Runs versus Lemons: Information Disclosure, Fiscal Capacity and Financial Stability

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Abstract

We study how information disclosure and fiscal backstops affect financial stability in an economy that is subject to runs and adverse selection. The planner in our economy faces a rich set of policy options: it can disclose information about banks’ assets, it can intervene to stop bank runs, recapitalize banks and intervene in credit markets. In any intervention, the planner faces a trade-off between mitigating adverse selection and causing inefficient bank runs. Reducing adverse selection increases welfare by increasing investment in positive NPV projects, but revealing information can trigger bank runs and inefficient liquidation. We find that the optimal policy depends on the fiscal capacity available to the planner. When capacity is ample, the planner chooses to reveal information and provide liquidity to banks that are run on; conversely, when capacity is low, the planner prefers to hide information and mitigate adverse selection by intervening in credit markets. Our model sheds light on optimal interventions and provides an explanation for the different choices that countries make in response to financial crises.

JEL: E5, E6, G1, G2.

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1 Introduction

Since the beginning of the financial crisis, the balance sheets of sovereigns and their financial institutions have become intertwined. This is in fact a generic feature of financial crises as argued in Reinhart and Rogoff (2009). Gorton (2012) shows that government interventions always play an important role in stopping financial panics. Governments use various tools to intervene during financial crises, but different government use different tools and with varying degrees of success. Our goal is to understand these choices and their consequences.

In October 2008, the US government decided to inject cash into banks under the Troubled Asset Relief Program. In May 2009, the Federal Reserve publicly reported the results of the Supervisory Capital Assessment Program (SCAP). The SCAP, known as the banking stress test, was an assessment of the capital adequacy, under adverse scenarios, of a large subset of US financial firms. The exercise is broadly perceived as having been successful in reducing uncertainty about the state of the US financial system and helping to restore calm to financial markets.

The Committee of European Banking Supervisors (CEBS) also conducted an EU-wide stress test from May-October 2009, the results of which were not made public. A year later the exercise was repeated, but the results of the stress test, including bank-by-bank results, were published. In both cases, the stress tests are regarded as having been ineffective in restoring confidence to the financial sector. 1

What explains this marked difference in the success of stress tests as a means of restoring financial stability? We propose a model that highlights the tradeoffs faced by a regulator in deciding how much information about the financial system to make public.

We study optimal interventions by a planner in an economy that features adverse selection in the spirit of Akerlof (1970) and Stiglitz and Weiss (1981) as well as bank runs as in Diamond and Dybvig (1983). Our economy is populated by short-term funded intermediaries that differ in the quality of their existing assets. 2 The quality of these legacy assets is private information to each bank. In order to invest in new projects with positive net present value, banks must raise additional funds from the credit market. Asymmetric information about the quality of existing assets creates adverse selection in the credit market, leading to inefficiently high interest rates and low investment in the decentralized equilibrium. The planner might be able to improve welfare by disclosing information about banks’ types. But runs make information disclosure potentially costly. If short term creditors (depositors) learn that a particular bank is bad, they might decide to run. Runs are inefficient for two reasons: there is a liquidation discount on the assets of banks that suffer a run, and liquidated banks cannot invest in new projects.

In this environment, a policy maker has a large set of tools to try to enhance welfare: asset quality reviews and stress tests, recapitalizations of “bad banks”, liquidity support and deposit insurance, among others. This paper provides a model through which the tradeoffs involved in the choice of these policies can be studied. We focus on combinations of two types of policies: information revelation (a ‘stress test’ or ‘asset quality review’) and fiscal

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1 Ong and Pazarbasioglu (2013) provide a thorough overview of the details and perceived success of SCAP and the CEBS stress tests.

2 We have in mind all short term runable liabilities: MMF, Repo, ABCP, and of course large uninsured deposits. In the model, for simplicity, we refer to intermediaries as banks and liabilities as deposits.
interventions by the planner. The motivation for this focus is the ongoing interest in the academic literature and among practitioners in the (de)merits and perceived effectiveness of bank stress tests.

We are particularly interested in the effect that the fiscal capacity available to a planner for the implementation of a given policy has on the optimal choice of policy. The planner in our model must pay for its interventions with distortionary taxation. It may also have pre-existing obligations that it must pay for in the future. The extent to which taxation is distortionary and the magnitude of pre-existing spending commitments determine fiscal capacity.

Our main result is that a planner’s fiscal capacity shapes the optimal policy. When fiscal capacity is high, it is optimal for the planner to reveal information in a transparent manner and provide liquidity to at least a subset of banks that suffer a run, such that these banks survive and are able to invest in profitable projects. When capacity is low, the planner prefers to avoid runs by not revealing each bank’s type, and then mitigate the resulting adverse selection in the credit market by providing loans and credit guarantees.

We study two extensions of our basic model. In one extension, we show that aggregate uncertainty reinforces our results. We find that government with low fiscal capacity are effectively risk averse, and this makes them unwilling to risk runs by disclosing information.

2 Related literature

Our work builds on the rich literature that studies asymmetric information, following Akerlof (1970), Spence (1974), and Stiglitz and Weiss (1981). If no information is revealed by the planner, our economy very closely resembles the one studied by Philippon and Skreta (2012) and Tirole (2012). The optimal policy in the case in which information is not fully revealed is similar to theirs.

Since we add bank runs to an economy with asymmetric information, we also build on the large literature started by Diamond and Dybvig (1983). Several recent papers study specifically the tradeoffs involved in revealing information about banks. Goldstein and Leitner (2013) focus on the trade-off between a market breakdown due to asymmetric information and the Hirshleifer (1971) effect: revealing too much information destroys risk-sharing opportunities between risk-neutral investors and (effectively) risk averse bankers. These risk-sharing arrangements play an important role in Allen and Gale (2000). Parlatore Siritto (2013) studies a Diamond and Dybvig (1983) type economy with aggregate risk in which more precise information about realizations of the aggregate state can lead to more bank runs. A simple way to think about disclosure in models of bank runs is to view disclosure as a way to break pooling equilibria. Whether disclosure is good or bad then simply depends on whether the pooling equilibria is desirable. If agents pool on the “no run” equilibrium then there is no reason to disclose information. And of course this is more likely to happen in good times as long as we consider “refined” equilibria à la Carlsson and van Damme (1993) and Morris and Shin (2000), where fundamentals matter. On the other hand, in bad times, agents might run on all the banks, in which case it is better to disclose information to save at least the good banks.
This is the basic result of Bouvard, Chaigneau, and de Motta (2012), who also consider ex-ante disclosure rules that allow pooling across macroeconomic states. Gorton and Metrick (2012) investigate how uncertainty about bank insolvency (and, implicitly, the quality of bank portfolios) leads to increases in repo haircuts that, along with declining asset values, cause several institutions to become insolvent. Shapiro and Skeie (2013) study reputation concerns by a regulator in an environment characterized by a trade-off between moral hazard and runs. None of these papers model new lending and borrowing by banks and therefore cannot address the trade-off between unfreezing credit markets and triggering bank runs. Gorton and Ordoñez (2014) consider a model where crises occur when investors have an incentive to learn about the true value of otherwise opaque assets. In our model, it is optimal to disclose starting with bad types. This is consistent with what 19th century clearing houses did to contain financial panics, and also with current regulatory practice. (Gorton, 2012)

Our paper relates to the theoretical literature on bank bailouts. Gorton and Huang (2004) argue that the government can bail out banks in distress because it can provide liquidity more effectively than private investors. Diamond and Rajan (2005) show that bank bailouts can backfire by increasing the demand for liquidity and causing further insolvency. Diamond (2001) emphasizes that governments should only bail out the banks that have specialized knowledge about their borrowers. Aghion, Bolton, and Fries (1999) show that bailouts can be designed so as not to distort ex-ante lending incentives. Farhi and Tirole (2012) examine bailouts in a setting in which private leverage choices exhibit strategic complementarities due to the monetary policy reaction. Corbett and Mitchell (2000) discuss the importance of reputation in a setting where a bank’s decision to participate in a government intervention is a signal about asset values, and Philippon and Skreta (2012) formally analyze optimal interventions when outside options are endogenous and information-sensitive. Mitchell (2001) analyzes interventions when there is both hidden actions and hidden information. Landier and Ueda (2009) provide an overview of policy options for bank restructuring. Philippon and Schnabl (2013) focus on debt overhang in the financial sector. Diamond and Rajan (2012) study the interaction of debt overhang with trading and liquidity. In their model, the reluctance to sell assets leads to a collapse in trading which increases the risks of a liquidity crisis.

Goldstein and Sapra (2014) review the literature on the disclosure of stress tests results. They explain that stress tests differ from usual bank examinations in four ways: (i) traditional exams are backward looking, while stress tests project future losses; (ii) the projections under adverse scenarios provide information about tail risks; (iii) stress tests use common standards and assumptions, making the results more comparable across banks; (iv) unlike traditional exams that are kept confidential, stress tests results are publicly disclosed. They list two benefits of disclosure: (i) enhanced market discipline; (ii) enhanced supervisory discipline. Our model is based on another benefit, namely the unfreezing of the credit market. They list four costs of disclosure: (i) disclosure might prevent risk sharing through Hirshleifer (1971)’s effect, which is the focus of Goldstein and Leitner (2013); (ii) improving market discipline is not necessarily good for ex-ante incentives; (iii) disclosure might trigger runs; (iv) disclosure might reduce the ability of regulators to learn from market prices, as in Bond, Goldstein, and Prescott (2010). Our
model is based on cost (iii).

3 Model

3.1 Technology and Preferences

The economy is populated by a continuum of households, a continuum mass 1 of financial intermediaries (banks), and a government. There are three dates, \( t = 0, 1, 2 \). Figure 1 summarizes the timing of decisions in the model, which are explained in detail below.

Households
Households are risk-neutral and their utility depends only on consumption at \( t = 2 \). They receive an endowment \( \omega \) at \( t = 1 \). At periods 0 and 1 they have access to a storage technology that pays one unit of consumption at time 2 per unit invested. There is no discounting. This allows us to treat total output at time 2 (which equals total consumption) as the measure of welfare that the government seeks to maximize.

Banks
Banks are indexed by \( j \in [0, 1] \) and may be of either good \((g)\) or bad \((b)\) type. A bank’s type is private information, but agents in the economy have common beliefs about the probability \( s_j \) that a bank \( j \) is of good type, for \( j \in [0, 1] \).\(^3\) The distribution of these beliefs in the population is summarized by the distribution function \( F(s) \), with \( \mathbb{E}[s] = \bar{s} \). We assume that \( \bar{s} \) is also the true (physical) fraction of good banks in the economy and is known to all agents.

Banks start with existing assets and liabilities, which can be thought of as any type of short-term demand liabilities (demand deposits, money market funds, repo, etc.), but which we refer to as deposits for simplicity. Legacy assets deliver a payoff \( a = A^i \) for \( i \in \{g, b\} \) at \( t = 2 \). The short-term demand liabilities entitle a depositor to \( D > 1 \) at \( t = 2 \) or their face value of 1 if withdrawn earlier. We impose the following ordering

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\(^3\)To economize on language we will refer to “the belief that agents have about bank \( j \)” as “bank \( j \)’s belief”. 
Assumption 1  Good banks are fundamentally safe, bad banks are fundamentally risky.

\[ A^g > D > A^b > 0, \]

This assumption implies that legacy assets of good banks are large enough to cover liabilities, but those of bad banks are not. Demand deposits are senior to any other claims on the bank, and may be withdrawn at any time. This induces a maturity mismatch problem, and makes banks vulnerable to runs.

Banks have access to a liquidation technology that yields \( \delta \in [0, 1] \) units of the consumption good per unit of asset liquidated. The liquidation value of assets is \( \delta A^i \) for \( i \in \{ g, b \} \). In the event of a run, banks use this liquidation technology to meet depositors’ demand for funds.

At \( t = 1 \), banks receive investment opportunities. All new investments cost the same fixed amount \( k \) and deliver random income \( v \) at \( t = 2 \), which does not depend on the type. Investment income is \( v = V \) with probability \( q \) and 0 with probability \( 1 - q \).

3.1.1 Government

The government in our model has access to three policies: a disclosure technology (via an asset quality review, for instance)\(^4\) that can reveal information about a bank’s type, and two types of fiscal intervention. The government can provide deposit insurance to prevent runs on banks, and it can provide loans directly to banks in the \( t = 1 \) credit market (equivalently, provide credit guarantees). To fund these fiscal interventions, the government borrows in international markets at the storage rate. At \( t = 2 \), borrowing is repaid in full and the government raises distortionary taxes to pay for the costs of programs.

We summarize the information set of the private sector after disclosure by a posterior distribution \( H(s) \) of beliefs about banks’ types. The advantage of disclosure is that changing beliefs about banks’ types may mitigate adverse selection in credit markets; as we explain below, this may come at the cost of triggering costly runs on banks. We assume the disclosure technology is available at \( t = 0 \).

Fiscal interventions are described in greater detail in section 6. To pay for costs arising from fiscal interventions, the government levies distortionary taxes at \( t = 2 \). We assume that the deadweight costs of taxation are quadratic and scaled by a parameter \( \gamma \). Denoting by \( \Psi \) the costs of fiscal interventions, the total welfare loss from taxation is \( \gamma \Psi^2 \).

3.1.2 Runs on Deposits at \( t = 0 \)

Demand depositors can withdraw their deposits from banks \( t = 0 \). Before \( t = 2 \), when asset payoffs are realized, banks have to liquidate assets in order to pay depositors that withdraw using an inefficient liquidation technology\(^4\) Note that without aggregate uncertainty there is no meaningful distinction between a stress test and an asset quality review.
that yields $\delta A^i$ per unit of asset liquidated. To simplify the analysis, we assume that banks that make use of this technology lose the investment opportunity at $t = 1$.

We denote by $\lambda$ the fraction of assets that is liquidated and $x$ the fraction of depositors in a given bank that run. If a fraction $\lambda$ of a bank’s assets are liquidated, the bank generates $\lambda \delta A^i$ at the time of liquidation, and $(1 - \lambda)A^i$ at $t = 2$.

We assume that, under a full run, good banks are safe and bad banks are not

**Assumption 2** Good banks are safe even under a full run, bad banks are not

\[ \delta A^g > 1, \quad \delta A^b < 1 \]

Consider the decision problem of a depositor in a bank that is known to be good. Withdrawing early yields 1 with certainty even if every other depositor runs. Waiting yields the minimum of the promised payment $D$ and a pro-rata share of the residual value of the bank:

\[ \min \left( D, \frac{(1 - \lambda)A^g}{1 - x} \right) \]

When a full run occurs, $x = 1$ and $\lambda = \frac{1}{\delta A^g} < 1$, so the above expression is always equal to $D$. The implication is that even if every other depositor runs, a depositor prefers to wait because $D > 1$, so the unique equilibrium for a bank known to be good is no run, $x = 0$ and $\lambda = 0$. For bad banks, we have that $\delta A^b < 1$, so $\lambda = 1$ when $x = 1$. That is, the bank has no assets left to repay depositors who decide to wait in case of a full run. This means that a full run is an equilibrium.

Since the type of a bank is private information, the run decision is a function of the belief about the quality of a bank. Clearly, if $s_j = 1$, no run is the only equilibrium. It is possible to derive a threshold belief $s^R$ above which no run is the unique equilibrium. This threshold must be such that depositors with this belief are indifferent between running and waiting even if every other depositor runs. This indifference condition is

\[ s + (1 - s) \delta A^b = s D \]

Rearranging,

\[ s^R = \frac{\delta A^b}{D + \delta A^b - 1} \]

For beliefs in the set $[0, s^R]$ multiple equilibria exist. We follow a common approach in the literature on equilibrium selection in models of bank runs (for example, Cooper and Ross (1998), ?) and use the realization of an exogenous sunspot variable as an equilibrium selection device. Let $\sigma \sim G$ with support $[0, 1]$ be a random variable that defines a bank $s^\sigma$ given by
such that all banks with beliefs $s_j < s^r$ suffer a full run and banks above this cutoff are spared from runs.

We do not explicitly model households with liquidity shocks that motivate the existence of deposit contracts in the first place. As a result, we cannot address the question of when a planner, assuming that it could, would choose to suspend convertibility. This is a well studied issue and the trade-offs are well understood (see Gorton (1985)). When liquidity demand is random, suspending convertibility is socially costly. So we assume that these costs are large enough that the government prefers to guarantee deposits. Note also that “deposits” in the model include short term wholesale funding, whose suspension would be difficult to implement in any case. As shown by Allen and Gale (2000), random liquidity shocks across regions/banks motivate the creation of an interbank market where banks can insure against these shocks. While the interbank market is welfare enhancing, it creates scope for the bank runs to become contagious across different banks.

### 3.1.3 Borrowing Contracts at $t = 1$

At $t = 1$, banks do not have any cash and need to borrow $l$ to take advantage of the investment opportunity. As is standard in the security design and corporate finance literature, we assume that only total income at time 2, $y = a + v$ is contractible

$$y(i) = a + i.v$$

where $i = 1$ if the bank invests and $i = 0$ otherwise. The amount that banks need to borrow to invest is $l = k \cdot i$. This new borrowing is junior to deposits. Letting $r$ denote the (gross) interest rate between periods 1 and 2, we have the following payoffs for long term debt holders (depositors), new lenders (at $t = 1$) and equity holders, respectively

$$y^D = \min(a + v \cdot i, D)$$
$$y^l = \min(a + v \cdot i, rl)$$
$$y^e = a + v \cdot i - y^l - y^D$$

Finally, we assume that new projects have positive NPV, and that households receive an endowment $\omega$ at time 1 that is enough to sustain full investment.

**Assumption 3** The investment project has positive NPV

$$\mathbb{E}[v] > k$$
Assumption 4  *Full investment is feasible*

\[ \omega > k \]

4  **Equilibrium**

In this section, we look at the credit market equilibrium at \( t = 1 \), which may feature adverse selection. We then proceed to describe first-best welfare, as well as welfare that results from the decentralized equilibrium without any sort of intervention.

4.1  **Equilibrium at time 1**

At \( t = 1 \), all banks with the belief \( s_j \) have the opportunity to borrow from a competitive and anonymous credit market.\(^5\) The equilibrium is characterized by a threshold belief \( s^I \) such that good banks with a lower belief do not participate in the credit market due to adverse selection, and all banks with higher beliefs borrow and invest.

Good banks find it profitable to borrow at rate \( r \) and invest if and only if\(^6\)

\[
A^g - D + qV - rk \geq A^g - D
\]

This constraint implies a maximum interest rate \( r^g \) above which good banks do not participate in the credit market:

\[
r \leq r^g \equiv \frac{qV}{k} \tag{2}
\]

Similarly, bad banks earn \( q(V - D + A^b - rk) \) if they invest, and 0 otherwise (since their assets are insufficient to repay senior depositors, in case of no investment or project failure), so they invest if and only

\[
q(V - D + A^b - rk) \geq 0
\]

This implies:

\[
V - D + A^b \geq rk,
\]

So if the equilibrium interest rate is such that bad banks find it profitable to invest, junior investors are repaid only if the project succeeds, which happens with probability \( q \). Since the outside option for junior investors is zero net return storage, the rate of return on the junior debt of bad banks is \( q^{-1} \).

Our interest is in studying situations where the information asymmetry in this economy induces adverse selection in the \( t = 1 \) credit market, creating a role for government interventions via information disclosure or credit market

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\(^5\)The credit market is competitive and anonymous for each \( s_j \); lenders can differentiate between banks with different beliefs, which can be thought of as accessing different credit markets.

\(^6\)We assume that \( A^g \) is large enough that the junior debt taken on by good banks to finance the investment opportunity is safe: \( A^g - D > rk \).
policies (as in Philippon and Skreta (2012)). This requires that the fair interest rate when only bad types invest, \( q^{-1} \), exceeds the maximum interest rate at which good types are willing to invest, which is equivalent to imposing \( q \leq \sqrt{\frac{k}{V}} \). Furthermore, since the liabilities of bad banks in our model are risky, it may be the case that, even in the absence of asymmetric information, underinvestment (by bad banks instead of good banks) occurs in equilibrium due a debt overhang problem as in Philippon and Schnabl (2013). For most of the paper we ensure that this is not the case by imposing \( q \geq \frac{k}{V - (D - Ab)} \).

**Assumption 5**  The equilibrium (without government intervention) of the \( t = 1 \) credit market features adverse selection but no debt overhang.

\[
\frac{k}{V - (D - Ab)} < q < \sqrt{\frac{k}{V}}
\]

Note that \( r = q^{-1} \) and \( i(g) = 0 \) is always a possible equilibrium of the credit market: equilibrium multiplicity is a feature of these models. We rule this out by assuming that, in case multiple equilibria exist, the best pooling equilibrium is selected.\(^7\)

If both good and bad types invest for a certain belief \( s_j \), the interest rate must satisfy the break-even condition for lender (whose outside option is zero net return storage)

\[
k = s_j r_j k + (1 - s_j) qr_j k
\]

this can be rearranged to yield

\[
r_j = \frac{1}{s_j + (1 - s_j) q}
\]

Note that for good types to invest, the interest rate must satisfy equation (2). Equating good banks’ participation constraint with lenders’ break-even constraint we can define a threshold posterior \( s^l \) such that above this threshold all banks invest and below it only bad banks do so:

\[
s^l = \frac{k}{s_j + (1 - s_j) q} - q
\]

To summarize, the credit market equilibrium can be characterized as follows

- For banks with \( s_j < s^l \), only bad types invest and the interest rate is \( r_j = \frac{1}{q} \)
- If \( s_j > s^l \), both good and bad types invest and the interest rate is \( r_j = \frac{1}{s_j + (1 - s_j) q} \)

\(^7\)When we consider credit market interventions, this assumption is without loss of generality because the government would always be able to costlessly implement the best pooling by setting the interest rate appropriately.
4.2 Welfare at time 2

At period 2, payoffs from long-term assets, investment, deposits and storage are realized. The government repays its \( t = 0,1 \) borrowing by levying distortionary taxes that entail a real resource cost.

Recall that all banks with \( s \geq s^I \), as defined in equation (3) invest regardless of their type, and all banks with \( s < s^\sigma \), as defined in equation (1) suffer a bank run. If \( s^\sigma \geq s^I \), then all banks that are potentially subject to adverse selection suffer a run – bank runs effectively clean the market from adverse selection. To prevent this from occurring, and to make the problem interesting, we make the following assumption

**Assumption 6** The threshold for bank runs is strictly smaller than the threshold for full investment.

\[
s^\sigma \leq s^R < s^I
\]

\[
\frac{\delta A^b}{D + \delta A^b - 1} < \frac{k - q}{1 - q}
\]

Since we assume that households are risk-neutral, aggregate welfare coincides with aggregate output. Given a posterior \( H \), sunspot \( \sigma \) and government intervention \( \Psi \), welfare is

\[
W(\sigma, \Psi) = \omega + \int_{s^I}^{1} [sA^g + (1 - s)A^b + qV - k] dH (s)
\]

\[
+ \int_{s^\sigma}^{s^I} [sA^g + (1 - s)A^b + (1 - s)(qV - k)] dH (s)
\]

\[
+ \int_{0}^{s^\sigma} \delta [sA^g + (1 - s)A^b] dH (s)
\]

\[
- \gamma \Psi^2
\]

The first term is households’ period 1 endowment. The second term corresponds to the total output generated by banks that do not suffer a run or adverse selection in the credit market. The third term corresponds to the output of banks that do not suffer a run but face suboptimally low investment due to adverse selection, and the fourth term is the value in liquidation of the assets of banks that suffer full runs. The final term is the deadweight loss of taxation.

4.2.1 First-Best Welfare

New projects have positive net present value, bank runs entail costly asset liquidation and taxation is distortionary. This means that in the first-best equilibrium, every bank invests and there is no distortionary taxation. First-best welfare is then

\[
W^{FB} = \omega + \bar{s}A^g + (1 - \bar{s})A^b + qV - k
\]

Proposition 1 summarizes the equilibrium of the model without government intervention.
Proposition 1. With no government intervention, the private equilibrium is characterized by a distribution of beliefs $H(s)$ and threshold beliefs $s^\sigma$ and $s^I$ such that:

1. All banks $j$ with beliefs $s_j < s^\sigma$ suffer full runs, liquidate their assets and do not invest at $t = 1$,

2. All banks with beliefs $s_j \in [s^\sigma, s^I)$ are spared from runs, but good banks with beliefs in this range do not invest in the new project due to adverse selection,

3. All banks with beliefs $s_j \in [s^I, 1]$ borrow and invest in the new project.

Welfare is described by equation (4).

5 Disclosure

The government may find it optimal to disclose information about banks’ quality, since reducing information asymmetries can mitigate adverse selection. Disclosure, however causes bank runs, and the net gains from this policy depend on the balance between the benefits of increasing investment and the costs of banks runs.

We model disclosure as the choice of a posterior distribution of beliefs $H(s)$. Specifically, since $s$ is a proportion, it is convenient to work with beta distributions. We characterize the prior $F(s)$ as a beta distribution with shape parameters $\alpha_0$ and $\beta_0$ and the posterior as having shape parameters $\alpha_1$ and $\beta_1$.

It is useful to characterize the full transparency and opacity benchmarks in our model. Agents in our model have a belief $s_j$ associated with each bank $j$. When information is perfect, investors know exactly which banks are good ($s_j = 1$) and which ones are bad, so $s_j = \{0, 1\}, \forall j$. In this case, the PDF $f(s)$ has mass only at 0 and 1. In particular, $F(0) = 1 - \bar{s}$, the mass of bad banks (and $F(1) - F(0) = \bar{s}$, the mass of good banks). In contrast, the maximum opacity benchmark is the situation where good and bad banks appear identical to outsiders. This situation is described by $s_j = \bar{s}, \forall j$, and the probability density function (PDF) is a mass point at $\bar{s}$.

As mentioned, we restrict our attention to beta posteriors. For the posterior to be consistent with Bayesian updating on the part of the depositors and lenders, it must satisfy the following condition:

\[
\int s dH(s) = \frac{\alpha_1}{\alpha_1 + \beta_1} = \frac{\alpha_0}{\alpha_0 + \beta_0} = \int s dF(s) = \bar{s}
\]

That is, the mean quality of a bank according to the posterior must coincide with the mean quality of a bank according to the prior, which in turn equals the true (physical) mean quality of a randomly sampled bank. This condition is very similar to the Bayesian Plausibility constraint that is featured in the Bayesian persuasion literature (see ? and Gick and Pausch (2013), for example). If this condition is violated, the posterior is not credible. This

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8This is done for expositional purposes only; it is also possible to characterize optimal policies in the case in which the regulator is free to choose any posterior distribution, see Appendix B.
condition imposes that any posterior chosen by the regulator must satisfy

\[ \alpha_1 = \frac{\bar{s}}{1 - \beta_1} \]

So that the optimal disclosure problem simplifies to choosing a single variable, \( \beta_1 \). In this environment, the full information benchmark is achieved by setting \( \beta_1 \to 0 \), while full opacity consists of setting \( \beta_1 \to \infty \).

Following the above logic, disclosure in our model has a simple geometric interpretation: through its choice of disclosure the government can reallocate mass (area below the curve) from the region \([s_R, s_I]\) into \([0, s_R]\) and \([s^I, 1]\). Without the restriction that the average belief under the posterior must equal \( \bar{s} \), the government would optimally put all the mass at or above \( s^I \), since this would eliminate runs and undo adverse selection. However in our model the government must trade off improving some banks’ beliefs and worsening others’ due to the Bayesian plausibility constraint in (6). This trade-off is illustrated in figure 2, that shows the shape of the belief distribution for different levels of disclosure and the cutoffs \( s_R \) and \( s^I \).

**Figure 2: Distribution of Beliefs \( s \)**

![Distribution of Beliefs](image)

*This figure illustrates the distribution of beliefs \( s \) for different values of disclosure \( d_1 \), for an economy with \( \bar{s} = 0.5 \). The investment and run cutoffs \( s^I \) and \( s_R \) are those implied by the parameterization presented in Appendix A.*
The government’s disclosure problem, before the sunspot \( \sigma \) has been realized, can be written as

\[
\max_{\beta_1 < \beta_0, \alpha_1 < \alpha_0} \mathbb{E}_\sigma[W(\sigma, 0)] = \omega + \int_0^1 [sA^g + (1 - s)A^b + qV - k] \, dH(s)
\]

\[
+ \int_0^1 \int_{s^\sigma}^{s^\sigma^*} [sA^g + (1 - s)A^b + (1 - s)(qV - k)] \, dH(s) \, dG(\sigma)
\]

\[
+ \int_0^1 \int_{s^\sigma}^{s^\sigma^*} \delta [sA^g + (1 - s)A^b] \, dH(s) \, dG(\sigma)
\]

subject to

\[H = \text{Beta}(\alpha_1, \beta_1)\]

\[\alpha_1 = \frac{s}{1 - \beta_1}\]

where \( F(s) \) is \( \text{Beta}(\alpha_0, \beta_0) \). To solve the problem, we focus on \( s = 0.5 \), so that \( \alpha_1 = \beta_1 \). For simplicity, we refer to \( d_1 = \beta_1^{-1} \) as the level of disclosure. \( d_1 \to 0 \) corresponds to zero disclosure, while \( d_1 \to \infty \) is full disclosure.

Figure 3 provides an example of the effect on the magnitude of runs, adverse selection and welfare of the government’s choice of \( d_1 \). In this example, we set the sunspot as following a two-point distribution: \( \sigma = 1 \) with probability \( p \) and \( \sigma = 0 \) with probability \( 1 - p \). The figure plots the expected number of banks suffering from underinvestment, and bank runs. As disclosure increases, the proportion of banks that suffer runs increases and the proportion of banks in the adverse selection region decreases, as we would expect. The behavior of panel suggests that the disclosure problem is non-convex: expected welfare as a function of disclosure is U-shaped, with the extreme cases of no disclosure and full disclosure being optimal.

6 Fiscal Interventions

In this section, we describe in greater detail the fiscal interventions that the government can use to mitigate adverse selection and bank runs. We start by analyzing pure fiscal interventions separately in the absence of any sort of information disclosure, and we proceed to analyze the two fiscal interventions jointly.

6.1 Credit Bailout: Optimal Intervention to Unfreeze Credit Market without Disclosure

In the region where banks do not suffer runs, but in which there is underinvestment due to adverse selection, \( s_j \in [s^\sigma, s^I] \), the government can promote full investment by offering a credit subsidy to banks. This consists in setting the interest rate \( r_j = r^g = \frac{qV}{k} \), so that good banks which would otherwise not do so are willing to invest. For any bank with belief \( s_j \), the policy consists of either setting \( r_j = r^g \) or doing nothing, since (as explained below) setting \( r \in \left[ r^g, \frac{1}{\bar{q}} \right] \) is costly on average for the government and does not contribute to mitigating adverse selection. Setting \( r_j < \frac{qV}{k} \) is also costly and cannot increase investment further.
This figure displays (the expectation over the run sunspot of) endogenous variables as a function of the level of disclosure, $d_1$ (log scale). The sunspot follows a two-point distribution, being equal to 1 w.p. $p$ and 0 w.p. $1 - p$. The first panel plots the expected number of banks that suffer from adverse selection, $E[H(s^I) - H(s^R)]$; the second panel plots the expected number of banks that are subject to runs, $E[H(s^R)]$; the third panel plots expected welfare as in (7).

For a particular class of banks with belief $s_j = s$, the cost of implementing the program is

$$s(k - r^g k) + (1 - s)(k - qr^g k) = s(k - qV) + (1 - s)(k - q^2 V)$$

the cost is strictly positive as long as $s \leq s^I$. The net marginal benefit of implementing this program is given by

$$s(qV - k)$$

Note that the benefit is increasing in $s$, while the costs are decreasing in $s$. Alternatively, the benefits are decreasing in $1 - s$ and the costs increasing in $1 - s$. This implies that the government will adopt a threshold rule $s^k$, implementing the program for all banks with beliefs in the set $[s^k, s^I]$. The total cost of such a program is

$$\Psi^k = \int_{\max(s^k, s^\sigma)}^{s^I} [k - qV(q(1 - s) + s)] \, dH(s)$$
And the welfare of implementing a given threshold $s^k$ is

$$W(\sigma, \Psi^k) = \omega + \int_{\max(s^k, s^\sigma)}^{1} \left[ sA^g + (1 - s)A^b + qV - k \right] dH(s) \tag{8}$$

$$+ \int_{\min(s^k, s^\sigma)}^{s^k} \left[ sA^g + (1 - s)A^b + (1 - s)(qV - k) \right] dH(s)$$

$$+ \int_{0}^{s^\sigma} \delta \, \left[ sA^g + (1 - s)A^b \right] dH(s) - \gamma(\Psi^k)^2$$

The government solves the problem

$$\max_{s^k} W(\sigma, \Psi^k)$$

And the following first order condition yields the threshold $s^k$

$$s^k (qV - k) = 2\gamma \Psi^k \left[ k - qV(s^k + q(1 - s^k)) \right]$$

If $s^k < s^\sigma$, the government offers credit guarantees to banks in $[s^\sigma, s^f]$ – interventions below $s^\sigma$ are ineffective since banks with belief $s_j < s^\sigma$ suffer full runs and lose the investment opportunity.

### 6.2 Deposit Guarantees

The government may also intervene to prevent liquidation by banks that are susceptible to runs (those with beliefs in $[0, s^\sigma]$). Preventing runs on these banks is desirable both because liquidation is costly in itself, and also because banks that are run on are unable to invest at $t = 1$.

To prevent runs, the government announces deposit guarantees for banks with beliefs $s$. These guarantees work as follows: the government guarantees to repay depositors of all banks in class $s$ the contractual deposit amount $D$ at $t = 2$. With probability $\rho$, the guarantee is ineffective, as the bank liquidates its assets, but the government still has to pay $D$ at $t = 2$. With probability $1 - \rho$, the guarantee succeeds and the government prevents bank asset liquidation. In either case, the government effectively purchases the deposit contract: it commits to pay $D$ to the depositors, and demands $D$ from the bank. As before, some banks may be unable to repay their senior debt, in which case the program is costly for the government.

The cost of guaranteeing deposits for bank with belief $s$ is then

$$\text{effective} : \quad (1 - \rho)[D - sD - (1 - s)(qD + (1 - q)A^b)]$$

$$\text{ineffective} : \quad + \rho [D - s \min(D, \delta A^g) - (1 - s)\delta A^b]$$

If the guarantee is effective, the government pays $D$ to depositors and, with probability $s$ the bank is good in which case the program breaks even (since good banks can always repay $D$ if they are not liquidated), while with
probability $1 - s$ the bank is bad and is only able to repay with probability $q$ (recall that bad banks always invest). If the guarantee is ineffective, the government still has to repay $D$; if the bank is good, the government is able to recover $\min(D, \delta A^g)$, while if the bank is bad, the government only recovers $\delta A^b < A^b < D$. This establishes that, for any $s$, the costs for this program are always non-negative. Furthermore, the costs are decreasing in $s$, or increasing in $1 - s$, meaning that banks with lower $s$ are more expensive to save.

The net benefit of guaranteeing bank with belief $s$, on the other hand, is

$$(1 - \rho)[(1 - \delta)(sA^g + (1 - s)A^b + (1 - s)(qV - k))]$$

where we use the fact that $s^R \leq s^I$, so that any bank that suffers a run is potentially subject to adverse selection.

We make the following technical assumption that ensures that benefits are increasing in $s$, or decreasing in $1 - s$,

**Assumption 7** *Marginal benefits of saving bank with belief $s$ are increasing in the belief $s$*

$$(1 - \delta)(A^g - A^b) \geq qV - k$$

Since costs are increasing, and benefits are decreasing in $1 - s$, it is optimal for the government to adopt a threshold policy $s^d$ such that all banks with $[s^d, s^\sigma]$ are supported by the deposit guarantee program and potentially saved from a run.

The total cost of the deposit guarantee policy is

$$\Psi^d = \int_{s^d}^{s^\sigma} (1 - \rho)(1 - q)(1 - s)(D - A^b) + \rho \left( s \max(D - \delta A^g, 0) + (1 - s)(D - \delta A^b) \right) \, dH(s)$$

The first term under the integral is the cost of a successful guarantee – in this case, only guarantees of bad banks are costly. If the guarantee fails, however, guaranteeing a good bank may also be costly if $\delta A^g < D$ (if the opposite is true, we assume the bank’s owners are entitled to the residual value in liquidation).

Welfare with the deposit guarantee is

$$W(\sigma, \Psi^d) = \omega + \int_{s^d}^{s^\sigma} \left[ sA^g + (1 - s)A^b + qV - k \right] \, dH(s)$$

$$+ \int_{s^d}^{s^\sigma} \left[ sA^g + (1 - s)A^b + (1 - s)(qV - k) \right] \, dH(s)$$

$$+ \int_{s^d}^{s^\sigma} (1 - \rho) \left[ sA^g + (1 - s)A^b + (1 - s)(qV - k) \right] + \rho \delta [sA^g + (1 - s)A^b] \, dH(s)$$

$$+ \int_{0}^{s^d} \delta \left[ sA^g + (1 - s)A^b \right] \, dH(s) - \gamma(\Psi^d)^2$$

where the first term is the endowment; the second term is the surplus generated by banks that do not suffer adverse
selection and are not subject to runs; the second line is surplus generated by banks that do not suffer runs, but are subject to adverse selection; the third line is the surplus generated by banks that are supported by the deposit guarantee program; while the final line is the surplus from non-supported banks and costs of intervention.

The government solves,

$$\max_{s^d} W(\sigma, \Psi^d)$$

The solution to this problem characterizes the optimal deposit insurance as a threshold $s^d$. The first-order condition is

$$(1 - \rho)[(1 - \delta)(s^dA^g + (1 - s^d)A^b) + (1 - s^d)(qV - k)]$$

$$= 2\gamma \Psi^d[(1 - \rho)(1 - q)(1 - s^d)(D - A^b) + \rho (s^d \max (D - \delta A^g, 0) + (1 - s^d) (D - \delta A^b))]$$

Due to increasing marginal costs and decreasing marginal benefits, a solution exists $s^d \in [0, s^\sigma]$.

6.3 Combining deposit insurance and credit guarantees

To complete our description of equilibrium with fiscal intervention, we characterize the welfare function when the government can use both policies.

$$W(\sigma, \Psi^{d+k}) = \omega + \mathcal{A}(s^\sigma, s^d) + \mathcal{I}(s^\sigma, s^k, s^d) - \gamma(\Psi^{d+k})^2$$

(10)

where $\mathcal{A}(s^\sigma, s^d)$ is the welfare component that pertains to surplus generated by assets and liquidation

$$\mathcal{A}(s^\sigma, s^d) = \int_{s^\sigma}^{1} [sA^g + (1 - s)A^b] \, dH(s)$$

$$+ \int_{s^\sigma}^{s^d} [1 - \rho(1 - \delta)][sA^g + (1 - s)A^b] \, dH(s)$$

$$+ \int_{0}^{s^d} \delta[sA^g + (1 - s)A^b] \, dH(s)$$

the first line corresponds to asset surplus generated by banks that suffer no run; the second line is asset surplus for banks supported by the deposit guarantee program; and the third line is the surplus generated by unsupported
banks that suffer a run. \( I(s^\sigma, s^k, s^d) \) is the welfare component related to surplus generated by investment

\[
I(s^\sigma, s^k, s^d) = \int_{\max(s^k, s^\sigma)}^{1} (qV - k) \, dH(s) + \int_{\min(s^k, s^\sigma)}^{s^k} (1 - s)(qV - k) \, dH(s) + \int_{\max(s^\sigma, s^d)}^{s^\sigma} (1 - \rho)(qV - k) \, dH(s) + \int_{s^d}^{s^\sigma} (1 - s)(1 - \rho)(qV - k) \, dH(s)
\]

the first line is investment by classes that suffer no adverse selection and no run; the second line is investment by classes that suffer adverse selection and no run; the third line is investment by classes that are supported by the deposit guarantee and face no adverse selection; the fourth line is investment by classes that are supported by the deposit guarantee and face adverse selection. Finally, total costs of intervention are given by

\[
\Psi^{d+k} = \int_{s^d}^{s^\sigma} (1 - \rho)(1 - q)(1 - s)(D - A^b) + \rho(s \max(D - \delta A^g, 0) + (1 - s)(D - \delta A^b)) \, dH(s)
\]

\[
+ \int_{s^d}^{s^d} [k - qV(q(1 - s) + s)] \, dH(s)
\]

\[
+ \int_{s^d}^{s^\sigma} (1 - \rho)[k - qV(q(1 - s) + s)] \, dH(s)
\]

where the first term is the cost of the deposit guarantee; the second line is the cost of the credit program for banks that are not supported by the deposit guarantee; and the third line is the cost of the credit program for banks that are also supported by the deposit guarantee.

The optimal fiscal policy is the solution to

\[
\max W(\sigma, \Psi^{d+k})
\]

The first-order conditions for the government are, for \( s^d \)

\[
1[s^d \leq s^\sigma](1 - \delta)(s^d A^g + (1 - s^d)A^b) + (1 - s^d)1[s^d \leq s^k](qV - k)
\]

and for \( s^k \)

\[
1[s^k \geq s^\sigma] + (1 - \rho)1[s^d \leq s^k \leq s^\sigma]2\gamma \Psi^{d+k} \left[ k - qV(q(1 - s^k) + s^k) \right]
\]

\[
= 1[s^k \geq s^\sigma] + (1 - \rho)1[s^d \leq s^k \leq s^\sigma](qV - k)s^k
\]
The government never finds it optimal to set $s^d > s^k$, and setting $s^d > s^\sigma$ is weakly dominated. We can then write the first-order conditions in a simpler form, as

$$
2\gamma \Psi^{d+k}[(1 - \rho) (1 - q) (1 - s^d) (D - A^l) + \rho (s^d \max (D - \delta A^g, 0) + (1 - s^d) (D - \delta A^l))] = (1 - \rho) [(1 - \delta)(s^d A^g + (1 - s^d) A^l) $$

where $\Psi^{d+k}$ is given in equation (11). The above system can then be solved for the two unknowns $(s^d, s^k)$. Figure 4 illustrates numerically the behavior of the optimal policies (both separately and jointly), for a uniform posterior, $H = \mathcal{U}(0, 1)$, as fiscal capacity decreases, $\gamma \uparrow$.

7 Disclosure with Fiscal Interventions

Finally, we consider the equilibrium with intervention when the government has access to disclosure and both fiscal policies.

In addition to the thresholds $s^d$ and $s^k$ the government also chooses posterior beliefs. As described above, we model disclosure as a choice of a single parameter of a beta distribution. Formally, the government solves

$$
\max_{d_1, d_1 > d_0} \mathbb{E}_{\sigma} \max_{s^d, s^k} W(\sigma, \Psi^{d+k})
$$

Where the welfare function is given in equation 10 and the posterior is restricted to be $H(s) = \text{Beta}(d^{-1}, d^{-1})$. Figure 5 illustrates comparative statics with respect to $\gamma$ for fixed levels of disclosure $d_1$, and optimal fiscal interventions. The lines represents different levels of disclosure: no disclosure $(d = 0)$, intermediate $(d = 1$, a uniform posterior) and full $(d = \infty)$. As before, full disclosure features no adverse selection, but a greater number of banks suffering a run, while no disclosure features no bank runs, but more adverse selection. The intermediate disclosure case features intermediate levels of bank runs and adverse selection. The final panel plots welfare: for the range of parameters that is considered, the disclosure problem is non-convex, and the optimal disclosure policy (the upper envelope of the three lines) consists of either no or full disclosure. For high levels of fiscal capacity, $\gamma \to 0$, full disclosure is optimal – this is a mechanical consequence of the fact that the deposit guarantee policy has a strictly positive failure rate, $\rho > 0$. As fiscal capacity is high, the government is able to achieve first-best welfare under no disclosure by using the credit policy. First-best is not achieved with full disclosure since the deposit guarantee policy fails in a fraction $\rho$ of the cases. For low levels of fiscal capacity, full disclosure is optimal, but as fiscal capacity decreases, $\gamma \uparrow$, no disclosure becomes the optimal policy. This is related to the marginal costs of each policy: deposit guarantees are, everything else constant, more costly per bank than credit assistance. This is because credit assistance is targeted at banks with intermediate beliefs $s$, while deposit guarantees tend to be targeted at banks that are more likely to be bad, with low $s$. For either policy, assisting bad banks is costlier (for different reasons). The benefits of the
This figure plots (the expectation over the size of the run of) several variables as a function of fiscal capacity $\gamma$, for different regimes: the solid line is the no fiscal policy benchmark, the dashed line corresponds to credit bailouts only, the dotted line corresponds to credit guarantees only, and the dashed-dotted line corresponds to simultaneous use of the two policies. For a fixed level of fiscal capacity $\gamma$, and a fixed prior (which is set as Uniform, $s \sim U[0,1]$), the government solves for optimal fiscal interventions (deposit guarantees and credit market interventions). The first panel plots the expected number of classes that suffer adverse selection, $\mathbb{E}[F(s^k) - F(s^d)]$; the second panel plots the expected number of classes that suffer a run $\mathbb{E}[F(s^d)]$; the third panel shows the expected number of classes that are supported by the credit policy, $\mathbb{E}[F(s^I) - F(s^k)]$; the fourth panel shows the expected number of classes that are supported by the deposit policy, $\mathbb{E}[F(s^R) - F(s^d)]$; the fifth panel plots expected total spending, $\mathbb{E}[\Psi^d+\Psi^k]$; and the sixth panel plots expected welfare, $\mathbb{E}[W(\sigma,\Psi^d+\Psi^k)]$. 

Figure 4: Fiscal Policy
deposit guarantee program are also greater: for high fiscal capacity, costs are irrelevant, and so it is preferable to disclose and offer deposit guarantees. As $\gamma$ increases, the costs become more welfare-relevant than the benefits, and the government prefers not to disclose and simply offer credit bailouts.

8 To-do

1. Introduce contagion in the model

2. Disclosure and commitment

All of the above analysis is valid in the presence of any type of aggregate shocks, provided that they are commonly observed (and the prior $F$ is common knowledge).

This is not true, however, if the government has some sort of private information about the physical distribution of banks. If the government had private information regarding the physical mean, $\bar{s}$, for example, then disclosure can become time inconsistent.
This figure plots (the expectation over the size of the run of) several variables as a function of fiscal capacity $\gamma$, for different levels of disclosure $d$. Disclosure is defined as the inverse of the Beta distribution parameter: $d = \infty$ corresponds to a Beta distribution with parameter equal to zero (two mass points, at 0 and 1), for example. For a fixed level of fiscal capacity $\gamma$, and disclosure $d$, the government solves for optimal fiscal interventions (deposit guarantees and credit market interventions). The first panel plots the expected number of classes that suffer adverse selection, $E[H(s^I) - H(s^k)]$; the second panel plots the expected number of classes that suffer a run $E[H(s^d)]$; the third and fourth panels plot the expected masses of classes that receive credit market and deposit assistance, $E[H(s^I) - H(s^k)]$ and $E[H(s^R) - H(s^d)]$, respectively; the fifth panel plots total expected spending with both fiscal programs $E[\Psi_{d+k}]$; the sixth and final panel plots expected welfare $E[W(\sigma, \Psi_{d+k})]$.
This figure plots (the expectation over the size of the run of) several variables as a function of fiscal capacity $\gamma$, for the optimal disclosure level $d$. Disclosure is defined as the inverse of the Beta distribution parameter: $d = \infty$ corresponds to a Beta distribution with parameter equal to zero (two mass points, at 0 and 1), for example. For a fixed level of fiscal capacity $\gamma$, and disclosure $d$, the government solves for optimal fiscal interventions (deposit guarantees and credit market interventions). The first panel plots the expected mass of banks that suffer adverse selection, $E[H(s^d) - H(s^k)]$; the second panel plots the expected mass of banks that suffer a run $E[H(s^d)]$; the third panel plots total expected spending with both fiscal programs $E[\Psi_d + k]$; final panel plots expected welfare $E[W(\sigma, \Psi_d + k)]$. The shaded area corresponds to values of $\gamma$ for which zero disclosure ($d = 0$) is optimal and the unshaded areas to values of $\gamma$ for which full disclosure is optimal ($d = \infty$).
A Parameters used in examples

To generate the figures, we use the parametrization in the table below.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
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<tr>
<td>$A^g$</td>
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</tr>
<tr>
<td>$A^b$</td>
<td>Bad Asset</td>
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</tr>
<tr>
<td>$D$</td>
<td>Deposits</td>
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<tr>
<td>$V$</td>
<td>Project Payoff</td>
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<tr>
<td>$q$</td>
<td>Prob. Success</td>
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</tr>
<tr>
<td>$k$</td>
<td>Investment Cost</td>
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<tr>
<td>$\delta$</td>
<td>Recovery Rate</td>
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<tr>
<td>$\rho$</td>
<td>Failure Rate</td>
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</tr>
<tr>
<td>$p$</td>
<td>Prob. of Run</td>
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</tr>
</tbody>
</table>

B Optimal Posterior

We consider the arbitrary choice of the posterior distribution $H$ in the absence of fiscal policy. The planner solves a problem similar to the one studied by ?, and chooses the posterior distribution $H$ to maximize \textit{ex-ante} welfare (i.e. before aggregate uncertainty is realized) under two constraints:

1. Physical plausibility: the posterior distribution must be a well-defined cumulative distribution function

\[
\int h(s)ds = 1
\]

2. Bayesian plausibility: the mean of the posterior distribution must coincide with the mean of the prior distribution

\[
\int sh(s)ds = \int sf(s)ds = \bar{s}
\]

Note that any deviation from the above constraints violates physical features of the problem/model, and leads to the resulting posterior being ignored by the agents.

A different version of this problem is studied in a much more simplified framework by Gick and Pausch (2013). The authors apply Bayesian persuasion techniques to an environment characterized by a regulator that can produce a signal (and a posterior distribution) on the true state, and investors whose utility depend on whether the actions coincide with the true state or not [elaborate on this].

For simplicity, we assume that there is full commitment: the regulator chooses the posterior before aggregate uncertainty $\mathcal{E}$ is realized \(^9\), and commits to transmit the same posterior regardless of the realization of the random variables in $\mathcal{E}$. Clearly, commitment can be an issue only in the presence of aggregate uncertainty.

\(^9\) $\mathcal{E}$ may contain several sources of uncertainty. The sunspot $\sigma$ is a single element of this set, and will be the unique element in most of our analysis.
B.1 Pure Disclosure (no fiscal policy)

We start with the simplest case in which there is no fiscal policy. As before, \( W(\sigma, \Psi) \) is welfare under shocks \( \sigma \). That is, welfare is as in equation (4) with \( \Psi = 0 \) since we are abstracting from fiscal policy for now. The government solves

\[
\max_{H \in \Delta[0,1]} \mathbb{E}_{\sigma,H}[W(\sigma,0)]
\]

subject to

\[
\int sh(s)ds = \int sf(s)ds
\]

\[
\int h(s)ds = 1
\]

Let \( \lambda \) and \( \mu \) denote the Lagrange multipliers on the first (physical) and second (Bayesian) plausibility constraints, respectively. The planner takes first-order conditions pointwise with respect to \( h(s) \) for each \( s \in [0,1] \), where \( \int_0^1 h(x)dx = H(s) \). We restrict our attention to the family of distribution functions in the \([0,1]\) interval. Focusing on a two-point distribution for the sunspot as in the main text, the first-order condition with respect to \( h(s) \) is

\[
[sA^g + (1 - s)A^b][1 - p(1 - \delta)1(0, s^R)] + (qV - k)[1 - s1(s^R, s^I) - 1(0, s^R)(p + s(1 - p))] - \lambda s - \mu \leq 0
\]

Giving rise to the three FOCs

\[
s \in [s^I, 1] : s(A^g - A^b - \lambda) + qV - k - \mu \leq 0
\]

\[
s \in [s^R, s^I] : s[A^g - A^b - (qV - k) - \lambda] + A^b + qV - k - \mu \leq 0
\]

\[
s \in [0, s^R] : s[(A^g - A^b)(1 - p(1 - \delta)) - (1 - p)(qV - k) - \lambda] + (1 - p(1 - \delta))A^b + (1 - p)(qV - k) - \mu \leq 0
\]

We proceed to characterize the choice of the optimal posterior through a series of intermediate claims. For expositional simplicity, we refer to the FOC in the \([0, s^R]\) interval as the “first FOC”, the \([s^R, s^I]\) interval as the “second FOC” and the final interval \([s^I, 1]\) as the “third FOC”. For what follows, we always assume that

\[
s^R < \bar{s} < s^I
\]

Claim 1. The optimal posterior consists of mass points only.

Proof. We follow by contradiction, assuming that the posterior is continuous in open sets in each of the intervals.

1. Assume first that \( h(s) > 0 \) for a measurable set in the third interval \([s^I, 1]\). The third interval FOC must then
bind for any $s$ in the interval. Since this FOC is linear, we must have that

$$\lambda = A^g - A^b$$
$$\mu = A^b + qV - k$$

This means that the second FOC is equal to $-s(qV - k) < 0, \forall s \in [s^R, s^I]$, so no solution exists in this interval. For the first interval, the FOC is also negative everywhere. Then, the posterior density is only positive in points in the third interval. However, since $s^I > \bar{s}$, this violates Bayesian Plausibility and is not feasible.

2. Assume that $h(s) > 0$ for a measurable set in the second interval. Once again, since the FOC is linear, it must bind for any $s$ in the interval, implying that

$$\lambda = A^g - A^b - (qV - k)$$
$$\mu = A^b + qV - k$$

But, then, the third interval FOC is positive everywhere, contradicting optimality.

3. If $h(s) > 0$ for a measurable set in the third interval, the above argument applies, and this cannot be an optimal solution.

This allows us to conclude that there exists no open set $\mathcal{O} \subseteq [0, 1]$ such that $h(s) > 0, \forall s \in \mathcal{O}$, and so the posterior must be composed of mass points only.

Since the posterior is composed only of mass points and the FOCs are linear, there are four candidates to mass points: \{0, $s^R$, $s^I$, 1\}. The optimal posterior will be composed of a subset of these. We proceed by proving that 1 is not optimal, and that $s^I$ must always be optimal.

**Claim 2.** 1 is not a mass point, and a mass point must exist in [$s^I$, 1], so $s^I$ is a mass point.

**Proof.** We prove each of the claims in sequence:

1. 1 is not a mass point - This follows by contradiction, assume that $h(1) > 0$. Then, the FOC for the third interval must be increasing, and binding at $s = 1$. This implies that the slope is positive and the intercept negative

$$\lambda < A^g - A^b$$
$$\mu > A^b + qV - k$$

Since the posterior is composed only of mass points and the FOCs are linear, there are four candidates to mass points: \{0, $s^R$, $s^I$, 1\}. The optimal posterior will be composed of a subset of these. We proceed by proving that 1 is not optimal, and that $s^I$ must always be optimal.

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$$\lambda < A^g - A^b$$
$$\mu > A^b + qV - k$$
Since the FOC binds at 1, we also have that

\[ \lambda + \mu = A^g + qV - k \]

implying the following restrictions on the parameters

\[ \lambda \in [0, A^g - A^b] \]
\[ \mu \in [A^b + qV - k, A^g + qV - k] \]

Since the FOC for the third interval is negative at \( s^I \), so must the FOC for the second interval be (since the slope is strictly smaller and the intercept is the same). So, either there is no solution in this interval, or \( s^R \) is the only solution. Assume first that no solution exists; then, knowing that the intercept is negative, this must also be the case for the first interval FOC. Furthermore, the slope of the second FOC is strictly greater than the slope of the first interval FOC if and only if

\[ (1 - \delta)(A^g - A^b) > qV - k \]

which we have previously assumed (Assumption 7). So, if there is no solution in the second interval, there cannot exist a solution in the first interval either, as the FOC is negative everywhere. This makes 1 the only solution, violating Bayesian Plausibility. This means that \( s^R \) must then be a solution. The second FOC then binds at this point, and the multipliers can be solved for as

\[ \lambda = A^g - A^b + \frac{s^R}{1 - s^R}(qV - k) \]
\[ \mu = A^b + (qV - k) \left( \frac{1 - 2s^R}{1 - s^R} \right) \]

But this contradicts \( \lambda \leq A^g - A^b \), and the fact that the third interval FOC is increasing in \( s \). So \( s = 1 \) cannot be a mass point.

2. Assume now that no mass points exist in the third interval, \([s^I, 1]\). Then, the third interval FOC does not bind for \( \{s^I, 1\} \) (and, by convexity, for any point in between). Since the slope of the second FOC is strictly smaller than the slope of the third FOC, this means that the second FOC does not bind for \( s^I \) either. So, if a solution exists in the second interval, it is \( s^R \). But this means that \( s^R \) is the greatest solution, and \( s^R < \bar{s} \), violating Bayesian Plausibility.

Since the FOC is linear, 1 cannot be a solution, and a solution must exist in \([s^I, 1]\), we conclude that \( s^I \) must be a solution.
Since $s^I$ is a solution, the third FOC must bind at this point, thus the slope is negative and the intercept is positive.

$$\lambda > A^g - A^b$$

$$\mu < A^b + qV - k$$

Claim 3. At least one other mass point in $\{0, s^R\}$ exists.

Proof. Existence of at least one other mass point, smaller than $\bar{s}$, is required by Bayesian Plausibility. Since the slope of the second FOC is strictly smaller than that of the third interval FOC, the second FOC has a strictly positive intercept and negative slope. Since it cannot bind at $s^I$, it either binds at $s^R$, or it does not bind anywhere in this interval. Assume first that it binds at $s^R$. This allows us to derive closed forms for the multipliers

$$\lambda = A^g - A^b + \frac{s^R}{s^I - s^R}(qV - k)$$

$$\mu = A^b + (qV - k)\left(1 - \frac{s^I s^R}{s^I - s^R}\right)$$

Since we must have that $\mu \geq 0$, this requires the following parameter restriction

$$s^R \leq s^I \frac{1 + \frac{A^b}{qV - k}}{s^I + 1 + \frac{A^b}{qV - k}}$$

If this restriction is violated, $s^R$ cannot be a solution. Assume first that the restriction is valid. Then, since both slope and intercept are strictly smaller for the first interval FOC, it cannot bind at $s^R$. At most, it binds at $s = 0$. The FOC evaluated at $s = 0$ is

$$A^b(1 - p(1 - \delta)) + (1 - p)(qV - k) - \mu$$

This FOC either binds, in which case 0 is a solution, or is strictly negative. The former is generically not satisfied (as it relies on a parametric restriction). The latter requires that

$$s^R \leq s^I \frac{p \left(1 + \frac{(1-\delta)A^b}{qV - k}\right)}{s^I + p \left(1 + \frac{(1-\delta)A^b}{qV - k}\right)}$$

Since this condition is more binding, it is the relevant condition for $s^R$ to be a solution.

Assume now that the original condition is not satisfied, then

$$s^R > s^I \frac{1 + \frac{A^b}{qV - k}}{s^I + 1 + \frac{A^b}{qV - k}}$$
This means that the second condition is not satisfied either, and so we must have that \( s = 0 \) is a solution, or

\[
\mu = A^b(1 - p(1 - \delta)) + (1 - p)(qV - k)
\]

We can then use a parametric restriction to characterize the optimal posterior:

1. If \( s_R > s_I \frac{p\left(\frac{1 - (1 - \delta)A^b}{qV - k}\right)}{s_t + p\left(\frac{1 - (1 - \delta)A^b}{qV - k}\right)} \), the optimal posterior consists of mass points \( \{0, s^I\} \). The masses can be determined using Bayesian and physical plausibility

\[
h(0) \times 0 + h(s^I)s^I = \bar{s} \\
\therefore h(0) + h(s^I) = 1
\]

yielding

\[
h(s^I) = \frac{\bar{s}}{s^I} \\
h(0) = 1 - \frac{\bar{s}}{s^I}
\]

2. If \( s_R = s_I \frac{p\left(\frac{1 - (1 - \delta)A^b}{qV - k}\right)}{s_t + p\left(\frac{1 - (1 - \delta)A^b}{qV - k}\right)} \), the optimal posterior consists of mass points \( \{0, s^R, s^I\} \), with masses given by

\[
h(s^R) = \frac{\bar{s} - h(s^I)s^I}{s_R} \\
h(0) = 1 + h(s^I) \left( \frac{s^I}{s_R} - 1 \right) - \frac{\bar{s}}{s_R}
\]

for the greatest feasible \( h(s^I) \).

3. If \( s_R < s_I \frac{p\left(\frac{1 - (1 - \delta)A^b}{qV - k}\right)}{s_t + p\left(\frac{1 - (1 - \delta)A^b}{qV - k}\right)} \), the optimal posterior consists of mass points \( \{s^R, s^I\} \), and masses are

\[
h(s^I) = \frac{\bar{s} - s^R}{s^I - s^R} \\
h(s^R) = \frac{s^I - \bar{s}}{s^I - s^R}
\]
References


