PERSONALITY TRAITS AND THE MARRIAGE MARKET

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Abstract. How many and which attributes are relevant for the sorting of agents in a matching market? This paper addresses these questions by introducing a novel technique called “Saliency Analysis” that is grounded in a continuous extension of the structural equilibrium model of Choo and Siow (2006). Saliency Analysis consists in estimating a quadratic joint utility function and performing its Singular Value Decomposition, which allows to approximate a multivariate sorting model using a lower number of dimensions. The methodology is applied on a survey of Dutch households containing information about education, height, BMI, health, attitude towards risk and personality traits of spouses. Three important empirical conclusions are drawn regarding sorting in the marriage market. First, our test rejects the hypothesis that sorting occurs on a single dimension: individuals face important trade-offs between the attributes of their spouses which are not amenable to a single-dimensional index. Second, although education explains the largest share of a couple’s observable joint utility, personality traits explain another important part. Third, different personality traits matter differently for men and for women.

Keywords: Multidimensional sorting, Saliency Analysis, marriage market, personality traits, continuous logit.

JEL Classification: D3, J21, J23 and J31.

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1. Introduction

Marriage, understood in a broad sense, is probably one of the most important factors for happiness. It also plays an important role in the generation of welfare and its redistribution across individuals. An in-depth understanding of marriage patterns is therefore of crucial importance for the study of a wide range of economic issues. A growing body of the economic literature studies the determinants of marriage, seen as a competitive matching market, both empirically and theoretically. This literature draws insights from the seminal model of the marriage market developed by Becker (1973). At the heart of Becker’s theory lies a two-sided assignment model with transferable utility where agents on both sides of the market (men and women) are characterized by a set of attributes only partly observed by the researcher. Each agent aims at matching with a member of the opposite sex so as to maximize his or her own payoff. This model is particularly interesting since under certain conditions, one can identify and estimate features of agents’ preferences. A central question in this market is which and how many attributes are relevant for the sorting of agents?

A large body of literature has focused on the identification and estimation of preferences in the marriage market and in other matching markets; however, it has been constrained by some methodological limitations regarding the quantitative methods available to identify and estimate features of the joint utility function. In the current state of the art, no estimation tool can handle sorting on multiple continuous attributes in a convenient manner. Until recently, most empirical literature assumed sorting occurs on a single continuous dimension, which is a single index aggregating the various attributes of the agents. The choice of this approach was strongly influenced by Becker’s seminal model of “positive assortive mating”, which is essentially single-dimensional. Due to this limitation, empirical

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1See e.g. Stutzer and Frey (2006) and Zimmermann and Easterlin (2006).
studies to date have therefore either focused on one attribute at a time, hence ignoring the effect of other attributes on sorting (see e.g. Charles et al., 2013), or assumed that all observed attributes matter but only through a single index of mutual attractiveness (see e.g. Wong, 2003, Anderberg, 2004 and Chiappori, Oreffice and Quintana-Domeque, 2012). More recently however, a new vein of the literature initiated by Choo and Siow (2006), and pursued by Fox (2010, 2011), Chiappori et al. (2013), Galichon and Salanié (2010, 2013) among others, has built on discrete choice theory and is therefore restricted to the case of discrete characteristics. It seems fair to assess that a standard procedure for the estimation of continuous multivariate matching models is still needed, in spite of recent attention on the matter. Another limitation of the current empirical literature is related to the set of observable attributes available in the data. Most studies solely have access to data on education and earnings, and only a few observe other dimensions such as anthropometric measures captured by height and BMI or self-assessed measures of health (Chiappori, Oreffice and Quintana-Domeque, 2012 and Oreffice and Quintana-Domeque, 2010 are notable exceptions).

In the present paper, we contribute to the literature on three accounts.

First, on the modeling front, we extend: (i) the Choo and Siow matching model to account for possibly continuous multivariate attributes, and (ii) Galichon and Salanié’s (2010, 2013) surplus estimator of the Choo and Siow model to the continuous case. Extending Choo and Siow’s model to continuous regressors is an important problem, which has been left open so far. Indeed, many attributes that appear in empirical studies on the marriage market are intrinsically continuous: income, wealth, height, body mass index and, as our paper illustrates, psychometric attributes such as personality traits. Even if measuring necessarily involves discretization, it remains desirable to have models which treat attributes

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3Recently, two papers have studied markets where sorting occurs on more than one dimension. Coles and Francesconi (2011) and Chiappori et al. (2012) study sorting on a single continuous index and a binary variable. Nesheim (2012) focuses on the identification of multivariate hedonic models without heterogeneity, and based on the observation of the price.

4None of these two papers allows for continuous observable characteristics.
as continuous. Using the Choo and Siow model directly on the discretized attributes to perform inference is problematic since changing the level of discretization of the data will imply modifying the assumptions of the model. To solve this problem, we make use of a continuous version of the logit choice framework, pioneered by Cosslett (1988) and Dagsvik (1994), which relies on extreme value stochastic processes. This ensures that our assumptions do not depend on the level of discretization of the data.

Second, on the data analysis front, we introduce a new technique, which we call “Saliency Analysis,” to determine the most relevant dimensions on which sorting occurs in a matching market. The starting point of this analysis requires inferring the strength of complementarities between men and women’s attributes. Using our structural model, we evaluate the intensity of assortativeness (positive or negative) between any pair of attributes, and we call the resulting matrix “affinity matrix.” Saliency Analysis consists in analysing the affinity matrix by means of a Singular Value Decomposition. This allows one to derive “indices of mutual attractiveness,” such that the joint utility of matching is a sum of mutually exclusive pairwise interaction terms. The first $k$ indices (for males and females) provide a convenient approximation of the joint utility by a model where attributes are vectors of only $k$ dimensions. As a consequence, one can perform inference on the number of dimensions that are required to explain the equilibrium sorting by testing how many singular values differ from zero.

Third, on the empirical front, we make use of a dataset that allows us to observe a wide range of attributes of both spouses. The set of attributes we observe in the data includes socio-economic variables such as education, anthropometric measures such as height and BMI, a measure of self-assessed health, as well as psychometric attributes such as risk aversion and the “Big Five” personality traits well-known in Psychology: conscientiousness, extraversion, agreeableness, emotional stability and autonomy. This paper is, to the extent of our knowledge, the first attempt to evaluate the importance of personality traits in the sorting of men and women in the marriage market. We will show that although education explains 28% of a couple’s observable joint utility, personality traits explain another 17% and different personality traits matter differently for men and for women. Our results relate to the literature showing the importance of personality traits in making economic
decisions (Borghans et al., 2008 for instance). Bowles et al. (2001) and Mueller and Plug (2006) among others have shown the importance of personality traits for earnings inequality. Closer to our focus, Lundberg (2012) studies the impact of personality traits on the odds in and out of a relationship (marriage and divorce) and finds empirical evidence that personality traits significantly affect the extensive margin in the marriage market. In particular, conscientiousness increases the probability of marriage at the age of 35 for men and extraversion increases the odds of marriage at the age of 35 for women. In the present work, we study the intensive margin, that is to whom conscientious men and extraverted women are the most attractive. We show among other things that conscientious men have preferences for conscientious women whereas extraverted women have preferences for autonomous and less agreeable men.

The outline of the rest of the paper is as follows. Section 2 presents an important extension of the model of Choo and Siow to continuously distributed observables. Section 3 deals with parametric estimation of the joint utility function in this setting. Section 4 presents a methodology for deriving indices of mutual attractiveness that determine the principal dimensions on which sorting occurs. The problem of inferring the number of dimensions on which sorting occurs is dealt with in section 5. Section 6 presents the data used for our empirical estimation and Section 7 discusses the results. Section 8 concludes.

2. The Continuous Choo and Siow model

2.1. The Becker-Shapley-Shubik model of marriage. The setting is a one-to-one, bipartite matching model with transferable utility. Men and women are characterized by vectors of attributes, respectively denoted $x \in X = \mathbb{R}^{d_x}$ for men and $y \in Y = \mathbb{R}^{d_y}$ for women. Matched men and women are by definition in equal number; we let $P$ and $Q$ be the respective probability distributions of their attributes. Throughout the paper, $P$ and $Q$ are treated as exogenous, except in Appendix D where we show that incorporating singles leaves the analysis unchanged while allowing us to identify reservation utilities. $P$ and $Q$
are assumed to have densities with respect to the Lebesgue measure denoted respectively $f$ and $g$. Without loss of generality, it is assumed throughout that $P$ and $Q$ are centered distributions, that is $\mathbb{E}_P [X] = \mathbb{E}_Q [Y] = 0$.

A matching is the probability density $\pi(x, y)$ of occurrence of a couple with characteristics $(x, y)$ from the matched population. Quite obviously, this imposes that the marginals of $\pi$ should be $P$ and $Q$. Write $\pi \in \mathcal{M}(P, Q)$, where

$$
\mathcal{M}(P, Q) = \left\{ \pi : \pi(x, y) \geq 0, \int_Y \pi(x, y) \, dy = f(x) \text{ and } \int_X \pi(x, y) \, dx = g(y) \right\}.
$$

Let $\Phi(x, y)$ be the joint utility generated when a man $x$ and a woman $y$ match, which is shared endogenously between them. Let $\Phi(x, \emptyset)$ and $\Phi(\emptyset, y)$ be the utility of man $x$ and woman $y$ respectively if they remain single; in Appendix D we shall show that $\Phi(x, \emptyset)$ and $\Phi(\emptyset, y)$ are identified if and only if the populations of singles are observed, but that the identification of $\Phi(x, y)$ is not impeded if singles are not observed. In the rest of the paper we shall assume that only the matched population is observed, so we will not focus on $\Phi(x, \emptyset)$ and $\Phi(\emptyset, y)$; as a result, the matching surplus $\Phi(x, y) - \Phi(x, \emptyset) - \Phi(\emptyset, y)$ will not be identified. Shapley and Shubik (1972) have shown that the equilibrium matching $\pi$ maximizes the total utility

$$
\max_{\pi \in \mathcal{M}(P, Q)} \mathbb{E}_\pi [\Phi(X, Y)]. \tag{2.1}
$$

Optimality condition (2.1) leads to very strong restrictions on $(X, Y)$, which are rarely met in practice. We need to incorporate some amount of unobserved heterogeneity in the model.

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5While we present the case with continuous distributions for $x$ and $y$, our framework easily extends to the case where some dimensions of $x$ and $y$ are discrete.

6In Appendix D this will be shown to be a consequence of the Independence of Irrelevant Alternatives (IIA) property of the logit model.

7A basic result in the theory of optimal transportation (Brenier’s theorem) implies that when $\Phi(x, y) = x' Ay$, the optimal matching is characterized by $(AY)_i = \partial V(X)/\partial x_i$, where $V$ is some convex function. Hence as soon as $A$ is invertible, the matching is pure, in the sense that no two men of the same type may marry women of different types. This is obviously never observed in the data.
2.2. **Adding heterogeneities.** Bringing the model to the data requires the additional step of acknowledging that sorting might also occur on attributes that are unobserved to the econometrician. In the case where men and women’s attributes are discrete, Choo and Siow (2006) introduced unobservable heterogeneities into the matching problem by considering that if a man \( m \) of attributes \( x_m = x \) and a woman \( w \) of attributes \( y_w = y \) match, they create a joint utility \( \Phi (x, y) + \varepsilon_m (y) + \eta_w (x) \), where \( \varepsilon_m (y) \) and \( \eta_w (x) \) are unobserved random “sympathy shocks” drawn by individuals. Assuming that \( (\varepsilon_m (y))_y \) and \( (\eta_w (x))_x \) have i.i.d. centered Gumbel (extreme value type I) distributions with scaling parameter \( \sigma / 2 \), Choo and Siow have shown that the joint utility \( \Phi (x, y) \) can be split into \( \Phi (x, y) = U (x, y) + V (x, y) \) such that the utility of man \( m \) matching with a woman of type \( y \) is given by

\[
U (x, y) + \varepsilon_m (y)
\]

with a similar expression for the utility of woman \( w \). An important implication of this setting is that at equilibrium, agents are indifferent between partners with same observable attributes: the matching utility of man \( m \) at equilibrium depends only on the observable attributes of that woman. As a consequence, each agent in the market solves a discrete choice problem.

In the Choo and Siow model, partners are assumed to have i.i.d. Gumbel sympathy shocks for the discrete attributes of the opposite side of the market. However, in many applied settings, these attributes are continuous random vectors, and even though the data that the analyst handles are obviously discretized, there is a strong need for a continuous framework. To illustrate, we shall take a setting where only the height of the partners is relevant, and assume that the precision of the measure is poor, say it is rounded at the nearest foot. A direct implication of the Choo and Siow assumptions is that individuals’ sympathy shocks are perfectly correlated within a foot bracket, and perfectly independent across feet. Suppose instead that height is measured at the nearest inch, Choo and Siow’s assumptions would now imply that individuals’ sympathy shocks are perfectly correlated within an inch bracket, and perfectly independent across inches, which of course comes at odds with the previous assumptions. So, while it is of course always possible to apply the
Choo and Siow setting to the discretized data, this implicitly leads to ad hoc assumptions which depend on the level of discretization of the available data.

In the present paper, we shall present an application where $x$ and $y$ measure height, BMI and various personality traits, which have a continuous multivariate distribution. Hence we need to model the random processes for $\varepsilon_m(x)$ and $\eta_w(y)$ accordingly. A legitimate candidate in the wake of Choo and Siow’s approach is the continuous logit model. Although very natural and particularly tractable, this setting has been surprisingly little used in economic modeling, with some notable exceptions. McFadden (1976) initiated the literature of continuous logit models by extending the definition of Independence of Irrelevant Alternatives (IIA) beyond finite sets. Ben-Akiva and Watanatada (1981) and Ben-Akiva, Litinas and Tsunekawa (1985) define continuous logit models by taking the limits of the discrete choice probabilities, with applications in particular to the context of spatial choice models. Cosslett (1988) and Dagsvik (1988) have independently suggested using max-stable processes to model continuous choice. We base our approach on their insights.

Assume that each man $m$ of type $x_m = x$ only knows a random subset of the total population of women we will call “acquaintances”, and that man $m$ only considers potential partners from his set of acquaintances. These acquaintances are indexed by $k \in \mathbb{N}$; and their observable attributes are represented by $y^m_k$. Each of these acquaintances is associated with a random “sympathy shock” $\varepsilon^m_k$ which enters additively into the man’s utility, so that the utility of a man $m$ who marries a woman of attributes $y^m_k$ can be written as

$$U(x, y^m_k) + \frac{\sigma^2}{2} \varepsilon^m_k,$$

(2.2)

where $U(x, y)$ is the “systematic” (in Choo and Siow’s term) part of the utility obtained by man $x$ matching with a woman with attributes $y$, whose existence and characterization will be provided in Theorem 1 below. Note that in contrast with the original setting of Choo and Siow described above, men do not have access to the whole population of women.

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8 As explained in Appendix A it will result from the distributional assumptions that each man draws an infinite but countable number of acquaintances almost surely, so that these can be indexed by the set of integers.
but only to their randomly selected set of acquaintances, which is a subset of the whole population.

We have yet to specify the distribution of \( y^m_k \) and \( \varepsilon^m_k \). Following Coslett and Dagsvik’s idea, we assume that \( \{(y^m_k, \varepsilon^m_k), k \in \mathbb{N}\} \) are the points of a Poisson process on \( \mathcal{Y} \times \mathbb{R} \) of intensity \( dy \times e^{-\varepsilon}d\varepsilon \). This means that: (i) the probability that man \( m \) has an acquaintance whose observable attributes are in a small set of infinitesimal size \( dy \) around \( y \) and with sympathy shock in a set of infinitesimal size \( d\varepsilon \) around \( \varepsilon \) is equal to \( e^{-\varepsilon}d\varepsilon dy \), and (ii) letting \( S \) and \( S' \) be two disjoint subsets of \( \mathcal{Y} \times \mathbb{R} \), the events “\( m \) has an acquaintance in \( S \)” and “\( m \) has an acquaintance in \( S' \)” are independent. According to the standard theory of Poisson point processes, this implies that, for \( S \) a subset of \( \mathcal{Y} \times \mathbb{R} \), the probability that man \( m \) has no acquaintance in set \( S \) is \( \exp \left( -\int_S e^{-\varepsilon}dyd\varepsilon \right) \). In Appendix A we show that this yields a continuous version of the multinomial logit choice model. As a result, the probability distribution of man \( m \) choosing a woman with attributes \( y \) is given by its density of probability

\[
\pi_{Y|X}(y|x) = \frac{\exp \frac{U(x,y)}{\sigma^2}}{\int_{\mathcal{Y}} \exp \left( \frac{U(x,y')}{\sigma^2} \right) dy'}
\]

which is clearly the extension of the logit formalism to the continuous choice setting. Similarly, the utility of a woman \( w \) with attributes \( y_w = y \) who marries a man with attributes \( x \) is

\[
V(x_w, y) + \frac{\sigma}{2} \eta_w,
\]

where \( V(x, y) \) is the systematic part of the utility, and \( \{ (x^w_l, \eta^w_l), l \in \mathbb{N} \} \) are the points of a Poisson process on \( \mathcal{X} \times \mathbb{R} \) of intensity \( dx \times e^{-\eta}d\eta \), so that the probability distribution of woman \( w \) choosing a man with attributes \( x \) is given by its density of probability

\[
\pi_{X|Y}(x|y) = \frac{\exp \frac{V(x,y)}{\sigma^2}}{\int_{\mathcal{X}} \exp \left( \frac{V(x',y)}{\sigma^2} \right) dx'}.
\]

The continuous logit framework inherits the structural assumptions of the discrete multinomial logit model. In particular, the independence property, which implies that the sympathy shock for women whose attributes are in a small set around \( y \) is perfectly uncorrelated with the sympathy shock for women whose attributes are in a small set around \( y' \neq y \). Hence the
logit framework (continuous or discrete) does not allow for a systematic sympathy shock, i.e. correlated sympathy shocks across observables. In the example where the attribute of interest is height, it may be desirable to accommodate a random sympathy shock for height (some men prefer taller women, some prefer shorter women, regardless of their own observable attributes). We conjecture, however, that if the amount of variation of the unobserved heterogeneity is small, the misspecification of the sympathy shocks has only a minor impact on the market outcome and the identification of the joint utility.

Taking the logarithm of Equations (2.3) and (2.5) respectively yields

\[ U_a = \left( \frac{\sigma}{2} \right) \log \pi \]

and

\[ V_b = \left( \frac{\sigma}{2} \right) \log \pi, \]

where

\[ a(x) = \frac{\sigma}{2} \log \int_y e^{-\frac{U(x,y')}{\sigma/2}} dy' \] and \[ b(y) = \frac{\sigma}{2} \log \int_x e^{-\frac{V(x',y)}{\sigma/2}} dx'. \] (2.6)

and since \( \Phi = U + V \), one obtains by summation

\[ \log \pi (x, y) = \frac{\Phi (x, y) - a(x) - b(y)}{\sigma}. \] (2.7)

We formalize this result in Theorem 1, which extends Galichon and Salanié (2010) to the continuous case.

**Theorem 1.** Under the assumptions stated above, the following holds:

(i) The equilibrium matching \( \pi \) maximizes the social gain

\[ \max_{\pi \in \mathcal{M}(P,Q)} \int_{\mathcal{X} \times \mathcal{Y}} \Phi(x,y) \pi(x,y) dxdy - \sigma \int_{\mathcal{X} \times \mathcal{Y}} \log \pi(x,y) \pi(x,y) dxdy. \] (2.8)

(ii) In equilibrium, for any \( x \in \mathcal{X}, y \in \mathcal{Y} \)

\[ \pi (x, y) = \exp \left( \frac{\Phi (x, y) - a(x) - b(y)}{\sigma} \right) \] (2.9)

where the potentials \( a(x) \) and \( b(y) \) are determined such that \( \pi \in \mathcal{M}(P,Q) \). They exist and are uniquely determined up to a constant.

(iii) A man \( m \) of attributes \( x \) who marries a woman \( k^* \) from his set of acquaintances obtains utility

\[ U(x,y_k^*) + \frac{\sigma}{2} \varepsilon_k^{m} = \max_k \left( U(x,y_k^*) + \frac{\sigma}{2} \varepsilon_k^{m} \right) \] (2.10)
where
\[ U(x, y) = \frac{\Phi(x, y) + a(x) - b(y)}{2}. \]  

(2.11)

Similarly, a woman \( w \) of attributes \( y \) who marries man \( l^* \) from her set of acquaintances obtains utility
\[ V(x^w_l, y) + \frac{\sigma^2}{2} \eta^w_l = \max_l \left( V(x^w_l, y) + \frac{\sigma^2}{2} \eta^w_l \right) \]

(2.12)

where
\[ V(x, y) = \frac{\Phi(x, y) - a(x) + b(y)}{2}. \]

(2.13)

As in Galichon and Salanié (2010; 2013), and independently, Decker et al. (2013), part (i) of this result expresses the fact that the equilibrium matching reflects a trade-off between sorting on the observed attributes (which tends to maximize the term \( \int \Phi(x, y) \pi(x, y) \, dxdy \)), and sorting on the unobserved attributes (which in turn tends to maximize the entropic term \( \int \log \pi(x, y) \pi(x, y) \, dxdy \)). The second term will therefore pull the solution towards the random matching, where partners are randomly assigned; the parameter \( \sigma \), which captures the intensity of the unobserved heterogeneity, measures the intensity of this trade-off. The smaller the \( \sigma \) (i.e. the less unobserved heterogeneity in the model), the closer the solution will be to the solution without heterogeneity. On the contrary, the higher the \( \sigma \), the larger the probabilistic independence between the observed attributes of men and women. As an illustration, we consider the simple toy example below, in which this phenomenon is explicit.

**Example 1.** When \( P \) and \( Q \) are the standard univariate Gaussian distribution \( \mathcal{N}(0,1) \), and \( \Phi(x, y) = -\frac{1}{2} (x - y)^2 \), the equilibrium matching \( \pi \) is such that \( \pi_{Y|X}(y|x) = \mathcal{N}(tx, 1 - t^2) \), where \( t = \sqrt{\frac{\sigma^2}{4} + 1 - \frac{\sigma^2}{2}} \). Hence, \( \sigma = 0 \) implies \( t = 1 \), in which case \( Y = X \) (sorting predominates and we have positive assortative matching), while \( \sigma \to \infty \) implies \( t \to 0 \), in the limit of which \( Y \) becomes independent from \( X \) (unobserved heterogeneity predominates, there is no sorting on observables). Closed-form formulae can also be provided in the multivariate case when \( P \) and \( Q \) are Gaussian and \( \Phi \) is quadratic, see Bojilov and Galichon (2013).
Part (ii) of Theorem 1 is an expression of the first order optimality conditions. The program is an infinite dimensional linear programming problem where \( a(x) \) and \( b(y) \) are the Lagrange multipliers corresponding to the constraints \( \int \pi(x,y) \, dy = f(x) \) and \( \int \pi(x,y) \, dx = g(y) \) respectively. Equation (2.9), or more precisely its logarithmic transform Equation (2.7), will be the basis of our estimation strategy. Together with the constraint \( \pi \in \mathcal{M}(P,Q) \), this equation provides a nonlinear system of equations in \( a(.) \) and \( b(.) \). In the applied mathematical literature it is known as the Schrödinger-Bernstein equation, or more commonly as the Schrödinger problem. Existence and uniqueness (up to a constant) are well studied under very general conditions on \( P \) and \( Q \), see for instance Rüschendorf and Thomsen (1993) and references therein. An efficient algorithm for the numerical determination of the solution based on a fixed point idea is studied in Rüschendorf (1995). For completeness it is further explained in Appendix C.

Part (iii) of Theorem 1 explains how the joint utility is shared at equilibrium. Unsurprisingly, just as in Choo and Siow (2006) and the ensuing literature, individuals do not transfer their sympathy shock at equilibrium, which is expressed by (2.10) and (2.12). Expressions (2.11) and (2.13) provide the formulae for the systematic part of the utility. As previously noted, \( a(x) \) and \( b(y) \) are the Lagrange multipliers of the scarcity constraints of men’s observable attributes \( x \) and women’s attributes \( y \). Hence a higher \( a(x) \) shall imply a higher relative scarcity for \( x \), and therefore a greater prospect for utility extraction.

Identification. From an identification perspective, note that equations (2.3) and (2.5) imply that the observation of \( \pi(x,y) \) leads to identification of \( U(x,y) \) up to an additive term \( c(x) \), and similarly, \( V(x,y) \) up to an additive term \( d(y) \) by

\[
U(x,y) = \frac{\sigma}{2} \left( \log \pi_{Y|X} (y|x) + c(x) \right) \quad \text{and} \quad V(x,y) = \frac{\sigma}{2} \left( \log \pi_{X|Y} (x|y) + d(y) \right)
\]

and thus

\[
\Phi(x,y) = \frac{\sigma}{2} \left( \log \pi_{Y|X} (y|x) + \log \pi_{X|Y} (x|y) + c(x) + d(y) \right)
\]

As a result, \( \Phi(x,y) \) is identified up to a separatively additive function since we restrict our attention to the matched population. Since \( \Phi(x,y) \) yields the same equilibrium

\[9\text{Appendix D explains how these results are extended when singles are observed.}\]
matching as $\Phi(x,y) + c(x) + d(y)$, the identified quantity is actually the cross-derivative $\partial^2 \Phi(x,y)/\partial x \partial y$, while neither $\partial \Phi(x,y)/\partial x$ nor $\partial \Phi(x,y)/\partial y$ can be identified, nor can their signs be identified, either. To illustrate, assume that there is only one dimension—education. It may be that men and women who are more educated generate more utility, which we call “absolute attractiveness,” and which translates into $\partial \Phi(x,y)/\partial x \geq 0$ and $\partial \Phi(x,y)/\partial y \geq 0$. However, this is not identifiable in our model, because models where the joint utility is $\Phi(x,y)$ are observationally undistinguishable from models where the joint utility is $\Phi(x,y) + c(x) + d(y)$, and the terms $c$ and $d$ might be strongly negatively correlated with education. Instead, the present framework allows us to determine whether education is mutually attractive in the sense that $\partial^2 \Phi(x,y)/\partial x \partial y \geq 0$, meaning not only that highly educated men and women attract each other, but also that lower educated men and women attract each other. Hence, our model allows us to measure the strength of mutual attractiveness (or assortativeness) on various dimensions, but not absolute attractiveness.

3. Parametric estimation

3.1. Specification of the matching utility. In this section, we shall specify a parametric form for the joint utility function, the estimation of which shall be discussed in the next section. While Choo and Siow’s estimator is fully nonparametric, the fact that the variables under study are continuous reinforces the need for a parametric estimator. For the purpose of this discussion, we shall look back at the illustrative example from the introduction where only both partners’ heights are observed. The Choo and Siow analysis provides a nonparametric estimator for the joint utility $\Phi(x,y)$ of a match between a man of height $x$ and a woman of height $y$. If heights were to be rounded at the nearest inch and individuals’ heights in inches ranged, say, from 50 to 90, then the dimension of vector $\Phi(x,y)$ would be $40 \times 40 = 1600$. Note that this would worsen significantly if several characteristics were observed. But even in the single-dimensional case, there would be a serious missing data problem, since the odds that one would observe data for every pair of heights are virtually zero. Moreover, even if one were lucky enough to obtain the full nonparametric estimator of $\Phi(x,y)$, one would have to heavily process this information before being able to draw any
stylized conclusion. This simple example highlights the need for a parametric estimation when considering continuous variables.

Throughout the rest of the paper, we shall assume a quadratic parametrization of $\Phi$: for $A$ a $d_x \times d_y$ matrix, we take

$$\Phi_A (x, y) = x' Ay,$$

where we call matrix $A$ the affinity matrix. One has

$$A_{ij} = \frac{\partial^2 \Phi (x, y)}{\partial x_i \partial y_j}.$$ 

The parameter $A_{ij}$ accounts for the strength of mutual attractiveness (which can be positive or negative) between dimensions $x_i$ and $y_j$. It measures how the (marginal) gain in joint utility from increasing the man’s $i^{th}$ attribute evolves as the woman’s $j^{th}$ attribute increases. It captures the intensity of the complementarity/substitutability between attribute $x_i$ of man $x$ and attribute $y_j$ of woman $y$ in the joint utility.

Two comments about this parametric choice are noteworthy. First, this parametric choice is arguably the simplest one which captures nontrivial complementarities between any pair of attributes. If $A_{ij} > 0$, $x_i$ and $y_j$ are complements, and (all things else being equal) high $x_i$ tend to match with high $y_j$. It reflects positive assortative matching across men’s $i$-th attribute and women’s $j$-th attribute. For instance, the level of education of one of the partners may be complementary with the risk aversion of the other partner. On the contrary, if $A_{ij} < 0$, then $x_i$ and $y_j$ are substitutes, there is negative assortative matching between $x_i$ and $y_j$. Note that attributes $x$ and $y$ should not be interpreted as an absolute quality (where a greater value of $x_i$, the $i$-th dimension of $x$, would be more socially desirable than a smaller value of $x_i$). In fact, the model is observationally undistinguishable from a model where $x$ is changed into $-x$ and $y$ is changed into $-y$.

Second, this quadratic setting where $\Phi$ is bilinear in $x$ and $y$ is less restrictive than it seems and can be extended to the case when the various observed attributes have nonlinear contributions to the joint utility\(^{10}\). For instance it may be plausible that extraverted men are indifferent about the education and the height of their wives, but that if a woman is

\(^{10}\)We thank a Referee for pointing this out.
tall, then men prefer her with a higher education. Our setting can easily be extended to incorporate such nonlinear effects. We assume no restrictions on the attributes that enter $x$ and $y$, so that the observables can be enriched by the addition of nonlinear functions of them, i.e. adding $x_i^2$, $x_i^3$ etc. and $x_i x_j$ as observable attributes for men and similarly for women. This will allow $\Phi(x, y)$ to be any polynomial function of $x$ and $y$. Thus, our setting can easily incorporate any utility function which is a polynomial expression of the observable attributes.

3.2. Inference. We turn to the estimation of the affinity matrix $A$. The technique we apply here was introduced by Galichon and Salanié (2010); we discuss this extension to the continuous case. By taking the cross-derivative of Equation (2.7), one has

$$ \frac{A_{ij}}{\sigma} = \frac{\partial^2 \log \pi(x, y)}{\partial x_i \partial y_j}. $$

(3.1)

A seemingly natural procedure would consist in estimating $\pi$ nonparametrically, and obtaining $A$ from the cross derivatives with respect to $x_i$ and $y_j$. While feasible, this procedure faces a number of issues both in theory and in practice. First, it requires a nonparametric estimation of the second derivatives of the loglikelihood, which is quite challenging: the “curse of dimensionality” would fully apply. Second, since equation (3.1) is valid at any point $(x, y)$, this equation is an over-identifying restriction to the estimation of $A$. The right hand side of (3.1) depends on $(x, y)$, while the left hand side does not. One may certainly take some averaging of the right hand side of Equation (3.1), but it is not quite obvious how to weigh each point optimally, and it would only partially offset the problems stemming from the curse of dimensionality. As a result, this procedure will be statistically inefficient.

Instead, we prefer to resort to a moment matching procedure, which is relatively simple while achieving asymptotic statistical efficiency as shown in Theorem 2 below. Let us provide intuition for this method. Each value of the matrix $A$ yields an equilibrium matching distribution, which we denote $\pi^A(x, y)$. As argued in Appendix C, $\pi^A$ can be computed

\[11\] In our application, both $x$ and $y$ have 10 dimensions, so $(x, y)$ is of dimension 20.
efficiently using a fixed point method. Recall that we have assumed that the distributions of $X$ and $Y$ have zero mean, and introduce the cross-covariance matrix

$$\Sigma_{XY} = (\mathbb{E}[X_i Y_j]_{ij} = \mathbb{E}[XY'])$$

(3.2)

which is observed in the data. The idea is to look for the value of $A$ such that for all $i$ and $j$, the covariances predicted by the model match the covariances observed in the data, that is

$$\mathbb{E}_{\pi^A} [X_i Y_j] = \mathbb{E} [X_i Y_j].$$

(3.3)

This yields a map $A \rightarrow (\mathbb{E}_{\pi^A} [X_i Y_j])_{ij}$ that is invertible. The inversion of this map (in order to estimate $A$) can be formulated as a convex optimization problem, thus making it easy to solve numerically. To see this, we shall recall that the equilibrium matching $\pi$ maximizes the social gain

$$W_\sigma (A) := \max_{\pi \in \mathcal{M}(P, Q)} \mathbb{E}_{\pi} [X'AY] - \sigma \mathbb{E}_{\pi} [\ln \pi (X, Y)],$$

(3.4)

and we see that models with parameters $(A, \sigma)$ and models with parameters $(A/\sigma, 1)$ are observationally equivalent, which translates mathematically into positive homogeneity $W_\sigma (A) = \sigma W_1 (A/\sigma)$. By the envelope theorem, the predicted covariance between $X_i$ and $Y_j$ coincides with the partial derivative of $W_\sigma$ with respect to $A_{ij}$, that is

$$\mathbb{E}_{\pi^A} [X_i Y_j] = \frac{\partial W_\sigma}{\partial A_{ij}} (A) = \frac{\partial W_1}{\partial A_{ij}} (A/\sigma),$$

(3.5)

which implies that, upon normalization $\sigma = 1$, the map $A \rightarrow (\mathbb{E}_{\pi^A} [X_i Y_j])_{ij}$ is invertible since $W_1$ is strictly convex (see Lemma 3). This led Galichon and Salanié (2013) to conclude, in a setting with discrete observable attributes, that $B = A/\sigma$ is identified as a solution to the following convex optimization program

$$\min_{B \in \mathcal{M}_{d_x d_y} (\mathbb{R})} \left\{ W_1 (B) - \sum_{ij} B_{ij} \Sigma_{XY}^{ij} \right\}$$

(3.6)

whose first-order conditions are precisely $\partial W_1 (B)/\partial B_{ij} = \Sigma_{XY}^{ij}$, that is $\mathbb{E}_{\pi^B} [X_i Y_j] = \Sigma_{XY}^{ij}$. In the present setting with continuous observable attributes, things work in an identical manner. Since the model is scale-invariant, only $A/\sigma$ is identified and we normalize $A$ so that $\|A\| = 1$, where $\|A\| = (\sum_{ij} A_{ij}^2)^{1/2}$. $A$ and $\sigma$ are then obtained by $A = B/\|B\|$ and
\[ \sigma = 1/\|B\|. \] Let us denote \( A^{XY} \) the (unique) solution to this problem, which will be our estimator of the affinity matrix \( A \). Affinity matrix \( A^{XY} \) is “dual” to cross-covariance matrix \( \Sigma_{XY} \) in the sense that there is a one-to-one correspondence between them by Equation (3.5). However, the former has a structural interpretation: it measures the strength of the interactions between pairs of attributes.

At this point, it is worth commenting on the relevance of the structural approach. Indeed, it does not suffice to just look at the variance-covariance matrix inside matches to infer the sign of complementarities, as illustrated on the following example. Imagine two observed characteristics, where the first dimension is education and the second dimension is risk aversion. Suppose we observe positive correlation in partners’ educations and in partners’ risk aversions (i.e., \( \Sigma_{11} > 0 \) and \( \Sigma_{22} > 0 \)). One might naively infer that there is positive complementarity both in education and in risk aversion (i.e., \( A_{11} > 0 \) and \( A_{22} > 0 \)). However, this is not necessarily the case; there could actually be negative complementarity in risk aversion \( (A_{22} < 0) \), but positive association between individuals’ education and risk aversion, if positive complementarity in education \( (A_{11} > 0) \) dominates the negative complementarity in risk aversion, thus leading to positive correlation in risk aversions inside matches. The structural approach allows to avoid this misinterpretation by allowing to control for the marginal distributions (e.g. control for the fact that there is positive association between individuals’ education and risk aversion).

Once the affinity matrix \( A^{XY} \) has been estimated, two questions arise. First, what is the rank of \( A^{XY} \)? This question is of importance since one would like to know the number of dimensions of \( x \) and \( y \) on which sorting occurs. Second, how can we construct “indices of mutual attractiveness” such that each pair of indices for men and women explains a mutually exclusive part of the matching utility? Many studies resort to a technique called “Canonical Correlation,” which essentially relies on a singular value decomposition of \( \Sigma^{XY} \). In Dupuy and Galichon (2012), we argue that this technique is not well suited for studying assortative matching, and that the resulting procedure is inconsistent. Instead, in Section 4 we propose a method we call “Saliency Analysis” in order to accurately answer these
two questions. This method is essentially based on the singular value decomposition of the affinity matrix $A^{XY}$ (instead of $\Sigma^{XY}$ as in Canonical Correlation). Testing the rank of the affinity matrix is equivalent to testing the number of (potentially multiple) singular values different from 0. Performing this decomposition allows one to construct the indices of mutual attractiveness that each explain a separate share of the joint utility.

4. Saliency Analysis

In this section we set out to determine the rank of the affinity matrix $A^{XY}$, and the principal dimensions in which it operates. For this, we introduce and describe a novel technique we call Saliency Analysis, which is similar in spirit to Canonical Correlation but does not suffer the pitfalls of the latter. Instead of performing a singular value decomposition of the (renormalized) cross-covariance matrix $\Sigma_{XY}$, we shall perform a singular value decomposition of the affinity matrix $A^{XY}$, properly renormalized. This idea is similar in spirit to the proposal of Heckman (2007), who interprets the assignment matrix as a sum of Cobb-Douglas technologies using a singular value decomposition in order to refine bounds on wages.

Recall that we have defined the cross-covariance matrix $\Sigma_{XY} = \mathbb{E}_\pi [XY']$, and let us introduce $S_X$ and $S_Y$ the diagonal matrices whose diagonal terms are respectively the variances of the $X_i$ and the $Y_j$, that is

$$S_X = \text{diag}(\text{var}(X_i), \ i = 1, ..., d_x), \ S_Y = \text{diag}(\text{var}(Y_j), \ j = 1, ..., d_y).$$

We shall work with the rescaled attributes $S_X^{-1/2}X$ and $S_Y^{-1/2}Y$, whose entries each have unit variance. By Lemma 1 in Appendix B.3, the affinity matrix between the rescaled attributes $S_X^{-1/2}X$ and $S_Y^{-1/2}Y$ is

$$\Theta = S_X^{1/2} A^{XY} S_Y^{1/2},$$

for which a singular value decomposition of $\Theta$ yields

$$\Theta = U' \Lambda V,$$
where $\Lambda$ is a diagonal matrix with nonincreasing elements $(\lambda_1, \ldots, \lambda_d)$, $d = \min(d_x, d_y)$ and $U$ and $V$ are orthogonal matrices. Define the vectors of *indices of mutual attractiveness*

$$\tilde{X} = US_x^{-1/2}X \text{ and } \tilde{Y} = VS_y^{-1/2}Y,$$

where each index is a weighted sum of the observed attributes. Let $A_{\tilde{X}\tilde{Y}}$ be the affinity matrix on the rescaled vectors of characteristics $\tilde{X}$ and $\tilde{Y}$. From Lemma 1, it follows that $A_{\tilde{X}\tilde{Y}} = \Lambda$, and as a result

$$\Phi_A(x, y) = \sum_{i=1}^{d_x} \sum_{j=1}^{d_y} A_{ij} x_i y_j = \sum_{i=1}^{d} \lambda_i \tilde{x}_i \tilde{y}_i.$$

Hence, the new indices $\tilde{x}$ and $\tilde{y}$ are such that $\tilde{x}_i$ and $\tilde{y}_j$ are complements for $i = j$, and neither complements, nor substitutes if $i \neq j$. In other words, there is positive assortative matching between $\tilde{x}_i$ and $\tilde{y}_j$ for $i = j$, and no assortativeness for $i \neq j$. This justifies the choice of terminology: $\tilde{x}_i$ and $\tilde{y}_i$ are “mutually attractive” because they are complementary with each other and only with each other. All things being equal, a man with a higher $\tilde{x}_i$ tends to match with a woman with a higher $\tilde{y}_i$.

The weights of each index of mutual attractiveness constructed by Saliency Analysis are given by the associated row of $US_x^{-1/2}$ for men and $VS_y^{-1/2}$ for women. The value $\lambda_i/(\sum_i \lambda_i)$ indicates the share of the observable matching utility of couples explained by the $i^{th}$ pair of indices. The fact that $U$ and $V$ are orthogonal implies strong restrictions on how $\tilde{x}$ and $\tilde{y}$ are obtained from $S_x^{-1/2}x$ and $S_y^{-1/2}y$. In particular, this mapping preserves distances between points; that is, the distance between $\tilde{x} \text{ and } \tilde{x}'$ is equal to the distance between $S_x^{-1/2}x \text{ and } S_x^{-1/2}x'$.

We observe that in contrast with Canonical Correlation analysis, a convenient feature of Saliency Analysis is that the results do not change when the attributes are measured using different measurement units, as expressed in Lemma 2. For instance, if the partner’s heights are measured in feet rather than in meters, the outcome of Saliency Analysis does not change.

For illustrative purposes, we give a stylized example of how Saliency Analysis operates in a simple two-dimensional situation.
Example 2. Assume that there are two dimensions on each side of the market, and that 
\( S_X = S_Y = Id \), so that \( \Theta = A \). Suppose that 
\[
A = \begin{pmatrix}
0 & 4 \\
-1 & 0
\end{pmatrix}.
\] (4.1)

Then the singular value decomposition of \( A \) is \( A = U' \Lambda V \), where 
\[
U = \begin{pmatrix}
1 & 0 \\
0 & 1
\end{pmatrix}, \quad \Lambda = \begin{pmatrix}
4 & 0 \\
0 & 1
\end{pmatrix} \quad \text{and} \quad V = \begin{pmatrix}
0 & 1 \\
-1 & 0
\end{pmatrix}
\]

The economic interpretation of this simple example is the following: if the joint utility is given by 
\( \Phi(x, y) = 4x_1y_2 - x_2y_1 \), then the indices of mutual attractiveness should be 
given by \( \tilde{x}_1 = x_1 \), \( \tilde{x}_2 = x_2 \) and \( \tilde{y}_1 = y_2 \), \( \tilde{y}_2 = -y_1 \). One has \( \Phi(\tilde{x}, \tilde{y}) = 4\tilde{x}_1\tilde{y}_1 + \tilde{x}_2\tilde{y}_2 \). The vectors \( \tilde{x} = (x_1, x_2) \) and \( \tilde{y} = (y_2, -y_1) \) can be interpreted as indices of mutual attractiveness, 
meaning that there is sorting between \( \tilde{x}_1 = x_1 \) and \( \tilde{y}_1 = y_2 \), and between \( \tilde{x}_2 = x_2 \) and 
\( \tilde{y}_2 = -y_1 \). If one was willing to approximate the model by a one-dimensional sorting model, 
then Saliency Analysis advocates to keep \( \tilde{x}_1 = x_1 \) as a proxy for the attributes of men and 
\( \tilde{y}_1 = y_2 \) as a proxy for the attributes of women. In this case the joint utility is approximated 
by \( \Phi(\tilde{x}, \tilde{y}) = 4\tilde{x}_1\tilde{y}_1 \).

Example 2 is the occasion to compare Singular Value Decomposition to another matrix 
decomposition, the Eigenvalue Decomposition, which economists may be more familiar with. 
Eigenvalue decomposition consists in writing, whenever possible, a square matrix \( M \) as 
\( M = R \Lambda R^{-1} \), with \( \Lambda \) diagonal and \( R \) invertible. In the context of Saliency Analysis, this 
decomposition cannot be performed on \( A \) as \( A \) is not necessarily a square matrix; further, 
as soon as \( A \) is not symmetric, this decomposition does not necessarily exist. In particular, 
when \( A \) is given by (4.1), it does not exist since \( A \) has no real eigenvalue.12

As Example 2 also illustrates, the observation of vector \( \Lambda \) will allow one to draw con-
clusions about the multivariate nature of the sorting, and on the number of dimensions

12However, the singular values of \( A \) can be interpreted as eigenvalues of a larger matrix. Indeed, letting 
\( H \) be the constant Hessian matrix of the map 2\( \Phi \), then \( H \) is a symmetric matrix of size \((d_x + d_y)\) written 
blockwise with two zero blocks on its diagonal and \( A \) and \( A' \) as off-diagonal blocks, and the eigenvalues of 
\( H \) are plus and minus the singular values of \( A \) (see Horn and Johnson, 1991, p. 135).
on which the sorting occurs. In particular, testing for multidimensional sorting versus unidimensional sorting is equivalent to testing whether at least two singular values \( \lambda_i \) are significantly larger than 0, as we shall elaborate in the next section.

5. INFERRING THE NUMBER OF SORTING DIMENSIONS

Assume that a finite sample of size \( n \) is observed. For the sake of readability, dependence in \( n \) of the estimators will be dropped from the notations. The vector of mutual attraction weights estimated on the sample is denoted \( \hat{\Lambda} \), while the vector of mutual attraction weights in the population is denoted \( \Lambda \). Similarly \( \hat{A} \) is the estimator of \( A \) (which was denoted \( A^{XY} \) in Section 3.2, where the construction of that estimator is described). Let \( \hat{S}_X, \hat{S}_Y \) and \( \hat{\Sigma}_{XY} \) be the sample estimators of \( S_X, S_Y \) and \( \Sigma_{XY} \), respectively. For a given quantity \( M \), we shall denote

\[
\delta M = \hat{M} - M, \tag{5.1}
\]

the difference between the estimator of \( M \) and \( M \).

Consider two important matrices associated to the large sample properties of the model. The Fisher Information matrix is defined by

\[
F_{ij}^{kl} = \mathbb{E}_\pi \left[ \frac{\partial \log \pi(X,Y)}{\partial A_{ij}} \frac{\partial \log \pi(X,Y)}{\partial A_{kl}} \right], \tag{5.2}
\]

where we note that the lines of \( F \) are indexed by pairs of integers \((ij)\), just as the columns of \( F \) are indexed by pairs \( kl \). (This is due to the fact that the parameter to be estimated, \( A_{ij} \), is not a vector but a matrix). Hence \( F \) is a “doubly-indexed matrix,” which we shall denote using bold font. Some basic formalism on doubly-indexed matrices is recalled in Appendix B, Section B.4.

Our next result expresses the asymptotic distribution of the estimators of \( A, S_X \) and \( S_Y \). It will be the main building block for testing the rank of the affinity matrix (and the number of sorting dimensions).

\footnote{In a discussion with one of the authors, Jim Heckman suggested the intuition of the approach proposed in this paper to test for multidimensional sorting.}
Theorem 2. The following convergence holds in distribution for \( n \to +\infty \):

\[
n^{1/2}(\delta A, \delta S_X, \delta S_Y) \implies \mathcal{N}\left(0, \begin{pmatrix} \mathbb{F}^{-1} & 0 & 0 \\ 0 & K_{XX} & K_{XY} \\ 0 & K'_{XY} & K_{YY} \end{pmatrix} \right)
\]

where \( \mathbb{F} \) has been defined in (5.2), \( K_{XY} \) is defined by

\[
(K_{XY})_{ij}^{kl} = 1_{\{i=j, k=l\}} \text{cov}_\pi (X_i X_j, Y_k Y_l)
\]

and we define similarly \( K_{XX} \) and \( K_{YY} \) by

\[
(K_{XX})_{ij}^{kl} = 1_{\{i=j, k=l\}} \text{cov}_\pi (X_i X_j, X_k X_l) \quad \text{and} \quad (K_{YY})_{ij}^{kl} = 1_{\{i=j, k=l\}} \text{cov}_\pi (Y_i Y_j, Y_k Y_l).
\]

Note that, as shown in Lemma 5 in Appendix B.3, \( \mathbb{F} \) can be evaluated numerically as the Hessian matrix of \( W_1 \). Theorem 2 implies in particular that the asymptotic variance-covariance matrix of our estimator of \( A \) is the inverse of the Fisher information matrix. As a result, our estimator attains asymptotic statistical efficiency.

Now denoting \( \hat{\Theta} = \hat{S}_X \hat{A} \hat{S}_Y \) the estimated counterpart of \( \Theta \) whose singular value decomposition is denoted \( \hat{\Theta} = \hat{U}' \hat{\Lambda} \hat{V} \), we show in Appendix B.4 that \( \hat{\Theta} \) is asymptotically normal, and we give an expression for its asymptotic variance-covariance matrix in Lemma 6. We use this asymptotic result to test the rank of the affinity matrix \( \Lambda \). Testing the rank of a matrix is an important issue with a distinguished tradition in econometrics (see e.g. Robin and Smith, 2000 and references therein). Here, we use results from Kleibergen and Paap (2006). One wishes to test the null hypothesis \( H_0 \): the rank of the affinity matrix is equal to \( p = 1, 2, \ldots, d - 1 \). Following Kleibergen and Paap, the singular value decomposition \( \hat{\Theta} = \hat{U}' \hat{\Lambda} \hat{V} \) is written blockwise

\[
\hat{\Theta} = \begin{pmatrix} \hat{U}'_{11} & \hat{U}'_{21} \\ \hat{U}'_{12} & \hat{U}'_{22} \end{pmatrix} \begin{pmatrix} \hat{\Lambda}_1 & 0 \\ 0 & \hat{\Lambda}_2 \end{pmatrix} \begin{pmatrix} \hat{V}_{11} & \hat{V}_{12} \\ \hat{V}_{21} & \hat{V}_{22} \end{pmatrix}
\]
where the blocks are dimensioned so that $\hat{\Lambda}_{11}$, $\hat{U}_{11}'$ and $\hat{V}_{11}$ are $p \times p$ square matrices. Define

$$\hat{T}_p = \left(\hat{U}_{22}'\hat{U}_{22}\right)^{-1/2} \hat{U}_{22}'\hat{\Lambda}_2\hat{V}_{22} \left(\hat{V}_{22}'\hat{V}_{22}\right)^{-1/2}$$

$$\hat{A}_{p\perp} = \left(\hat{U}_{21}'\hat{U}_{22}\right) \left(\hat{U}_{22}'\hat{U}_{22}\right)^{-1} \left(\hat{U}_{22}'\hat{U}_{22}\right)^{1/2}$$

$$\hat{B}_{p\perp} = \left(\hat{V}_{22}'\hat{V}_{22}\right)^{1/2} \hat{V}_{22}^{-1} \left(\hat{V}_{21} \hat{V}_{22}\right)$$

so that our next result provides a test for the number of sorting dimensions.

**Theorem 3.** Under the null hypothesis that the rank of the affinity matrix is $p$, the quantity $n^{1/2}\hat{T}_p$ is asymptotically normally distributed, and the expression of its variance-covariance matrix $\Omega_p$ is given in Appendix B.4, formula (B.18). As a result, the test-statistic

$$n\hat{T}_p'\hat{\Omega}_p^{-1}\hat{T}_p$$

converges under the null hypothesis to a $\chi^2((d_x - p)(d_y - p))$ random variable.

6. The data

6.1. The dataset. In this paper, we use the waves 1993-2002 of the DNB Household Survey (DHS) to estimate preferences in the marriage market. For a thorough description of the setup and the quality of this data we refer the reader to Nyhus (1996). This data is a representative panel of the Dutch population with respect to region, political preference, housing, income, degree of urbanization, and age of the head of the household among others. The DHS data was collected via on-line terminal sessions and each participating family was provided with a PC and a modem if necessary. The panel contains on average about 2,200 households in each wave.

This data includes three main features that are particularly attractive for our purposes. First, within each household, all persons aged 16 or over were interviewed. This implies that the data contains detailed information not only about the head of the household but about all individuals in the household. In particular, the data identifies “spouses” and “permanent partners” of the head of each household. This information reveals the nature
of the relationship between the various individuals of each household and allows us to reconstruct “couples”.

Second, this data contains very detailed information about individuals. This rich set of information includes socio-demographic variables such as birth year and education, as well as variables about the anthropometry of respondents (height and weight), a self-assessed measure of health, and, above all, information about personality traits and risk attitude, which are included in the waves 1993-2002.

Finally, as for most panel data, the DHS data suffers from attrition problems. The attrition of households is on average 25% each year, cf. Das and van Soest (1999) among others. To remedy attrition, refreshment samples were drawn each year, such that, over the period 1993-2002, about 7,700 distinct households were interviewed at least once. Since the methodology implemented in this paper relies essentially on the availability of a cross-section of households, attrition and its remedy is in fact an asset of this data as it allows us to have access to a rather large pool of potential couples.

Note that our methodology could be applied on other panel datasets (such as GSOEP, for instance) that also include supplementary questionnaires enabling one to construct measures of personality traits and risks aversion together with socio-demographic and morphological variables. However, the main asset of the DNB dataset is that it allows us to measure all relevant variables in a single wave whereas in GSOEP, one would have to use the panel structure to match measures of BMI (from wave 2008 and 2009) and measures of personality traits (from waves 2006 and 2007).

6.2. Variables. Educational attainment is measured from the respondent’s reported highest level of education achieved. The respondents could choose among 13 categories (7 in the later waves), ranging from primary to university education. The reduction to 7 categories in the later waves implies that only three broad educational categories can be consistently constructed. We coded responses as follows:

(1) Lower [kindergarten, primary, elementary secondary] education,
(2) Intermediate [secondary, pre-university, vocational] education,
(3) Higher [university] education.
The respondents were also asked about their height and weight. The answers of these questions allowed us to calculate the Body Mass Index of each respondent as the weight in kilograms divided by the square of the height measured in meters. The respondents were also asked to report their general health. The phrasing of the question was: “How do you rate your general health condition on a scale from 1, excellent, to 5, poor?” We make use of the panel structure to deal partly with nonresponses on socio-economic and health variables. When missing values for height, weight, education, year of birth etc. were encountered, values reported in adjacent years were imputed. We defined our measure of health by subtracting the answer to this question to 6.

In Appendix E.2, we recall the methodology of Nyhus and Webley (2001) which we followed in order to construct five factors of personality traits. These factors were labeled as:

- Emotional stability: a high score indicates that the person is less likely to interpret ordinary situations as threatening, and minor frustrations as hopelessly difficult,
- Extraversion (outgoing): a high score indicates that the person is more likely to need attention and social interaction,
- Conscientiousness (meticulous): a high score indicates that the person is more likely to be self-disciplined and plan his/her actions,
- Agreeableness (flexibility): a high score indicates that the person is more likely to be pleasant with others and go out of their way to help others,
- Autonomy (tough-mindedness): a high score indicates that the person is more likely to be direct, rough and dominant.

6.3. Couples. Our definition of a couple is a man and a woman living in the same household and reporting being either head of the household, spouse of the head or a permanent partner (not married) of the latter. To construct our dataset of couples, we first pool all the selected waves (1993-2002). We then keep only those respondents that report being head of the household, spouse of the head or permanent (not married) partner of the head.

Note that using the subsample of legally married couples does not affect the three main results of our analysis mentioned in the abstract. These results are available from the authors upon request.
This sample contains roughly 13,000 men and women and identifies about 7,700 unique households. We then split this sample and create two datasets, one containing women and one containing men. Each dataset identifies about 6,500 different men and women. We then merge the men dataset to the women dataset using the household identifier. We identify 5,500 unique couples while roughly 1,250 men and 1,250 women remain unmatched.

Given the aim of our main analysis, we further restrict our sample to relatively newly formed couples. In the absence of information about when couples actually formed, following the literature (see Chiappori et al. 2012, for instance), we select only couples whose wives are younger than 40 years old.

Table 1 reports the number of identified young couples and the number of young couples for which we have complete information on the various dimensions. For nearly all couples we have information on both spouses’ educational attainment. However, out of 2,897 couples only 1,595 provide complete information on education, height, health and BMI. We lose another 337 couples for which personality traits are not fully observed. Another 100 couples are lost when attitude towards risks is additionally taken into consideration. Our working dataset therefore contains 1158 young couples.

Table 2 presents summary statistics for men and women. On average, in our sample, men are 3 years older than women, slightly more educated, taller by 13 centimeters, have a BMI of 1Kg/m² higher, are less conscientious (meticulous), less extraverted but more emotionally stable and more risk averse. On average, in our sample, men and women have similar (good) health and a comparable degree of agreeableness and autonomy.

Oreffice and Quintana-Domeque (2010) estimate features of the (observed) matching function between men and women in the marriage market using the PSID data for the US. Their strategy consists in regressing each attribute of men on all attributes of women and vice versa. This procedure can easily be replicated with our data in an attempt to compare features of the matching function in both datasets (US versus Dutch marriage.

\footnote{Note that the mean age at first marriage is relatively high in the Netherlands (30.1 and 32.8 for women and men respectively, source: United Nations Economic Commission for Europe, 2010 Statistical Database) compared to the USA (26.1 and 28.2) which is reflected in the relative high mean age of men and women in our sample of couples.}
Interestingly enough, we find very similar results as those obtained by Oreffice and Quintana-Domeque (2010). For instance, these authors find that an additional unit in the husband’s BMI is associated with a 0.4 additional unit in the wife’s BMI. Using our sample, our estimate is also significant and of similar magnitude even after controlling for personality traits, i.e. 0.25. Furthermore, they find that an additional inch in the husband’s height is associated with an additional 0.12 inch in the wife’s height. Here too, our estimate is significant and of similar magnitude, i.e. 0.15. Yet, Oreffice and Quintana-Domeque (2010) find that richer men (higher educated men in our case) tend to be married with wives of lower BMI (an increase of 10% in the husband’s earnings is associated with a decrease of 0.21 points in his wife’s BMI). In our sample, we find that higher educated men (interpreting education as permanent income) are matched with women of lower BMI, i.e. a man with one additional level of education is matched with a woman whose BMI is 0.56 units lower.

7. Empirical results

We apply the Saliency Analysis, outlined in the previous section, on our sample of couples. The procedure requires first to estimate the affinity matrix $A$. This is done by applying the technique presented in section 3. The estimation results are reported in Table 3. It is important to note that the estimates reported in the table are obtained using standardized attributes rather than the original ones. The main advantage of using standardized attributes is that the magnitude of the coefficients is directly comparable across attributes, allowing a direct interpretation in terms of comparative statics.

The estimates of the affinity matrix reveal four important and remarkable features:

(1) **On-diagonal**: education is the single most important attribute in the marriage market. The largest coefficient of the affinity matrix is indeed observed on the diagonal for education. This coefficient is more than twice as large as the second largest coefficient obtained on the diagonal for the variable BMI. Loosely speaking, this means that increasing the education of both spouses by 1 standard deviation increases the
couple’s joint utility by 0.56 units. To achieve a similar increase in utility, the BMI of both spouses should be increased by 1.67 standard deviations each.

(2) **Off-diagonal:** the table clearly indicates the importance of cross-gender interactions between the various attributes as many off-diagonal coefficients of the affinity matrix are significantly different from 0. This implies that important trade-offs take place between the various attributes. For instance, men’s emotional stability interacts positively with women’s conscientiousness, i.e. 0.20. Stated otherwise, this means that increasing the husband’s emotional stability increases the joint utility of couples whose wives are relatively conscientious. Other examples are noticeable: men’s autonomy interacts negatively with women’s conscientiousness, i.e. $-0.11$ but positively with women’s extraversion, i.e. 0.12. Conversely, men’s agreeableness interacts positively with women’s conscientiousness, i.e. 0.14, but negatively with women’s extraversion, i.e. -0.15.

(3) **Asymmetry:** the affinity matrix is not symmetric indicating that preferences for attributes are not similar for men and women. For instance, increasing a wife’s conscientiousness by 1 standard deviation increases the joint utility of couples with more agreeable men relatively more (significant coefficient of magnitude 0.14) while increasing the husband’s conscientiousness by 1 standard deviation has the same impact on a couple’s joint utility, indifferently of how agreeable his wife is.

(4) **Personality traits:** personality traits matter for preferences, not only directly (terms on the diagonal are significant for conscientiousness, and risk aversion and of respective magnitude, 0.15 and 0.11) but mainly indirectly through their interactions with other attributes. For instance, the single most important interaction between observable attributes of men and women is found between the emotional stability of husbands and the conscientiousness of women, i.e. 0.20, a magnitude that matches with the direct effect of BMI. Also, personality traits interact not only with other personality traits but also with anthropometry. The emotional stability of men interacts negatively with women’s BMI, i.e. -0.11.

Using the estimated affinity matrix, we then proceed to the Saliency Analysis as introduced in the previous section. This enables us i) to test whether sorting is unidimensional,
i.e. occurs on a single-index and ii) to construct pairs of indices of mutual attractiveness for men and women.

We first test the dimensionality of the sorting in the marriage market. For $p = 1$, that is testing against the null hypothesis that sorting occurs on a single index, we find that $n\hat{T}_1\hat{\Omega}_1^{-1}\hat{T}_1 = 272.34$ which is significant at the 1% level. This implies that sorting in the marriage market does not occur on a single index as has been assumed in most of previous literature. In fact, our test-statistic never becomes insignificant. Even for $p = 9$ we have $n\hat{T}_9\hat{\Omega}_9^{-1}\hat{T}_9 = 12.76$ which is still significant at the 1% level. This suggests that the affinity matrix has full rank and that sorting occurs on at least 10 observed indices. Our results therefore clearly highlight that sorting in the marriage market is multidimensional and individuals face important trade-offs between the attributes of their spouses.

Each pair of indices derived from Saliency Analysis explains a mutually exclusive part of the total observable matching utility of couples. The share explained by each of our 10 indices is reported in Table 4. The table shows that the share of the first 8 pairs of indices is significantly different from 0 at the 1% level.

As for the Principal Component Analysis, the labeling of each dimension is subjective and becomes increasingly difficult to interpret as one considers more dimensions. Table 5 therefore only contains the 3 pairs of indices explaining most of the joint utility. Together these 3 pairs of indices explain about 60% of the total matching utility. The first pair, indexed $I_1$, explains about 28% of the joint utility. These indices load heavily (in bold weight $\geq 0.5$) on education and the weights on education are of similar magnitude for men and women. This confirms that education plays the most important role in sorting in the marriage market. However, the second pair of indices, which explains another 17% of the joint utility, loads heavily on personality traits (i.e. emotional stability for men and conscientiousness for women). Personality traits play a strong role in the sorting in the marriage market. Interestingly enough, while conscientiousness only matters for the attractiveness of women, emotional stability only matters for the attractiveness of men. The third pair explains another 14% of the joint utility and loads on BMI and extraversion for women and agreeableness for men. This result corroborates Chiappori, Oreffice and
Quintana-Domeque’s (2012) finding that BMI is important for the sorting in the marriage market.

8. Summary and Discussion

This paper has introduced a novel technique to test for the dimensionality of the sorting in the marriage market, and derive indices of mutual attractiveness, namely Saliency Analysis. This technique is grounded in the structural equilibrium model of Choo and Siow (2006) which we have extended to the continuous case in this paper. Indices of mutual attractiveness derived in Saliency Analysis, in contrast to Canonical Correlation for instance, have a structural interpretation and are therefore informative about agents’ preferences.

Saliency Analysis has been performed on a dataset of Dutch households containing information about education, height, BMI, health, attitude towards risk and five personality traits of both spouses. The empirical results of this paper reveal two important features of the marriage market. First, our results clearly show that sorting occurs on multiple indices rather than just on a single one, as assumed in most of current literature. This implies that individuals face important trade-offs between the attributes of their potential spouse. For instance, in the dataset we studied, more conscientious men prefer more conscientious women (0.15), but more autonomous men prefer less conscientious women (-0.11). Hence, women face a trade-off between being attractive for more conscientious men and being attractive for more autonomous men. Similarly, more conscientious women prefer more agreeable men (0.14) but more extraverted women prefer less agreeable men (-0.15). Men therefore face a trade-off between being attractive for more conscientious women and being attractive for more extraverted women.

Second, personality traits and attitude towards risk matter for the sorting of spouses in the marriage market. In fact, although education explains the largest share (28%) of the observable joint utility of spouses, personality traits explain a rather large share too (17%). Interestingly enough, different traits matter differently for men and women. For instance, women find emotionally stable men more attractive. Yet, men prefer conscientious women but are indifferent about the emotional stability of women.
The analysis presented in this paper opens up interesting possibilities for further research. In particular, our analysis could be applied on other markets besides the marriage one, such as the market for CEOs. A recent literature led by Bertrand and Schoar (2003), Falato, Li and Milbourn (2012) and Custodio, Ferreira and Matos (2013), acknowledges the multidimensionality of CEO's talent, but assumes that sorting occurs on a single index. Our setting can then be used to extend the seminal contributions of Terviö (2008) and Gabaix and Landier (2008), who calibrate a single-dimensional multiplicative sorting model in order to explain CEO compensation. An important difference in the CEO compensation literature is that transfers (i.e. salaries) are typically observed, unlike in the case of the marriage market considered in the present paper. The observation of the transfers has interesting consequences for identification. Assume that $x$ is a CEO’s vector of characteristics (say, track record, education, political inclinations, cultural affinities) and $y$ is a vector of firm’s characteristics. Let $\alpha(x, y)$ be the nonpecuniary utility of CEO $x$ working with firm $y$, and let $\gamma(x, y)$ be the productivity (in monetary units) of CEO $x$ if hired by firm $y$. In the case where transfers are unobserved, only the joint utility $\Phi = \alpha + \gamma$ is identified. However, in the case where transfers are observed, it is possible to identify separately $\alpha$ and $\gamma$. Hence when CEO compensation data is available, the results of the present paper can be easily extended to identify simultaneously the CEO’s productivity and his/her nonpecuniary utility for working with a given firm.

Lastly, we observe that the Poisson process approach which appears in the framework of this paper may provide the “missing link” between search models and matching with unobserved heterogeneity. Indeed, Poisson processes are central to search models, and the fact that they also play an important role in our model suggests that they may provide an interesting connection. The key difference, of course, comes from the fact that in search models, agents are faced with an optimal stopping problem: agents cannot know what their opportunities will be in advance, and they cannot retain offers, while in our framework they are fully aware of all their opportunities from the start. While we briefly elaborate on the formal connection in Appendix A, we leave a full exploration of the matter for future work.
Appendix A. Continuous logit formalism

In this paragraph, we expound the main ideas of Cosslett (1988) and Dagsvik (1994) who show how to obtain a continuous version of the multinomial logit model. Assume that \( \{(y_k^n, \varepsilon_{mk}^n), k \in \mathbb{N}\} \) are the points of a Poisson point process on \( \mathcal{Y} \times \mathbb{R} \) of intensity \( dy \times e^{-\varepsilon}d\varepsilon \).

We recall that this implies that for \( S \) a subset of \( \mathcal{Y} \times \mathbb{R} \), the probability that man \( m \) has no acquaintance in set \( S \) is \( \exp \left( -\int_S e^{-\varepsilon}dyd\varepsilon \right) \). From (2.2), man \( m \) chooses woman \( k \) among his acquaintances such that his utility is maximized, that is, man \( m \) solves

\[
\max_k \left\{ U(x, y_m^k) + \varepsilon_{mk}^n \right\}.
\]

Letting \( Z \) be the value of this maximum, one has for any \( c \in \mathbb{R} \)

\[
\Pr(Z \leq c) = \Pr(U(x, y_m^k) + \varepsilon_{mk}^n \leq c \forall k)
\]

which is exactly the probability that the Poisson point process \((y_k, \varepsilon_{mk}^n)\) has no point in \( \{(y, \varepsilon) : U(x, y) + \varepsilon > c\} \), thus

\[
\log \Pr(Z \leq c) = -\int_{\mathcal{Y} \times \mathbb{R}} 1(U(x, y) + \varepsilon > c)dye^{-\varepsilon}d\varepsilon = -\int_{\mathcal{Y}} \int_{c-U(x,y)} e^{-\varepsilon}d\varepsilon dy
\]

\[
= -\int_{\mathcal{Y}} e^{-c+U(x,y)}dy = -\exp \left( -c + \log \int_{\mathcal{Y}} \exp U(x,y) dy \right),
\]

hence \( Z \) is a \( \left( \log \int_{\mathcal{Y}} \exp U(x,y) dy, 1 \right) \)-Gumbel. In particular,

\[
\mathbb{E}\left[ \max_k \left\{ U(x, y_m^k) + \varepsilon_{mk}^n \right\} \right] = \log \int_{\mathcal{Y}} \exp U(x,y) dy
\]
and the choice probabilities are given by their density with respect to the Lebesgue measure

\[ \pi(y|x) = \exp(U(x,y)) / \int \exp(U(x,y')) \, dy'. \]

The same logic also implies that \( \{\varepsilon_k : k \in \mathbb{N}\} \) has a Gumbel distribution. Indeed, the probability that this Poisson point process has no element in the set \( \{\varepsilon : \varepsilon > c\} \) is equal to

\[ \exp\left(-\int_c^{+\infty} e^{-\varepsilon} \, d\varepsilon\right) = \exp(-\exp(-c)) \]

which is equivalent to say that \( \Pr(\max_{k \in \mathbb{N}} \varepsilon_k \leq c) = \exp(-\exp(-c)) \). Finally, note that a similar argument would show that \( m \) has almost surely an infinite, though countable, number of acquaintances, as announced.

Note that an interesting connection remains to be explored with the search literature (Shimer and Smith, 2000, Atakan, 2006). Assume that each man \( m \) draws a Poisson sample of acquaintances \( (y_k^m, \varepsilon_k^m) \), where \( y_k^m \) is the type of partner of index \( k \), and \( \varepsilon_k^m \) is now the time at which this acquaintance is met. Assume that it has been agreed that if \( x \) matches with \( y \), \( x \) will receive utility \( U(x,y) \) out of the joint utility \( \Phi(x,y) \). In the spirit of Atakan (2006), assume unmatched agents pay a utility cost equal to \( \sigma \) per unit of time while unmatched, such that if \( x \) matches with \( y \) at time \( \varepsilon \), his utility is \( U(x,y) - \sigma \varepsilon \). If agents could perfectly foresee their opportunities (i.e., know the full sample \( (y_k^m, \varepsilon_k^m) \) in advance), they would choose opportunity \( k \) so as to maximize the quantity \( U(x^m,y_k^m) - \sigma \varepsilon_k^m \) exactly as in the present paper. The difference, of course, comes from the fact that in search models, agents are faced with an optimal stopping problem: agents cannot know what their opportunities will be in advance, and they cannot retain offers. At each time \( t \) they know only what opportunities have already been received up to time \( t \), and they do not know about the set of \( k \)'s such that \( \varepsilon_k^m > t \). This is an optimal stopping problem with a Poisson process, well-studied in Probability Theory and Operations Research following seminal work by Elfving (1967). The basic idea is as follows: there exists a function \( \psi : \mathbb{R} \to \mathbb{R} \) such that the partner chosen by \( m \) is the first partner (in terms of meeting time) such that \( U(x^m,y_k^m) \) exceeds \( \psi(\varepsilon_k^m) \). \( \psi \) can be characterized as a solution to an Ordinary Differential Equation, and in some cases, can be expressed analytically.
Appendix B. Proofs

B.1. Proof of Theorem 1

Proof. (i) The first part of the argument extends Galichon and Salanié (2010) to the continuous case; the argument is decomposed in four steps which are now briefly commented. In Step 1, we shall show that the expression of the social welfare is given by

\[
\min_{U,V} \int_{\mathcal{X}} G_x(U(x,.)) f(x) \, dx + \int_{\mathcal{Y}} H_y(V(.,y)) g(y) \, dy
\]

s.t. \( U(x,y) + V(x,y) \geq \Phi(x,y) \)

where \( U(x,y) \) (resp. \( V(x,y) \)) is the share of the systematic joint utility going to man \( x \) (resp. woman \( y \)), and \( G_x(U) \) (resp. \( H_y(V) \)) is the ex-ante indirect utility of a man of type \( x \) (resp. a woman of type \( y \)), namely

\[
G_x(U(x,.)) = \mathbb{E} \left[ \max_k \left\{ U(x,y^m_k) + \frac{\sigma^m}{2} \varepsilon^m_k \right\} \right]
\]

and

\[
H_y(V(.,y)) = \mathbb{E} \left[ \max_l \left\{ U(x^w_l,y) + \frac{\sigma^w}{2} \varepsilon^w_l \right\} \right].
\]

Welfare expression (B.1) has a straightforward interpretation in terms of equilibrium. The constraint \( U + V \geq \Phi \) is a stability condition, and the minimization of the sum of the individual ex-ante indirect utility function expresses the absence of rents.

In step 2, we shall express the dual of variational problem (B.1) as

\[
W = \sup_{\pi \in \mathcal{M}(P,Q)} \int \Phi d\pi - I(\pi)
\]

where

\[
I(\pi) = \sup_U \left( \int_{\mathcal{X} \times \mathcal{Y}} U(x,y) d\pi(x,y) - \int_{\mathcal{X}} G_x(U(x,.)) dP(x) \right)
\]

\[
+ \sup_V \left( \int_{\mathcal{X} \times \mathcal{Y}} V(x,y) d\pi(x,y) - \int_{\mathcal{Y}} H_y(V(.,y)) dQ(y) \right).
\]

In step 3, we shall show that under the distributional assumptions made on the heterogeneities, the expression of \( I \) is given by

\[
I(\pi) = \sigma \int_{\mathcal{X} \times \mathcal{Y}} \log \frac{\pi(x,y)}{\sqrt{f(x)g(y)}} \pi(x,y) \, dx \, dy
\]
In step 4, we shall show that as a result, the social welfare, which is the value of variational problem (B.1), can be expressed up to irrelevant constants as

$$\max_{\pi \in \mathcal{M}(P,Q)} \int_{\mathcal{X} \times \mathcal{Y}} \Phi(x, y) \pi(x, y) \, dx \, dy - \sigma \int_{\mathcal{X} \times \mathcal{Y}} \log \pi(x, y) \pi(x, y) \, dx \, dy$$

(B.3)

which will establish (i).

**Step 1.** Introduce $\varepsilon_m(\cdot)$ a stochastic process on $\mathcal{Y}$ defined by

$$\varepsilon_m(y) = \frac{\sigma}{2} \max_k \{\varepsilon_k^m : y_k = y\}$$

if the set $\{k : y_k = y\}$ is nonempty, $\varepsilon_m(y) = -\infty$ otherwise. Similarly, introduce $\eta_w(x)$ a stochastic process on $\mathcal{X}$ defined by

$$\eta_w(x) = \frac{\sigma}{2} \max_l \{\eta_l^w : x_l = x\}$$

if the set $\{l : x_l = x\}$ is nonempty, $\eta_w(x) = -\infty$ otherwise. By the results of Shapley and Shubik (1972), extended to the continuous case by Gretsky, Ostroy and Zame (1992), the equilibrium matching solves the dual transportation problem which expresses the social welfare

$$W = \inf_{u_m + v_w \geq \Phi(x_m, y_m) + \varepsilon_m(y) + \eta_w(x)} \int u_m \, dm + \int v_w \, dw$$

(B.4)

now, the constraint can be rewritten as

$$U(x, y) + V(x, y) \geq \Phi(x, y)$$

where $U$ and $V$ have been defined as

$$U(x, y) = \inf_m (u_m - \varepsilon_m(y)) \quad \text{and} \quad V(x, y) = \inf_w (v_w - \eta_w(x))$$

which implies that $u_m$ and $v_w$ can be expressed in $U(x, y)$ and $V(x, y)$ by

$$u_m = \sup_{y \in \mathcal{Y}} (U(x, y) + \varepsilon_m(y)) \quad \text{and} \quad v_w = \sup_{x \in \mathcal{X}} (V(x, y) + \eta_w(x)).$$

(B.5)

Therefore, replacing $u_m$ and $v_w$ by their expression in $U$ and $V$, (B.4) rewrites as (B.1), with $G_x$ and $H_y$ given by (B.2).
Step 2. Rewrite (B.1) as a saddlepoint problem

\[ W = \inf_{U,V} \sup_\pi \left( \int_{X \times Y} \Phi d\pi + \int X G(U(x,.)) dP(x) - \int_{X \times Y} U d\pi \right) \]

or in other words

\[ W = \sup_\pi \int \Phi d\pi - I(\pi) \]

where

\[ I(\pi) = \sup_U \left( \int_{X \times Y} U d\pi - \int_X G_x(U(x,.)) dP(x) \right) + \sup_V \left( \int_{X \times Y} V d\pi - \int_Y H_y(V(.,y)) dQ(y) \right). \]

Step 3. From the derivation in Appendix A, we get that

\[ G_x(U(x,.)) = \frac{\sigma}{2} \log \int_Y \exp \frac{U(x,y)}{\sigma/2} dy \quad \text{and} \quad H_y(V(.,y)) = \frac{\sigma}{2} \log \int_X \exp \frac{U(x,y)}{\sigma/2} dx \]

Now, in order to get an expression for \( I(\pi) \) it remains to compute

\[ \sup_{U(x,y)} \int_{X \times Y} U(x,y) \pi(x,y) dxdy - \int G_x(U(x,.)) f(x) dx \quad (B.6) \]

and the similar expression on the other side of the market.

By F.O.C.,

\[ \pi(x,y) = \frac{f(x) \exp \frac{U(x,y)}{\sigma/2}}{\int_Y \exp \frac{U(x,y)}{\sigma/2} dy} \]

which implies that the value of the problem is infinite unless \( \int \pi(x,y) dy = f(x) \), in which case it is

\[ (\sigma/2) \int_{X \times Y} \pi(x,y) \log \frac{\pi(x,y)}{f(x)} dxdy \]

which is the value of (B.6). A symmetric expression is obtained for the other side of the market, and finally \( I(\pi) \) obtains as

\[ I(\pi) = \sigma \int_{X \times Y} \log \frac{\pi(x,y)}{\sqrt{f(x)g(y)}} \pi(x,y) dxdy \]

if \( \pi \in \mathcal{M}(P,Q) \), while \( I(\pi) = +\infty \) otherwise.
Step 4. One has

\[
\mathcal{I}(\pi) = \sigma \int_{X \times Y} \log \pi(x, y) \pi(x, y) \, dx \, dy \\
- \left(\sigma/2\right) \int_{X} \log f(x) \, f(x) \, dx - \left(\sigma/2\right) \int_{Y} \log g(x) \, g(x) \, dx
\]

the last two terms do not depend on the particular matching \(\pi \in \mathcal{M}(P, Q)\), thus are irrelevant in the expression of the social welfare, which establishes \((B.3)\) and point (i).

(ii) Letting

\[
a(x) = -\frac{\sigma}{2} \log \frac{f(x)}{\int_{Y} \exp U(x, y) \, dy} \quad \text{and} \quad b(y) = -\frac{\sigma}{2} \log \frac{g(y)}{\int_{X} \exp V(x, y) \, dx},
\]

one has

\[
\log \pi(x, y) = \frac{U(x, y) - a(x)}{\sigma/2} \quad \text{and} \quad \log \pi(x, y) = \frac{V(x, y) - b(y)}{\sigma/2}
\]

and by summation

\[
\pi(x, y) = \exp \left( \frac{\Phi(x, y) - a(x) - b(y)}{\sigma} \right).
\]

(iii) One has

\[
U(x, y) = \frac{\sigma \log \pi(x, y)}{2} + a(x) = \frac{\Phi(x, y) + a(x) - b(y)}{2}
\]

and similarly

\[
V(x, y) = \frac{\Phi(x, y) - a(x) + b(y)}{2}.
\]

By \((B.3)\), one sees that if man \(m\) of type \(x\) marries a woman of type \(x\), he gets utility

\[
u_m = \sup_{y' \in Y} \left( U(x, y') + \varepsilon_m(y') \right) = U(x, y) + \varepsilon_m(y).
\]
B.2. Useful lemmas. We state several useful lemmas which are useful in Sections 4 and 5, and in the proof of Theorem 2. First, we need a formula which expresses the affinity matrix of the rescaled attributes as a function of the affinity matrix between $X$ and $Y$. This is given in the following:

**Lemma 1.** For $M$ and $N$ two invertible matrices, one has:

$$A^{MX,NY} = (M')^{-1} A^{XY} N^{-1}. \quad (B.7)$$

This result should be compared with the expression of the cross-covariance matrix between $MX$ and $NY$, namely $\Sigma_{MX,NY} = M \Sigma_{XY} N'$. A quick dimensionality check is coherent, as the unit of $A^{XY}$ is the inverse of the product of the units of $X$ and $Y$, while the unit of $\Sigma_{XY}$ is the product of the units of $X$ and $Y$.

**Proof of Lemma 1**. Recall that every affinity matrix $A^{XY}$ is characterized by the fact that:

$$\frac{\partial W_{P,Q}}{\partial A_{ij}} (A^{XY}) = \Sigma_{XY}^{ij}. \quad (B.8)$$

Let $P_M$ (resp. $Q_N$) be the distribution of $MX$ (resp $NY$). We therefore have

$$\frac{\partial W_{P,M,Q_N}}{\partial A_{ij}} (A^{MX,NY}) = \Sigma_{MX,NY}^{ij} = M \Sigma_{XY}^{ij} N' = M \frac{\partial W_{P,Q}}{\partial A_{ij}} (A^{XY}) N', \quad (B.9)$$

where the second equality follows by definition and the third by using (B.8). A simple calculation shows that

$$W_{P,M,Q_N} (A^{MX,NY}) = W_{P,Q} (M' A^{MX,NY} N).$$

Taking the derivative with respect to $A$, yields

$$\frac{\partial W_{P,M,Q_N}}{\partial A} (A^{MX,NY}) = M \frac{\partial W_{P,Q}}{\partial A} (M' A^{MX,NY} N) N'. \quad (B.10)$$

And, by comparing (B.9) and (B.10), one gets

$$\frac{\partial W_{P,Q}}{\partial A} (M' A^{MX,NY} N) = \frac{\partial W_{P,Q}}{\partial A} (A^{XY}).$$
From the strict convexity of $W^{P,Q}$, we therefore have $M'A^{MX,NY}N = A^{XY}$, and given that $M$ and $N$ are invertible, it follows that

$$A^{MX,NY} = (M')^{-1} A^{XY} N^{-1}.$$  

QED.  

As a consequence of Lemma 1, we are able to state that the results of Saliency Analysis are invariant with respect to a (linear) change in the measurement units.

**Lemma 2.** For $\zeta_i$ and $\xi_j$ two vectors of positive scalars, let

$$\hat{X}_i = \zeta_i X_i \text{ and } \hat{Y}_j = \xi_j Y_j$$

be the values of partners’ attributes measured under different measurement units. Then the outcome of Saliency Analysis under the new measurement units coincides with the outcome under the former.

**Proof.** Saliency Analysis consists in determining the Singular Value Decomposition of $\Theta = \sigma_X A^{X,Y} \sigma_Y$ under the old units, and of $\hat{\Theta} = \sigma_{\hat{X}} A^{\hat{X},\hat{Y}} \sigma_{\hat{Y}}$ under the new units. Letting $D_\zeta = \text{diag}(\zeta_i)$ and $D_\xi = \text{diag}(\xi_j)$, one has

$$A^{\hat{X},\hat{Y}} = D_\zeta^{-1} A^{X,Y} D_\zeta^{-1}, \sigma_{\hat{X}} = \sigma_X D_\zeta \text{ and } \sigma_{\hat{Y}} = D_\xi \sigma_Y,$$

thus $\hat{\Theta} = \Theta$.  

The next lemma shows that $W_1(A)$ is strictly convex.

**Lemma 3.** The map $A \to W_1(A)$ is strictly convex.

**Proof.** Consider two matrices $A$ and $\hat{A}$. Let $\pi$ be the matching associated to $\Phi (x, y) = x' Ay$, and $\tilde{\pi}$ be the matching associated to $\Phi (x, y) = x' \tilde{A} y$ (uniqueness of $\pi$ and $\tilde{\pi}$ follows from the uniqueness of the solution to the Schrödinger problem, see Rüschendorf and Thomsen 1993, Theorem 3). Then convexity of $W_1$ implies

$$W_1 (\hat{A}) \geq W_1 (A) + \langle \nabla W_1 (A), \hat{A} - A \rangle$$  

(B.11)
where, by the Envelope Theorem, $\nabla W_1 (A) = \mathbb{E}_\pi [XY']$. In order to show strict convexity, we need to show that equality in (B.11) implies $A = A'$. Assume (B.11) holds as an equality. One has

$$W_1 (\tilde{A}) = W_1 (A) + \left< \nabla W_1 (A), \tilde{A} - A \right>$$

$$= \mathbb{E}_\pi [X'\tilde{A}Y] - \mathbb{E}_\pi [\ln \pi (X,Y)]$$

This implies that $\pi$ is optimal for the matching problem associated to $\Phi (x,y) = x'\tilde{A}y$. Again by the uniqueness of the solution to the Schrodinger problem mentioned above, it follows that $\pi = \tilde{\pi}$, and hence that

$$A = \partial^2 \ln \pi (x,y) / \partial x \partial y = \partial^2 \ln \tilde{\pi} (x,y) / \partial x \partial y = \tilde{A}.$$
thus
\[ \mathbb{E}_{\pi} \left[ \frac{\partial \log \pi(X,Y) | X = x}{\partial A_{ij}} \right] = 0 \]
which proves (B.12). (B.13) then follows directly. ■

The final lemma in this section shows that the Hessian of \( W \) coincides with the Fisher information matrix \( \mathcal{F} \).

**Lemma 5.** The Hessian of \( W_1 \) is given by
\[ \frac{\partial^2 W_1}{\partial A_{ij} \partial A_{kl}} = \mathcal{F}^{ij}_{kl} \]
where the expression of \( \mathcal{F} \) is given by 5.2.

**Proof of Lemma 5.** By the envelope theorem,
\[ \frac{\partial W_1}{\partial A_{ij}} = \int x_i y_j \pi_A(x,y) \, dx \, dy. \]
Thus,
\[ \frac{\partial^2 W_1}{\partial A_{ij} \partial A_{kl}} = \int x_i y_j \frac{\partial \log \pi_A(x,y)}{\partial A_{kl}} \pi_A(x,y) \, dx \, dy = \mathcal{F}^{ij}_{kl}, \]
where the second equality follows from Lemma 4. ■

**B.3. Proof of Theorem 2.** The proof of Theorem 2 relies on the auxiliary results derived in the previous paragraph. In the sequel, we assume \( \sigma = 1 \); by positive homogeneity, this is without loss of generality.

**Proof of Theorem 2.** Let
\[ \hat{\pi}(x,y) = \frac{1}{n} \sum_{k=1}^{n} \delta(x - X_k) \delta(y - Y_k) \]
be the distribution of the empirical sample under observation, and \( \pi_A \) is the equilibrium matching computed for matching utility function \( \Phi_A \) (we shall drop the subscript \( A \) when
there is no ambiguity). Recall that the (population) affinity matrix $A$ and its sample estimator $\hat{A}$ are respectively characterized by

$$\frac{\partial W_1(A)}{\partial A_{ij}} = \Sigma_{XY}^{ij} \quad \text{and} \quad \frac{\partial W_1(\hat{A})}{\partial A_{ij}} = \hat{\Sigma}_{XY}^{ij}.$$ 

By the Delta method, we get

$$\left(\mathbb{F} \cdot \delta A\right)^{ij} = \int \frac{\partial \log \pi_A}{\partial A_{ij}} (\tilde{\pi} - \pi) \, dx \, dy + o_D(n^{-1/2}) \quad \text{(B.14)}$$

where $\mathbb{F}$ is the Hessian of $W_1$ at $A$, whose expression is

$$\mathbb{F}_{kl}^{ij} = \mathbb{E}_\pi \left[ \frac{\partial \log \pi_A (X,Y)}{\partial A_{ij}} \frac{\partial \log \pi_A (X,Y)}{\partial A_{kl}} \right]$$

where $\pi \in \mathcal{M}(P,Q)$ is the equilibrium matching computed for the joint utility function $\Phi_A$.

Further,

$$(\delta S_X)^{ij} = 1_{i=j} \int x_i x_j (\tilde{\pi} - \pi) \, dx \, dy + o_D(n^{-1/2})$$

$$(\delta S_Y)^{kl} = 1_{k=l} \int y_i y_j d\pi (\tilde{\pi} - \pi) \, dx \, dy + o_D(n^{-1/2})$$

hence

$$\mathbb{E} \left[ (\mathbb{F} \cdot \delta A)^{ij} (\delta S_X)^{kl} \right] = \text{cov} \left( \frac{\partial \log \pi}{\partial A_{ij}}, X_k X_l \right) 1_{(k=l)} = 0,$$

where we have used (B.12), and similarly, $\mathbb{E} \left[ (\delta A)^{ij} (\delta S_Y)^{kl} \right] = 0$. This proves the asymptotic independence between $\delta A$ and $(\delta S_X, \delta S_Y)$. The conclusion follows by noting that the asymptotic variance-covariance matrix of $\delta A$ is $\mathbb{F}^{-1}$, and that of $(\delta S_X, \delta S_Y)$ is

$$\begin{pmatrix}
K_{XX} & K_{XY} \\
K_{XY}' & K_{YY}
\end{pmatrix}.$$ 

### B.4. Proof of Theorem 3

In order to give asymptotic distributions of matrix estimators, it is convenient to represent matrices as vectors, an operation which is called *vectorization* in matrix algebra. Linear operators acting on these vectorized matrices will therefore be called *doubly-indexed matrices*, for which we shall use the bold notation to distinguish them from simply-indexed matrices. If $\mathbb{R}$ is a doubly-indexed matrix, its general term will be denoted $\mathbb{R}_{ij}^{kl}$, where $ij$ indexes the lines and $kl$ indexes the columns of $\mathbb{R}$. Then $\mathbb{R} \cdot M$ will
denote the (simple) matrix $N$ such that $N_{ij} = \sum_{kl} R_{kl}^{ij} M^{kl}$. We recall the definition of the Kronecker product: for two matrices $A$ and $B$, $A \otimes B$ is the doubly-indexed matrix $R$ such that

$$R_{ij,kl} = A_{ik} B_{jl}.$$  

**Lemma 6.** The following convergence holds in distribution for $n \to +\infty$:

$$n^{1/2} \left( \hat{\Theta} - \Theta \right) \Rightarrow \mathcal{N}(0, \mathbb{V})$$

where

$$\mathbb{V} = \mathbb{T}_{XY} \mathbb{F}^{-1} \mathbb{T}'_{XY} + \mathbb{T}_X \mathbb{K}_X \mathbb{T}'_X + \mathbb{T}_Y \mathbb{K}_Y \mathbb{T}'_Y + \mathbb{T}_X \mathbb{K}_X \mathbb{T}'_Y + \mathbb{T}_Y \mathbb{K}_Y \mathbb{T}'_X.$$

**Proof of Lemma 6**  As

$$\delta \Theta = \left( S^{1/2}_Y \otimes S^{1/2}_X \right) \delta A + \left( S^{1/2}_Y A' \otimes I \right) \delta S^{1/2}_X + \left( I \otimes S^{1/2}_X A \right) \delta S^{1/2}_Y$$

one has

$$\delta \Theta = \mathbb{T}_{XY} \delta A + \mathbb{T}_X \delta S_X + \mathbb{T}_Y \delta S_Y,$$

where

$$\mathbb{T}_X = \left( S^{1/2}_Y A' \otimes I \right) \left( S^{1/2}_X \otimes I + I \otimes S^{1/2}_X \right)^{-1} \quad (B.15)$$

$$\mathbb{T}_{XY} = S^{1/2}_Y \otimes S^{1/2}_X \quad (B.16)$$

$$\mathbb{T}_Y = \left( I \otimes S^{1/2}_X A \right) \left( S^{1/2}_Y \otimes I + I \otimes S^{1/2}_Y \right)^{-1}, \quad (B.17)$$

The proof of Theorem 3 follows as an easy consequence.

**Proof of Theorem 3**  Let

$$\Omega_p = (B_{p \perp} \otimes A'_{p \perp})' \vee (B_{p \perp} \otimes A'_{p \perp})'.' \quad (B.18)$$

By Kleibergen and Paap, Theorem 1, the convergence

$$n^{1/2} \hat{T}_p \Rightarrow \mathcal{N}(0, \Omega_p)$$
Appendix C. Computation

Let $a$ and $b$ be the solutions of equation (2.7), and introduce

$$\tilde{a}(x) = \exp \left( -a(x)/\sigma \right) \text{ and } \tilde{b}(y) = \exp \left( -b(y)/\sigma \right)$$

so equation (2.7) rewrites

$$\pi(x,y) = \tilde{a}(x) \tilde{b}(y) K(x,y) \quad (C.1)$$

where $K(x,y) = \exp \left( \Phi(x,y)/\sigma \right)$, and the system of equations formed by the constraints on the marginals rewrites

$$\tilde{a}(x) = f(x) \left( \int_{\mathcal{Y}} \tilde{b}(y) K(x,y) \, dy \right)^{-1} \quad (C.2)$$

$$\tilde{b}(y) = g(y) \left( \int_{\mathcal{X}} \tilde{a}(x) K(x,y) \, dx \right)^{-1} \quad (C.3)$$

Note that by (C.3), $\tilde{b}$ can be expressed as a function of $\tilde{a}$. Then $\tilde{a}$ rewrites as a fixed point equation $\tilde{a} = F(\tilde{a})$, where $F$ is given by

$$F(\tilde{a})(x) = f(x) \left( \int_{\mathcal{Y}} \left( \int_{\mathcal{X}} \tilde{a}(x') K(x',y) \, dx' \right)^{-1} K(x,y) \, dy \right)^{-1}.$$

The Iterative Projection Fitting Procedure (IPFP) consists in starting with some proper choice of $\tilde{a}_0(x)$ that ensures integrability of $x \to \tilde{a}(x) K(x,y)$, and iteratively applying $\tilde{a}_{k+1} = F(\tilde{a}_k)$. Details and proof of convergence are provided in Rüschendorf (1995); convergence is very quick in practice.

Appendix D. Incorporating singles

Throughout this appendix, the symbol $\emptyset$ stands for singlehood; this enlarges the sets of marital choices of men and women, which we denote $\mathcal{X}_0 = \mathcal{X} \cup \{\emptyset\}$ and $\mathcal{Y}_0 = \mathcal{Y} \cup \{\emptyset\}$. Let $\bar{f}(x)$ be the density of mass of men of type $x$, $f_0(x)$ be the density of mass of single men of type $x$, and, as in the rest of the paper, $f(x)$ is the density of mass of matched men of type $x$, so that $\bar{f}(x) = f_0(x) + f(x)$. Introduce similar notations on the other side of the
market: \( g(y) = g_0(y) + g(y) \), and note that the total mass of men and women no longer needs to coincide, i.e. in general one has

\[
\int_X f(x) \, dx \neq \int_Y g(y) \, dy.
\]

The set of acquaintance of man \( m \) is now expanded to include singlehood: \( \{(y_m^k, \varepsilon_m^k), k \in \mathbb{N}\} \) are now the points of a Poisson process on \( Y_0 \times \mathbb{R} \) of intensity \( \lambda_0 \times e^{-\varepsilon}d\varepsilon \), where for \( B \subseteq Y_0 \)

\[
\lambda_0(S) = 1 \{\emptyset \in B\} + \lambda(B \setminus \emptyset)
\]

where \( \lambda \) is the Lebesgue measure on \( Y \). As in Appendix A the utility of a man \( m \) matching with acquaintance \( k \) is determined at equilibrium by

\[
U(x, y_m^k) + \frac{\sigma}{2} \varepsilon_m^k,
\]

but \( y_m^k \) can now take value \( \emptyset \), in which case \( U(x, \emptyset) = \Phi(x, \emptyset) \). The indirect utility of man \( m \) is thus given by

\[
Z = \max_k \{U(x, y_m^k) + \frac{\sigma}{2} \varepsilon_m^k\},
\]

and one has

\[
\log \Pr(Z \leq c) = -\int \int_{Y_0 \times \mathbb{R}} 1 \left(U(x, y) + \frac{\sigma}{2} \varepsilon > c\right) d\lambda_0(y) e^{-\varepsilon}d\varepsilon
\]

so that

\[
f_0(x) = \frac{\exp \frac{\Phi(x, \emptyset)}{\sigma/2}}{\exp \frac{\Phi(x, \emptyset)}{\sigma/2} + \int Y \exp \frac{U(x, y)}{\sigma/2} \, dy + \int X \exp \frac{V(x, y)}{\sigma/2} \, dx}
\]

and

\[
g_0(y) = \frac{\exp \frac{\Phi(\emptyset, y)}{\sigma/2}}{\exp \frac{\Phi(\emptyset, y)}{\sigma/2} + \int Y \exp \frac{U(x, y)}{\sigma/2} \, dy + \int X \exp \frac{V(x, y)}{\sigma/2} \, dx}
\]

while

\[
\pi(y | x) = \frac{\exp \frac{U(x, y)}{\sigma/2}}{\int Y \exp \frac{U(x, y')}{\sigma/2} \, dy'} \quad \text{and} \quad \pi(x | y) = \frac{\exp \frac{V(x, y)}{\sigma/2}}{\int X \exp \frac{V(x', y)}{\sigma/2} \, dx'}
\]

hence we see that the observation of \( \pi \) identifies \( U(x, y) \) up to an additive term \( c(x) \), and \( V(x, y) \) up to an additive term \( d(y) \), hence \( U \) and \( V \) are identified by

\[
U(x, y) = \frac{\sigma}{2} (\log \pi(y | x) + c(x)), \quad V(x, y) = \frac{\sigma}{2} (\log \pi(x | y) + d(y))
\]

and

\[
\Phi(x, y) = \frac{\sigma}{2} (\log \pi(y | x) + \log \pi(x | y) + c(x) + d(y))
\]

where \( c(x) \) and \( d(y) \) are undetermined. This is precisely the identification achieved in Section 2.2. The crucial conclusion is that the observation of singles does not change anything in the identification of \( U \) and \( V \). This is a consequence of the independence of irrelevant alternatives (IIA) of the logit model: indeed, the incentive for remaining single does not
affect the odd ratios of the choices of the partners types. As a result, the distributions of matched men and women \( f(x) \) and \( g(y) \) may be treated as exogenous.

Once \( U \) and \( V \) have been identified, one has

\[
\frac{f_0(x)}{f(x)} = \frac{\exp \frac{\Phi(x, \emptyset)}{\sigma/2}}{\exp \Phi(x, \emptyset) + \exp c(x)} \quad \text{and} \quad \frac{g_0(y)}{g(y)} = \frac{\exp \frac{\Phi(\emptyset, y)}{\sigma/2}}{\exp \Phi(\emptyset, y) + \exp d(y)}
\]

hence by inversion

\[
\Phi(x, \emptyset) = \frac{\sigma}{2} \left( \log \frac{f_0(x)}{f(x)} + c(x) \right) \quad \text{and} \quad \Phi(\emptyset, y) = \frac{\sigma}{2} \left( \log \frac{g_0(y)}{g(y)} - g_0(y) + d(y) \right)
\]

which implies that the observation of single individuals allows one to identify the reservation utilities. As a result, the utility surplus from matching \( \Phi(x, y) - \Phi(x, \emptyset) - \Phi(\emptyset, y) \) is identified in the data by

\[
\log \left( \frac{\pi(y|x)(\bar{f}(x) - f_0(x))}{f_0(x)} \frac{\pi(x|y)(\bar{g}(y) - g_0(y))}{g_0(y)} \right) \quad \text{(D.1)}
\]

and the ex-ante expected utility surpluses of men of type \( x \) and women of type \( y \) are given just as in Choo and Siow by

\[
u(x) = \log \frac{\bar{f}(x)}{f_0(x)} \quad \text{and} \quad v(y) = \log \frac{\bar{g}(y)}{g_0(y)}.
\quad \text{(D.2)}
\]

These formulae are the continuous extensions of the formulae given in Choo and Siow (2006), where the surplus from matching is identified by \( \log \left( \frac{\mu_{xy}^2}{\mu_{x0}\mu_{0y}} \right) \), where \( \mu_{x0} \) and \( \mu_{0y} \) are respectively the number of single men and women of type \( x \) and \( y \) respectively, and \( \mu_{xy} \) is the number of \( xy \) pairs.

**APPENDIX E. FURTHER DETAILS ON THE DATASET**

E.1. **Questionnaire about personality and attitudes**\(^\text{16}\) *Personality traits, the 16PA scale.*

Now we would like to know how you would describe your personality. Below we have mentioned a number of personal qualities in pairs. The qualities are not always opposites. Please indicate for each pair of qualities which number would best describe your personality.

\(^\text{16}\)The following website: http://www.centerdata.nl/en/TopMenu/Databank/DHS_data/Codeboeken/ provides a link to the complete description of the questionnaire.
If you think your personality is equally well characterized by the quality on the left as it is by the quality on the right, please choose number 4. If you really don’t know, type 0 (zero).

Scale: 1 2 3 4 5 6 7

TEG1: oriented towards things oriented towards people.

TEG2: slow thinker quick thinker.

TEG3: easily get worried not easily get worried.

TEG4: flexible, ready to adapt myself stubborn, persistent.

TEG5: quiet, calm vivid, vivacious.

TEG6: carefree meticulous.

TEG7: shy dominant.

TEG8: not easily hurt/offended sensitive, easily hurt/offended.

TEG9: trusting, credulous suspicious.

TEG10: oriented towards reality dreamer.

TEG11: direct, straightforward diplomatic, tactful.

TEG12: happy with myself doubts about myself.

TEG13: creature of habit open to changes.

TEG14: need to be supported independent, self-reliant.

TEG15: little self-control disciplined.

TEG16: well-balanced, stable irritable, quick-tempered.

Attitude towards risk.

The following statements concern saving and taking risks. Please indicate for each statement to what extent you agree or disagree, on the basis of your personal opinion or experience.

totally disagree 1 2 3 4 5 6 7 totally agree

SPAAR1: I think it is more important to have safe investments and guaranteed returns, than to take a risk to have a chance to get the highest possible returns.
SPAAR2: I would never consider investments in shares because I find this too risky.

SPAAR3: if I think an investment will be profitable, I am prepared to borrow money to make this investment.

SPAAR4: I want to be certain that my investments are safe.

SPAAR5: I get more and more convinced that I should take greater financial risks to improve my financial position.

SPAAR6: I am prepared to take the risk to lose money, when there is also a chance to gain money.

E.2. Construction of the “Big Five” personality factors. The DHS panel contains three lists of items that would allow one to assess a respondent’s personality traits.

(1) The first list contains 150 items and refers to the Five-Factor Personality Inventory measure, developed by Hendriks et al. (1999). This list was included in a supplement to the 1996 wave.

(2) The second list refers to the 16 Personality Adjective (16PA) scale developed by Brandstätter (1988) and was included in the module “Economic and Psychological Concepts” from 1993 until 2002.

(3) From 2003 on, the panel replaced the 16PA scale by the International Personality Item Pool (IPIP) developed by Golberger (1999). The 10-item list version of the IPIP scale is used except for the 2005 wave where the 50-item list was implemented.

Of the three scales, the 16PA scale covers the largest sample of individuals. For that reason, the 16PA scale was chosen to measure personality traits. This scale offers the respondents the opportunity to locate themselves on 16 personality dimensions. Each dimension is represented by two bipolar scales so that the full scale contains 32 items. Nyhus and Webley (2001) show that this scale distinguishes 5 factors. They labeled these factors

Using the 1996 wave that contains both the FFPI module and the 16PA module, Nyhus and Webley (2001) checked the correlation between the 5 factors identified by the 16PA scale and the (big) five factors identified by the FFPI. The correlation is generally high though not perfect. This suggests that both sets of factors assess slightly different aspects of the latent factors. We followed Nyhus and Webley and use a slightly less general wording for the various dimensions identified from the 16PA scale.
as: Emotional stability, Extraversion, Conscientiousness, Agreeableness, and Autonomy. Of the 32 items associated with the 16PA measure, the first half was asked in 1993, 1995 and each year between 1997 and 2002 while the other half was asked in 1994 and 1996 only. Constructing the full scale, therefore, requires losing all respondents but those who responded in two successive years between 1993 and 1996. To avoid throwing out too many observations, we constructed the five dimensions using only those 16 items included in waves 1993, 1995 and 1997-2002. Since answers given to the same item by the same person in different waves are strongly correlated (see Nyhus and Webley, 2001), we simply collapse the data by individual using the person’s median answer to each item.

We have constructed our five factors by adding the (standardized) items identified by Nyhus and Webley (2001) for the respective scales. In other words, “Emotional stability” is constructed using items:

- “oriented toward reality”/“dreamer”,
- “happy with myself”/“doubtful”,
- “need to be supported”/“independent”,
- “well-balanced”/“quick-tempered”,
- “slow-thinker”/“quick-thinker” and,
- “easily worried”/“not easily worried”.

“Agreeableness” is constructed using items:

- “creature of habit”/“open to changes”,
- “slow thinker”/“quick thinker”,
- “quiet, calm”/“vivid, vivacious”.

“Autonomy” is constructed based on:

- “direct, straightforward”/“diplomatic”,
- “quiet, calm”/“vivid, vivacious” and,
- “shy”/“dominant”.

“Extraversion” is based on:

- “oriented towards things”/“towards people”,
- “talkative”/“withdrawn”,
- “cheerful”/“melancholy” and,
“Conscientiousness” is constructed using:

- “little self-control”/“disciplined”,
- “carefree”/“meticulous” and,
- “not easily hurt”/“easily hurt, sensitive”.

As a robustness check, we constructed the full scale using the 1993, 1994, 1995 and 1996 waves. We followed Nyhus and Webley (2001) and constructed the five factors using Principal Component Analysis and varimax rotation on the five main factors. The correlation between each of the factors we constructed using only 16 items and the corresponding factor using the full scale varies between 0.42 for agreeableness and 0.76 for emotional stability.
Appendix F. Tables

Table 1. Number of identified young couples and number of young couples
with complete information for various subset of variables.

<table>
<thead>
<tr>
<th></th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td>Identified couples</td>
<td>2,897</td>
</tr>
<tr>
<td>Couples with complete information on:</td>
<td></td>
</tr>
<tr>
<td>Education</td>
<td>2,883</td>
</tr>
<tr>
<td>The above + Health, Height and BMI$^a$</td>
<td>1,595</td>
</tr>
<tr>
<td>The above + Personality traits (Big 5)</td>
<td>1,258</td>
</tr>
<tr>
<td>The above + measure of risk aversion</td>
<td>1,158</td>
</tr>
</tbody>
</table>

Notes: (1) We have excluded all individuals taller than 210cm or shorter than 145cm and all individuals lighter than 40kg, no one is heavier than 200kg in our data. These exclusions represent less than 1 percent of the sample of adults in the source data. (2) The selected sample for our analysis is the one from the last row.

$^a$: Excluding health produces exactly the same number of couples at this stage.

Source: DNB. Own calculation.
Table 2. Sample of young couples with complete information: summary statistics by gender.

<table>
<thead>
<tr>
<th></th>
<th>Husbands</th>
<th></th>
<th>Wives</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>N</td>
<td>mean</td>
<td>S.E.</td>
<td>N</td>
</tr>
<tr>
<td>Age</td>
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<td>35.52</td>
<td>6.01</td>
<td>1158</td>
</tr>
<tr>
<td>Educational level</td>
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<td>2.01</td>
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<td>1158</td>
</tr>
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<td>Height</td>
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<td>182.33</td>
<td>7.20</td>
<td>1158</td>
</tr>
<tr>
<td>BMI</td>
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<td>24.53</td>
<td>2.94</td>
<td>1158</td>
</tr>
<tr>
<td>Health</td>
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<td>3.21</td>
<td>0.66</td>
<td>1158</td>
</tr>
<tr>
<td>Conscientiousness</td>
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<td>-0.25</td>
<td>0.64</td>
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<td>0.06</td>
<td>0.68</td>
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### Table 3. Estimates of the Affinity matrix: quadratic specification (N = 1158).

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<tbody>
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<td>-0.01</td>
<td>-0.03</td>
<td>-0.03</td>
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<td>0.00</td>
<td>-0.03</td>
<td>0.05</td>
<td>0.01</td>
<td>0.03</td>
<td>0.02</td>
<td>0.02</td>
</tr>
<tr>
<td>BMI</td>
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<td>0.01</td>
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<td>-0.01</td>
<td>0.04</td>
<td>-0.04</td>
<td>0.01</td>
<td>0.01</td>
</tr>
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<td>Health</td>
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<td>-0.06</td>
<td>0.14</td>
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<td>-0.07</td>
<td>-0.01</td>
<td>0.15</td>
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<td>0.02</td>
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Note: Bold coefficients are significant at the 5 percent level.
Table 4. Share of observed joint utility explained.

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<th>I3</th>
<th>I4</th>
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<th>I7</th>
<th>I8</th>
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<td>28.04***</td>
<td>16.50***</td>
<td>13.95***</td>
<td>10.40***</td>
<td>9.35***</td>
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*** significant at 1 percent
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<th>M</th>
<th>W</th>
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<td>-0.11</td>
<td></td>
</tr>
</tbody>
</table>

Note: M means men and W means women. Bold coefficients indicate coefficients larger than 0.5.
REFERENCES


[37] Heckman, J.J. (2007), Notes on Koopmans and Beckmann’s “Assignment Problems and the Location of Economic Activities”, lecture notes, the University of Chicago.


