure of stress tests). By making such information more readily available, the hope is that prices will more fully reflect this information, leading to increased price informativeness. However, as we show, such actions can exacerbate the problem by making prices less accurate and efficient.

As transparency does not enter either the efficiency or accuracy measures directly, but only through the optimal precisions chosen by the investor, we can apply Theorem 2 for this analysis. Instead of considering the change in precisions as a result of changes in general transparency (i.e., $\frac{d\kappa}{dh}$), we apply the conditions to changes in targeted transparency along dimension $i$ (i.e., $\frac{d\kappa}{d\kappa_i}$). It is clear that the direct effect of increasing transparency along a particular dimension is to induce the investor to learn more precisely along this dimension. When there is no learning about feedback traders, the implications of this direct effect are clear: accuracy and efficiency always fall when we make learning about noise traders (specifically, $u_2$) easier, whereas accuracy and efficiency always rise when transparency about fundamental information increases.

In our setting, however, this is not the end of the story. Because learning about feedback trading introduces complementarities across signals, learning more along the “cheaper” dimension makes learning along the other dimensions more valuable, creating an indirect effect of targeted transparency. The relevant metric, as Theorem 2 makes clear, is how much these precisions change relative to each other, or how strong the direct effect is relative to the indirect effect. As a result, increasing fundamental transparency can have counterintuitive implications. We illustrate these using the log-linear and entropy-based cost specifications from the previous subsection.

First, we consider an example using the log-linear cost specification of (42) illustrated in Figure 7. As fundamental transparency (i.e., $h_f$) increases, efficiency increases but accuracy decreases. This is due to the multiplier term (37) in the price that is generated due to learning about feedback trading. Specifically, recall from Proposition 3 that accuracy can decrease in $\kappa_f$ when the feedback effect is sufficiently large (i.e., $3(pcb\kappa_b)^2 > 1$). As a result, when learning about feedback trading is sufficiently easy, either because overall transparency is high, or information about other traders is easy to acquire, an increase in fundamental transparency can lead to a decrease in price accuracy.

However, the complementarity in learning can generate more surprising results. Note that since an increase in fundamental transparency leads to an increase in $\kappa_f$, it can also lead to an
Figure 7: Log-linear cost of precision and targeted transparency about $\phi$

This figure plots (a) optimal choice of precisions, (b) efficiency, and (c) accuracy, as a function of targeted transparency $h_f$, when the cost of precision is given by (42). The other parameters are set to the following values: $\rho = 0.6$, $c = 1$, $b = 1$, $\sigma_f^2 = 1$, $\sigma_{u,2}^2 = \sigma_{u,1}^2 = 0.4$, $h = 4$, $h_u = 1$, $h_b = -3$. The choice of precisions in panel (a) are given by: $\kappa_f$ (solid), $\kappa_u$ (dashed) $\kappa_b$ (dot-dashed). The dotted lines in panels (b) and (c) mark the level of maximum efficiency and accuracy, respectively, for the parameter range plotted.

Incorporation of targeted transparency can move in the opposite direction as well: if making it easier to learn about feedback trading causes the investor to increase $\kappa_b$, then it can also cause her to learn more about $\phi$. Standard intuition suggests that when learning about feedback trading becomes cheaper, investors choose to learn more along this dimension, which should decrease efficiency and accuracy. However, in our model, learning about feedback trading makes it more valuable to learn about fundamentals. Consequently, as Figure 9 illustrates, an increase in transparency about feedback trading can result in an increase in efficiency and accuracy when the precision of the investor’s fundamental information is sufficiently sensitive. With further increases in targeted transparency, accuracy begins to fall since the investor learns sufficiently about feedback trading (i.e., $\kappa_b$ is large enough), which leads the multiplier effect to dominate. However, efficiency can still be increasing in this region.

From a policy perspective, these effects change the calculus on the value of improving access to financial data. If investors simply divert their resources to learning about non-fundamental factors, price efficiency can fall. This is more likely to be true in economies in which (i) investors are relatively sophisticated and (ii) fundamental information is currently widely available, if not
Figure 8: Entropy based cost of precision and targeted transparency about $\phi$

This figure plots (a) optimal choice of precisions, (b) efficiency, and (c) accuracy, as a function of targeted transparency $h_f$, when the cost of precision is given by (48). The other parameters are set to the following values: $\rho = 0.6$, $c = 1$, $b = 1$, $\sigma_f^2 = 2$, $\sigma_{u,2}^2 = 0.4$, $\sigma_{u,1}^2 = 0$, $h = 0$, $h_u = 4$, $h_b = 0$. The choice of precisions in panel (a) are given by: $\kappa_f$ (solid), $\kappa_u$ (dashed) $\kappa_b$ (dot-dashed). The dotted lines in panels (b) and (c) mark the level of maximum efficiency and accuracy, respectively, for the parameter range plotted.

Figure 9: Log-linear cost of precision and targeted transparency about $\beta$

This figure plots (a) optimal choice of precisions, (b) efficiency, and (c) accuracy, as a function of targeted transparency $h_b$, when the cost of precision is given by (42). The other parameters are set to the following values: $\rho = 0.6$, $c = 1$, $b = 1$, $\sigma_f^2 = 1$, $\sigma_{u,2}^2 = 0$, $\sigma_{u,1}^2 = 0$, $h = 0$, $h_f = 3$. The choice of precisions in panel (a) are given by: $\kappa_f$ (solid), $\kappa_u$ (dashed) $\kappa_b$ (dot-dashed). The dotted lines in panels (b) and (c) mark the level of maximum efficiency and accuracy, respectively, for the parameter range plotted.
perfectly transparent. This is especially true in light of the estimated cost of these regulations (see, for instance, Iliev (2010)). Similarly, decreasing the availability or timeliness of information about other participants, which is currently a popular proposal to mitigate the negative impact of high frequency trading (e.g., see Harris (2013)), can lead to a decrease in price informativeness by decreasing the incentives of sophisticated investors to acquire fundamental information.

3.5 When feedback trading can respond to transparency

Our analysis so far has focused on the case when feedback trading is unaffected by changes to the information environment. While endogenizing the behavior of the noise traders is beyond the scope of this paper, in this section, we consider the effect of allowing the intensity of feedback trading to respond to transparency. In particular, the following result characterizes conditions under which efficiency and accuracy fall in transparency, when the parameter $b$ responds (sufficiently smoothly) to changes in the investor’s access to information.

**Theorem 3.** Suppose $b$ is a continuous and differentiable function of $h$. Let $\lambda = \rho c \kappa_b b$, and suppose the investor’s optimal choice of precisions, as characterized in Proposition 2, is given by $\kappa_f(h), \kappa_u(h)$ and $\kappa_b(h)$, where $\kappa_i(h) \equiv \kappa_i(h; b(h))$ so that $\frac{d\kappa_i}{dh} = \frac{\partial \kappa_i}{\partial h} + \frac{\partial \kappa_i}{\partial b} \frac{db}{dh}$. Then,

(i) Efficiency $E$ decreases in transparency (i.e., $h$) if and only if

$$\frac{2}{\kappa_f} \frac{d\kappa_f}{dh} - \frac{\sigma_f^2}{\sigma_y^2} \left( \frac{d\kappa_f}{dh} + \alpha \frac{d\kappa_u}{dh} \right) - \frac{\rho c}{1-G(\lambda)} \frac{dG(\lambda)}{d\lambda} \left( b \frac{db}{dh} + \kappa_b \frac{db}{dh} \right) < 0. \quad (51)$$

(ii) Accuracy $A$ decreases in transparency (i.e., $h$) if and only if

$$\frac{d\kappa_f}{dh} - \left[ \alpha \frac{d\kappa_u}{dh} + \lambda^2 \left( 3 \frac{d\kappa_f}{dh} + \alpha \frac{d\kappa_u}{dh} \right) \right] - \frac{2\lambda \left( 1 + 3\lambda^2 \right) \kappa_f + \lambda^2 \left( 3 + \lambda^2 \right) \left( \alpha \kappa_u + \omega \right)}{(1-\lambda)^2} \rho c \left( b \frac{db}{dh} + \kappa_b \frac{db}{dh} \right) < 0. \quad (52)$$

Since higher transparency allows the investor to learn about, and trade against, feedback traders more cheaply, a natural specification would be to allow feedback traders to cut back on their trading as transparency increases. If we expect the intensity of feedback trading to decrease in response to increased transparency, then $\frac{db}{dh} \leq 0$. The above expressions are analogous to the conditions in Theorem 2, but account for the fact that a change in $b$ in response to changes in transparency has an additional effect on the optimal choice of precisions — the total change in precision $\frac{d\kappa_i}{dh}$ depends not only on $\frac{\partial \kappa_i}{\partial h}$ but also on $\frac{\partial \kappa_i}{\partial b} \frac{db}{dh}$.

Since learning about noise trading is less valuable when the intensity of feedback trading is lower, efficiency and accuracy should decrease more slowly (or even increase) with transparency. Figure 10 presents an instance of this. We assume that the intensity of feedback trading $b$ is given by $b = b_0 e^{-rh}$. Comparing the plots to those in Figure 4 (since both assume a log-linear cost of precision and the same parameter values), note that since the intensity of
Figure 10: Log-linear cost of precision with slowly decreasing $b$

This figure plots (a) feedback intensity $b$, (b) optimal choice of precisions, (c) efficiency, and (d) accuracy, as a function of general transparency $h$, when the cost of precision is given by (42) and feedback intensity is given by $b = b_0 e^{-rh}$. The other parameters are set to the following values: $\rho = 0.6$, $c = 1$, $b_0 = 1$, $\sigma^2_f = 1$, $\sigma^2_{f,2} = 0.4$, $h_f = 4$, $h_u = 0$, $h_b = -4$ and $r = 0.1$. The choice of precisions in panel (b) are given by: $\kappa_f$ (solid), $\kappa_u$ (dashed) $\kappa_b$ (dot-dashed). The dotted lines in panels (c) and (d) mark the level of maximum efficiency and accuracy, respectively, for the parameter range plotted.

feedback trading decreases with transparency, the investor has a lower incentive to learn about noise trading. In fact, the investor no longer learns about $\beta$ in the plotted parameter range. Moreover, while efficiency and accuracy still decrease when transparency is sufficiently high, this only happens at a much higher level of transparency.

However, the dependence of learning about fundamentals on the intensity of feedback trading gives rise to a more counterintuitive result. If feedback trading decreases quickly enough with an increase in transparency, this can lead the investor to learn less about fundamentals, even though learning becomes cheaper. Figure 11 provides an illustration of this effect. Compared to the previous example, the only parameter change is the rate at which feedback trading decreases with $h$ (i.e., $r = 0.75$ instead of $r = 0.1$). The plots suggest that when the decrease in feedback trading is fast enough, learning along all three dimensions — and, in particular, about fundamentals — can decrease as transparency increases.

This result arises because the investor only chooses to acquire information in order to speculate against the feedback traders. In more general settings, if investors choose to learn about
fundamentals for other motives, the result is not likely to be as dramatic. However, the example does highlight the importance of understanding why investors choose to acquire information in the first place. If investors are motivated to learn about fundamentals in order to speculate against other traders, increasing transparency can have unintended (and even counterproductive) consequences through its effect on the behavior of these other traders.

4 Discussion and concluding remarks

In the presence of price-dependent liquidity demand, we show that learning about asset fundamentals and the trading behavior of other investors is complementary. In this setting, we characterize the optimal information acquisition decision or a risk-neutral investor who faces transaction costs and liquidation risk, and show how this affects the extent to which prices reflect fundamentals. Importantly, we show that improving transparency, even if targeted to fundamental information, can make the price less informative. In this section, we discuss some
features and implications of our model and possible extensions left for future work.

**Information-processing constraint.** The signals considered in our model are costly, directly reducing the investor’s utility. A commonly-used alternative is to impose an information-processing constraint on the investor’s learning problem. For instance, one could study the information acquisition decision subject to a capacity constraint which limits the reduction in total entropy for the investor. Such a budget constraint introduces a new channel through which some of our effects may be amplified. Consider the effect of increasing fundamental transparency. By making fundamental information cheaper, the investor learns more about φ — this is analogous to the substitution effect in a portfolio allocation problem. However, since her budget is fixed, this also frees up capacity to learn about noise traders — analogous to a wealth effect. Given the complementarity in learning about fundamentals and feedback, this additional channel can serve to amplify the decrease in efficiency and accuracy as a result of increased fundamental transparency.

**Direct provision of information.** In addition to changing the cost of acquiring information, policymakers and insiders can often directly provide public information about fundamentals to market participants (e.g., forward guidance by central banks, earnings guidance by firms). In the context of our model, this can be analyzed as a reduction in the prior uncertainty, \( \sigma_f^2 \), about fundamentals. The direct effect of such a change is to increase efficiency and accuracy. However, our model highlights an indirect effect: the reduction in prior uncertainty reduces the incentives of the investor to acquire information about fundamentals, which might lead to a decrease in efficiency and accuracy. While this crowding out effect is not unique to our setting (see Section 1), it suggests that the benefit of providing information to market participants may be much less than anticipated. This is an especially important consideration when information generation or provision of such public information is costly.

**Empirical Implications.** Testing our model directly presents a number of empirical challenges, including identifying proxies for both the informativeness of prices and the costs of information processing, as well as isolating the impact of price-dependent noise on prices. However, our model’s implications for the impact of general transparency provides an explanation for the puzzling evidence documented by Bai, Philippon, and Savov (2014). They find that while price informativeness has increased for the large stocks in the S&P 500 since the 1990’s, it has steadily declined for the full sample of CRSP stocks, even though one would expect the increased availability and ease of access to firm-specific information should have improved informativeness more for the smaller firms. Our analysis implies that if transactions costs (i.e., \( c \)) or uncertainty about price-dependent liquidity (i.e., \( b \)) is larger for small stocks, then increased transparency is more likely to decrease price informativeness for them. Our predictions on the impact of fundamental transparency are also consistent with evidence documented by Jorion,
Liu, and Shi (2005), Duarte, Han, Harford, and Young (2008) and others, which suggests the introduction of Reg FD may have decreased the information content of stock prices, at least for a subset of firms.

The importance of our mechanism for explaining variation in returns (and price efficiency) is likely to differ across assets and over time. For instance, our results are more applicable to securities that are susceptible to price pressure and for which investors are able to obtain fundamental information that is not reflected in the price, provided the likelihood of positive feedback trading is high enough. This suggests that our mechanism should have the biggest impact for small-cap, illiquid stocks with low analyst coverage. Our mechanism is most relevant when (ex-ante) fundamental uncertainty is high and market liquidity is low. Such conditions commonly arise during periods of market stress or financial crises, which is often when new regulatory policy is introduced to improve price efficiency.\textsuperscript{23}

Our analysis also recommends against the construction of naive price signals to infer changes in fundamentals, since prices combine information about fundamentals and other traders nonlinearly. We show that an increase in transparency is more likely to decrease price informativeness when investors are already sufficiently informed about fundamentals. Given recent technological and regulatory changes that improve access to information, the mechanism we describe is increasingly relevant and may help explain the rising importance of sophisticated institutional investors that focus on learning about the behavior of others (e.g., statistical arbitrageurs, high frequency traders).\textsuperscript{24}

**Heterogeneity, Asymmetric Information, and Dynamics.** We have kept the model as parsimonious as possible for tractability and to highlight clearly the intuition for our results. Introducing asymmetric information by allowing for heterogeneity across investors is a natural extension, as is generalizing the analysis to multiple periods. A general analysis of either extension requires characterizing how investors update their beliefs using the information in prices, however, and since the equilibrium price depends nonlinearly on the information about fundamentals and noise trading, this class of model does not immediately appear to be analytically tractable. We hope to explore the feasibility of such extensions in future work.

\textsuperscript{23}If such periods also coincide with higher intensity of feedback trading (as suggested by Cohen and Shin (2003)), the mechanism we describe may be amplified even further.

\textsuperscript{24}Along these lines, in their Concept Release on Equity Market Structure (Federal Register Volume 75, Issue 13, 2010), the Securities and Exchange Commission (SEC) specifically requested comment on “two types of directional strategies that may present serious problems in today’s market structure – order anticipation and momentum ignition,” and their role in price discovery. Order anticipation strategies seek to “ascertain the existence of one or more large buyers (sellers) in the market and to buy (sell) ahead of the large orders with the goal of capturing a price movement in the direction of the large trading interest (a price rise for buyers and a price decline for sellers).” A momentum ignition strategy involves initiating “a series of orders and trades... in an attempt to ignite a rapid price move either up or down.”
Appendix

Proof of Proposition 2. The optimization problem is a standard constrained optimization problem. The conditions given in equation (26) are the standard Kuhn tucker conditions. If the objective function is concave, the Kuhn tucker conditions are both necessary and sufficient to guarantee a global maximum. The hessian of the objective function is given by

\[
H = \begin{bmatrix}
-C_{ff} & 0 & V_{fb} \\
0 & -C_{uu} & V_{ub} \\
V_{bf} & V_{bu} & V_{bb} - C_{bb}
\end{bmatrix}
\]

If the hessian matrix is negative definite, the objective function is concave and will have a unique global maximum. The determinant of the above matrix is given by

\[
\text{det}(H) = C_{ff}C_{uu}(V_{bb} - C_{bb}) + V_{fb}^2C_{uu} + V_{bb}^2C_{ff}
\]

The determinant of hessian matrix H is negative if \(V_{bb} - C_{bb} < 0\) i.e., if the cost function is sufficiently convex. Note that this is a sufficient condition but not a necessary condition. □

Proof of Theorem 1. Let \(\lambda = \rho c \kappa b\) and \(\sigma_y^2 = \kappa_f \sigma_f^2 + \rho^2 c^2 \kappa_u \sigma_u^2 + c^2 \left(1 + \rho^2\right) \sigma_u^2\). Efficiency is

\[
\mathcal{E} = -\mathbb{E} \left( (\phi - \mathbb{E}_1 [\phi | P_1])^2 \right) = -\mathbb{E} \left[ \text{var} (\phi | P_1) \right]
\]

(53)

\[
= -\mathbb{E} \left[ \mathbb{E} \left[ \text{var} (\phi | P_1, S_b) | P_1 \right] - \mathbb{E} \left( \mathbb{E} (\phi | P_1, S_b) | P_1 \right) \right].
\]

(54)

Conditional on \(P_1\) and \(S_b\), the price is a linear signal about fundamentals, and so

\[
\mathbb{E} [\phi | P_1, S_b] = \frac{\kappa \sigma^2_f}{\sigma^2_y} (1 - \rho c \kappa b S_b) P_1, \quad \text{var} [\phi | P_1, S_b] = \sigma^2_f \left(1 - \frac{\kappa \sigma^2_f}{\sigma^2_y} \kappa_f\right).
\]

(55)

Moreover, conditional on \(P_1\), the probability that \(S_b = b\) is given by

\[
\pi = \frac{1}{\left(1 - \rho c \kappa b\right)^2} \phi \left( \sqrt{\frac{1}{\left(1 - \rho c \kappa b\right)^2}} \right) \phi \left( \sqrt{\frac{1}{\left(1 - \rho c \kappa b\right)^2}} \right) + \frac{1}{\left(1 + \rho c \kappa b\right)^2} \phi \left( \sqrt{\frac{1}{\left(1 + \rho c \kappa b\right)^2}} \right) = \frac{2 \rho c \kappa b \pi^2}{\left(1 - \rho c \kappa b\right) e^{\frac{2 \rho c \kappa b \pi^2}{\left(1 - \rho c \kappa b\right)^2}}}
\]

(56)

where \(\phi (\cdot)\) is the pdf of the standard normal. Given these probabilities, we have

\[
\mathcal{E} = -\sigma^2_f \left(1 - \frac{\kappa \sigma^2_f}{\sigma^2_y} \kappa_f\right) - 4 \left(\frac{\kappa \sigma^2_f}{\sigma^2_y}\right)^2 \rho^2 c^2 \kappa_u^2 b^2 \mathbb{E} \left[ \pi (1 - \pi) P_1^2 \right]
\]

(57)

\[
= -\sigma^2_f + \frac{\kappa^2 \sigma^2_f^2}{\sigma^2_y} \left(1 - \frac{1}{\pi^2} \rho^2 c^2 \kappa_u^2 b^2 \mathbb{E} \left[ \pi (1 - \pi) P_1^2 \right] \right)
\]

(58)

Let \(y = \kappa_f S_f + \rho \kappa u S_u + c (1 + \rho^2) u_1\) and note that \(s \equiv \frac{y}{\sqrt{\sigma_y^2}} \sim N(0, 1)\). Then, we can express

\[
\mathbb{E} \left[ \pi (1 - \pi) P_1^2 \right] = \mathbb{E} \left[ \left(\frac{\pi(1 - \pi)}{1 - \rho c \kappa u S_u}\right)^2 y^2 \right] = \sigma_y^2 \mathbb{E} \left[ \left(\frac{\pi(1 - \pi)}{1 - \rho c \kappa u S_u}\right)^2 s^2 \right].
\]

(59)

Moreover, note that

\[
\pi = \begin{cases} 
\frac{(1 - \lambda)e^{(1 - \lambda)^2 \sigma_y^2}}{1 + \lambda + (1 - \lambda)e^{(1 - \lambda)^2 \sigma_y^2}} & \text{when } S_b = b \\
\frac{(1 - \lambda)e^{(1 + \lambda)^2 \sigma_y^2}}{1 + \lambda + (1 - \lambda)e^{(1 + \lambda)^2 \sigma_y^2}} & \text{when } S_b = -b 
\end{cases}
\]

(60)
The first Kuhn-Tucker condition implies $\kappa$ as transparency ($\beta$). They are given by

\[ \text{and at this transparency, (66) holds for} \]

\[ \text{Proof of Corollary 1. The Kuhn-Tucker conditions for the optimization problem are given by:} \]

\[ \frac{\partial c}{\partial \kappa_f} = \frac{\partial v}{\partial \kappa_f} - e^{-h-h_f} \kappa_f \frac{1}{1-\kappa_f} \leq 0; \kappa_f \geq 0; \kappa_f \frac{\partial c}{\partial \kappa_f} = 0 \]

\[ \frac{\partial c}{\partial \kappa_u} = \frac{\partial v}{\partial \kappa_u} - e^{-h-h_u} \kappa_u \frac{1}{1-\kappa_u} \leq 0; \kappa_u \geq 0; \kappa_u \frac{\partial c}{\partial \kappa_u} = 0 \]

\[ \frac{\partial c}{\partial \kappa_b} = \frac{\partial v}{\partial \kappa_b} - e^{-h-h_b} \kappa_b \frac{1}{1-\kappa_b} \leq 0; \kappa_b \geq 0; \kappa_b \frac{\partial c}{\partial \kappa_b} = 0 \]

where the marginal values of learning along each dimension are given by:

\[ V_f \equiv \frac{\partial v}{\partial \kappa_f} = (1 - \rho) \frac{c}{2} \frac{\lambda_2}{(1-\lambda_f)^2} \beta^2 \sigma_f^2, \]

\[ V_u \equiv \frac{\partial v}{\partial \kappa_u} = (1 - \rho) \frac{c}{2} \left( \frac{\lambda_2}{(1-\lambda_u)^2} \beta^2 \rho^2 c^2 + \frac{2\lambda^2}{1-\lambda_u} \right) \sigma_u^2, \]

\[ V_b \equiv \frac{\partial v}{\partial \kappa_b} = (1 - \rho) \beta^2 (2 \lambda (3+\lambda^2) \sigma_f^2 (\kappa_f + \alpha u + \sigma_u) + 2 (1-\lambda^2) (\kappa_u \sigma_u^2 + (1+\rho^2) \sigma_u^2) \right) \]

The first Kuhn-Tucker condition implies $\kappa_f$ is either $=0$ or $>0$. Suppose $\kappa_f = 0$. Then the first Kuhn-Tucker equation cannot be satisfied since $\frac{\partial c}{\partial \kappa_f} > 0$, which is a contradiction. Similarly, we can rule out the case that $\kappa_u = 0$. The third Kuhn Tucker condition implies $\kappa_b$ is either $=0$ or $>0$. So, the only possible choices are $\{\kappa_f, \kappa_u, \kappa_b\} > 0$ or $\{\kappa_f, \kappa_u\} > 0$ and $\kappa_b = 0$. For low values of transparency, $h$, investors only learn about $\{\kappa_f, \kappa_u\}$ and for high values of $h$, agent learns about all 3 dimensions. Let the transparency at which investor start increasing $\kappa_b$ be denoted $\tilde{h}$. At this transparency, all three Kuhn Tucker conditions hold with equality and $\kappa_b = 0$. For $h < \tilde{h}$, the optimal precisions are solved using the first two Kuhn Tucker conditions and $\kappa_b = 0$. They are given by

\[ \kappa_f(h) = \frac{(1-\rho) \beta^2 \sigma_f^2 \kappa_f + h_f}{1+(1-\rho) \beta^2 \sigma_f^2 \kappa_f + h_f}, \quad \kappa_u(h) = \frac{\alpha (1-\rho) \beta^2 \sigma_f^2 \kappa_f + h_f + \alpha u}{1+(1-\rho) \beta^2 \sigma_f^2 \kappa_f + h_f + \alpha u}, \]

As transparency $(h)$ increases, optimal $\kappa_f$ and $\kappa_u$ increases and this increases the marginal value of learning about feedback traders($\beta$). The cutoff $\tilde{h}$ is the point at which investor starts learning about feedback traders and at this transparency, (66) holds for $\kappa_b \rightarrow 0$. This implies

\[ e^{-\tilde{h}+\kappa_b} = \lim_{\kappa_b \rightarrow 0} \frac{\partial v}{\partial \kappa_b} = (1 - \rho) \frac{c}{2} \beta^2 \rho^2 \left( \kappa_f (\tilde{h}) \sigma_f^2 + \kappa_u (\tilde{h}) \rho^2 \sigma_u^2 \right) + 4 \kappa_u (\tilde{h}) \sigma_u^2 \]

The cutoff $\tilde{h}$ solves above equation. As $h$ increases, left side of the equation decreases and right side increases, which implies a unique solution. For $h > \tilde{h}$, the optimal signal precisions solve the Kuhn Tucker conditions with equality.
Proof of Proposition 4. Denote $z = (1 - \rho)\frac{b^2}{2}\sigma_f^2$. Then the optimal signal precisions at $h = \bar{h}$ can be rewritten as

$$\kappa_f(\bar{h}) = \frac{ze^{h + \bar{f}}}{1 + ze^{h + \bar{f}}}, \quad \kappa_u(\bar{h}) = \frac{\alpha ze^{h + h_u}}{1 + \alpha ze^{h + h_u}}, \quad \kappa_b(\bar{h}) = 0$$

and $\bar{h}$ solves the equation

$$e^{-(\bar{h} + h_u)} = (1 - \rho)\frac{b^2}{2}\sigma_f^2 \left( 6\bar{h}^2\sigma_f^2 \left( \frac{ze^{h + \bar{f}}}{1 + ze^{h + \bar{f}}} + \frac{\alpha ze^{h + h_u}}{1 + \alpha ze^{h + h_u}} \right) + 4\frac{\alpha ze^{h + h_u}}{1 + \alpha ze^{h + h_u}} \sigma^2_{u,2} \right)$$

which simplifies to

$$= z^2 \left( 6\bar{h}^2b^2 \left( \frac{ze^{\bar{h} + \bar{f}}}{b} \right) + (6\bar{h}^2b^2 + 4) \frac{\alpha ze^{h + h_u}}{1 + \alpha ze^{h + h_u}} \right)$$

(71)

Since $\kappa_u$ increases smoothly at $h = \bar{h}$, we can show that the optimal $\kappa_f(h)$ and $\kappa_u(h)$ are continuous functions of $h$. This implies

$$\frac{d}{dh} \mathcal{E} = \frac{\kappa_f^2(\sigma_f^2)^2}{\sigma_f^4} (-G'(\rho c_k b) \rho c_k b) + \frac{\sigma_f^2 \kappa_f(\sigma_f^2)^2 - \kappa_u^2(\sigma_f^2)^2}{\sigma_u^2} \left( \kappa_f - \alpha \kappa_u \right) (1 - G(\rho c_k b))$$

Using the facts that $\kappa_u' > 0$ and $G'(\cdot) > 0$, we can write

$$\frac{d}{dh} \mathcal{E} < \frac{\sigma_f^2 \kappa_f'(\sigma_f^2)^2 - \kappa_u^2(\sigma_f^2)^2}{\sigma_u^2} \left( \kappa_f - \alpha \kappa_u \right) (1 - G(\rho c_k b))$$

So, the sufficient condition for price efficiency to decrease with transparency is $(\kappa_f + 2\alpha \kappa_u) \kappa_f' < \alpha \kappa_f \kappa_u'$. Also,

$$\kappa_f'(\bar{h}) = \frac{ze^{h + \bar{f}}}{(1 + ze^{h + \bar{f}})^2}, \quad \kappa_u'(\bar{h}) = \frac{\alpha ze^{h + h_u}}{(1 + \alpha ze^{h + h_u})^2}$$

At $h = \bar{h}$, price efficiency decreases in transparency if

$$(\frac{ze^{h + \bar{f}}}{1 + ze^{h + \bar{f}}} + 2\frac{\alpha ze^{h + h_u}}{1 + \alpha ze^{h + h_u}}) \frac{1}{1 + ze^{h + \bar{f}}} < \frac{\alpha ze^{h + h_u}}{(1 + \alpha ze^{h + h_u})^2}$$

From equation (71), observe that as $h_u$ decreases, the cutoff $\bar{h}$ increases and $h_u + \bar{h}$ tends to a constant. As cost of learning about feedback increases, investors have less incentive to learn about it. Given this, the price efficiency decreases in transparency if $(1 + 2\alpha) < e^{h_f - h_u}$. This condition is true for sufficiently high $h_f$ or sufficiently low $h_u$. Hence there exists a cutoff for $h_u$ below which price efficiency decreases with transparency at $h = \bar{h}$.

Price efficiency at $h = \bar{h}$ and $h = \infty$ are given by

$$\mathcal{E}_h = -\sigma_f^2 \left( 1 - \frac{\kappa_f(\bar{h})^2}{\kappa_f(h) + \alpha \kappa_u(h)} \right) \quad \mathcal{E}_\infty = -\sigma_f^2 \left( \frac{\alpha + G(\rho c_k b)}{\alpha + 1} \right)$$

We want the conditions under which $\mathcal{E}_h > \mathcal{E}_\infty$. This can be rewritten as

$$\frac{(\frac{ze^{h + \bar{f}}}{1 + ze^{h + \bar{f}}} + 2\frac{\alpha ze^{h + h_u}}{1 + \alpha ze^{h + h_u}})^2}{\frac{ze^{h + \bar{f}}}{1 + ze^{h + \bar{f}}} + \frac{\alpha ze^{h + h_u}}{1 + \alpha ze^{h + h_u}}} > \frac{1 - G(\rho c_k b)}{1 + \alpha}$$

From equation (71), as $h_u$ decreases, the cutoff $\bar{h}$ increases and $h_u + \bar{h}$ tends to a constant. The above condition reduces to $\frac{1}{1 + \alpha} > \frac{1 - G(\rho c_k b)}{1 + \alpha}$ which is obviously true. This implies that there exists a cutoff for $h_u$ below which price efficiency decreases with transparency. 

△
References


