Identification in multidimensional hedonic models

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What is a hedonic model?

- Hedonic models are models of markets for differentiated products.
- They are used to analyse a wide range of markets including those for a) workers, b) houses, c) automobiles and many others.
- In a hedonic market, each product is described by \( z \), a vector of observable (and sometimes unobservable) characteristics that affect either costs or utility.
- Hedonic models are used to forecast prices for new products, to adjust consumer price indexes for changes in product quality, and to estimate how consumer demand and consumer utility depend on \( z \).
Nearly all empirical applications of hedonic models involve products with multidimensional characteristics.

- Labor markets: wages depend on a vector of skills, occupational characteristics, risk of injury/death.
- Housing: housing quality is measured by its location vector, size of house, size of lot, quality of materials, age of dwelling, etc.
- Computers: prices depend on processing cores, speed, memory, brand, display, weight, ...
Nonseparable hedonic models: the identification and estimation problem

- **Nonseparable** models:
  - Hedonic demand \( z = d(x, \varepsilon) \) for vector \( z \) implicitly defined by system of first-order conditions
  \[
  D_z p(z, m) = D_z u(z, x, \varepsilon).
  \]
  - Prices and demand generated from matching equilibrium.
  - Identification and estimation problem?
    - Given data on \((z, x, p, m)\), identify and estimate \((p, u, F_\varepsilon)\).
  - Nonparametric identification results for scalar nonseparable hedonic models worked out in Heckman, Matzkin, Nesheim (2010).
  - This paper extends this work to multidimensional hedonic models.
  - Related work in Chernozhukov, Galichon, and Henry (2014).
Fully **nonseparable** hedonic model is point identified:

- Requires policy invariant normalizations.
- Requires data from multiple markets with “rich” price variation.
- Requires observable multidimensional aggregate supply or demand shifters.

Estimation based on nonparametric estimation of the joint distribution $F_{Z|XM}$ and $p(z, m)$.

Identified structural functions can be used to estimate the impact of counterfactual policy changes or changes in the economic environment.
Results: single market data

1. Additivity restrictions
   - Additive specification is **over-identified** (up to normalizations)
     \[ D_z u (z, x, \varepsilon) = u_0 (z) + v (x) + \varepsilon. \]
   - Resulting econometric model is a multidimensional transformation model
     \[ D_z T (z) = v (x) + \varepsilon. \]

2. Work in progress
   - Simulations of two dimensional model and estimation.
   - Applications to 1) wage data and injury/death risk, 2) housing market data and location quality are in progress.
Outline

1. Model.
2. Equilibrium.
   - Existence, uniqueness and purity, differentiability of price function.
3. Identification: single market data.
4. Identification: multiple market data.
Products (jobs, houses) have payoff relevant characteristics $z \in \mathbb{Z} \subseteq \mathbb{R}^2$.

Traded in many markets indexed by $m \in \mathbb{M}$.

Price in market $m$ is $p(z, m)$.

Buyers and sellers cannot move between markets.
  - If they do, we are in the single market case.
Assumption 1: Distribution of buyers

- Set of buyers with characteristics $(x, \varepsilon) \in \mathbb{X} \times \mathbb{E} \subseteq \mathbb{R}^{n_x} \times \mathbb{R}^2$.
- Random variables $(X, M, \varepsilon)$ continuously distributed on their supports.
- With $\varepsilon \perp (X, M)$.
- Density functions $(f_{XM}, f_\varepsilon) \in C^2 (\mathbb{X} \times \mathbb{M}) \times C^2 (\mathbb{E})$ with $0 < b_{XL} \leq (f_{XM}, f_\varepsilon) \leq b_{XH} < \infty$.
- Can allow for discrete $X$; potential loss of point identification.
Assumption 2: Buyer utility

- Buyers have reservation utility $u_0(x, \varepsilon, m) = 0$.
- Buyer $(x, \varepsilon)$ in $m$ who chooses $z$ obtains $u(z, x, \varepsilon) - p(z, m)$.
- Utility function $u \in C^5(\mathbb{Z} \times \mathbb{X} \times \mathbb{E})$. 


Assumption 3: Distribution of sellers

- Set of sellers with characteristics \((y, \eta) \in \mathbb{Y} \times \mathbb{H} \subseteq \mathbb{R}^{ny} \times \mathbb{R}^2\).
- Random variables \((Y, M, \eta)\) continuously distributed on their supports.
- With \(\eta \perp (Y, M)\).
- Density functions \((f_{YM}, f_{\eta}) \in C^2 (\mathbb{Y} \times \mathbb{M}) \times C^2 (\mathbb{H})\) and with \(0 < b_{YL} \leq (f_{YM}, f_{\eta}) \leq b_{YH} < \infty\).
Assumption 4: Seller profits

- Sellers have reservation profit $\pi_0(y, \eta, m) = 0$.
- Seller $(y, \eta)$ in $m$ who chooses $z$ obtains $p(z, m) - c(z, y, \eta)$.
- Cost function $c \in C^5(Z \times Y \times H)$.
- Alternate (easier) case: endowment economy
  - Distribution of sellers is $F_{ZM}$ and profits are $p(z, m)$.
- Alternate (harder) case: Monopoly case
Buyer’s problem

- In market $m$, problem of buyer $(x, \varepsilon)$ is to find $z$ to solve
  \[
  v(x, \varepsilon, m) = \max_{z \in Z} \left\{ u(z, x, \varepsilon) - p(z, m) \right\}.
  \]

- First-order condition is
  \[
  D_z u(z, x, \varepsilon) - D_z p(z, m) = 0.
  \]

- Solution defines demand function
  \[
  z = d(x, \varepsilon, m).
  \]

- For the moment, assume second-order condition is satisfied.
- Analogous problem for seller.
Definition (Equilibrium)

Let $\alpha$ be a measure on the space of buyers, sellers and products.

A pair $(\alpha, p)$ is an equilibrium if

1. **Markets clear**: The marginals of $\alpha$ equal the marginals of buyers and sellers respectively.

2. **Agents optimise**: If $(z, x, \varepsilon, y, \eta)$ is in the support of $\alpha$, then $z$ is optimal for $(x, \varepsilon)$ and for $(y, \eta)$. 
Assumption 5: Generalized single-crossing

5a. For almost all \((x, \varepsilon, y, \eta)\), the problem

\[
s(x, \varepsilon, y, \eta) = \max_{z \in \mathbb{Z}} \{ u(z, x, \varepsilon) - c(z, y, \eta) \}
\]

has a unique interior optimizer, \(z = h(x, \varepsilon, y, \eta)\).

5b. The matrix \(M + M^T\) is positive definite where

\[
M = \begin{bmatrix}
D_{xy}^2 & D_{x\eta}^2 \\
D_{xy}^2 & D_{x\eta}^2
\end{bmatrix}.
\]

5c. Moreover, \(D_{z\varepsilon}^2 u > 0\) and \(D_{z\eta}^2 c < 0\).
Find equilibrium by matching buyers and sellers together to maximise social welfare.

Social welfare maximisation problem is

\[
\max \int s(x, \varepsilon, y, \eta) \, d\gamma
\]  \hspace{1cm} (1)

subject to \( \gamma \in \Gamma(\mathcal{F}_X, \varepsilon, \mathcal{F}_Y, \eta) \).
Equilibrium

Theorem 1 (Existence, uniqueness and purity of equilibrium)

An equilibrium exists and is unique. In this equilibrium, matching is pure.

- Each buyer matches with a unique optimal seller.
- Second-order conditions are satisfied for everyone who enters the market.
- If \( s(x, \varepsilon, y, \eta) > 0 \) almost surely, everyone enters the market almost surely.
Equilibrium

Proof.
Under the conditions stated:
- Social welfare maximisation problem is an optimal transport problem with a unique solution.
- Matching is pure.
- Multipliers on constraints can be used to construct prices that decentralise the optimal allocation.
- Prices are unique for trades that occur in equilibrium.
- See Chiappori, McCann and Nesheim (2010).
Assumption 6: Curvature of surplus

- To ensure smoothness of equilibrium payoffs, fourth derivatives cannot be too big.
- Specifically, there exists a constant $c_0 > 0$ such that for any $(x, \varepsilon) \in \mathbb{X} \times \mathbb{E}$, $(y, \eta) \in \mathbb{Y} \times \mathbb{H}$, and $\xi \in \mathbb{X} \times \mathbb{E}$, $\zeta \in \mathbb{Y} \times \mathbb{H}$, $\xi \perp \zeta$

$$\sum_{i,j,k,l,p,q,r,s} (s^{p,q} s_{ij,p} s_{q,rs} - s_{ij,rs}) s^{r,k} s^{s,l} \xi_{i} \xi_{j} \zeta_{k} \zeta_{l} \leq -c_0 |\xi|^2 |\zeta|^2.$$ 

- See Ma, Trudinger and Wang (2005).
Assumption 7: $s$-convexity of spaces of buyers and sellers

- If optimal map from buyers to sellers hits boundary of space, can lead to failure of smoothness.
- To avoid this, require spaces of buyers and sellers to be $s$-convex and $s^*$-convex (these are generalisations of convexity).
- For all $(y, \eta) \in Y \times H$, the set
  \[
  (D_{y\eta}s)^{-1}(X \times E, y, \eta)
  \]
  is convex.
- For all $(x, \epsilon) \in X \times E$, the set
  \[
  (D_{x\epsilon}s)^{-1}(x, \epsilon, Y \times H)
  \]
  is convex.
Differentiability of price function

Theorem 2 (Differentiability of price function)

\( p(z) \) is twice continuously differentiable in \( z \).

1. Envelope theorem implies \( s(x, \varepsilon, y, \eta) \in C^4 \).
2. Thus, under assumptions 1 - 7, Ma, Trudinger and Wang (2005) Theorem 2.1 implies \( \nu(x, \varepsilon) \in C^3 \) and \( \pi(y, \eta) \in C^3 \).
3. Price function satisfies

\[
p(z) = u(x, z, \varepsilon) - \nu(x, \varepsilon).
\]

4. So, using envelope theorem again

\[
D_z p(z) = D_z u + (D_x u - D_x \nu) D_z x
\]

\[
D_z p(z) = D_z u.
\]
Single market estimation problem

- Data on \((x_i, z_i, \tilde{p}_i)_{i=1}^N\).
- Price function
  \[ \tilde{p} = p(z) + \zeta \]
  where \(\zeta\) is measurement error.
- Demand function \(z = d(x, \varepsilon)\) satisfies
  \[ D_z p(z) = D_z u(z, x, \varepsilon). \]
- Want to estimate \((p, u, F_\varepsilon)\).
- Data provide direct estimates of \(p\) and \(F_{Z|X}\).
Failure of point-identification in a single market

- Even if $F_\varepsilon$ is known, multiple models for $d$ are consistent with data.
- Suppose $\varepsilon$ is bivariate independent uniform.
- A demand model consistent with data is
  \[
  d_1^A(x, \varepsilon_1) = \left( F_{Z_1|x} \right)^{-1}(x, \varepsilon_1)
  \]
  \[
  d_2^A(x, \varepsilon_1, \varepsilon_2) = \left( F_{Z_2|Z_1x} \right)^{-1}\left[ x, \left( F_{Z_1|x} \right)^{-1}(x, \varepsilon_1), \varepsilon_2 \right] .
  \]
- An observationally equivalent demand model is
  \[
  d_1^B(x, \varepsilon_1, \varepsilon_2) = \left( F_{Z_1|Z_2x} \right)^{-1}\left[ x, \left( F_{Z_2|x} \right)^{-1}(x, \varepsilon_2), \varepsilon_1 \right]
  \]
  \[
  d_2^B(x, \varepsilon_2) = \left( F_{Z_2|x} \right)^{-1}(x, \varepsilon_2) .
  \]
- Neither model is structural. Can be used to predict distribution of demand if price held fixed but not otherwise.
- Not a normalisation in a single market.
• Even if \( d(x, \varepsilon) \) were identified, still \( u \) is not.
• The first-order condition is

\[
D_z p \left( d \left( x, \varepsilon \right) \right) = D_z u \left( d \left( x, \varepsilon \right), x, \varepsilon \right).
\]

• \( u \) is a function of \( n_x + 4 \) variables, but, in a single market, we only observe its values on a \( n_x + 2 \) dimensional manifold.
• Without further structure, it is impossible to estimate

\[
D_z u \left( d \left( x, \varepsilon \right) + \Delta z, x, \varepsilon \right).
\]

• Need data from multiple markets or further restrictions on \( u \).
Point-identification using additive marginal utility restriction

- Suppose utility takes an additive form

\[ D_z u(z, x, \varepsilon) = u_0(z) + v(x) + \varepsilon. \]

- First-order conditions become

\[ D_z p(z) = u_0(z) + v(x) + \varepsilon. \]
Rewrite this as

\[ D_z T(z) = v(x) + \varepsilon \]

where

\[ D_z T(z) = D_z p(z) - u_0(z). \]

This is a bivariate "transformation" model with the restriction that \( D_z T(z) \) is the gradient of a convex function and \( T(z) \) is one-to-one and onto.
Implications of transformation model

- Density of data is

  \[ f_{Z|X}(z, x) = f_\varepsilon(D_z T(z) - \nu(x)) |\det(D_{zz'} T(z))| . \]

- CDF of data is

  \[ F_{Z|X}(z_1, z_2, x) = \int_{-\infty}^{z_1} \int_{-\infty}^{z_2} f_\varepsilon(D_z T(s) - \nu(x)) |\det(D_{zz'} T(s))| ds \]

  \[ = \int_{D_z T(A) - \nu(x)} f_\varepsilon(\varepsilon) d\varepsilon. \]

  where

  \[ A = \{ Z \in \mathbb{Z} : Z_1 \leq z_1 \text{ and } Z_2 \leq z_2 \} \]

  \[ D_z T(A) - \nu(x) = \{ \varepsilon : \varepsilon = T(z) - \nu(x) \text{ for some } z \in \mathbb{Z} \} . \]

- Multivariate formula for change of variable.

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Derivative w.r.t. $z$

$$D_{z_i} F_Z | \chi (z_1, z_2, x) = \int_{\partial D_z T(A) - \nu(x)} f_\varepsilon (\varepsilon) \left( \frac{\partial^2 T (z)}{\partial z_1 \partial z_i} d\varepsilon_2 - \frac{\partial^2 T (z)}{\partial z_2 \partial z_i} d\varepsilon_1 \right)$$

$$= \int_{D_z T(A) - \nu(x)} D_{\varepsilon_1} f_\varepsilon (\varepsilon) \frac{\partial^2 T}{\partial z_1 \partial z_i}$$

$$+ \int_{D_z T(A) - \nu(x)} D_{\varepsilon_2} f_\varepsilon (\varepsilon) d\varepsilon \frac{\partial^2 T}{\partial z_2 \partial z_i}$$

$$= \left( \int_{D_z T(A) - \nu(x)} D_{\varepsilon} f_\varepsilon (\varepsilon) d\varepsilon \right)^T \frac{\partial (D_z T)}{\partial z_i}.$$
Derivative w.r.t. $x$

\[
\frac{\partial F_{z|x}(z_1, z_2, x)}{\partial x_i} = - \frac{\partial}{\partial x_i} \left( \int_{D_z T(A) - v(x)} f_\varepsilon(\varepsilon) \, d\varepsilon \right)
= - \int_{\partial D_z T(A) - v(x)} \left( f_\varepsilon(\varepsilon) \left[ \frac{\partial v_1}{\partial x_i} d\varepsilon_2 - \frac{\partial v_2}{\partial x_i} d\varepsilon_1 \right] \right)
= - \int_{D_z T(A(z)) - v(x)} (D_\varepsilon f_\varepsilon)^T D_{x_i} v.
\]
Restrictions of separability are

\[ D_z F_{Z|X} = (D_{zz'})^T \int D_\varepsilon f_\varepsilon d\varepsilon \]  \hspace{1cm} (2)

\[ D_x F_{Z|X} = - (D_x v) \int D_\varepsilon f_\varepsilon d\varepsilon. \]  \hspace{1cm} (3)
Normalization

• For some $x_0$, normalize $D_x \nu (x_0) = I$ and define

$$ F^0_{Z|X} = F_{Z|X} (z, x_0) $$

$$ F^0_\varepsilon = F_\varepsilon (T (A) - \nu (x_0)) . $$

• Then, using (2) and (3)

$$ D_z F^0_{Z|X} = - (D_{zz'} T) D_x F^0_{Z|X} . $$

• Linear system of PDE’s in unknown $D_{zz'} T$. 

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Final result

- Restriction on $T$ is
  \[
  \left( -\frac{F_{z_1}^0}{F^0_{x_1}} + \frac{F^0_{x_2} F_{z_2}^0}{(F^0_{x_1})^2} \right) + \frac{(F^0_{x_2})^2}{(F^0_{x_1})^2} T_{22} = T_{11}.
  \]

- Linear second order elliptic differential equation in the unknown $T$.
- Identified up to boundary conditions. For example, one could impose that for all $z \in \mathbb{Z}$,
  \[
  D_{z_1} T(z) = \Phi^{-1}(z_1) \\
  D_{z_2} T(z) = \Phi^{-1}(z_2)
  \]

- Once $T$ is known, equation (3) is a system of first-order linear PDE’s in $D_x v$. Looking at two different values of $z$, this can be solved for $v(x)$ with an initial condition such as $v(0) = 0$.
- With $T$ and $v$, known, recover $f_\varepsilon$ from change of variables formula.
Single market estimation

\[
\max_{\{f, T, \nu_1, \nu_2\}} \left\{ \sum_i (\ln f(\varepsilon_{1i}, \varepsilon_{2i}) + \ln \det(D_{zz} T(z_i)) \right\}
\]

subject to

\[
\varepsilon_{1i} = D_{z1} T(z_i) - \nu_1(x_i)
\]
\[
\varepsilon_{2i} = D_{z2} T(z_i) - \nu_2(x_i)
\]
\[
D_{z1} T(z) = \Phi^{-1}(z_1) \forall z \in \partial Z
\]
\[
D_{z2} T(z) = \Phi^{-1}(z_2) \forall z \in \partial Z
\]
\[
\nu_1(0) = 0
\]
\[
\nu_2(0) = 0
\]
Recall the FOC

\[ D_z p (z, m) = D_z u (z, x, \varepsilon) . \]

Let \( z^*_m = d (x, \varepsilon, m) \). In single market, it is impossible to learn the precise value of

\[ D_z u (z^*_m + \Delta z, x, \varepsilon) \]

because outcome \( z' = z^*_m + \Delta z \) is never observed.

With multiple markets, can learn marginal utility at outcome \( z' \) by finding a market \( m' \) with price \( p (z, m') \) such that

\[ z' = d (x, \varepsilon, m') = z^*_m + \Delta z . \]
Equilibrium implies that marginal price varies across markets due to observable cost or demand shifters.

That is, variation in distributions of observables $F_Y|M$ and/or $F_X|M$.

Use this variation to estimate $p(z, m)$ and $F_Z|XM$.

Need support and price variation conditions.

Also need normalisation on distribution $F_\varepsilon$. 
Assumptions 8 and 9: Full support and price variation

- Assumption 5 implies that for each \((z, x)\), \(D_z u(z, x, \varepsilon) : \mathbb{E} \rightarrow \mathbb{V}\) is one-to-one where \(\mathbb{V}_{zx} = \text{Range} \ (D_z u)\).

- **Assumption 8: Range of \(D_z u\)**
  - Assume that \(\text{Range} \ (D_z u)\) is independent of \((z, x)\).

- Define the random variable \(v = D_z p(z, m)\) and let \(\mathbb{V} = \text{Range} \ (D_z u)\).

- **Assumption 9: Distribution of \(V|ZX\)**
  - Suppose that \((V, Z, X)\) has support on \(\mathbb{V} \times \mathbb{Z} \times \mathbb{X}\).
Normalization (1): Distribution of unobservables

- Vector $\mathbf{\varepsilon}$ are not observed and have no natural units.
- Enter model in non-additive fashion.
- Natural normalization: each element of $\mathbf{\varepsilon}$ measures percentile rank of consumer in some dimension.
  - For example, $\varepsilon_1$ may be the percentile rank of $z_1$ conditional on $(x, m)$ and $\varepsilon_2$ may be the percentile rank of $z_2$ conditional on $(x, m)$ and $\varepsilon_1$.
- Choose units of measurement so that elements of $\mathbf{\varepsilon}$ are independent uniform random variables.
The identified set

- There is a family of demand functions, distributions $F_{\varepsilon}$ and utility functions $D_{z}u$ that are consistent with the data.
- Let
  \[ d(x, m, u) = \begin{bmatrix} Q_{Z_{1}|XM}(u_{1}, x, m) \\
  Q_{Z_{2}|Z_{1}XM}(u_{2}, Q_{Z_{1}|XM}(u_{1}, x, m), x, m) \end{bmatrix}. \]
- The identified set of demand functions includes all functions of the form
  \[ \tilde{d}(x, m, \varepsilon) = d(x, m, R[F_{\varepsilon}(\varepsilon), x]) \]
  where $F_{\varepsilon}$ is a distribution function and $R$ is any invertible measure preserving transformation satisfying $\det \left( \frac{\partial R}{\partial u} \right) = 1.$
Identification of $D_z u$

- Suppose I choose $F_\varepsilon \sim \text{Uniform}$ and $R(\varepsilon, x) = \varepsilon$. Then

$$D_z u(z, x, \varepsilon) = D_z p \left( d(x, m^*, \varepsilon), m^* \right)$$

where $m^*$ solves $z = d(x, m^*, \varepsilon)$.

- If instead I choose an alternative $\tilde{F}_\varepsilon$, $\tilde{R}$, and $\tilde{d}$, then

$$D_z \tilde{u}(z, x, \tilde{\varepsilon}) = D_z p \left( \tilde{d}(x, m^*, \tilde{\varepsilon}), m^* \right)$$

where $m^*$ solves $z = \tilde{d}(x, m^*, \tilde{\varepsilon})$.

- Observationally equivalent models, different interpretations of $\varepsilon$ since $\varepsilon = \tilde{R} \left( \tilde{F}_\varepsilon(\varepsilon), x \right)$.
Normalization (2)

- Choice between $d$ and $\tilde{d}$ is a normalisation.
- Both predict same distribution of demand in each market $m$.
- Demand of consumer $(x, \varepsilon)$ predicted by $d(x, \varepsilon)$ is same as demand of consumer $(x, \tilde{\varepsilon})$ under $\tilde{d}$ where $\tilde{\varepsilon} = \tilde{R} \left[ \tilde{F}_\varepsilon(\varepsilon), x \right]$.
- By construction, equilibrium price in market $m'$ using $d$ is the same as that obtained using $(\tilde{d}, R, F_\varepsilon)$.
- Conditions under which this is NOT a normalisation:
  - Change in economic environment changes $u$.
  - Change in $F_\varepsilon$.
  - If $\varepsilon$ is NOT independent of $x$.
  - If support conditions are not met.
Conclusion

- Multimarket data required for point identification of unrestricted model.
  - Estimation based on nonparametric estimation of a multidimensional distribution function.
  - Computationally simple but highly demanding of data.
- Restrictions that reduce dimension required for point identification using single market data.
- Applications in progress
  - Willingness-to-pay for locational quality and housing quality using English Housing Survey and American Housing Survey.
  - Willingness to accept risk of injury and death on the job using US CPS.
Monopoly case

- Equilibrium price $p(z, m)$ in market $m$ could be determined by monopolist or by oligopoly competition.
- For the monopoly case, given $(u, F_{xM}, F_\varepsilon)$ and product cost $c(z, m)$, monopolist chooses $p(z)$.
- Monopolist solves

$$\max \left\{ p \right\} \int (p(d(x, \varepsilon)) - c(d(x, \varepsilon))) f_{xm}(x, m) f_\varepsilon(\varepsilon) \, dx \, d\varepsilon$$

subject to

$$d(x, \varepsilon) \in \arg \max_{z} \{u(z, x, \varepsilon) - p(z)\}.$$
Or, equivalently, choose \((v, d)\) to maximize profits subject to participation and incentive constraints (Rochet and Chone, 1998, Carlier, 2001 or Figalli, McCann and Kim, 2011).

That is,

\[
\max_{\{v,d\}} \int \left[ u(d, x, \varepsilon) - v(x, \varepsilon) - c(d(x, \varepsilon)) \right] f_x(x) f_\varepsilon(\varepsilon) \, dx \, d\varepsilon
\]

subject to

\[
\begin{align*}
D_x v &= D_x u(d, x, \varepsilon) \\
D_\varepsilon v &= D_\varepsilon u(d, x, \varepsilon) \\
v &\geq 0
\end{align*}
\]

\[
v(x, \varepsilon) = \sup_{\{z', x', \varepsilon'\}} \left\{ u(z', x, \varepsilon) - \left[ u(z', x', \varepsilon') - v(x', \varepsilon') \right] \right\}
\]

Solution \(p(z)\) depends on \(c(z, m), F_{XM}, F_\varepsilon\) and \(u\).
Example: Distribution of buyers

Density of $x$ vs. $(x_1, x_2)$
Example: Distribution of sellers

Density of $y$ vs. $(y_1, y_2)$

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Example: Optimal matching