Efficiency and Information Transmission in Bilateral Trading

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Abstract

We study optimal pairwise trading mechanisms when both the buyer and seller have private information about the value of an asset. We are particularly interested in the role played by limited commitment, in which either trader can walk away from a proposed trade when he learns the trading price. When each trader’s signal is relevant for the other trader’s value of the asset, optimal trading arrangements may conceal the traders’ signals and do so by introducing noise into the trading process, so that the transaction price is not a function of the traders’ signals. While limited commitment itself may not be costly, it shapes how prices transmit information.

1 Introduction

A buyer and seller meet to exchange an asset for money. Each trader has private information that may affect both traders’ valuations of the asset. While they bargain over the price, each is allowed to walk away from a proposed trade based on his private information and any information gleaned from the trading process, including any price proposals. We examine how prices are formed and how much of each trader’s information is transmitted to the other in a Pareto optimal trading mechanism.

We approach this problem using mechanism design. Formally, we look at the set of veto-incentive-compatible (hereafter, veto-IC) mechanisms (Forges, 1999). The buyer and

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seller receive private signals and the value of the asset to each depends on both signals. They simultaneously make a report to a mechanism, which recommends either trading at some price or not trading. If the mechanism recommends not trading, the seller keeps the asset and no money changes hands. If the mechanism recommends a price, either trader may still opt not to trade. Note that at this stage, a trader knows his own signal, his report to the mechanism, the recommended price, and the structure of the mechanism. This information may allow him to learn something about the other trader’s signal, which he then uses when deciding whether to accept the recommended price. We ask whether Pareto optimal trading mechanisms transmit each trader’s information to the other through prices. We also ask whether prices depend only on fundamentals—traders’ private information and the mapping from that information into valuations—or whether prices may be noisy.

One might expect that Pareto optimal trading mechanisms would induce traders to reveal their information to each other and would not obscure the information content of prices with noise. After all, a reduction in asymmetric information alleviates the lemons problem (Akerlof, 1970). This conclusion would be warranted if there were policies that could costlessly force traders to reveal their information to each other. In our environment, however, information revelation is endogenous. It is possible to construct trading mechanisms that induce traders to reveal their information to each other, but we find that Pareto optimal mechanisms typically do not have this property.

To understand why, assume that the buyer always values the asset more than the seller. Suppose that the buyer learns through the trading mechanism that the seller expects the asset to have a low value. This lowers the maximum price that he is willing to pay for the asset, which in turn makes it more difficult to persuade the seller to reveal that she has a low valuation for the asset, i.e. it tightens the seller’s incentive constraint. To satisfy the incentive constraint, it is necessary to reduce the probability of trade when the seller has a high valuation for the asset, which is inefficient. A symmetric argument explains why it may not be optimal to let the seller learn that the buyer expects that the asset has a high value.

But suppose instead that the buyer does not learn all of the seller’s information. This makes the buyer’s willingness to pay less sensitive to the seller’s information, reducing the necessary comovement between prices and private information. This moderates the seller’s desire to misrepresent her information and therefore moderates the need to reduce trading probabilities. If the gains from trade are large relative to the extent of the information problem, we find that prices which are completely insensitive to information, i.e. constant, are feasible and can achieve the first best. Otherwise, prices optimally reflect
some of the trader’s information but typically not all of it. Trade may not occur even when it is common knowledge that the buyer’s valuation exceeds the seller’s.

One way to hide information is through randomization. The price recommended by the mechanism is sometimes a noisy function of the reported signals. Different combinations of signals result in the same combination of prices, but with different probabilities. This reduces the ability of one trader to deduce the other trader’s signal from the reports. This leads to our conclusion that Pareto optimal trading mechanisms may conceal each trader’s information from the other and may do so by making prices stochastic given fundamentals.

We also explore the value of commitment. Suppose traders could commit to trade at the time they report their signals to the mechanism, before they receive any information about the other trader’s signal. This additional commitment enlarges the set of attainable payoffs by relaxing the constraints on the mechanism. It also eliminates any interest in the question of whether prices reveal information, since information revelation is costless with commitment. But surprisingly we find in some important cases, such as when the buyer and seller each have two signals, that the maximum feasible gains from trade is unaffected by whether traders have this commitment technology. This may explain why real-world trading arrangements allow traders to back away from a trade even after they start the bargaining process.

We do not view this paper as offering an actual proposal for a mechanism that will improve the efficiency of trade, although it may be possible to use our approach to construct such a mechanism. Instead, we are interested in understanding what a pair of traders can accomplish on their own and what features real-world trading outcomes might have. Our main conclusions are that we should not expect that pairwise optimal trading mechanisms will induce information revelation, nor should we expect that equilibrium prices will depend only on fundamentals, nor should we be surprised by the ability of either party to walk away from a trade based on the information they learn through bargaining.

The remainder of this paper proceeds as follows. In section 2, we review the most relevant literature. Section 3 sets up our environment and defines both the standard and the veto-IC mechanism design problems. Section 4 establishes some general properties of such mechanism, including the fact that they randomize over only a finite set of prices when the number of possible signals is finite. Section 5 discusses the special case of private values in which each trader’s valuation depends only on his own signal. We show that there is no role for randomization nor any cost to full information revelation. We then turn in Section 6 to the case of common values, when these conclusions no longer hold. We illustrate our results through a parameterized example.
2 Related Literature

Many previous papers have examined the information content of prices. Grossman (1976) considers a large market in which each trader’s information is negligible relative to the information held by other traders. He finds that prices naturally aggregate information into a fully revealing rational expectations equilibrium. We are interested in environments in which each trader holds a non-trivial portion of the information available to traders as a whole. The simplest environment that captures this is one with only two traders, each of whose valuations depends on both of their information.

Golosov, Lorenzoni and Tsyvinski (2013) explore an economy composed of a large number of traders, some of whom are informed and some of whom are uninformed about the relative quality of two assets. Traders trade bilaterally, with one trader making a take-it-or-leave-it offer to the other. They find that in the long run information diffuses through the economy, with traders learning all the relevant information from their trades. Our environment is simpler, since there is only one buyer and one seller. But in addition to looking at bargaining equilibria, we also study optimal trading mechanisms. Our results suggest that full information diffusion may not be optimal.

Our results on inefficiency of trade in pairwise exchange markets recall Myerson and Satterthwaite (1983). That paper looks at an environment with independent private values and shows that if there is a positive probability that the buyer values an asset less than the seller, then the first best level of trade cannot be achieved through any mechanism. We allow for a more general information structures, including correlated private values, common values, and affiliated values Milgrom and Weber (1982).

With correlated private values, McAfee and Reny (1992) show that the Myerson and Satterthwaite (1983) inefficiency result disappears. The buyer and seller optimally set up a mechanism in which they make a side payment to an intermediary whenever the ex ante likelihood of their joint reports is small and receive a side payment from the intermediary when their reports are more consistent. Compte and Jehiel (2009) show that this result depends on the ability of the three market participants to commit to the mechanism. If each trader retains the right to veto a proposed trade based on the information available to him, the intermediary can never be induced to make a side payment and the first best is unobtainable. This motivates our desire to explore veto-IC mechanisms.

In an important recent paper Gerardi, Hörner and Maestri (2013) study veto-IC mechanism design with affiliated values and one-sided private information. They compare this environment to a standard mechanism design problem, in which traders are locked in to a mechanism after they make their reports. They prove that the set of veto-IC allocations is
equivalent to the standard set of feasible allocations which satisfy some additional linear constraints. In the case with pure common value, these additional constraints are slack, and so the solutions to the two problems coincide. We find that with two-sided private information, this is not generally true. Veto-IC constraints reduce the set of attainable payoffs even in the pure common values case.

An alternative approach would be to write down a particular bargaining game and study the set of equilibria of that game. For example, Morris and Shin (2012) study an environment with two-sided asymmetric information and common values. They show that seemingly small amounts of private information may lead to market breakdowns. This contrasts with our results, where the maximum trading probability is continuous in the extent of private information. The difference is that they study a particular trading mechanism with a fixed market price, set at the expected value of the asset, while we study optimal trading mechanisms. Following Kennan and Wilson (1993), we view the mechanism design approach as informative about what traders may accomplish through any bargaining mechanism, but we impose the additional veto-IC constraints to capture a realistic limited commitment problem in many trading environments.

Finally, there is a large recent literature on private information in asset markets. A non-exhaustive list includes Akerlof (1970); Eisfeldt (2004); Daley and Green (2012); Tirole (2012); Kurlat (2013); Chari, Shourideh and Zetlin-Jones (2011); Chiu and Koeppel (2011); Chang (2011); Camargo and Lester (2012); Guerrieri and Shimer (forthcoming, 2013).

3 Environment

3.1 Preferences and Information

There is a single asset and two traders. One of the traders, whom we call the seller, initially owns the asset. The other trader, the buyer, has some cash that he could use to purchase the asset. Each trader is risk-neutral and their valuations for the asset depend on signals that each receives. Let \( b \in \{1, \ldots, N^B\} \) denote the buyer’s signal and \( s \in \{1, \ldots, N^S\} \) denote the seller’s. Let \( \pi_{bs} \) denote the ex ante joint probability that the buyer receives signal \( b \) and the seller receives signal \( s \). Without loss of generality, \( \sum_b \pi_{bs} > 0 \) for all \( s \) and...
\[ \sum_s \pi_{bs} > 0 \text{ for all } b; \text{ otherwise we could drop one of the signals.} \]

Both traders are risk-neutral. The buyer’s (seller’s) expected value for the asset is \( v^B_{bs} \) (\( v^S_{bs} \)) when the buyer’s signal is \( b \) and the seller’s signal is \( s \). If the seller gives the asset to the buyer with probability \( q \) in return for a certain cash transfer of \( \tau \), the buyer’s expected utility is \( q v^B_{bs} - \tau \) and the seller’s is \( \tau - q v^S_{bs} \), where we normalize the utility from no trade to zero. When \( q > 0 \), we call \( \tau / q \) the price of the asset, but this notation allows for cash transfers even in situation where the asset is not transferred.

The buyer privately observes his signal \( b \) and the seller privately observes her signal \( s \). In the private values case, \( v^B_{bs} \) depends only on \( b \) and \( v^S_{bs} \) depends only on \( s \), so the willingness of a trader to accept a transfer \( \tau \) in exchange for trade with probability \( q \) depends only on her own signal. But more generally, each trader’s willingness to trade at any particular price \( \tau / q \) depends on both traders’ signals. Different trading mechanisms allow a trader to refuse to trade based on different information sets.

### 3.2 Interim Mechanism

We start by recalling the standard mechanism design problem. A trader observes his own signal \( t \) and submits a report \( \hat{t} \) to the mechanism. The mechanism then determines a transfer \( \tau_{bs} \) and a trading probability \( q_{bs} \) as a function of the two reports \((\hat{b}, \hat{s})\). Following the revelation principle, we restrict attention to mechanisms that satisfy participation and incentive constraints. The participation constraints state that a buyer with any signal \( b \) has nonnegative expected profits when both traders truthfully reports their signals, and similarly for a seller with any signal \( s \):

\[
\sum_s (q_{bs} v^B_{bs} - \tau_{bs}) \pi_{bs} \geq 0 \text{ for all } b \text{ and } (1a)
\]

\[
\sum_b (\tau_{bs} - q_{bs} v^S_{bs}) \pi_{bs} \geq 0 \text{ for all } s. \quad (1b)
\]

Note that the left hand side of the first inequality is the buyer’s expected gain from trade given the signal \( b \) times the probability of receiving the signal \( b \), and similarly for the second inequality.

The incentive constraints state that a buyer with any signal \( b \) weakly prefers to truth-
fully report his signal when the seller truthfully reports her signal, and vice versa:

\[ \sum_s (q_{bs} v^B_{bs} - \tau_{bs}) \pi_{bs} \geq \sum_s (q_{bs} v^B_{bs} - \tau_{\hat{b}}) \pi_{bs} \text{ for all } b \text{ and } \hat{b} \]  

(2a)

\[ \sum_b (\tau_{bs} - q_{bs} v^S_{bs}) \pi_{bs} \geq \sum_b (\tau_{bs} - q_{bs} v^S_{bs}) \pi_{bs} \text{ for all } s \text{ and } \hat{s}. \]  

(2b)

A feasible interim mechanism is a pair of \( N_B \times N_S \) matrices \( \{\tau, q\} \) with typical element \( \{\tau_{bs}, q_{bs}\} \) that satisfy conditions (1) and (2). The buyer’s and seller’s expected payoffs under a feasible interim mechanism are

\[ V^B = \sum_{b,s} (q_{bs} v^B_{bs} - \tau_{bs}) \pi_{bs} \text{ and} \]  

(3a)

\[ V^S = \sum_{b,s} (\tau_{bs} - q_{bs} v^S_{bs}) \pi_{bs} \text{.} \]  

(3b)

and so a Pareto optimal interim mechanism maximizes \( \phi V^B + (1 - \phi) V^S \) across the set of feasible interim mechanism for some \( \phi \in [0, 1] \). We will sometimes be interested in the utilitarian solution with \( \phi = \frac{1}{2} \), in which case the objective is equivalent to maximizing the expected gains from trade \( \sum_{b,s} q_{bs} (v^B_{bs} - v^S_{bs}) \pi_{bs} \).

### 3.3 Veto-IC Mechanism

In a veto-IC mechanism, a trader observes his own signal \( t \) and submits a report \( \hat{t} \) to the mechanism. The mechanism then determines a probability distribution over transfers and trading probabilities as a function of the two reports and informs the traders of this outcome. Either trader can then refuse to trade, knowing her true signal, her report, the outcome recommended by the mechanism, and the structure of the mechanism itself. This constraint captures the idea that traders may use the mechanism’s recommendation to infer something about the other trader’s signal and hence about her own desire to trade.

We prove in Section 4.1 that any payoff achievable with a general veto-IC mechanism can also be satisfied by a veto-IC mechanism that satisfies two properties. First, it always recommends either to trade at a price \( p \) or not to trade, rather than a more general probability distribution over transfers and trading probabilities. And second, the mechanism relies on a finite number of prices, where the proposition gives a bound on that number. We refer the reader to that section and the associated propositions for a general statement of the set of veto-IC mechanisms, and simplify the exposition here by taking advantage of those results.

A veto-IC mechanism consists of a set of prices \( p_n, n \in \{1, \ldots, N\} \) and probabilities
\( \omega_{n|\hat{b}\hat{s}} \) such that the mechanism recommends trading at price \( p_n \) with probability \( \omega_{n|\hat{b}\hat{s}} \) when the buyer reports signal \( \hat{b} \) and the seller reports signal \( \hat{s} \). Naturally \( \omega_{n|b's} \geq 0 \) for all \((b,s)\). We allow that \( \sum_n \omega_{n|bs} < 1 \), in which case \( 1 - \sum_n \omega_{n|bs} \) is the probability that the mechanism recommends no trade following the reports \((b,s)\).

We restrict attention without loss of generality to mechanisms that satisfy ex post participation and veto-IC constraints. The ex post participation constraint states that a buyer with signal \( b \) has nonnegative expected profits when both traders truthfully report their signals and the mechanism recommends trading at any price \( p_n \):

\[
\sum_s (v_{bs}^B - p_n) \omega_{n|bs}\pi_{bs} \geq 0 \quad \text{for all } b \text{ and } n \quad \text{(4a)}
\]

\[
\sum_b (p_n - v_{bs}^S) \omega_{n|bs}\pi_{bs} \geq 0 \quad \text{for all } s \text{ and } n. \quad \text{(4b)}
\]

To understand these equations, first suppose that for some buyer signal \( b \), \( \omega_{n|bs} = 0 \) for all seller signals \( s \). Then the buyer knows that the mechanism will never recommend trading at price \( n \) and so the condition is trivially satisfied. Otherwise, the probability that the seller’s signal is \( s \) given that the buyer’s signal is \( b \) and the mechanism recommends trading at price \( p_n \) is determined by Bayes rule:

\[
\frac{\omega_{n|bs}\pi_{bs}}{\sum_{s'} \omega_{n|bs'}\pi_{bs'}}.
\]

The numerator is the probability that buyer’s signal is \( b \), the seller’s signal is \( s \), and the price is \( p_n \), while the denominator is the probability that the the buyer’s signal is \( b \) and the price is \( p_n \), with an arbitrary seller signal. The expression in condition (4a) is therefore proportional to the buyer’s expected profit conditional on her truthfully reported signal \( b \) and the recommendation to trade at price \( p_n \), where the proportionality constant is just the probability of receiving this signal and recommendation. The construction of the seller’s ex post participation constraint is identical.

The veto-IC constraints are constructed like the ex post participation constraints, but ensure that each trader prefers to truthfully report his signal given that the other trader truthfully reports hers, rather than lie about his signal and possibly opt not to trade:

\[
\sum_{n,s} (v_{bs}^B - p_n) \omega_{n|bs}\pi_{bs} \geq \sum_n \max \left\{ \sum_s (v_{bs}^B - p_n) \omega_{n|bs'}\pi_{bs'}, 0 \right\} \quad \text{for all } b \text{ and } b' \quad \text{(5a)}
\]

\[
\sum_{n,b} (p_n - v_{bs}^S) \omega_{n|bs}\pi_{bs} \geq \sum_n \max \left\{ \sum_b (p_n - v_{bs}^S) \omega_{n|bs'}\pi_{bs'}, 0 \right\} \quad \text{for all } s \text{ and } s' \quad \text{(5b)}
\]
We start again with the buyer’s incentive constraint. The left hand side is his expected profit if he truthfully reports the signal \( b \) times the probability of receiving the signal \( b \). Since the left hand side of condition (4a) is equal to the buyer’s expected profit when he receives signal \( b \) and the mechanism recommends trading at price \( p_n \), multiplied by the probability of this event, we simply sum this across prices to obtain the buyer’s expected profit. Also note that the buyer’s expected profit is nonnegative conditional on each price recommendation, by condition (4a). The right hand side is similar, except that a buyer who receives signal \( b \) and reports some other \( b' \) has a different posterior belief over the seller’s signal upon seeing the price \( p \). In addition, this posterior may induce him not to buy at some prices, a possibility that we capture through the nonnegativity constraint on profits conditional on the true signal \( b \), the misreported \( b' \), and the recommendation to trade at price \( p_n \). Once again, the construction of the seller’s participation constraint is similar.

A feasible veto-IC mechanism \( \{ p, \omega \} \) is an \( N \) vector of prices \( p_n \) and a \( N^b \times N^s \times N \) matrix of probabilities \( \omega_n|bs \) satisfying \( \sum_n \omega_n|bs \leq 1 \) that satisfy conditions (4) and (5). The buyer’s and seller’s expected payoffs under a feasible veto-IC mechanism are

\[
V^B = \sum_{b,s,n} (v^B_{bs} - p_n) \omega_n|bs \pi_{bs} \quad \text{and} \\
V^S = \sum_{b,s,n} (p_n - v^S_{bs}) \omega_n|bs \pi_{bs}.
\]

These are simply the sums of the left hand side of the incentive constraint (5). Finally, a Pareto optimal veto-IC mechanism maximizes \( \phi V^B + (1 - \phi) V^S \) across the set of feasible veto-IC mechanisms for some \( \phi \in [0, 1] \). The utilitarian mechanism sets \( \phi = \frac{1}{2} \) and so maximizes the gains from trade, which here is equivalent to maximizing \( \sum_{b,s,n} (v^B_{bs} - v^S_{bs}) \omega_n|bs \pi_{bs} \).

Take any feasible veto-IC mechanism \( \{ p, \omega \} \). Let

\[
q_{bs} = \sum_n \omega_n|bs \quad \text{and} \quad \tau_{bs} = \sum_n p_n \omega_n|bs.
\]

Then it is immediate that \( \{ \tau, q \} \) is a feasible interim mechanism. Condition (4) collapses to (1), condition (5) collapses to (2), and payoffs (6) collapses to (3). We therefore treat the problem of finding a Pareto efficient interim mechanism as a relaxed version of the problem of finding a Pareto efficient veto-IC mechanism.

The reverse possibility, finding a feasible veto-IC mechanism that achieves the same payoffs as a feasible interim mechanism, is not trivial and in some cases not possible. Our
analysis explores the relationship between these two problems in depth.

3.4 Fully-Revealing and Deterministic Mechanisms

A fully-revealing veto-IC mechanism is a feasible veto-IC mechanism in which prices fully reveal the other trader’s signal: if $\omega_{n|bs} > 0$, $\omega_{n|b's} = \omega_{n|b's'} = 0$ for all $b, s, n, b' \neq b$ and $s' \neq s$. This means that a trader can always deduce the other trader’s report from her own report and the price. In a fully-revealing veto-IC mechanism, any trade fully reveals each investor’s information to the other through the price.

A deterministic veto-IC mechanism is a feasible veto-IC mechanism in which prices are a function of the traders’ signals: if $\omega_{n|bs} > 0$, $\omega_{n'|bs} = 0$ for all $b, s, n, n' \neq n$. In a deterministic veto-IC mechanism, prices depend only on investor’s information, not on extraneous events like the outcome of a lottery.

A standard concept of informational efficiency in asset markets is that “security prices fully reflect all available information” (Fama, 1991). A standard indicator of inefficiency is that prices vary too much relative to fundamentals (Shiller, 1981), which here are simply $b$ and $s$. Of course, both Fama (1991) and Shiller (1981) focus on exchanges and other markets with many participants, while we look at markets with two privately-informed traders, and so the analogies we develop here are inexact. Nevertheless, one might conjecture from the existing literature that Pareto optimal veto-IC mechanisms would be both fully-revealing and deterministic. For example, although investors might be able to profit from private information, one might expect that the resulting market power would generally be Pareto inefficient. In this paper, we find that neither hypothesis is generally valid in pairwise exchange. Restricting attention to mechanisms are either fully-revealing or deterministic will generally lead to Pareto inferior allocations.

4 General Results

4.1 Characterization of Optimal Mechanisms

We are interested in understanding the set of payoffs that the buyer and seller may jointly achieve through an arbitrary trading mechanism. Following the Revelation Principle (My-

\footnote{We allow $\omega_{n|bs} > 0$ and $\omega_{n'|b's'} > 0$ for some $b' \neq b$ and $s' \neq s$, since this does not interfere with the signal extraction problem. Imposing the additional restriction that $\omega_{n|bs} > 0 \Rightarrow \omega_{n|b's'} = 0$ for all $b, s, n, b' \neq b$ and $s' \neq s$ would further constrain trading possibilities.}

\footnote{We allow a deterministic mechanism to randomize between trade and no-trade, $\omega_{n|bs} \in (0,1)$. Imposing the additional restriction that $\omega_{n|bs} \in \{0,1\}$ would further constrain trading possibilities.}
erson, 1979), we focus without loss of generality on a direct revelation mechanism. The two traders simultaneously report their signals to a machine, which then recommends a lottery to them, a joint probability distribution over a cash payment from the buyer to the seller and asset transfers from the seller to the buyer. Finally, either trader may reject the lottery. If both accept it, the outcome of the lottery is realized and any specified transfers take place.

We prove two results in this section. The first is that any payoffs that can be achieved with a veto-IC mechanism using a lottery can also be achieved with veto-IC mechanism whose outcome is deterministic, either “trade at a price $p$” or “don’t trade.” It will be convenient to denote “don’t trade” by $p = \emptyset$. Note that we do not claim that the mapping from the reports $(\hat{b}, \hat{s})$ into the outcome is deterministic, only that the mechanism recommends a deterministic outcome.

**Proposition 1.** Take any feasible veto-IC mechanism. There exists a feasible veto-IC mechanism with the same trading probabilities and expected payoffs conditional on any signals $(b, s)$, in which the recommendation is always of the form $p \in \mathbb{R}_+ \cup \emptyset$.

Intuitively, risk-neutrality implies that both traders evaluate any lottery using the expected cash transfer from the buyer to the seller $\tau$ and the trading probability $q$, rather than the probability distribution over transfers and trades. Therefore any lottery can trivially be reduced to a simpler choice of $(q, \tau) \in [0, 1] \times \mathbb{R}$. Next, if $q = 0$, veto-IC requires $\tau = 0$, since the buyer would veto any $\tau > 0$ and the seller would veto any $\tau < 0$. Finally, any $(q, \tau)$ with $q \in (0, 1)$ can be implemented with a lottery prior to the recommendation, setting a price $p = \tau / q$ with probability $q$ and implementing no trade otherwise. This has no direct effect on either the standard incentive constraints or on the veto-IC constraints, since it is payoff equivalent. It may, however, conceal information from one or both traders, if some other reports can lead to this price recommendation, in which case this mechanism relaxes the veto-IC constraints.

The second result provides a finite upper bound on the number of prices that the mechanism may use.

**Proposition 2.** Take any feasible veto-IC mechanism. There exists a feasible veto-IC mechanism with the same trading probabilities and expected payoffs conditional on any signals $(b, s)$, in which the recommendation is always of the form $p \in \{p_1, p_2, \ldots, p_N, \emptyset\}$, where $N \equiv (N^B)^2 + (N^S)^2 + N^B N^S + 1$.

Recall that $N^B$ and $N^S$ are the number of buyer and seller signals. Our proof shows that any feasible veto-IC mechanism is completely characterized by the payoff of a trader.

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who receives signal \( t \) and reports signal \( t' \) and by the trading probabilities conditional on the signal pair \((b, s)\). This outcome is an object of dimension \( N - 1 \) and is linear in the probability measure over prices. We can therefore express the outcome as the weighted average of at most \( N \) extreme points of this set, i.e. by putting weight onto at most \( N \) such prices, in addition to the option not to trade.

### 4.2 Local Incentive Constraints (To be done)

Regularity conditions which ensure that only local incentive constraints bind. This result is useful for two reasons. First, it reduces the number of constraints that we need to check to see if a veto-IC mechanism is feasible. Second, the logic of Proposition 2 yields a tighter bound on the number of prices, \( N^B N^S + 3(N^B + N^S) - 4 \), a tighter bound if either \( N^B > 2 \) or \( N^S > 2 \).

For now we focus on the case with \( N^B = N^S = 2 \), so all incentive constraints are local.

### 4.3 Computational Algorithm

We again allow for an arbitrary report-conditional measure over prices \( \mu_{bs} \). A Pareto optimal, feasible veto-IC mechanism solves

\[
\max_{\{\mu_{bs}\}} \int_{\mathbb{R}^+} \sum_{b, s} \left( \phi (v_{bs}^B - p) + (1 - \phi) (p - v_{bs}^S) \right) \pi_{bs} d\mu_{bs}(p)
\]

for some \( \phi \in [0, 1] \), subject to

\[
\int_{\mathbb{R}^+} \sum_{s} (v_{bs}^B - p) \pi_{bs} d\mu_{bs}(p) \geq 0 \text{ for all } b \text{ and } P \subset \mathbb{R}^+, \quad (1,)
\]

\[
\int_{\mathbb{R}^+} \sum_{b} (p - v_{bs}^S) \pi_{bs} d\mu_{bs}(p) \geq 0 \text{ for all } s \text{ and } P \subset \mathbb{R}^+, \quad (2)
\]

\[
\int_{\mathbb{R}^+} \sum_{s} (v_{bs}^B - p) \pi_{bs} d\mu_{bs}(p) \geq \int_{\mathbb{R}^+} \max \left\{ \sum_{s} (v_{bs}^B - p) \pi_{bs} d\mu_{bs'(p)}, 0 \right\} \text{ for all } b \text{ and } b', \quad (3)
\]

\[
\int_{\mathbb{R}^+} \sum_{b} (p - v_{bs}^S) \pi_{bs} d\mu_{bs}(p) \geq \int_{\mathbb{R}^+} \max \left\{ \sum_{b} (p - v_{bs}^S) \pi_{bs} d\mu_{bs'(p)}, 0 \right\} \text{ for all } s \text{ and } s', \quad (4)
\]

\( d\mu_{bs}(p) \geq 0 \), and \( \mu_{bs}(\mathbb{R}^+) \leq 1 \). Here \( \phi \) represents the buyer’s Pareto weight and \( 1 - \phi \) is the seller’s. A Pareto optimal mechanism fixes report-conditional measures over prices that satisfy versions of the ex post participation constraints (4) and the veto incentive compatibility constraints (5), extended to allow for a possibly-continuous distribution of
prices. Note that this problem is linear (and hence convex).

Letting \( \lambda \) denote the multipliers, we form the Lagrangian

\[
L_\lambda \equiv \int a_\lambda \cdot d\mu - \int \max \{ b_\lambda \cdot d\mu, 0 \}
\]

subject to \( c_{j,\lambda} \cdot d\mu \geq 0 \), where \( a_\lambda \) is a function that combines the objective, the left hand side of the incentive constraints and the integrability condition, \( b_\lambda \) combines the right hand side of the incentive constraints, and \( c_{j,\lambda} \) captures the \( j^{th} \) nonnegativity and participation constraint. Here \( \lambda \) is just a set of multipliers of dimension equal to the number of incentive and integrability constraints, \( N^B(N^B - 1) + N^S(N^S - 1) + N^B \times N^S \).

This is a cell problem, i.e. it can be solved separately for each price \( p \). Define

\[
L_\lambda(p) = \max_{\{c_{j,\lambda}(p) \cdot d\mu(p) \geq 0\}} (a_\lambda(p) \cdot d\mu(p) - \max \{ b_\lambda(p) \cdot d\mu(p), 0 \})
\]

By homogeneity it must be the case that \( L_\lambda(p) = 0 \) for all \( p \), otherwise we would have \( L_\lambda(p) = \infty \), a contradiction with necessary Lagrangian theorems. Note that \( L_\lambda(p) = 0 \) may obtain either by \( d\mu(p) = 0 \) or \( d\mu(p) \neq 0 \). To distinguish these two cases it is useful to define

\[
\hat{L}_\lambda(p) = \max_{\{c_{j,\lambda}(p) \cdot \gamma \geq 0\} | \gamma | = 1} (a_\lambda(p) \cdot \gamma - \max \{ b_\lambda(p) \cdot \gamma, 0 \})
\]

for values of \( p \) for which the constraint set is non-empty; otherwise we set \( \hat{L}_\lambda(p) = -\infty \). We have \( L_\lambda(p) = \max \{ 0, \hat{L}_\lambda(p) \} \). Then if \( \hat{L}_\lambda(p) < 0 \) then \( d\mu(p) = 0 \). Conversely, suppose \( d\mu(p) = 0 \) whenever \( \hat{L}_\lambda(p) < 0 \) and that the mechanism is feasible and satisfies the complementary slackness conditions; then this mechanism is efficient. Thus it is easy to verify whether a given set of multipliers \( \lambda \) are consistent with the solution to our problem and, if they are, to find the solution.

## 5 Private Values

Assume \( v^B_{bs} = v^B_{bs}', \) and \( v^S_{bs} = v^S_{bs}' \) for all \( b, b', s, \) and \( s' \). In this case, we say that the buyer and seller have private values and denote those values as \( v^B_b \) and \( v^S_s \). Note that in general the values may be correlated through the joint density \( \pi_{bs} \). Nevertheless, once a trader knows his own signal, his willingness to pay a price \( p \) is unaffected by the other trader’s signal.

In general, there are costs and benefits to stochastic mechanisms. The cost of a stochastic mechanism is that the variability in prices increases the number of relevant partici-
pation constraints, since the buyer must be induced to buy at the highest prices he may observe and the seller induced to sell at the lowest prices. In addition, stochastic mechanisms tighten the incentive compatibility constraints, since a trader has the option to misreport his type and then only opt to trade with some probability. The benefit is that a stochastic mechanism hides information from the traders so that each trader is less certain about his own valuation. This relaxes the participation constraints.

With private values, neither trader’s ex post participation constraint is affected by information revealed through the trading process, which eliminates the benefits to randomization. As a result, random prices are never optimal:

**Proposition 3.** Assume private values. Take any feasible veto-IC mechanism. There exists a feasible veto-IC mechanism with the same trading probabilities and expected payoffs conditional on any signals \((b, s)\) which is deterministic.

The proof constructs a deterministic mechanism with the same expected transfers as the veto-IC mechanism and shows that the use of a deterministic mechanism relaxes both the participation and incentive constraints.

In many cases we also find that an optimal mechanism is fully-revealing under private values; however, it is possible to construct examples in which one trader is unable to deduce the other trader’s value from the price. For example, suppose \(v_b^B > v_s^S\) for all \(b\) and \(s\). Then a feasible mechanism is to set a price \(p \in [\max_s v_s^S, \min_b v_b^B]\) with probability 1, achieving the first best outcome. Such a mechanism does not reveal either trader’s report to the other, but conversely there is no cost to revealing those reports. This last result is true more generally. With private values, there is no cost to revealing each trader’s report to the other, along with the recommended price, since the revelation will affect neither the participation nor the incentive constraint.

### 6 Common Values

#### 6.1 An Example

The remainder of the paper focuses on a particular example with common values. We use this example to illustrate some properties of the model, but note that our simulations indicate that these properties are more general. We assume that each trader’s signal is
binary, \( N^B = N^S = 2 \), and the two signals are correlated:

\[
\pi_{bs} = \begin{cases} 
\frac{1}{2} (\alpha^2 + (1 - \alpha)^2) & \text{if } b = s \\
\alpha (1 - \alpha) & \text{if } b \neq s,
\end{cases}
\]

where \( \frac{1}{2} < \alpha < 1 \). The seller’s payoff is

\[
v^S_{bs} = \begin{cases} 
\frac{(1-\alpha)^2}{\alpha^2 + (1-\alpha)^2} & b = s = 1 \\
\frac{\alpha^2}{\alpha^2 + (1-\alpha)^2} & b = s = 2,
\end{cases}
\]

while the buyer’s payoff is \( v^B_{bs} = \gamma + v^S_{bs} \) for all \((b, s)\) where \( \gamma > 0 \).

As motivation for the payoff structure, suppose that the asset has a binary payoff \( \delta \in \{0, 1\} \), taking on each value with equal probability. The seller values the asset at \( \delta \), while the buyer values it at \( \gamma + \delta \), reflecting some extrinsic gain from trade. Neither trader knows the true value of \( \delta \), but instead each receives a binary signal. The signals are independent conditional on the asset’s payoff, but they are only imperfectly correlated with the payoff. In particular, the buyer (seller) receives the “accurate” signal \( b = \delta + 1 \) \((s = \delta + 1)\) with probability \( \alpha \) and otherwise receives the inaccurate signal \( b = 2 - \delta \) \((s = 2 - \delta)\). One can easily verify that these signals give rise to the correlation structure and signal-conditional expected values defined above.

### 6.2 Trade Maximization

We start by describing the veto-IC mechanism that maximizes the gains from trade \( V^B + V^S \). Since \( v^B_{bs} > v^S_{bs} \) for all \((b, s)\), this is equivalent to maximizing the probability of trade and so is a natural benchmark. The following proposition characterizes the optimal mechanism:

**Proposition 4.** A trade-maximizing feasible veto-IC mechanism uses three prices, \( v^B_{11}, \frac{1}{2} (1 + \gamma) \), and \( v^S_{22} \).

1. Assume \( \gamma \geq 2\alpha - 1 \). The trade-maximizing veto-IC mechanism is non-revealing. Trade occurs at price \( \frac{1}{2} (1 + \gamma) \) for sure.

2. Assume

\[
2\alpha - 1 > \gamma > \frac{\alpha (\alpha (3 - 2\alpha) - 1)}{(1 - \alpha (1 - \alpha))(1 - 2\alpha (1 - \alpha))}.
\]

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Then the trade-maximizing veto-IC mechanism is partially-revealing and stochastic:

(a) If \( b = s = 1 \), there is trade at price \( v_{B11}^B \) with probability \( \mu \) and otherwise at price \( 1 + \gamma / 2 \).
(b) If \( b = s = 2 \), there is trade at price \( v_{S22}^S \) with probability \( \mu \) and otherwise at price \( 1 + \gamma / 2 \).
(c) If \( (b,s) = (2,1) \), there is trade at price \( 1 / 2 (1 + \gamma) \) for sure.
(d) If \( (b,s) = (1,2) \), there is trade at price \( 1 / 2 (1 + \gamma) \) with probability \( \lambda \) and otherwise no trade.

3. Assume

\[
\frac{\alpha (\alpha (3 - 2\alpha) - 1)}{(1 - \alpha (1 - \alpha))(1 - 2\alpha(1 - \alpha))} \geq \gamma > 0.
\]

Then the trade-maximizing veto-IC mechanism is fully-revealing and deterministic:

(a) If \( b = s = 1 \), there is trade at price \( v_{11}^B \) with probability \( \lambda \) and otherwise no trade.
(b) If \( b = s = 2 \), there is trade at price \( v_{22}^S \) with probability \( \lambda \) and otherwise no trade.
(c) If \( (b,s) = (2,1) \), there is trade at price \( 1 / 2 (1 + \gamma) \) for sure.
(d) If \( (b,s) = (1,2) \), there is no trade.

Figure 1 shows the three regions of the parameter space. When the gains from trade are large, \( \gamma \geq 2\alpha - 1 \), the first best can be attained by trading at a constant price. In this case, a buyer who knows that he received the signal \( b \) is willing to accept the price \( 1 / 2 (1 + \gamma) \) if he does not know the seller’s signal, and similarly for the seller with signal \( s \) who does not know the buyer’s signal. A constant price successfully keeps this information hidden from each trader. Note, however, that if a buyer had the low signal and knew the seller had the low signal, she would refuse to trade at this price whenever \( v_{11}^B < 1 / 2 (1 + \gamma) \) or \( \gamma < \frac{a^2(1-\alpha)^2}{a + (1-\alpha)^2} \). Similarly a seller with the high signal would refuse to sell to a buyer with the high signal if he knew this information whenever \( v_{22}^S > 1 / 2 (1 + \gamma) \), which reduces to the same inequality. Therefore hiding information is critical to the structure of this mechanism.

For intermediate gains from trade, the mechanism necessarily transmit some information to the buyer and seller, since each knows the other’s signal whenever the mechanism recommends trading at the extreme prices \( v_{22}^S \) or \( v_{11}^B \). Nevertheless, randomization serves to moderate the amount of information transmitted by the mechanism. When the traders are instructed to trade at the intermediate price \( 1 / 2 (1 + \gamma) \), neither is sure about the other’s signal. Nevertheless, a buyer with the low signal is sufficiently confident that the seller has the high signal that he is willing to pay a high price, and conversely for a seller with the high signal.
Finally, with low gains from trade, the mechanism is deterministic and fully-revealing. In this case, the probability of trade is low but hiding information from the traders, while feasible, turns out to be counterproductive.

Our proof Proposition 4 is constructive. First we verify that each of the mechanisms describes a feasible veto-IC mechanism. To do so, we define probabilities $\mu$ and $\lambda$ and compute the ex ante probability of trade. We then solve the problem of maximizing the trading probability across all interim mechanisms and show that the trading probability is the same. Since any feasible veto-IC mechanism is associated with a feasible interim mechanism, this proves that the mechanism we constructed is optimal in the narrower class of veto-IC mechanisms.

The finding that veto-IC mechanisms achieve the same probability of trade as interim mechanisms is surprising. This result does not appear to be particular to our particular example. In extensive numerical work, we find that the utilitarian maximum is the same with veto-IC or interim mechanisms whenever each of the buyer and seller has two possible signals for any values of $\pi$, $v^B$, and $v^S$.\footnote{We have not explored models with more signals extensively. Proving this result is high on our agenda,}

Figure 1: Regions of the parameter space in which different types of veto-IC mechanisms are trade-maximizing. The details of the mechanisms are given in Proposition 4.
constraints are not binding and may be useful for understanding why real-world trading mechanisms allow traders to walk away after learning the transaction price. This lack of commitment may be costless.

At the same time, this conclusion does not imply that the veto-IC constraints are irrelevant. With interim mechanisms, the revelation of information within the mechanism is of no consequence. In contrast, with veto-IC mechanisms, information revelation may preclude trade, since agents may realize that the trade is not beneficial for them. Thus, the mechanism may have to conceal some information, placing questions of information transmission in center stage.

To see this last point, suppose we restrict attention to fully-revealing mechanisms. With very large or very small gains from trade, there are no losses from these restrictions. First, suppose

$$\gamma \geq \frac{2\alpha - 1}{\alpha^2 + (1 - \alpha)^2}.$$  

Then \(v_{bs}^S \leq v_{b's}^B\) for all \(b, b', s, \) and \(s'\). A constant price in \(\frac{1}{2}(v_{22}^S + v_{11}^B)\) achieves the first best, always trading, even if both traders know the other trader’s signal. Second, suppose condition (8) holds. Then the optimal veto-IC mechanism is fully revealing according to Proposition (4).

But at intermediate values of the gains from trade,

$$\frac{2\alpha - 1}{\alpha^2 + (1 - \alpha)^2} > \gamma > \frac{a(a(3 - 2\alpha) - 1)}{(1 - a(1 - \alpha))(1 - 2a(1 - a))},$$

trade maximization necessarily entails hiding information. If the buyer knew when both traders received the low signal, he would refuse to pay more than \(v_{11}^B\); and if the seller knew when both traders received the high signal, he would refuse to accept less than \(v_{22}^S\). The constrained optimal mechanism would look like the one in the second part of Proposition 4, with the additional constraint that \(\mu = 1\). Since expected prices are more sensitive to reports, the incentive constraint would have to reduce the probability of trading following the reports \((b, s) = (1, 2)\). For example, when \(\gamma = 2\mu - 1\), the probability of trade is 1 under the optimal mechanism but could fall to \(\frac{11}{12}\) under the best fully-revealing mechanism. Figure 2 shows the increased efficiency of a partially-revealing mechanism when \(\alpha = 0.75\).

A similar logic shows that a restriction to deterministic mechanisms is costless either when \(\gamma \geq 2\alpha - 1\) or condition (8) holds. But in the intermediate region, it is critical that the trading price depends on the two reports and on the outcome of a lottery. Restricting in part because a proof may offer some intuition for our finding.

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attention to deterministic mechanisms would have the same adverse impact as restricting attention to fully-revealing mechanisms.

### 6.3 Pareto Frontier

We close by characterizing the set of feasible payoffs, and in particular the Pareto frontier, for our example. We compare the standard interim problem (constraints 1 and 2) with the veto-IC problem (constraints 4 and 5). In general, we find that, away from the trade maximizing mechanism, the solution to the interim and veto-IC problems do not coincide. The following Proposition explains how veto incentive compatibility imposes additional constraints on transfers and trading probabilities that are not present in the interim problem. The proposition applies somewhat more generally than our particular example.

**Proposition 5.** Assume $N^B = N^S = 2$ and $v^B_{bs}$ and $v^S_{bs}$ are nondecreasing in $b$ and $s$. Consider any feasible veto-IC mechanism $\{p, \omega\}$. Let $q_{bs} \equiv \sum_n \omega_n|_{bs}$ and $\tau_{bs} \equiv \sum_n p_n \omega_n|_{bs}$ denote the trading probability and expected transfer conditional on the reports in this mechanism. Then

$$\tau_{bs} \in [v^S_{1s}q_{bs}, v^B_{bs}q_{bs}] \text{ for all } (b, s) \in \{1, 2\}^2.$$  

(9)
In addition, if $v_{22}^S q_{22} > \tau_{22}$,

$$\frac{v_{12}^B q_{12} - \tau_{12}}{v_{22}^S q_{22} - \tau_{22}} \geq \frac{\pi_{22} (v_{12}^B q_{22} - \tau_{22})}{\pi_{12} (\tau_{22} - v_{12}^S q_{22})},$$

(10)

and if $\tau_{11} > v_{11}^B q_{11}$,

$$\frac{\tau_{12} - v_{12}^S q_{12}}{\tau_{11} - v_{11}^S q_{11}} \geq \frac{\pi_{11} (\tau_{11} - v_{12}^S q_{11})}{\pi_{12} (v_{12}^B q_{11} - \tau_{11})}.$$

(11)

The first set of constraints are intuitive. For example, suppose $\tau_{12} < v_{12}^S q_{12}$. A seller who receives the high signal believes the value of the asset is at least $v_{12}^S$, regardless of the buyer’s signal, and so would never accept a price less than $v_{12}^S$, regardless of her beliefs. But any mechanism that implements the same allocation as the interim problem must have $\sum_n \omega_n q_{12} = q_{12}$ and $\sum_n p_n \omega_n q_{12} = \tau_{12}$. This implies $\sum_n p_n \omega_n q_{12} / \sum_n \omega_n q_{12} < v_{12}^S$, so $\omega_n q_{12}$ must put positive weight on some $p_n < v_{12}^S$, which is impossible. Similarly, if $\tau_{12} > v_{12}^B q_{12}$, a buyer who receives the low signal would refuse to pay the necessary average price, regardless of her beliefs.

It is easy to construct examples of economies in which these constraints bind. That is, there are feasible payoffs in the interim problem which can only be obtained by setting $\tau_{bs} \notin [v_{0s}^S q_{bs}, v_{b1}^B q_{bs}]$ for some $(b, s)$. Figure 3 illustrates one such case. The lightest shaded region illustrates the set of payoffs that can be obtained using a feasible fully-revealing veto-IC mechanism. The intermediate shaded region is the set of payoffs that can additionally be obtained using an arbitrary veto-IC mechanism. And the darkest shaded region is the set of payoffs that can be obtained using an interim mechanism.

The constraints (10) and (11) are more subtle but also play a role in the construction of Figure 3. The basic problem is that a mechanism in the interim problem may set a higher transfer relative to the trading probability when the buyer reports the low signal than when he reports the high signal, conditional on the seller reporting the high signal: $\tau_{12} / q_{12} > \tau_{22} / q_{22}$. To implement this, the average trading price must be lower when the buyer has the high signal than when he has the low signal. But this means that the seller with can infer from a low trading price that the buyer most likely got a high signal, raising the minimum price he is willing to accept above $v_{12}^S$. This places an additional restriction on the relationship between $\tau_{12}$, $\tau_{22}$, $q_{12}$, and $q_{22}$.

In the example in Figure 3, the constraints (10) and (11) sometimes bind. For example, suppose we want to maximize the seller’s utility subject to giving the buyer at least 0.09. In the interim problem, we find $q_{11} = q_{22} = q_{21} = 1$ and $q_{12} = 0.875$, with $\tau_{11} = 0.512$, $\tau_{22} = 0.543$, $\tau_{21} = 0.708$, and $\tau_{12} = 0.622$, giving the seller utility 0.214. But this policy violates the constraint (11) for the buyer (and none of the other constraints). Instead, the
veto-IC problem implies $q_{11} = q_{22} = q_{21} = 1$ and $q_{12} = 0.830$, with $\tau_{11} = 0.564$ and $\tau_{22} = 0.616$, both higher, and $\tau_{21} = 0.625$ and $\tau_{12} = 0.520$, both lower. The reduction in the trading probability implies that the seller’s utility is lower, 0.212. Note that in this example, it is still the case that the expected trading price $\tau_{bs}/q_{bs}$ is higher when one of the buyer or seller has the high signal, $b = 1 - s$, than when both have the high signal, $b = s = 1$, but the gap is smaller than in the interim mechanism.

Conversely, our numerical simulations suggest that there are no other binding constraints in the interim problem. That is, consider a feasible interim mechanism $\{\tau_{bs}, q_{bs}\}$ that satisfies the participation constraints (1), the incentive constraints (2), and the three additional constraints in Proposition 5. Then there exists an equivalent mechanism $\{p, \omega\}$ with $q_{bs} \equiv \sum_n \omega_{n|bs}$ and $\tau_{bs} \equiv \sum_n p_n \omega_{n|bs}$ satisfying the participation constraints (4) and
incentive constraints (5).

In any case, Figure 3 illustrates the important role played by concealing information in veto-IC mechanisms. The smallest shaded region indicates the set of payoffs obtainable by a fully-revealing veto-IC mechanisms. With fully-revealing mechanisms, a trader always knows his trading partner’s signal, and so the veto-IC constraints require that prices are sensitive to information. To offset this, the incentive constraints reduce the probability of trade when the buyer reports the low signal or the seller reports the high signal, lowering overall efficiency.

An open question is the conditions under which any payoff obtainable from an interim mechanisms is also obtainable from a veto-IC mechanisms. Our numerical investigations suggest that if the two signals are independent, so $\pi_{bs} = \pi_b^B \pi_s^S$ for all $b$ and $s$, then any payoff attainable from a feasible interim mechanism is attainable with a feasible veto-IC mechanism. This recalls the result in McAfee and Reny (1992) that with a third party, the first best is attainable in an interim feasible mechanism whenever the two signals are correlated. With independent signals, third parties do not expand the set of payoffs and veto incentive compatibility does not further restrict them.

References


A Proofs

Proof of Proposition 1. To be completed

Proof of Proposition 2. A deterministic veto-IC mechanism is a report-conditional measure $\mu_{bs}$ over sets of prices. In particular, for any $P \subset \mathbb{R}_+$, let $\mu_{bs}(P)$ denote the probability that the mechanism recommends trading at a price $p \in P$ conditional on the reports $(b, s)$, with $1 - \mu_{bs}(\mathbb{R}_+)$ denoting the conditional probability of no trade.

Take any measures $\{\mu_{bs}\}$ that satisfy the ex-post participation and nonnegative constraints:

$$\int_P \sum_b (v_b^B - p) \pi_{bs} d\mu_{bs}(p) \geq 0 \text{ for all } b \text{ and } P \subset \mathbb{R}_+,\n$$

$$\int_P \sum_b (p - v_s^S) \pi_{bs} d\mu_{bs}(p) \geq 0 \text{ for all } s \text{ and } P \subset \mathbb{R}_+,\n$$

and $\int_P d\mu_{bs}(p) \geq 0$ for all $P \subset \mathbb{R}_+$. Define

$$V_{bb'}^B \equiv \int_{\mathbb{R}_+} \max \left\{ \sum_s (v_s^B - p) \pi_{bs} d\mu_{bs}(p), 0 \right\},$$

$$V_{ss'}^S \equiv \int_{\mathbb{R}_+} \max \left\{ \sum_b (p - v_b^S) \pi_{bs} d\mu_{bs}(p), 0 \right\},$$

and $Q_{bs} \equiv \int_{\mathbb{R}_+} d\mu_{bs}(p)$.

These are the value of a buyer who receives signal $b$ and reports $b'$, the value of a seller who receives signal $s$ and reports $s'$, and the probability of trade when the buyer reports...
signal \(b\) and the seller reports signal \(s\). The outcome

\[
x \equiv (\{\bar{V}_{bb'}^B\}, \{\bar{V}_{ss'}^S\}, \{Q_{bs}\}),
\]

a vector of dimension \((N^B)^2 + (N^S)^2 + N^B N^S\), contains all the information necessary to evaluate a mechanism’s feasibility and compute the welfare for the buyer and seller. In particular, the measures are a feasible veto-IC mechanisms if and only if

\[
\bar{V}_{bb}^B \geq \bar{V}_{bb'}^B, \quad \bar{V}_{ss}^S \geq \bar{V}_{ss'}^S, \quad \text{and} \quad 1 \geq Q_{bs}
\]

for all \(b, b', s,\) and \(s'\); and the expected payoffs from the mechanism are \(V^B = \sum_b \bar{V}_{bb}^B\) and \(V^S = \sum_s \bar{V}_{ss}^S\).

Now define the set of prices \(P\) such that \(p \in P \iff \{d\mu_{bs}(p)\} \neq 0\). For each \(p \in P\), define

\[
\tilde{x}(p) \equiv (\{\bar{V}_{bb'}^B(p)\}, \{\bar{V}_{ss'}^S(p)\}, \{\gamma_{bs}\}),
\]

where

\[
\bar{V}_{bb'}^B(p) \equiv \max \left\{ \sum_s (v_{bs}^B - p) \pi_{bs} \gamma_{b's}(p), 0 \right\},
\]

\[
\bar{V}_{ss'}^S(p) \equiv \max \left\{ \sum_b (p - v_{bs}^S) \pi_{bs} \gamma_{bs'}(p), 0 \right\},
\]

and \(\gamma_{bs}(p) = \frac{d\mu_{bs}(p)}{\sum_{b's} d\mu_{b's}(p)}\).

Note that for each \(p \in P\), either \(\bar{V}_{bb'}^B(p) = \frac{\sum_s (v_{bs}^B - p) \pi_{bs} d\mu_{b's}(p)}{\sum_{b's} \sum_s d\mu_{b's}(p)}\) or \(\bar{V}_{bb'}^B(p) = 0\), and likewise for \(\bar{V}_{ss'}^S(p)\). This implies that

\[
x = \int_P \tilde{x}(p) d\mu(p),
\]

where \(\mu(P) \equiv \sum_{bs} \mu_{bs}(P)\) for all \(P \subset \bar{P}\). Thus \(\tilde{x}\) is a Radon-Nikodym derivative and the outcome \(x\) is obtained by choosing a measure \(\mu\) over the vectors \(\tilde{x}\) indexed by \(p \in P\). In other words, \(x\) is in the convex cone generated by the vectors \(\{\tilde{x}(p)\}_{p \in P}\). It follows by Carathéodory’s Theorem that there are \(N \equiv (N^B)^2 + (N^S)^2 + N^B N^S + 1\), such that

\[
x = \sum_{n=1}^{N} \tilde{x}(p_n) \omega_n
\]
for $\omega_n \geq 0$ and $\sum_{n=1}^{N} \omega_n = 1$. \hfill $\square$

**Proof of Proposition 3.** Take any feasible mechanism $\{p, \omega\}$. We construct a deterministic feasible mechanism with the same trading probabilities and expected payoffs. It will be useful to define $\omega_{n_{bs}} = (b - 1)N^S + s \in \{1, \ldots, NB^N S\}$, a unique indicator for the traders’ reports. Let $\tilde{\omega}_{n_{bs}|bs} = \sum_n \omega_{n_{bs}}$ and $\tilde{\omega}_{n_{bs}} = 0$ if $n \neq n_{bs}$. In addition, if $\tilde{\omega}_{n_{bs}|bs} > 0$, let $\tilde{p}_{n_{bs}} = \sum_n p_n \omega_{n_{bs}} / \tilde{\omega}_{n_{bs}|bs}$. Then $\{\tilde{p}, \tilde{\omega}\}$ is a deterministic mechanism: if the reports $(b, s)$ result in trade with positive probability, trade occurs at the price $\tilde{p}_{n_{bs}}$.

To prove that $\{\tilde{p}, \tilde{\omega}\}$ is feasible, we show that it satisfies the buyer’s participation and incentive constraints. The proof for the seller is symmetric. Fix $(b, s)$. If $\tilde{\omega}_{n_{bs}|bs} = 0$ or $\pi_{bs} = 0$, the participation constraints (4) trivially hold, so assume that both are positive. Then

$$\tilde{p}_{n_{bs}} = \frac{\sum_n p_n \omega_{n_{bs}}}{\sum_n \omega_{n_{bs}}} \leq \max_{n|\omega_{n_{bs}} > 0} p_n.$$  

The buyer’s participation constraint (4) for the mechanism $\{p, \omega\}$ implies

$$\sum_s (v^B_b - p_n) \omega_{n_{bs}} \pi_{bs} \geq 0,$$

so $v^B_b \geq p_n \geq \tilde{p}_{n_{bs}}$. This proves that

$$\sum_s (v^B_b - \tilde{p}_{n'_{bs}}) \tilde{\omega}_{n'_{bs}|bs} \pi_{bs} \geq 0,$$

for all $b$ and $n'_{bs}$, so the buyer’s participation constraint holds for the deterministic mechanism $\{\tilde{p}, \tilde{\omega}\}$ as well.

Turn next to the incentive constraints (5). With private values, the buyer’s incentive constraint for the mechanism $(p, \omega)$ reduces to

$$\sum_{n, s} (v^B_b - p_n) \omega_{n_{bs}} \pi_{bs} \geq \sum_n \max \left\{ \sum_s (v^B_b - p_n) \omega_{n_{bs}} \pi_{bs}, 0 \right\}$$

$$= \sum_{n, s} \max \left\{ (v^B_b - p_n) \omega_{n_{bs}} \pi_{bs}, 0 \right\}$$

$$\geq \sum_s \max \left\{ \sum_n (v^B_b - p_n) \omega_{n_{bs}}, 0 \right\}.$$  

The first inequality is the veto incentive constraint. The first equality holds because the sign of $(v^B_b - p_n) \omega_{n_{bs}} \pi_{bs}$ does not depend on $s$. The last inequality may be strict because the sign of the same term may depend on $n$. Now using the definitions of $\tilde{p}$ and $\tilde{\omega}$, this
reduces to
\[
\sum_s (v^B_b - \bar{p}_{nbs}) \tilde{\omega}_{nbs} \pi_{bs} \geq \sum_s \max \left\{ (v^B_b - \bar{p}_{n's}) \tilde{\omega}_{n's|b's} \pi_{bs}, 0 \right\},
\]
so the buyer’s incentive constraint holds for the deterministic mechanism \(\{\bar{p}, \tilde{\omega}\}\) as well.

**Proof of Proposition 4.** We divide the parameter space into three regions. In each region, we first verify that the proposed mechanism is feasible and then prove that it achieves the same trading probability as the trade-maximizing interim mechanism.

**Non-revealing Region:** Assume \(\gamma \geq 2\alpha - 1\). Under the proposed mechanism, the participation constraint of a buyer with the low signal \(b = 1\) and a seller with the high signal \(s = 2\) hold if and only if \(\gamma \geq 2\alpha - 1\). The remaining participation constraints (4) always hold, and the incentive constraints (5) hold trivially. This mechanism is therefore feasible. It obtains the unconstrained optimal amount of trade and hence is trade-maximizing.

**Partially-Revealing Region:** Next assume condition (7) holds. We first prove that the mechanism in the statement of the problem is feasible and find the probabilities \(\mu, \lambda\). These are pinned down by two equations. The first is the the binding participation constraint (4) of the buyer who receives the low signal and is instructed to trade at \(\frac{1}{2}(1 + \gamma)\):
\[
(v^B_{11} - \frac{1}{2}(1 + \gamma))(1 - \mu)\pi_{11} + (v^B_{12} - \frac{1}{2}(1 + \gamma))\lambda\pi_{12} = 0.
\]
Because of the symmetry of the mechanism, the participation constraint of the seller who receives the high signal and is instructed to trade at \(\frac{1}{1+\gamma}\) is identical. By construction, the buyer who receives the low signal and is instructed to buy at \(v^B_{11}\) is just indifferent about participating, and similarly for the seller who receives the high signal and is instructed to sell at \(v^S_{22}\). The remaining participation constraints are slack.

The second equation is the binding incentive constraint (5) of a seller who receives the low signal:
\[
(v^B_{11}\mu + \frac{1}{2}(1 + \gamma)(1 - \mu) - v^S_{11})\pi_{11} + (\frac{1}{2}(1 + \gamma) - v^S_{21})\pi_{21} = \\
(\frac{1}{2}(1 + \gamma) - v^S_{11})\lambda\pi_{11} + (v^S_{22}\mu + \frac{1}{2}(1 + \gamma)(1 - \mu) - v^S_{21})\pi_{21}.
\]
The incentive constraint of a buyer who receives the high signal is identical and the other two incentive constraints are slack. We can therefore solve these two equations for the probabilities \(\mu\) and \(\lambda\) and verify that under the condition (7), both lie strictly between 0 and 1.
Next solve the interim trade-maximization problem in this region. In the solution to this problem, the incentive constraints of the seller with a high signal and a buyer with a low signal are slack, as are the participation constraint of the other traders, the seller with a low signal and the buyer with a high signal. The remaining constraints bind as well as the constraints on trading probabilities $q_{00} \leq 1$, $q_{11} \leq 1$, and $q_{10} \leq 1$. This gives seven equations in eight unknowns. Generically there are a continuum of mechanisms that implement the trade-maximizing allocation, each with a different distribution of profits between the buyer and seller. Our veto-IC mechanism implements a particular one, where the buyer and seller earn equal profits.

In the symmetric trade-maximizing interim mechanism, $q_{11} = q_{21} = q_{22} = 1$ and $q_{12} < 1$. The value of $q_{12}$ is pinned down from three observations. First, symmetry implies the gains from trade are distributed equally, so $\tau_{11} + \tau_{22} = 1 + \gamma$ and $\tau_{12}/q_{12} = \tau_{21} = \frac{1+\gamma}{2}$. Second, the participation constraint (1) of the buyer with the low signal binds, as does the (symmetric) participation constraint of the seller with the high signal. Third, the incentive constraint (2) of the buyer with the high signal binds, as does the (symmetric) incentive constraint of the seller with the low signal. Together these results imply

\[
(v_{11}^B - \tau_{11}) \pi_{11} + (v_{12}^B q_{12} - \tau_{12}) \pi_{12} = 0
\]

\[
(\tau_{11} - v_{11}^S) \pi_{11} + (\tau_{21} - v_{21}^S) \pi_{21} = (\tau_{12} - v_{12}^S q_{12}) \pi_{11} + (\tau_{22} - v_{22}^S) \pi_{21}.
\]

The solution to this pair of equations uniquely defines $\tau_{11}$ and $q_{12}$, with $q_{12} \in [0, 1]$ under condition (7). Finally, these equations imply $\lambda = q_{01}$. Therefore the proposed veto-IC mechanism achieves the same trade probability as the interim trade-maximizing mechanism and so is trade-maximizing among all veto-IC mechanisms.

**Fully-Revealing Region:** Now assume condition (8) holds. The proposed mechanism is fully revealing and so trivially satisfies the ex post participation constraint (4). The binding incentive constraint of the seller who receives the low signal pins down the parameter $\lambda$:

\[
(v_{11}^B - v_{11}^S) \lambda \pi_{11} + (\frac{1}{2}(1 + \gamma) - v_{21}^S) \pi_{21} = (v_{22}^S - v_{21}^S) \lambda \pi_{21}.
\]

Under condition (8), this defines $\lambda \in (0, 1]$.

Next turn to the interim problem. Once again, the incentive constraints of the seller with $s = 2$ and the buyer with $b = 1$ are slack, as are the participation constraint of the seller with $s = 1$ and the buyer with $b = 2$. The remaining constraints bind, as well as the constraints $q_{12} \geq 0$ and $q_{21} \leq 1$. Again, there are generically a continuum of mechanisms that implement the trade-maximizing allocation, each with a different distribution of profits between the buyer and seller. Our veto-IC mechanism implements
In the symmetric trade-maximizing mechanism, \( q_{21} = 1, q_{12} = 0, \) and \( q_{11} = q_{22} < 1. \) The value of \( q_{11} = q_{22} \) is pinned down from three observations. First, symmetry implies \( \tau_{11} + \tau_{22} = q_{11}(1 + \gamma), \tau_{12} = 0, \) and \( \tau_{21} = \frac{1}{2}(1 + \gamma). \) Second, the participation constraint (1) of the buyer with the low signal binds, as does the (symmetric) participation constraint of the seller with the high signal. Third, the incentive constraint (2) of the buyer with the high signal binds, as does the (symmetric) incentive constraint of the seller with the low signal. Together these results imply

\[
(v_{11}^B q_{11} - \tau_{11}) \pi_{11} = 0, \\
(\tau_{11} - v_{11}^S q_{11}) \pi_{11} + (\tau_{21} - v_{21}^S) \pi_{21} = (\tau_{22} - v_{21}^S q_{22}) \pi_{21}.
\]

The solution to this pair of equations uniquely defines \( q_{11} = q_{22} = \lambda, \) which again proves that the proposed veto-IC mechanism is trade-maximizing among all veto-IC mechanisms.

**Proof of Proposition 5.** A seller who receives signal \( s \) knows that the value of the asset is at least \( v_{1s}^S \) and so the participation constraint (4) ensures that \( \omega_{n|bs} = 0 \) for all \( p_n < v_{1s}^S. \) Then either \( q_{bs} = 0, \) in which case \( \tau_{bs} = 0, \) or \( q_{bs} > 0 \) and \( \tau_{bs}/q_{bs} \geq v_{1s}^S. \) This proves \( \tau_{bs} \geq v_{1s}^S q_{bs}. \) Similarly, a buyer who receives signal \( b \) knows that the value of the asset is no more than \( v_{b2}^B \) and so the participation constraint (4) ensures that \( \omega_{n|bs} = 0 \) for all \( p_n > v_{b2}^B. \) Then either \( q_{bs} = 0, \) in which case \( \tau_{bs} = 0, \) or \( q_{bs} > 0 \) and \( \tau_{bs}/q_{bs} \leq v_{b2}^B. \) This proves \( \tau_{bs} \leq v_{b2}^B q_{bs}. \)

Now suppose \( q_{22} v_{22}^S > \tau_{22}. \) Define

\[
\phi^S(p) \equiv \begin{cases} 
\frac{\pi_{22}(v_{22}^S - p)(v_{12}^B - p)}{\pi_{12}(p - v_{12}^S)} & v_{12} < p \leq \min\{v_{22}^S, v_{12}^B\} \\
0 & p > \min\{v_{22}^S, v_{12}^B\}.
\end{cases}
\]

Observe that \( \phi^S \) is convex at any \( p \geq v_{12}^S. \) The seller’s participation constraint when he has the signal \( s = 2 \) and is instructed to trade at the price \( p_n \) is

\[
(p_n - v_{12}^S) \omega_{n|12} \pi_{12} + (p_n - v_{22}^S) \omega_{n|22} \pi_{22} \geq 0.
\]

This implies \( \omega_{n|12} = 0 \) for all \( p_n < v_{12}^S, \omega_{n|22} = 0 \) for all \( p_n \leq v_{12}^S, \) while for \( p_n \geq v_{12}^S, \)

\[
(v_{12}^B - p) \omega_{n|12} \geq \phi^S(p) \omega_{n|22}.
\]
Then

$$
\tau_{12} = \sum_{n | v_{12}^S \leq p_n \leq v_{12}^B} p_n \omega_{n|12}
= v_{12}^B q_{12} - \sum_{n | v_{12}^S \leq p_n \leq v_{12}^B} (v_{12}^B - p_n) \omega_{n|12}
\leq v_{12}^B q_{12} - \sum_{n | v_{12}^S < p_n \leq v_{12}^B} \phi^S(p_n) \omega_{n|22}
= v_{12}^B q_{12} - \sum_{n | v_{12}^S < p_n \leq v_{12}^B} \phi^S(p_n) \omega_{n|22}
$$

$$
\leq v_{12}^B q_{12} - \phi^S \left( \frac{\sum_n | v_{12}^S < p_n \leq v_{22}^B | p_n \omega_{n|22}}{\sum_n | v_{12}^S < p_n \leq v_{22}^B \omega_{n|22}} \right) \sum_n | v_{12}^S < p_n \leq v_{22}^B \omega_{n|22}
= v_{12}^B q_{12} - \phi^S (\tau_{22} / q_{22}) q_{22}
$$

The first equation is the definition of $\tau_{12}$. The second equation rewrites the sum in a more convenient form. The first inequality uses condition (12) and $\omega_{n|22} = 0$ if $p_n = v_{12}^S$. The third equality uses the definition of $\phi^S$, together with the fact that $\omega_{n|12} = \omega_{n|22} = 0$ for $p_n \in (v_{12}^B, v_{22}^S]$ if $v_{12}^B < v_{22}^S$. The second inequality uses Jensen’s inequality, since $\phi^S$ is convex. The final equality uses the definitions of $\tau_{22}$ and $q_{22}$. Now using the definition of $\phi^S$, we obtain condition (10).

Similarly, suppose $\tau_{11} > v_{11}^B q_{11}$. Define

$$
\phi^B(p) \equiv \begin{cases}
\frac{\pi_{11} (p - v_{11}^B) (p - v_{12}^B)}{\pi_{12} (v_{12}^B - p)} & \text{max}\{v_{11}^B, v_{12}^S\} \leq p < v_{12}^B \\
0 & p < \text{max}\{v_{11}^B, v_{12}^S\}.
\end{cases}
$$

Observe that $\phi^B$ is convex at any $p < v_{12}^B$. The buyer’s participation constraint when he has signal $b = 1$ and is instructed to trade at the price $p_n$ is:

$$(v_{11}^B - p_n) \omega_{n|11} \pi_{11} + (v_{12}^B - p_n) \omega_{n|12} \pi_{12} \geq 0.$$ 

This implies $\omega_{n|11} = 0$ for all $p_n > v_{12}^B$, $\omega_{n|12} = 0$ for all $p_n \geq v_{12}^B$, while for $p \leq v_{12}^B$,

$$(p - v_{12}^S) \omega_{n|12} \geq \phi^B(p) \omega_{n|11}. \quad (13)$$
Then

\[
\tau_{12} = \sum_{n\mid v_{12}^s \leq p_n \leq v_{12}^b} p_n \omega_{n|12} = v_{12}^s q_{12} + \sum_{n\mid v_{12}^s \leq p_n \leq v_{12}^b} (p_n - v_{12}^s) \omega_{n|12} \\
\geq v_{12}^s q_{12} + \sum_{n\mid v_{11}^s \leq p_n < v_{12}^b} \phi_B(p_n) \omega_{n|11} = v_{12}^s q_{12} + \sum_{n\mid v_{11}^s \leq p_n < v_{12}^b} \phi_B(p_n) \omega_{n|11} \\
\geq v_{12}^s q_{12} + \phi_B \left( \frac{\sum_{n\mid v_{11}^s \leq p_n < v_{12}^b} p_n \omega_{n|11}}{\sum_{n\mid v_{11}^s \leq p_n < v_{12}^b} \omega_{n|11}} \right) \sum_{n\mid v_{11}^s \leq p_n < v_{12}^b} \omega_{n|11} = v_{12}^s q_{12} + \phi_B(\tau_{11} / q_{11}) q_{11}
\]

The steps exactly parallel the previous step, with the inequalities exploiting conditions (13) and convexity of \( \phi^B \). Using the definition of \( \phi^B \), we obtain condition (11). \( \square \)