Financial Distress and Endogenous Uncertainty

preliminary

François Gourio*

September 8, 2014

Abstract

What is the macroeconomic effect of having a substantial number of firms close to default? This paper studies financial distress costs in a model where customers, suppliers and workers suffer losses if their employer goes bankrupt. I show that this mechanism generates amplification of fundamental shocks through procyclical TFP and countercyclical labor wedge. Because the strength of this amplification depends on the share of firms that are in financial distress, it operates mostly in recessions, when equity values are low. This leads macroeconomic volatility to be endogenously countercyclical. The cross-sectional distribution of firms’ equity values affects directly aggregate macroeconomic volatility. Empirical evidence consistent with the model is provided.


Keywords: time-varying uncertainty, uncertainty shocks, distance-to-default, leverage.

1 Introduction

Economic recessions and financial crises sometimes lead a large number of firms close to default. This arises either because of contraction in credit supply, or because equity values fall and become more volatile, reducing the cushion protecting solvency.1 Intuitively, the fact that a nontrivial share of the nonfinancial corporate sector is close to default would seem to have significant negative macroeconomic effects. This paper is concerned with the modeling and measuring the macroeconomic costs of such financial distress.

In particular, the paper focuses on a specific channel: when a firm becomes close to default, it becomes harder to find and retain employees, suppliers and customers, as they worry that they would suffer losses if the firm goes under. The anticipation of these losses make it more costly for firms to operate, and

*Federal Reserve Bank of Chicago. Address: 230 South LaSalle Street, Chicago IL 60604. Email: francois.gourio@chicfrb.org, phone: (312) 322 5627. The views expressed here are those of the author and do not necessarily represent those of the Federal Reserve Bank of Chicago or the Federal Reserve System. I thank Gadi Barlevy, Jeff Campbell, Simon Gilchrist, Joao Gomes, Alejandro Justiniano, Amir Yaron, and Tao Zha for discussions as well as participants in several presentations for their feedback. I am especially grateful to Egon Zakrajsek for sharing his data.

1 See Atkeson, Eifeldt and Weil (2013) for a recent empirical analysis of the business cycle variation in the distance to insolvency.
hence leads directly to lower production and employment. The losses include direct missed payments, but also capture more broadly the costs of the loss of a working relationship - employees, customers and suppliers must look for another match, and they may lose some relationship-specific capital. Hence, the ex-ante perception of a risk of bankruptcy leads to a “default wedge” that disconnects marginal revenue from marginal cost. This wedge is endogenous, depend on the distribution of default risk in the economy, and is time-varying, becoming larger when more firms are close to default.

This financial distress mechanism stands in contrast to alternative mechanisms where default risk affects the economy chiefly by creating investment wedges. For instance, well-studied mechanisms include limited pledgeability - a smaller distance to default makes it more costly to raise external finance- or debt-overhang - a smaller distance to default reduces incentives for investment on the part of equityholders as debtholders stand to reap the gains of investment. The investment wedge, however, mostly affects investment. A key challenge in macroeconomic models is to generate large variation in output and employment, making the “default wedge” wedge more appealing than the investment wedge. In particular, I show that it can generate the appareance of a labor wedge, so that an econometrician that looks at the data through the lens of the neoclassical model would find an excessive employment contraction, making it look as if labor income taxes were countercyclical. As argued by Mulligan (1997), Hall (1997), and Chari, Kehoe and McGrattan (2005), this is a desirable feature of the data. Furthermore, the mechanism also reduces aggregate total factor productivity by allocating labor and inputs across firms in part based on their default likelihood rather than their productivity. As a result of these two wedges, financial distress amplifies macroeconomic fluctuations.

Moreover, this amplification effect operates only under certain circumstances, leading to nonlinearities. In goods times, when productivity or equity values are high, a small aggregate shock does not change substantially the likelihood of default of most firms, which remains very low. But in bad times, the same-size shock could be sufficient to increase significantly the likelihood of default of many firms, with larger aggregate effects. The time-varying elasticity implies time-varying macroeconomic volatility even if the underlying, fundamental shock is homoskedastic. As a result, financial distress can generate endogenously time-varying second moments. This is of interest in light of the large recent literature on uncertainty shocks, which for the most part takes the variation of uncertainty as exogenous.

There is a lot of anecdotal evidence that customer/supplier and employee relationships are deteriorated when firms become close to default. For instance, in November 2008, Circuit City, the second largest electronics retailer in the U.S., was forced to file for bankruptcy when news of its deteriorating financial position led its suppliers (such as Samsung, Sony, and other big electronic manufacturers) to...

---

2For instance, in the United States, employee wages have a priority in bankruptcy up to 10,000$, if they were earned in the past six months. The excess over 10,000$ is treated as a regular unsecured claim, payable in proportion to the assets recovered, after administrative expenses are paid, and possibly with a substantial delay. Furthermore, bonuses, sick and vacation days are lost if they were earned over 180 days ago, and may be hard to recover otherwise. Finally, while company pension plans benefit from the backstop of the Pension Benefit Guarantee Corporation, this guarantee is incomplete (e.g. capped).

3For some models with shocks to aggregate uncertainty, see Bloom (2009), Fernandez-Villaverde et al. (2010, 2013), or Gourio (2012). Studies that attempt to endogenize volatility include, among others, Bachmann and Moscarini, Berger and Vavra, Brunnermeier and Sannikov, Bianchi and Mendoza.
refuse extending credit for deliveries. These phenomena are likely frequent and important, because trade credit is large: in Compustat, the median account receivables/assets is around 0.15, versus 0.21 for debt/assets. The impact of bankruptcy risk on customers was an important element for Chrysler and GM - there was widespread concern that consumers would stop buying these manufacturers’ cars once they realized the companies were likely to file for bankruptcy, since product warranties and auto parts might not be available later. This motivated the U.S. Federal government to introduce the warranty guarantee program (see Hortacsu et al. (2012)). More systematic evidence has been provided by the corporate finance literature. For instance, Brown and Matsa (2012) find that (financial) firms in financial distress during the recent recession received fewer applications to job openings than did financially healthy firms. Moreover, the quality of the applicants was worse.

A key challenge for the mechanism is to generate significant macroeconomic impact in spite of the fairly low observed default rates. Two observations are important here. First, this mechanism applies not only to firms that are actually in default or restructuring their debt, but rather to all firms that have some significant likelihood of default. Second, for some purposes, the relevant default rate is the exit rate. For instance, an employee who is let go faces a loss from the firm discontinuing its operations, even if the firm exits without financial default. For instance, even if his wages and benefits are paid, the worker will spend time searching for a new job, which might not be as good, and there is some lost firm-specific human capital.

To illustrate the cyclicality of default and bankruptcies, figure 1 presents the time series of business chapter 7 and chapter 11 filings in the United States. Figure 2 presents the S&P corporate bond default rate.

1.1 Related Literature

A large literature in corporate finance studies the costs of financial distress (e.g., Opler and Titman (2004)). A motivation for this literature is that it is difficult to understand the low observed leverage of most nonfinancial firms in light the large tax savings that firms could generate, and the relatively small likelihood of default or loss in the event of default. Or to put it a different way, most models require a loss upon default of nearly 50% to be roughly consistent with observed leverage choices and default probabilities. This deadweight loss seems too large in light of estimates of actual bankruptcy costs of 10% or less. This suggests there must be some other costs to being in financial distress. A natural interpretation is that some costs arise prior to default, in the form of higher costs of financing, inefficiencies driven by debt overhang, or the “default wedge” studied in this paper (and which is not original to this paper). Despite the large empirical work in corporate finance, there is less work in the macroeconomic literature that studies in general equilibrium the effects of these frictions. Moreover, the social and private costs of these frictions could be quite different. One limitation of some studies (including, to some extent, the Brown and Matsa study discussed above) is that it is difficult to disentangle the effects of economic from financial distress.

---

4See for instance http://online.wsj.com/news/articles/SB122632305224313513
5Unfortunately, these are the raw numbers of filings, i.e. they are not weighted by the firm size or the debt in bankruptcy. These data do not appear to exist.
Figure 1: US business bankruptcy filings. Note: series are smoothed using a 6-month moving average. The 2005 spike due to the bankruptcy reform is removed.

Figure 2: S&P default rate. Note: this is the percentage of S&P rated issuers that defaulted in a given year.
There has been some recent work focusing on the effect of financial frictions on the demand for labor (e.g. Chugh (2013); Petrovsky-Nadeau (2014); Quadrini et al. (2013)).

Also to discuss = domino effects and bankruptcy waves (Caballero and Simsek)

Large working capital literature, but rather ad-hoc.

2 Model

This section first lays out the model. We then

a simple model without capital, that demonstrates and evaluates quantitatively the key mechanism.

2.1 Technology

In order to study the effect on default risk between customers and suppliers, one needs to incorporate intermediate inputs; I follow the simple “roundabout” production structure of Basu (1995). I assume that there is a continuum of industries (of arbitrary measure $M$), that each produce a different good. These goods are then combined using a constant return to scale, perfectly competitive technology to produce the final good, which is used either for consumption or as intermediate input in each industry. (There is no investment or capital for simplicity.) Each industry, indexed by $j$, is competitive, and there is a measure one of firms in each industry, indexed by $i$. Hence, the total output of industry $j$ is

$$Y_j = \int_0^1 y_{ij} \, di,$$

and the total final goods production is

$$Y = M^{1-\sigma} \left( \int_0^M Y_j^{1-\sigma} \, dj \right)^{\frac{1}{1-\sigma}}.$$

In equilibrium, all industries make identical choices, so that $Y_j = Y$. The relative price of all these goods is one, i.e. the industry output price equals the final goods price, which is the numeraire.

Going now to study the problem faced by each firm in industry $j$, we assume a Cobb-Douglas production function with decreasing return to scale,

$$y_{it} = z_{it} x_{it} n_i^\alpha m_i^\beta,$$

where $m$ is the input, $n$ is labor, $z$ is aggregate productivity, which follows an exogenous stochastic process, and $x$ is idiosyncratic productivity, which follows an exogenous stationary stochastic process with invariant distribution $\mu(x)$.

2.2 Entry and Exit

The entry and exit process is kept simple to simplify aggregation. Each period, firms’ owner have to decide whether to pay some exogenous costs and remain in control of the firm, or default. These costs can be interpreted either as a debt repayment, or as fixed costs of production. These costs take two parts: first, there is a fixed number $b$. (A future version of this paper will endogenize the choice of $b$, for instance by using the trade-off model of debt.) Second, there is a random cost shock $\eta$, with mean 0 and
This shock is iid over time and across firms. The randomness captures pure profit windfall, that could be either positive or negative (e.g., a lawsuit that must be settled, or a one-time capital gain or profit from special circumstances). Each period, some firms will decide to default rather than continue operating and pay both costs. These exiters are replaced with new firms, which we assume have exactly the same idiosyncratic productivity $x$ as the old firms. The most natural interpretation is that these firms are immediately restructured, with creditors taking over the firm. This assumption keeps the analysis very simple since the cross-sectional distribution of firms according to $x$ is constant as a result, equal to $\mu(x)$, i.e. there is no truncation of the firm distribution. Firms will turn out to be heterogeneous only in this dimension.

### 2.3 Timing and Default Costs

The timing of the model is as follows. At the beginning of each period, the productivity shocks $z$ and $x$ are realized. Firm $x$ then decides to hire $n(x)$ workers and $m(x)$ units of the final good as input. The cash flow shock $\eta$ is then realized. Two possibilities arise: either the owners decide to continue running the firm; in this case they obtain as dividend the profits from today’s production, less $\eta + b$. This dividend may be negative, which means that there is no financial friction preventing recapitalization (i.e. equity issuance). The second possibility is that the firm owners decide not to repay $\eta + b$; in this case, they lose control of the firm and obtain 0, i.e. they exercise their limited liability option to walk away from the firm. For clarity, we assume that this default has no direct effect on the firm’s operations - workers provide labor, inputs are delivered, and output is produced and sold. (The net proceeds from these operations go to the new owners of the firm, which could be the creditors, or even the government, if it is the ultimate insurer; the identity of the new owner is irrelevant since this only affects the distribution of wealth among agents, which has no effect as we will assume a representative agent economy.)

The central assumption of the paper is that workers and suppliers bear a cost if the firm goes into default. Specifically, I assume that the cost borne by each worker is $\theta_w w(x)$ where $w(x)$ is their wage, and $\theta_w$ is a parameter; and similarly for suppliers the cost is $\theta_p p(x)$ where $p(x)$ is the input price. There are different interpretations for this reduced form. A literal interpretation is that workers or suppliers do not receive the payments (e.g. wages or account payables) to which they are entitled. For instance, the new firm owners might renegotiate salaries and benefits; or there might be no new owners, and the government might only partially insure for unpaid wages and account payables; or it might take time and effort, or be costly to recover wages. A broader interpretation (that goes outside the model) is that there are other costs of default that are borne by workers or suppliers, such as time spent unemployed or looking for new customers, or long-term wage losses because of displacement, or loss of relationship capital.

---

6Empirically, EBITDA contains large very transitory component.
2.4 The firm owner’s problem

The firm owner has to choose how much of each input to hire, and when to default. His problem is summarized by the following Bellman equation:

\[ V(x, \omega) = \max_{n, m \geq 0} E_{\eta} \max \{0, \pi - b - \eta + E_{\omega', \omega'} M(\omega, \omega')V(x', \omega') \} \]

Here \( V \) is the equity value, \( \omega \) is the aggregate state of this economy, to be discussed below, \( M(\omega, \omega') \) is the stochastic discount factor, which in equilibrium equals the marginal rate of substitution of the household, and \( \pi \) is the operating profit:

\[ \pi(n, m; x, \omega) = zxn^{\alpha}m^{\beta} - w(x, \omega)n - p(x, \omega)m. \]

Note that the input prices \( w(x, \omega) \) and \( p(x, \omega) \) depend on the firm’s characteristics, i.e. its productivity \( x \): workers and suppliers care about the likelihood of default of their employer or customer, hence they will charge different prices depending on the default risk.\(^7\) This Bellman equation reflects that, as discussed above, the equityholder has a default option that allows him to walk away from the firm following the observation of \( \eta \). Dividends \( \pi - b - \eta \) are allowed to be negative, so that equityholders can freely inject funds in the firm if it is profitable. Hence, we do not model “liquidity driven defaults” i.e. situations where firms end up in default for lack of cash, despite positive net present value.\(^8\)

In this problem, the firm takes as given the wage \( w(x, \omega) \) and prices \( p(x, \omega) \), which will be determined in equilibrium (in the next section). From equation (??), we obtain the optimal choices \( n^*(x, \omega) \), \( m^*(x, \omega) \) as well as the default threshold \( \eta^*(x, \omega) \). First, it is immediate that the firm defaults if \( \eta \geq \eta^*(x, \omega) \), where

\[ \eta^*(x, \omega) = zxn(x, \omega)^{\alpha}m(x, \omega)^{\beta} - w(x, \omega)n(x, \omega) - p(x, \omega)m(x, \omega) - b + E_{\omega', \omega'} (M(\omega, \omega')V(x', \omega')) \],

and the probability of default is hence

\[ PD(x, \omega) = 1 - H(\eta^*(x, \omega)). \]

To find input demands, rewrite the problem as:

\[
V(x, \omega) = \max_{n, m \geq 0} \int_{-\infty}^{\eta^*(x, \omega)} zxn^{\alpha}m^{\beta} - w(x, \omega)n - p(x, \omega)m - b - \eta + E_{\omega', \omega'} (M(\omega, \omega')V(x', \omega')) dH(\eta) \\
= \max_{n, m \geq 0} \left( H(\eta^*(x, \omega)) \left( zxn^{\alpha}m^{\beta} - w(x, \omega)n - p(x, \omega)m - b + E_{\omega', \omega'} (M(\omega, \omega')V(x', \omega')) \right) \right) \\
- \int_{-\infty}^{\eta^*(x, \omega)} \eta dH(\eta)
\]

This implies that the standard first-order conditions hold:

\[ \alpha zxn(x, \omega)^{\alpha-1} m(x, \omega)^{\beta} = w(x, \omega), \]

\[ \beta zxn(x, \omega)^{\alpha} m(x, \omega)^{\beta-1} = p(x, \omega). \]

---

\(^7\)By analogy with the sovereign or corporate debt literature, the reader might prefer to define the input prices as equilibrium schedules, e.g. \( w(b; x, \omega) \) with \( \partial w/\partial b > 0 \). However, given that equityholders try to maximize the firm value, it is easy to show that in the model \( \partial w/\partial b = 0 \) in equilibrium, so that the two equilibrium concepts coincide.

\(^8\)Liquidity driven defaults can lead to an alternative interesting mechanism, with “negative spirals”: as the firm becomes closer to default, its profitability deteriorates because of the default wedge, leading to lower cash flows and further increases in the likelihood of default.
We can use these equations to obtain the input demand as a function of the prices:

\[ n(x, \omega) = \left( \frac{\alpha}{w(x, \omega)} \right)^{\frac{1-\beta}{1-\alpha-\beta}} \left( \frac{\beta}{p(x, \omega)} \right)^{\frac{\beta}{1-\alpha-\beta}} (zx) \frac{1}{1-\alpha}, \]

\[ m(x, \omega) = (zx)^{\frac{1}{1-\alpha}} \left( \frac{\alpha}{w(x, \omega)} \right)^{\frac{1-\beta}{1-\alpha-\beta}} \left( \frac{\beta}{p(x, \omega)} \right)^{\frac{\beta}{1-\alpha-\beta}}, \]

To find the equilibrium prices \( w(x, \omega) \) and \( p(x, \omega) \), we turn to the household problem.

### 2.5 Households

There is a continuum of measure one of workers, all of whom have utility \( U(c) - \gamma n \). Following Hansen (1985) and Rogerson (1988), assume indivisible labor: \( n \in \{0, 1\} \). In the interest of simplicity, we also assume that there is perfect risk-sharing, so the household decisions can be thought of as arising from a big family perspective: the household head decides to allocate family members to work or leisure, and to work at different firms \( x \) with wages \( w(x, \omega) \).

As usual, consumption is equalized across household members given the separability of preferences. The household problem is to send \( n^*(x, w) \) people to work at firms with productivity \( x \), taking into account the wages offered by these firms and that there will be a loss in the event that the firm goes under. Summarizing,

\[
\max_{\{n^*(x)\}} U(C(\omega)) - \gamma \int_0^\infty n^*(x, w) d\mu(x),
\]

\[
s.t. \quad C(\omega) = \Pi(\omega) + \int_0^\infty n^*(x, w) w(x, w) (1 - \theta_w PD(x, \omega)) d\mu(x),
\]

where \( \Pi(\omega) \) are firm profits, rebated lump-sum to the household. Agents can also trade shares of the different firms that exist in this economy, however this trade “washes out” since there is no real investment, and it is thus omitted for simplicity.

This problem leads directly to the first-order conditions:

\[
\forall x \geq 0, \frac{\gamma}{U'(C(\omega))} = w(x, \omega)(1 - \theta_w PD(x, \omega)). \tag{1}
\]

This equation simply equates the marginal cost of working (the left hand-side, which is the standard expression for the marginal rate of substitution) and the benefit to working for a firm \( x \) (the right-hand side), which reflects the expected costs of default. This equation also reflects a compensating differential: workers require higher wages to work in more risky firms. In any competitive equilibrium, they must be indifferent between working at two firms with different productivities \( x \) and \( x' \):

\[
w(x, \omega)(1 - \theta_w PD(x, \omega))) = w(x', \omega)(1 - \theta_w PD(x', \omega))). \tag{2}
\]

The other first-order condition with respect to asset holdings shows that the household prices all assets in this economy, leading to the usual expression for the stochastic discount factor,

\[
M(\omega, \omega') = \frac{\beta U'(C(\omega'))}{U'(C(\omega))}. \]

Finally, we note that the same reasoning determines the input price \( p(x, \omega) \). The supplier (the producer of the final good) expects to receive \( p(x, \omega) \) with probability \( 1 - PD(x, \omega) \), but with probability
he will receive only \((1 - \theta_p)p(x, \omega)\). The total expected reward from selling is thus \(p(x, \omega)(1 - \theta_p PD(x, \omega))\). This must equal the reward from selling the good to consumers (who are assumed not to default), at price one. Hence,

\[
p(x, \omega) = \frac{1}{1 - \theta_p PD(x, \omega)}.
\]

### 2.6 Resource constraints and aggregation

There is no capital in this economy, hence the aggregate resource constraint for goods reads:

\[
C(\omega) + M(\omega) = Y(\omega) - \zeta DC(\omega),
\]

where \(DC(\omega)\) are default costs, and \(\zeta \in [0, 1]\) is a parameter that measures the fraction of default losses that are real resource costs (rather than transfers). Default costs include both the (standard) losses born by bondholders, and the (novel to this paper) losses born by workers and suppliers. Mathematically,

\[
DC(\omega) = \theta_w \int_0^\infty w(x, \omega)n(x, \omega)PD(x, \omega)\mu(x)
\]

\[
+ \theta_p \int_0^\infty p(x, \omega)m(x, \omega)PD(x, \omega)\mu(x)
\]

\[
+ \theta_b \int_0^\infty A(x, \omega)PD(x, \omega)\mu(x),
\]

and \(A(x, \omega)\) is the enterprise value of the firm.\(^9\) Since bonds are not priced ex-ante in this economy, the only effect of bondholder losses (\(\theta_b < 1\)) is to generate a wealth effect. In the interest of clarity, I will focus on the case where \(\zeta = 0\). This implies that there is no wealth effect of default losses, and

\[
C(\omega) + M(\omega) = Y(\omega),
\]

so that GDP \(Y(\omega) - M(\omega)\) equals consumption.

The aggregate resource constraint for labor reads

\[
N(\omega) = \int_0^\infty n(x, \omega)\mu(x).
\]

### 2.7 State space and computation

The key remark is that the only aggregate state is \(z : \omega = z\). This results from (i) net worth is not a state variable for firm since there is no constraint on equity issuance; (ii) the distribution of \(x\) is constant. This simplifies substantially the numerical computation. It is important in particular given the focus of the paper on nonlinearities which require a precise solution.

Intuitively, to solve the model we need factor demand from firms, which depend on prices, which depend on default risk; and default risk depends on the firm values, which depend on profits which depend on factor demand. Hence, we need to solve jointly for all policy functions. Specifically, we need to find firm-specific individual factor demand and prices, \(n(x, \omega), m(x, \omega), p(x, \omega),\) and \(w(x, \omega),\) firm-specific values, default cutoff and probabilities \(V(x, \omega), \eta^*(x, \omega), PD(x, \omega)\) as well as aggregates

\(^9\)It is given by the recursion

\[
A(x, \omega) = \pi(x, \omega) - \eta + E_{x', \omega'} M(\omega, \omega') A(x', \omega').
\]
$Y(\omega), M(\omega), C(\omega), N(\omega), M(\omega, \omega')$. The algorithm is as follows: we make an initial guess for default cutoffs $\eta^*(x, \omega)$ (e.g. a very high number i.e. no default) and for consumption $C(\omega)$; we deduce the default probability $PD(x, \omega) = 1 - H(\eta^*(x, \omega))$; given this, we find firm-specific prices and input demands:

$$p(x, \omega) = \frac{1}{1 - \theta_p PD(x, \omega)},$$

$$w(x, \omega) = \frac{\gamma C(\omega)}{1 - \theta_w PD(x, \omega)},$$

$$n(x, \omega) = \left(\frac{\alpha}{w(x, \omega)}\right)^{\frac{1-\beta}{1-\alpha}} \left(\frac{\beta}{p(x, \omega)}\right)^{\frac{\beta}{1-\alpha}} (zx)^{\frac{1-\alpha}{1-\beta}},$$

$$m(x, \omega) = (zx)^{\frac{1-\alpha}{1-\beta}} \left(\frac{\alpha}{w(x, \omega)}\right)^{\frac{1-\beta}{1-\alpha}} \left(\frac{\beta}{p(x, \omega)}\right)^{\frac{\beta}{1-\alpha}},$$

and we similarly obtain $y(x, \omega)$. We can then calculate the value function

$$V(x, \omega) = H(\eta^*(x, \omega))\eta^*(x, \omega) - R(\eta^*(x, \omega))$$

where $R(x) = \int_{-\infty}^{x} \eta dH(\eta)$, as well as aggregate output, inputs and labor:

$$Y(\omega) = \kappa_y z^{\frac{1-\alpha}{1-\beta}} \overline{w}(z)^{-\frac{\alpha}{1-\alpha}} \int_0^{\infty} x^{\frac{1}{1-\beta}} (1 - \theta_w PD(x, \omega))^{\frac{-\alpha}{1-\beta}} (1 - \theta_p PD(x, \omega))^{\frac{-\beta}{1-\alpha}} \mu(x),$$

$$M(\omega) = \kappa_m z^{\frac{1-\alpha}{1-\beta}} \overline{w}(z)^{-\frac{\alpha}{1-\alpha}} \int_0^{\infty} x^{\frac{1}{1-\beta}} (1 - \theta_w PD(x, \omega))^{\frac{-\alpha}{1-\beta}} (1 - \theta_p PD(x, \omega))^{\frac{-\beta}{1-\alpha}} \mu(x),$$

$$N(\omega) = \kappa_n z^{\frac{1-\alpha}{1-\beta}} \overline{w}(z)^{-\frac{\alpha}{1-\alpha}} \int_0^{\infty} x^{\frac{1}{1-\beta}} (1 - \theta_w PD(x, \omega))^{\frac{-\alpha}{1-\beta}} (1 - \theta_p PD(x, \omega))^{\frac{-\beta}{1-\alpha}} \mu(x),$$

where $\kappa_y = (\alpha)^{\frac{-\alpha}{1-\alpha}} (\beta)^{\frac{\beta}{1-\alpha}}$, $\kappa_m = (\alpha)^{\frac{-\alpha}{1-\alpha}} (\beta)^{\frac{-\beta}{1-\alpha}}$ and $\kappa_n = (\alpha)^{\frac{-\alpha}{1-\alpha}} (\beta)^{\frac{-\beta}{1-\alpha}}$, and $\overline{w}(\omega) = \gamma C(\omega)$ is the hypothetical risk-free wage.

We can finally verify that the guesses are consistent, i.e. that $C(\omega) = Y(\omega) - M(\omega)$, and that $\eta^*(x, \omega) = \pi(x, \omega) - b + E_{\omega'} (M(\omega, \omega')V(x', \omega'))$. If not, we update our guesses using some relaxation. This procedure was found to converge fairly rapidly.\(^{10}\)

### 3 Theoretical Results

I first discuss how the model generates a labor and a TFP wedge. I then discuss under which conditions the model admits multiple equilibria.

#### 3.1 Labor and TFP Wedges

Equation (1) clearly suggests that a labor wedge will arise in this economy, and will depend on the likelihood of default. However this equations relates the MRS to the wage offered by one firm rather than a measure of aggregate wage or aggregate MPL. To conform to the macroeconomic literature, I hence define the labor wedge as

$$1 - \tau_t = \frac{U_2(C_t, N_t) / U_1(C_t, N_t)}{\alpha Y_t / N_t},$$

\(^{10}\)We also discretized $x$ and $z$ for simplicity using the Rouwenhorst algorithm.
which given the preferences simplifies to\textsuperscript{11} 

\[ 1 - \tau_t = C_t \frac{Y_t}{N_t}. \]

Given that \( C_t = Y_t \) in equilibrium, and using the expression for labor demand obtained in section 2.7, it is easy to find the expression for the labor wedge in the model:

\[
1 - \tau(\omega) = \frac{\int_{0}^{\infty} x^{1-\alpha-\beta} (1 - \theta_{w}PD(x,\omega))^{1-\alpha-\beta} (1 - \theta_{p}PD(x,\omega))^{1-\alpha-\beta} d\mu(x)}{\int_{0}^{\infty} x^{1-\alpha-\beta} (1 - \theta_{w}PD(x,\omega))^{1-\alpha-\beta} (1 - \theta_{p}PD(x,\omega))^{1-\alpha-\beta} d\mu(x)}.
\]

While apparently complicated, this formula is quite useful, as it shows that the aggregate labor wedge is the ratio of two productivity weighted averages of firm-specific wedges, which are directly related to default risk. In some special cases, we can simplify this further. First, if \( \theta_{w} = 0 \) (and regardless of the value of \( \theta_{p} \)), or if \( PD(x,\omega) = 0 \) for all \( x \), we have \( \tau(\omega) = 0 \): there is no labor wedge, since there is no distortion to the exchange of labor. This is obviously at odds with the data.

Second, if all firms have the same default risk, \( PD(x,\omega) = PD(\omega) \),\textsuperscript{12} then \( \tau(\omega) = \theta_{w}PD(\omega) \). We now have a positive labor wedge, and one that is higher in times when the default likelihood is higher, i.e. in recessions. The magnitude of the labor wedge further depends directly on the employee losses \( \theta_{w} \) as well as the likelihood of default. Given the sharp movements in default risk, and the significant losses to workers of losing their job, this variation could be important.

More generally, when the default likelihood depends both on \( x \) and \( \omega \), the wedge will be different across firms. This in turn implies some “misallocation” of the workforce: there will be “too many” people working in high productivity firms, which have low default risk, and “too few” in low productivity firms, with high default risk. This in turn reduces aggregate total factor productivity, because decreasing returns make it inefficient to have too many workers in high \( x \) firms.

Specifically, one can define aggregate TFP (the Solow residual) implied by the model as \( TFP = \frac{GDP}{N^{\alpha}} \), where \( GDP = Y - M \):\textsuperscript{13}

\[
TFP = z^{1-\alpha} \frac{\int_{0}^{\infty} x^{1-\alpha-\beta} (1 - \theta_{w}PD(x,\omega))^{1-\alpha-\beta} (1 - \theta_{p}PD(x,\omega))^{1-\alpha-\beta} d\mu(x)}{\left(\int_{0}^{\infty} x^{1-\alpha-\beta} (1 - \theta_{w}PD(x,\omega))^{1-\alpha-\beta} (1 - \theta_{p}PD(x,\omega))^{1-\alpha-\beta} d\mu(x)\right)^{\frac{\beta}{\beta-\alpha}}},
\]

Again this is a ratio of productivity-weighted default wedges. It is easy to see that if there is no default risk, \( PD(x,\omega) = 0 \), then \( TFP = z^{1-\alpha} E(x^{1-\alpha-\beta})^{\frac{1-\beta}{1-\beta-\alpha}} = TFP^* \). This formula reflects the usual aggregate productivity, adjusted for (i) reallocation of inputs across firms, (ii) the “multiplier” effect of inputs on productivity. Second, if default risk is the same for all firms, \( PD(x,\omega) = PD(\omega) \), then

\[ TFP = TFP^* (1 - \theta_{p}PD(\omega))^{\frac{\beta}{\beta-\alpha}}, \]

so that measured \( TFP \) falls as default risk increases. The logic is that the economy uses too little inputs because of the default risk to suppliers. Finally, if \( PD(x,\omega) \) depends on \( x \), there will be a further

\textsuperscript{11} Some studies (e.g. Shimer (2010)) calculate the labor wedge assuming that the econometrician does not know the parameters \( \gamma \) and \( \alpha \). These are then picked to generate an “average” labor wedge of 40%, similar to the data. Here instead we assume that the econometrician knows the \( \alpha \) and \( \gamma \). This has little effect on the results; the alternative approach typically generates a proportional, but more volatile labor wedge.

\textsuperscript{12} This is the case, for instance, if the standard deviation of \( x \) is small relative to the standard deviation of \( \eta \).

\textsuperscript{13} There is a question of which prices are used for calculating the aggregate value of inputs.
misallocation as high productivity firms become “too big” because their low default risks allows them to hire more inputs and workers. The logic here is similar to Restuccia and Rogerson (2008).

3.2 Multiple equilibria (incomplete)

For some parameter values, this model admits multiple equilibria. Intuitively, if people expect a firm to default, they will require high wages and high prices to supply labor or inputs, which will reduce the firms’ profits and hence makes the firm more likely to default. The numerical solution focuses on the equilibrium with lowest default rate. This section discusses more precisely the conditions under which multiple equilibria arise. (To be finished.)

4 Numerical Results

This section first discusses the model parameters, then illustrates the implications of the model by solving it numerically.

4.1 Parametrization

The parametrization of the model is still in progress; the results here are preliminary and should be understood as “illustrative” more than “realistic”. The time period is one quarter. The discount factor is \( \beta = .995 \). The labor share in gross output is \( \alpha = .3 \) while the input share is \( \beta = .3 \). Preferences are assumed to have a unit IES to consumption and a Frisch elasticity equal to 4. The process for aggregate productivity \( z \) is an AR(1) with \( \sigma_z = 0.7\% \) and \( \rho_z = .98 \). The process for idiosyncratic productivity is an AR(1) with \( \sigma_x = 13\% \), and \( \rho_x = .85 \). The parameters \( b \) and \( \sigma_q \) are set to target a mean default rate of 1% per quarter with a standard deviation of 0.5%. The default costs are assumed to be \( \theta_w = \theta_p = 1 \).

4.2 Amplification

Figure 3 calculates the response of the economy as aggregate total factor productivity \( z \) varies. The blue line depicts the response of the model without worker losses (i.e. \( \theta_w = 0 \)); in this case, the model collapses to the standard RBC model (without capital). Given log utility, employment is independent of \( z \). Output responds hence one-for-one, aggregate TFP is measured without error, and there is no variation in the labor wedge. The blue line shows the case where \( \theta_w > 0 \) and \( \theta_p > 0 \); in this case, as \( z \) falls, the number of firms close to default becomes larger, which leads to lower employment and output than would otherwise be, as shown in the top two graphs. The bottom graphs establish that aggregate TFP falls further than \( z \) itself implies, and that there is a significant countercyclical labor wedge. This figure demonstrates that the model generates amplification of TFP shocks.

4.3 Time-varying sensitivity to macroeconomic shocks

To now illustrate that the response to the same shock is larger when many firms are close to default, I depict in figures 4 and 5 the elasticity of output and employment to a TFP shock, as a function of
the underlying level of TFP. The key results from these pictures are (a) without default wedges, this elasticity is constant, as the model is indeed log-linear; (b) with positive default wedges, the response of the economy to a TFP shock (positive or negative) is indeed bigger when TFP is low to start with, i.e. when more firms are close to default.

4.4 Countercyclical volatility of output and employment

The time-varying sensitivity to macroeconomic shocks demonstrated in the previous subsection implies a time-varying volatility of employment and output. To illustrate this in a more empirically oriented way, figures 6 and 7 present the result from a long model simulation. In these figures, each dot represents a time period, with the associated level of TFP today on the x-axis, and the associated macro volatility, proxied as the standard deviation of the growth rate of employment or output over the next 20 quarters. Note that the z shock is homoskedastik, so the standard deviation of the growth of z is constant. However, when z is low, more firms are close to default, the sensitivity to a macro shock is larger, and consequently employment and output become more volatile.

4.5 Alternative shocks

As a simple model extension, I introduce time-varying risk aversion in the model. To do so in a clean way, I use Epstein-Zin preferences:

\[ V_t = \left( (1 - \beta)u(C_t, N_t)^{1-\sigma} + \beta E_t \left( V_{t+1}^{1-\gamma_c} \right)^{\frac{1-\sigma}{\gamma_c}} \right) \frac{1}{1-\sigma} , \]
Figure 4: Elasticity of GDP to a TFP shock, as a function of the initial level of TFP. Blue line = model without default wedges; red line = model with default wedges.

Figure 5: Elasticity of employment to a TFP shock, as a function of the initial level of TFP. Blue line = model without default wedges; red line = model with default wedges.
Figure 6: Rolling standard deviation of employment growth in the next 20 quarters, as a function of the current level of TFP, with a fitted regression line.

Figure 7: Rolling standard deviation of GDP growth in the next 20 quarters, as a function of the current level of TFP, with a fitted regression line.
and assume that $\gamma_t$ follows a Markov chain that approximates an AR(1) process. A shock to risk aversion is a shortcut, frequently used in the finance literature or even financial press, to proxy for a “panic” whereby agents suddenly dislike risky assets. In a standard business cycle model, such as the current model with $\theta_w = \theta_p = 0$, a shock to risk aversion $\gamma_t$ has no effect on output or employment.

In contrast, in the model with default wedges, an increase in risk aversion reduces output and employment. Higher risk aversion reduces equity values, which increases the risk of default, and hence the associated wedges. As a result, output and employment contract. This channel connects asset prices and the macroeconomy in a different way than the standard Q-theory, which emphasizes that lower asset prices lead to lower investment.

This result is illustrated in figure 8, which depicts the level of output, employment, measured TFP, and the measured labor wedge, as a function of the level of risk aversion $\gamma$. As risk aversion (stochastically) rises, the economy contracts, partly because of lower TFP, and partly because of a higher labor wedge. The magnitudes, however, remain fairly small in the current calibration.

4.6 Summary of model implications

To summarize, this section has demonstrated three model implications: (1) the effect of a fundamental shock (such as a $z$ shock) is larger if there are financial distress costs (amplification); (2) the amplification effect of financial distress costs depends on the state of the economy and is larger when more firms are close to default; (3) more generally, any shock that affects equity value or likelihood of default will generate a contraction in economic activity. Overall, we expect the labor wedge, economic activity, the share of firms with high leverage, and macroeconomic volatility to be correlated.
5 Empirical Evidence

This section is a first attempt at assessing the mechanism discussed in the previous section. I proceed in three steps. First, panel data regressions provide some support for the basic idea that firms with higher leverage have sales and employment more sensitive to aggregate fluctuations. Second, I construct the cross-sectional distribution of (market) leverage and find that it varies substantially over time. In particular, the number of firms “close to default” (e.g. with a leverage above a threshold value) is strongly procyclical. This result holds for various definitions of leverage and various thresholds. In contrast, the median leverage is not clearly cyclical. Third, I investigate the time series relations between (i) aggregate uncertainty, (ii) the labor wedge, (iii) measures of economic activity, and (iv) the share of firms that are close to default. While some of the correlations are consistent with the model, others are not. However, an interesting finding is that the share of firms that are close to default appears to be a useful statistical predictor of economic activity - more so than the average or median leverage that is typically used in empirical studies such as Kollman and Zeugner (2013).

5.1 Cross-sectional evidence

A key empirical implication of the theory is that firms with high leverage or high default risk are more sensitive to aggregate fluctuations. This section provides some simple reduced-form evidence consistent with this implication (see also Sharpe (1994) for related evidence). I discuss below some potential endogeneity concerns.

Using firm-level annual panel data from Compustat, I estimate the regression,

\[
\Delta \log S_{it} = \delta_t + \gamma \text{HighLev}_{i,t-1} + \beta \text{HighLev}_{i,t-1} \Delta \log GDP_t + \varepsilon_{it},
\]

where \(S_{it}\) is real sales of firm \(i\) in year \(t\), \(\text{HighLev}_{i,t-1}\) is a dummy variable equal to one if a firm has “high leverage”, \(\Delta \log GDP_t\) is the growth rate of real GDP, and \(\delta_t\) include a full set of time effects.\(^{14}\) The coefficient of interests are \(\gamma\), which measures by how much “high leverage” reduces sales growth, and especially \(\beta\), which measures the extra sensitivity to GDP growth of high leverage firms.

I also estimate a similar regression using employment growth as the dependent variable,

\[
\Delta \log N_{it} = \delta_t + \gamma \text{HighLev}_{i,t-1} + \beta \text{HighLev}_{i,t-1} \Delta \log GDP_t + \varepsilon_{it}.
\]

The Compustat sample used is fairly standard; it includes all non-financial, non-utilities, domestic firms, with a December fiscal-year, from 1970 to 2011. Leverage is defined as the ratio of total debt (long-term debt plus short-term) over equity market value (i.e. \((\text{dltt+dlc})/(\text{csho}\ast\text{prcc} \_\text{f})\)), but we also consider alternative definitions including net-of-cash leverage (substracting cash and short-term investments from debt on the denominator \((\text{dltt+dlc-che})\)) and net-of-cash and trade credit leverage (substracting receivables and adding receivables, \((\text{dltt+dlc+ap-rectp-che})\)). A high leverage firm is defined as a firm with leverage above 0.45. The results continue to hold, however, if one uses other thresholds.

\(^{14}\)As a result, it is not necessary to include \(\Delta \log GDP_t\) on the right-hand side.
<table>
<thead>
<tr>
<th>Dependent variable: Δ log S_t</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Leverage definition</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(debt)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(debt-cash)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(debt-cash-trade)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>γ</td>
<td>-0.075</td>
<td>-0.068</td>
<td>-0.063</td>
<td>-0.051</td>
<td>-0.057</td>
<td>-0.055</td>
</tr>
<tr>
<td>t-stat</td>
<td>14.3</td>
<td>10.5</td>
<td>12.4</td>
<td>8.1</td>
<td>10.8</td>
<td>8.2</td>
</tr>
<tr>
<td>β</td>
<td>0.52</td>
<td>0.50</td>
<td>0.49</td>
<td>0.49</td>
<td>0.38</td>
<td>0.37</td>
</tr>
<tr>
<td>t-stat</td>
<td>3.6</td>
<td>3.4</td>
<td>3.5</td>
<td>3.4</td>
<td>2.6</td>
<td>2.5</td>
</tr>
<tr>
<td>Firm fixed-effects</td>
<td>n</td>
<td>y</td>
<td>n</td>
<td>y</td>
<td>n</td>
<td>y</td>
</tr>
<tr>
<td>Observations</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Observations</td>
<td>69377</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 1: Sales growth sensitivity as a function of leverage. Robust standard errors.

Table 1 reports the result. First, estimates of γ are consistently negative and highly significant. Being high leverage is associated with lower sales growth of 5-8% going forward. Obviously, this coefficient does not measure the “effect” of high leverage, since leverage is endogenous: negative shocks to expected sales drive equity value lower and leverage higher, so the causation runs “both ways”, and we expect a negative coefficient even in the absence of any financial distress cost. On the other hand, one might have expected some mean-reversion of sales for firms that had negative shocks the previous year.

Second, the coefficient β is positive and significant. The typical firm in Compustat has a sensitivity of sales growth to GDP around 2; a coefficient β of 0.5 reflects that high leverage firms have a sensitivity around 2 instead. This is economically important. Finally, note that both coefficients β and γ are fairly stable across different definitions of leverage.

Table 2 turns to the employment results, which are very similar overall. In Compustat, the typical firm has a sensitivity of employment to GDP around 1.5. Hence the estimated β is, if anything, even more economically important for employment than for sales.

I also report results using a different measure of default risk, the Merton distance to default,

\[ DD = \frac{\log \frac{\text{firm value}}{\text{debt}} + \mu_V - \frac{\sigma_V^2}{2}}{\sigma_V}, \]

where \( \sigma_V \) is inferred from equity volatility and leverage. I define “close to default” as a distance to default less than 4. For comparison, recall the following mapping between credit ratings and distance

15In this case too, a potential source of bias is that firms with different cyclical sensitivities in their real activities (in finance language, firms that have high asset beta) may choose a different leverage. For instance, a firm with high sensitivity may choose a lower leverage. This would tend to bias our estimated β towards zero. On the other hand, in a business cycle downturn, the market values of more cyclical firms may fall more, leading them to have higher leverage ex-post, which would bias our estimated β up. Hence, the overall effect is unclear.
Dependent variable: $\Delta \log N_{it}$ (1) (2) (3) (4) (5) (6)

<table>
<thead>
<tr>
<th></th>
<th>(debt)</th>
<th>(debt-cash)</th>
<th>(debt-cash-trade)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma$</td>
<td>-0.062</td>
<td>-0.060</td>
<td>-0.055</td>
</tr>
<tr>
<td>t-stat</td>
<td>15.1</td>
<td>11.3</td>
<td>13.2</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.44</td>
<td>0.51</td>
<td>0.44</td>
</tr>
<tr>
<td>t-stat</td>
<td>3.7</td>
<td>4.2</td>
<td>3.7</td>
</tr>
</tbody>
</table>

Firm fixed-effects | n | y | n | y | n | y |

Observations: 69,377

**Table 2:** Employment growth sensitivity as a function of leverage.

Dependent variable: $\Delta \log S_{it}$ (1) (2) (3) (4)

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma$</td>
<td>-.10</td>
<td>-.12</td>
<td>-.12</td>
<td>-.11</td>
</tr>
<tr>
<td>s.e.</td>
<td>(.01)</td>
<td>(.01)</td>
<td>(.01)</td>
<td>(.01)</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.89</td>
<td>0.69</td>
<td>0.57</td>
<td>0.62</td>
</tr>
<tr>
<td>s.e.</td>
<td>(0.23)</td>
<td>(0.23)</td>
<td>(0.24)</td>
<td>(0.24)</td>
</tr>
</tbody>
</table>

Firm fixed effects | n | y | y | y |
Industry controls | n | n | y | y |
Firm controls | n | n | n | y |

Observations: 60,631

**Table 3:** Sales growth sensitivity as a function of distance to default. Robust standard errors.

to default

<table>
<thead>
<tr>
<th>S&amp;P long-term rating</th>
<th>Median DD</th>
</tr>
</thead>
<tbody>
<tr>
<td>AAA or AA</td>
<td>11.3</td>
</tr>
<tr>
<td>A</td>
<td>8.6</td>
</tr>
<tr>
<td>BBB</td>
<td>6.3</td>
</tr>
<tr>
<td>BB</td>
<td>4.4</td>
</tr>
<tr>
<td>B</td>
<td>2.7</td>
</tr>
<tr>
<td>CCC or below</td>
<td>0.7</td>
</tr>
</tbody>
</table>

I estimate the similar regression

$$\Delta \log S_{it} = \alpha + \delta \Delta \log GDP_t + \gamma Z_{i,t-1} + \beta Z_{i,t-1} \Delta \log GDP_t + X_{it} \rho + \varepsilon_{it},$$

and similarly for employment.

**5.2 Business cycle variation in the cross-sectional distribution of leverage**
Dependent variable: $\Delta \log N_{it}$

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma$</td>
<td>-.08</td>
<td>-.12</td>
<td>-.11</td>
<td>-.10</td>
</tr>
<tr>
<td>s.e.</td>
<td>(.01)</td>
<td>(.01)</td>
<td>(.01)</td>
<td>(.01)</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.70</td>
<td>0.71</td>
<td>0.59</td>
<td>0.60</td>
</tr>
<tr>
<td>s.e.</td>
<td>(0.16)</td>
<td>(0.16)</td>
<td>(0.16)</td>
<td>(0.16)</td>
</tr>
<tr>
<td>Firm fixed effects</td>
<td>y</td>
<td>y</td>
<td>y</td>
<td>y</td>
</tr>
<tr>
<td>Industry controls</td>
<td>n</td>
<td>n</td>
<td>y</td>
<td>y</td>
</tr>
<tr>
<td>Firm controls</td>
<td>n</td>
<td>n</td>
<td>n</td>
<td>y</td>
</tr>
<tr>
<td>Observations</td>
<td>60,631</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 4: Employment growth sensitivity as a function of distance to default. Robust standard errors.

A second key piece of the mechanism is that the number of firms close to default varies substantially over time. To provide some light on this topic, Figures 9 and figure 10 provide the distribution of net-of-cash leverage in 2006Q1 and 2009Q1. First, note that there is a wide distribution, with the typical firms having essentially zero net leverage. But the fanning out of the distribution during the recession is impressive: there were many more firms with high net leverage in early 2009 than in early 2006. There were also significantly more firms with negative net leverage. This result also continues to hold for the alternative definitions of leverage.

One possible interpretation of these figures is that for the typical firm, financial distress is not a concern. But it is a concern for a small numbers of firms in “good times” like 2006, and for a significant fraction of firms in “bad times” like 2009. Hence, one really wants to track the time series of the number of firms with high leverage. Figure 11 plots the time series, which clearly exhibit a strong countercyclicality. This series remains very similar if we weight firms by sales rather than just counting firm units, as shown in figure 12; the main difference is some additional noise, as some large firms may go above or below the threshold.

An interesting fact is that this share variable behaves quite differently from the median or average leverage, depicted in figure 13. This suggests that there is some interesting information in the “tail” of the leverage distribution.

### 5.3 Time series evidence

The model makes strong prediction regarding the association of the following variables: (i) the share of firms close to default, (ii) the labor wedge, (iii) macroeconomic uncertainty, and (iv) economic activity. This section discusses the empirical correlation between these time series.
Figure 9: Distribution of net-of-cash leverage in 2006q1 in Compustat (nonfinancial firms). Net-of-cash leverage is (debt-cash)/(market value of equity).

Figure 10: Distribution of net-of-cash leverage in 2009q1 in Compustat (nonfinancial firms). Net-of-cash leverage is (debt-cash)/(market value of equity).
Figure 11: Share of firms with (debt-cash)/(market value of equity) ≥ 1. Compustat, nonfinancial firms.

Figure 12: Share of firms with (debt-cash)/(market value of equity) ≥ 1, both weighted by sales and unweighted. Compustat, nonfinancial firms.
5.3.1 Data construction

The share of firms close to default is constructed as in the previous section. The labor wedge is constructed as in Shimer (2010). Specifically, assuming a representative agent with utility

$$ E \sum_{t=0}^{\infty} \beta^t \left( \log(c_t) - \gamma \frac{n_t^{1+\phi}}{1+\phi} \right), $$

and a production function $y_t = k_t^\alpha (z_t n_t)^{1-\alpha}$, the first-order condition for labor implies

$$ w_t (1 - \tau_t) = (1 - \alpha) \frac{y_t}{n_t} (1 - \tau_t) = \gamma c_t n_t^\phi, $$

so that

$$ \tau_t = 1 - \frac{\gamma c_t}{1 - \alpha y_t} n_t^{1+\phi}. $$

Using nondurable per-capita consumption, hours worked per-capita, and real GDP, we construct the right-hand-side for given value of $\phi$. The coefficient $\frac{\gamma}{1-\alpha}$ is picked so that $\tau_t$ is on average equal to 0.4. Obviously, the elasticity of labor supply (the inverse of the parameter $\phi$) matters for this construction, but the labor wedge is a puzzle regardless of the value of $\phi$. Figure 14 depicts the labor wedge implied by different values of $\phi$. This series is highly countercyclical with respect to employment, and quite countercyclical with respect to output. Figure 15 illustrates this by plotting together HP filtered log labor wedge and HP filtered log hours.

We consider two macroeconomic uncertainty measures. First, we use the stock market volatility, constructed as the standard deviation of realized daily returns within a quarter. Second, we use the uncertainty measure constructed by Jurado, Ludvigson and Ng (2013). They use a large dataset of macro and financial indicators and estimate the average standard deviation of the unforecastable component.
Figure 14: Labor wedge, constructed as in Shimer (2010) and as explained in the text.

Figure 15: HP filtered log labor wedge and HP filtered log hours. US data.
of these time series. This uncertainty measure is depicted in figure 16. It has spikes in 2008, 1981, and 1975, but not much action in the 1991 or 2001 recessions.

5.3.2 Relations between time series (very preliminary and incomplete)

This section currently presents some reduced-form relations

Correlations

Table 3 presents the correlation between the key macroeconomic time series studied here. Interesting, and consistent with the model, uncertainty is strongly correlated with the share of firms with high leverage (0.48 or 0.46 depending on the measure of uncertainty). In contrast, the median or average leverage have much weaker correlation (from -0.03 to 0.33). Moreover, the labor wedge is significantly correlated with macro volatility (0.42). The correlation between leverage and the labor wedge is weaker however, and the share of firms with high leverage does not outperform here the median or average leverage.

Forecasting GDP growth

Kollman and Zeugner (2012) show that average leverage forecasts negatively GDP growth. I show that this relation is significantly stronger if one uses as measure of leverage not the average leverage, but the share of firms with high leverage. To show this, run the regression

$$\Delta \log GDP_{t+1} = a + b \Delta \log GDP_t + cZ_t + \varepsilon_{t+1},$$
Table 5: Correlations. 1980q1-2011q4.

<table>
<thead>
<tr>
<th></th>
<th>100×c</th>
<th>t-stat</th>
<th>( R^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average (unweighted)</td>
<td>-0.0046</td>
<td>-0.66</td>
<td>.235</td>
</tr>
<tr>
<td>Average (weighted)</td>
<td>-0.0024</td>
<td>-0.27</td>
<td>.233</td>
</tr>
<tr>
<td>Median (unweighted)</td>
<td>-0.0052</td>
<td>-0.92</td>
<td>.238</td>
</tr>
<tr>
<td>Median (weighted)</td>
<td>-0.0024</td>
<td>-0.27</td>
<td>.233</td>
</tr>
<tr>
<td>Share (unweighted)</td>
<td>-0.0330</td>
<td>-2.20</td>
<td>.262</td>
</tr>
<tr>
<td>Share (weighted)</td>
<td>-0.0276</td>
<td>-2.47</td>
<td>.269</td>
</tr>
</tbody>
</table>

Table 6: Forecasting GDP growth using its own lag and measures of leverage.

where \( Z_t \) is either average or median leverage, or the share of firms with high leverage. Table 5 summarizes the results. Clearly, the share of high-leverage firms has more explanatory power than just the average or median.

6 Conclusion

TBA

7 References (to be completed)


Review of Financial Studies.


Boissay, Frederic. “Credit chains and the propagation of financial distress”, ECB working paper

Kiyotaki, Nobuhiro and John Moore. “Credit chains”


Bigio, Saki and Jennifer Lao. ”


8 Appendix

8.1 Sample construction in Compustat

TBA

8.2 Model solution

The equilibrium is characterized by the following equations in $C(\omega), Y(\omega), M(\omega), N(\omega); \text{and } g(x, \omega), m(x, \omega), n(x, \omega), \eta^*(x, \omega)$

$$C(\omega) = Y(\omega) - M(\omega) - DC(\omega)$$

$$\alpha zxn(x, \omega)^{\alpha-1} m(x, \omega)^{\beta} = w(x, \omega)$$

$$\beta zxn(x, \omega)^{\alpha} m(x, \omega)^{\beta-1} = p(x, \omega)$$

$$g(x, \omega) = zxn(x, \omega)^{\alpha} m(x, \omega)^{\beta}$$
\[
\begin{align*}
Y(\omega) &= \int_{0}^{\infty} y(x, \omega) d\mu(x) \\
M(\omega) &= \int_{0}^{\infty} m(x, \omega) d\mu(x) \\
N(\omega) &= \int_{0}^{\infty} n(x, \omega) d\mu(x)
\end{align*}
\]

\[
\gamma C(\omega) = \mathfrak{m}(\omega)
\]

\[
\eta^*(x, \omega) = \pi(x, \omega) - b + E_{\omega'} (M(\omega, \omega') V(x', \omega'))
\]

\[
V(x, \omega) = H(\eta^*(x, \omega)) \eta^*(x, \omega) - R(\eta^*(x, \omega))
\]

\[
R(x) = \int_{-\infty}^{x} \eta dH(\eta)
\]

We can simplify this and obtain the input factor demand and output supply for each firm:

\[
n(x, \omega) = \left( \frac{\alpha}{w(x, \omega)} \right)^{\frac{1-\alpha}{\alpha}} \left( \frac{\beta}{p(x, \omega)} \right)^{\frac{\beta}{\alpha}} (z \gamma)^{\frac{1-\gamma}{\alpha}} \\
m(x, \omega) = (z \gamma)^{\frac{1}{\alpha}} \left( \frac{\alpha}{w(x, \omega)} \right)^{\frac{1-\alpha}{\alpha}} \left( \frac{\beta}{p(x, \omega)} \right)^{\frac{\beta}{\alpha - \alpha}} \\
y(x, \omega) = (z \gamma)^{\frac{1}{\alpha}} \left( \frac{\alpha}{w(x, \omega)} \right)^{\frac{1-\alpha}{\alpha}} \left( \frac{\beta}{p(x, \omega)} \right)^{\frac{\beta}{\alpha}}.
\]

We next define \( \kappa_n = (\alpha)^{\frac{\alpha}{\alpha - \alpha}} (\beta)^{\frac{\beta}{\alpha - \alpha}} \), \( \kappa_m = (\alpha)^{\frac{\alpha}{\alpha - \alpha}} (\beta)^{\frac{\beta}{\alpha - \alpha}} \) and \( \kappa_y = (\alpha)^{\frac{\alpha}{\alpha - \alpha}} (\beta)^{\frac{\beta}{\alpha - \alpha}} \).

Note that

\[
\frac{\kappa_y}{\kappa_n} = \frac{(\alpha)^{\frac{\alpha}{\alpha - \alpha}} (\beta)^{\frac{\beta}{\alpha - \alpha}}}{(\alpha)^{\frac{\alpha}{\alpha - \alpha}} (\beta)^{\frac{\beta}{\alpha - \alpha}}} = \alpha,
\]

and

\[
\frac{\kappa_y - \kappa_m}{\kappa_n} = \left( \frac{\alpha}{\alpha - \alpha} \right)^{\frac{\alpha}{\alpha - \alpha}} \left( \beta \right)^{\frac{\beta}{\alpha - \alpha}} - \left( \frac{\alpha}{\alpha - \alpha} \right)^{\frac{\alpha}{\alpha - \alpha}} \left( \beta \right)^{\frac{\beta}{\alpha - \alpha}}
\]

\[
= \frac{(\alpha)^{\frac{\alpha}{\alpha - \alpha}} \left( \beta \right)^{\frac{\beta}{\alpha - \alpha}}}{(\alpha)^{\frac{\alpha}{\alpha - \alpha}} \left( \beta \right)^{\frac{\beta}{\alpha - \alpha}}}
\]

\[
= 1 - \beta
\]

and

\[
\frac{\kappa_y - \kappa_m}{\kappa_n} = \left( \frac{1-\beta}{\alpha} \right)^{\frac{\alpha}{\alpha - \alpha}} \left( \beta \right)^{\frac{\beta}{\alpha - \alpha}} - \left( \frac{1-\beta}{\alpha} \right)^{\frac{\alpha}{\alpha - \alpha}} \left( \beta \right)^{\frac{\beta}{\alpha - \alpha}}
\]

\[
= \frac{(1-\beta)^{\frac{\alpha}{\alpha - \alpha}} \left( \beta \right)^{\frac{\beta}{\alpha - \alpha}}}{(1-\beta)^{\frac{\alpha}{\alpha - \alpha}} \left( \beta \right)^{\frac{\beta}{\alpha - \alpha}}}
\]

Note that

\[
p(x, \omega) = \frac{1}{1 - \theta_P D(x, \omega)}
\]

\[
w(x, \omega) = \frac{\bar{w}(z)}{1 - \theta_w D(x, \omega)}
\]
We obtain formulas for aggregate output, inputs and labor:

\[ Y(\omega) = \kappa_0 \gamma^{\frac{1-\sigma}{1-\sigma - \alpha}} \tilde{w}(\omega)^{-\frac{\alpha}{1-\sigma - \alpha}} \int x^{\frac{1}{1-\sigma}} (1 - \theta_w PD(x, \omega))^{\frac{\alpha}{1-\sigma - \alpha}} (1 - \theta_p PD(x, \omega))^{\frac{\beta}{1-\sigma - \alpha}} d\mu(x), \]

\[ M(\omega) = \kappa_m \gamma^{\frac{1-\sigma}{1-\sigma - \alpha}} \tilde{w}(\omega)^{-\frac{\alpha}{1-\sigma - \alpha}} \int x^{\frac{1}{1-\sigma}} (1 - \theta_w PD(x, \omega))^{\frac{\alpha}{1-\sigma - \alpha}} (1 - \theta_p PD(x, \omega))^{\frac{1-\alpha}{1-\sigma - \alpha}} d\mu(x), \]

\[ N(\omega) = \kappa_n \gamma^{\frac{1-\sigma}{1-\sigma - \alpha}} \tilde{w}(\omega)^{-\frac{\alpha}{1-\sigma - \alpha}} \int x^{\frac{1}{1-\sigma}} (1 - \theta_w PD(x, \omega))^{\frac{1-\alpha}{1-\sigma - \alpha}} (1 - \theta_p PD(x, \omega))^{\frac{\beta}{1-\sigma - \alpha}} d\mu(x). \]

### 8.3 Model solution for more general preferences

Suppose preferences are

\[ E \sum_{t=0}^{\infty} \beta^t U(C_t, N_t). \]

As a result,

\[ \tilde{w}(\omega) = \frac{u_2(C(\omega), N(\omega))}{u_1(C(\omega), N(\omega))}. \]

The SDF is

\[ M(\omega, \omega') = \beta \frac{U_1(C(\omega'), N(\omega'))}{U_1(C(\omega), N(\omega))}. \]

The other equations of the model are unchanged.

In the particular case \( U(C, N) = C^{1-\sigma} - B N^\gamma \), we have \( \tilde{w}(\omega) = BN(\omega)\gamma C(\omega) \), and if \( U(C, N) = \log C - \frac{N^{1+\frac{1}{\gamma}}}{1+\frac{1}{\gamma}} \), we have \( \tilde{w}(\omega) = \gamma N^{\frac{1}{\gamma}} C(\omega) \).

### 8.4 Algebra with Epstein-Zin preferences

Suppose preferences are now given by

\[ W = \left( u(C, N)^{1-\sigma} + \beta E (W^{1-\gamma})^{\frac{1-\sigma}{1-\gamma}} \right)^{1-\gamma}, \]

Here \( N \) is total employment, and \( u \) is the felicity function of the big family. For instance, \( u(C, N) = C^\gamma (1 - N)^{1-\gamma} \). The parameter \( \eta \) is related to the IES while the parameter \( \gamma \) reflects risk aversion over static gambles.

The real risk-free wage is given by the usual formula,

\[ \tilde{w}(\omega) = \frac{u_2(C(\omega), N(\omega))}{u_1(C(\omega), N(\omega))}. \]

For instance, if \( u(C, N) = C^\gamma (1 - N)^{1-\gamma} \), then

\[ \tilde{w}(\omega) = \frac{1-\gamma}{\gamma} \frac{C(\omega)}{1 - N(\omega)}. \]

The SDF is

\[ M(\omega, \omega') = \beta \frac{U_1(C(\omega'), N(\omega'))}{U_1(C(\omega), N(\omega))} \frac{U(C(\omega'), N(\omega'))^{-\sigma}}{U(C(\omega), N(\omega))^{-\sigma}} \frac{W(\omega)^{\sigma-\gamma}}{E(W(\omega')^{1-\gamma})^{\frac{\sigma-\gamma}{1-\gamma}}}. \]

The other equations of the model are unchanged. The only added complexity is that the value \( W(\omega) \) needs to be found as part of the solution.

29