Primary Elections and the Provision of Public Goods*

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Abstract  

We develop a model of electoral competition in which candidates from two parties compete in a primary and general election. There are three groups of voters, two representing “core” supporters for each party and one “swing” group. In the primary election, each party’s core voters choose a candidate to run in the general election. Candidates within a party share a fixed ideological platform and can promise to distribute a unit of public spending across private goods for the three groups of voters and a public good that distributes benefits evenly across all voters. In a benchmark world without primary elections, candidates promise only the public good when its benefit level is sufficiently high, and only private goods when this benefit is low. Relative to this benchmark, primaries do not always increase the provision of public goods, but do reduce the threshold value of public goods needed for candidates to promise only the public good when the swing group is small. The model also predicts that primary elections shift platforms toward benefits for swing voters as core voters become more ideologically extreme, and that public good provision is non-monotonic in ideological polarization.

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1 Introduction

How do political institutions shape public policy? This is a fundamental question in political economy. One important sub-question is: What features of the political system provide incentives for politicians or parties to spend government funds on public goods that benefit the vast majority of citizens, rather than goods targeted at narrow groups? This has been the subject of a number of recent theoretical and empirical studies, including Lancaster (1986), Persson and Tabellini (1999, 2003, 2004a, 2004b), Lizzeri and Persico (2001, 2005), Milesi-Ferretti, Perotti and Rostagno (2002), Persson, Roland and Tabellini (2000, 2007), and Blume et al. (2009).

These studies focus on key features of electoral systems and the separation of powers, such as plurality rule vs. proportional representation systems, district magnitude, parliamentary vs. presidential systems. One of the main arguments is that the “winner-take-all” structure of electoral outcomes under plurality rule with single-member-districts implies that the minimum winning coalition of voters to gain a majority in the legislature is smaller than under proportional representation. Thus, plurality rule induces politicians to target small but pivotal constituencies in individual electoral districts with local public goods and specific transfers. In contrast, under proportional representation “every vote counts” no matter where it is cast, and additional votes always translate directly into additional seats, providing incentives for politicians to seek the support of voters across the country. Proportional representation therefore induces politicians to favor policies benefiting large groups of voters such as general public goods and broad-based transfer programs that affect voters in many electoral districts.¹

One important political factor omitted from these analysis is internal party structure. In particular, there is no treatment of the various ways candidates are nominated. A variety of nomination methods are used around the world, including national, state, and local party conventions, caucuses, meetings restricted to small party elites, and — as in the U.S. and

¹Persson and Tabellini (1999, 2004b), Milesi-Ferretti, Perotti, and Rostagno (2002), Gagliarducci, Nannicini and Naticchioni (2011) find empirical support for these predictions. See also Lancaster and Patterson (1990), Stratmann and Baur (2002), and Kunicova and Remington (2005) for other evidence that is broadly consistent with the underlying incentives facing politicians. Many other features of the political system have also been studied, including bicameralism, vetoes, confidence procedures, party organization, and federalism. See, for example, Inman and Fitts (1990), Diermeier and Feddersen (1998), McCarty (2000a, 2000b), Bradbury and Crain (2002), Lockwood (2002), Ansolabehere, Snyder, and Ting (2003), Kalandrakis (2004), Cutrone and McCarty (2006), Berry (2008, 2009), Berry and Gersen (2009), Primo and Snyder (2010), Tergiman (2013), and Parameswaran (2012).
many Latin American countries — direct primary elections.\(^2\) Hirano, Snyder and Ting (2009) show that nomination systems may have significant effects on the allocation of distributive government spending. In particular, when electoral outcomes are uncertain, direct primaries may provide strong incentives for politicians to offer transfers to “core supporters” in addition to “swing voters.”\(^3\)

This paper shows that direct primaries may also provide incentives for politicians to supply public goods that benefit all voters, rather than distributive goods or narrowly targeted transfers that only benefit specific constituencies. The logic is straightforward. If there are no primary elections and candidates simply maximize their probability of winning in the general election, then they are driven to compete mainly for swing voters. Thus, when deciding between public goods and targeted goods, candidates are biased toward choosing targeted goods — and targeting them at swing voters. They will only choose public goods if public goods have an extremely high ratio of social benefits to costs. Under primaries, however, candidates may offer a more even distribution of targeted goods, aimed both at swing voters and core voters, since they must win both swing voters in the general election, and core voters in the primary election. Thus, they have incentives to offer public goods even when public goods have a relatively modest (but still favorable) ratio of benefits to costs.

Our theory is based on the models developed by Lindbeck and Weibull (1987) and Dixit and Londregan (1995, 1996). In their work, office-minded candidates from two parties compete for votes by promising distributions of particularistic spending across a large number of ideologically heterogeneous groups. Parties have fixed ideological positions, and are therefore advantaged in certain districts. We simplify this framework by considering only three groups of voters, corresponding to a swing group and core groups for each party. We further assume that core groups are “off limits” to the opposition in the sense that there is no incentive for the opposing party to offer particularistic goods in the hope of gaining votes. We add two important features. First, there are simultaneous primary elections in each party that determine which of its candidates will proceed to the general election. The pivotal voter in each primary is a member of the corresponding core group. Second, in addition to promising allocations of particularistic spending at the group level, candidates can commit to spending

\(^2\)For background on U.S. primary elections, see Merriam and Overacker (1928) and Ware (2002).

\(^3\)There is an extensive literature on the policy consequences of primaries or the incentives for adopting primary systems, including Aronson and Ordeshook (1972), Coleman (1972), Owen and Grofman (2006), Caillaud and Tirole (1999), Jackson, Mathevet, and Mattes (2007), Adams and Merrill (2008), Castanheira, Crutzen, and Sahuquet (2010), and Serra (2011). Most of these models are concerned with outcomes in a spatial or valence framework.
some portion of the budget on a public good that benefits all voters evenly.

As intuition might suggest, public goods will be attractive as platforms only when the value of the public good is high. Candidates from both parties choose private goods when that value is sufficiently low, and the public good when that value is sufficiently high. For intermediate values, parties will typically adopt platforms that combine the public good and private goods for either the swing group or their core group. Thus, equilibrium platforms never promise the public good and private goods to both groups. It is also possible in some cases for only one party to choose a positive level of public goods.

It is useful to illustrate the logic of platform selection in the model by considering the incentives of core groups when the value of the public good is relatively high. In choosing who wins their primary election, core groups must trade off between the probability of winning and the benefits received conditional upon victory. Extreme core voters are more inclined to give up private goods than moderates, because they are more concerned about ideological payoffs and private goods for swing voters raise a party’s probability of election. Consequently, private goods will go to the swing group when a party’s core group is extreme, and to the core group when it is moderate. In between these ideological extremes, public goods are a useful compromise between the two types of private goods. While the equilibrium of the general model can be quite complex, these relationships are examined in more detail in a variant of the model where the factions are symmetric with respect to core group size and ideology. We show that under these conditions, the level of public goods provision is non-monotonic: it increases initially in ideological polarization, and then abruptly drops to zero as candidates use only private goods to chase the swing group.

The model generates a number of predictions about the level of provision of private goods. Somewhat surprisingly, compared to a world without primary elections, public goods are not always better provided under a primary system. The reason for this is that when a core group is small, the per capita value of private goods can be so high that candidates will have to offer some in order to win the primary election. Compared to a non-primary system, the threshold social benefit at which public goods will be offered exclusively by all candidates is lower under a primary system when the swing group is the smallest group. Thus primaries can be said to encourage the exclusive production of public goods when core groups in society are both large and somewhat extreme. On another metric, however, primaries can do well in encouraging public goods. The threshold social benefit for candidates to promise some positive level of public goods is lower than the threshold for offering only public goods. Thus the range of parameter values under which some public goods are offered will typically be
larger under a primary system. But despite this, the model also predicts that this threshold is strictly above the level at which public goods are socially efficient.

The paper proceeds as follows. The next section describes the model. Section 3 derives the results for the model with and without primary elections. Section 5 concludes.

## 2 Model

Our model considers electoral competition between two parties, X and Y. There are two main variants of the model. In the first, there are no primary elections, and in the second, we introduce primaries within both parties. All elections are decided by plurality rule.

Voters are divided into three groups, indexed $i = 1, 2, 3$. The relative size of each group is $n_i$, with $\sum_{i=1}^{3} n_i = 1$. Also, no group is an outright majority, so $n_i < 1/2$ for $i = 1, 2, 3$. Group membership is important to the model because candidates are able to offer transfers that are targeted specifically toward a group. Within each group, members enjoy the benefits of a targeted transfer equally. Let the candidates in each party be denoted $a$ and $b$. Then $x^j = (x^j_0, x^j_1, x^j_2, x^j_3)$ is the offer of candidate $j \in \{a, b\}$ in party $X$, where $x^j_0 \geq 0$ is the amount allocated to a public good that is enjoyed by all citizens and for $i > 0$, $x^j_i \geq 0$ is targeted toward group $i$. Similarly, $y^k = (y^k_0, y^k_1, y^k_2, y^k_3)$ is the offer of candidate $k \in \{a, b\}$ in party Y. Offers must satisfy the budget constraints $x^j_0 + \sum_i n_i x^j_i = 1$ and $y^k_0 + \sum_i n_i y^k_i = 1$.

The transfers of private goods are “per capita,” while for the public good they are multiplied by a parameter $s > 0$ that measures the efficiency of the public good.

Candidates care only about winning office. Voters care about a “fixed” policy issue, candidate valence, and monetary transfers. All voters in each group have the same preference on the fixed issue. For each group $i = 1, 2, 3$, let $\gamma_i$ denote the members’ relative preferences for party $X$’s position on the fixed issue. Groups 1 and 3 consist of “extremists” and group 2 consists of “moderates.” We assume $\gamma_1 > K$ and $\gamma_3 < -K$, where $K = \max\{1/n_1, 1/n_2, 1/n_3, s\}$. Among other things, this guarantees that party $X$ can never buy the support of group 3 voters, and party $Y$ can never buy the support of group 1 voters. Thus, group 2 is the only swing group.

The preferences of group 2 voters on the fixed issue are stochastic. Specifically, $\gamma_2$ is a random variable whose value is revealed after the primary election and before the general election. This could represent a utility shock from the general election campaign that only

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4 The results of the model also hold for any larger number of candidates in each party. The important assumption is that candidates are ex ante identical to voters.
group 2 voters cared about. For simplicity, we assume that $\gamma_2$ is distributed uniformly on the interval $[-\theta/2, \theta/2]$. So, the density of $\gamma_2$ is $1/\theta$ for $\gamma_2 \in [-\theta/2, \theta/2]$ and 0 otherwise, and the c.d.f. is $F(\gamma_2) = 0$ for $\gamma_2 < -\theta/2$, $F(\gamma_2) = \gamma_2/\theta + 1/2$ for $\gamma_2 \in [-\theta/2, \theta/2]$, and $F(\gamma_2) = 1$ for $\gamma_2 > \theta/2$. We also allow party $X$ to have a party-specific electoral advantage, by giving group 2 voters $\alpha \in [0, \theta/2]$ in valence from either party $X$ candidate.\(^5\)

Voter utility is linear in income. So, if candidate $k$ from party $Y$ wins, the voter receives a payoff of $sy_k^0 + y_k^i$. If candidate $j$ from party $X$ wins the general election, then a voter from group $i = 1, 3$ receives a payoff of $sx_0^j + x_i^j + \gamma_i$, and a voter from group 2 receives $sx_0^j + x_2^j + \gamma_2 + \alpha$. A voter from group $i = 1, 3$ then votes for party $X$’s candidate in the general election if $\gamma_i > y_k^i - x_i^j + s(y_0^k - x_0^j)$. From our assumptions on $\gamma_1$ and $\gamma_3$, this always results in a vote for the ideologically proximate party. Similarly, a voter from group 2 votes for party $X$’s candidate in the general election if:\(^6\)

$$\gamma_2 > y_k^j - x_2^j + s(y_0^k - x_0^j) - \alpha.$$  

The sequence of play is as follows. In both games, candidates begin play by offering transfer vectors $x^a$, $x^b$, $y^a$, and $y^b$ to the voters. These platforms are binding policy commitments and cannot be changed. In the game without primaries, two candidates are chosen exogenously, one for each party, and these two compete in the general election. The simplest assumption is that each candidate is chosen to be their party’s nominee with probability $1/2$, but this is not necessary. With primaries, the two candidates within each party first compete for the party’s nomination. The electorate in the party $X$ primary is group 1 (the party that favors party $X$’s ideological position) and half of group 2. Likewise, the electorate in the party $Y$ primary is group 3 and half of group 2. Our results hold under any assumption about the distribution of group 2 voters’ participation in the primaries (such as complete abstention), as long as they are a minority in both primaries. The two primary winners then compete in the general election. In both variants, $\gamma_2$ is revealed immediately before the general election.

We derive subgame perfect equilibria in undominated voting strategies. An equilibrium consists of transfer announcements for each candidate and voting strategies for each voter at each election.

\(^5\)The logic and qualitative results of the model hold for a large class of symmetric, unimodal distributions.

\(^6\)The linearity assumption is made for simplicity. The logic and qualitative results of the model hold for any strictly increasing, concave utility function.
3 Results

We begin by deriving a general expression for party $X$’s probability of winning the general election. We will use this frequently in the analysis that follows. For any transfer vectors $(\vec{x}, \vec{y})$, all voters in group 1 vote for the party $X$ candidate and all voters in group 3 vote for the party $Y$ candidate. Since no group constitutes a majority, the party $X$ candidate wins if $\gamma_2 > y^k_2 - x^j_2 + s(y^k_0 - x^j_0) - \alpha$. Thus, at an interior solution, the probability that the party $X$ candidate wins is:

$$1 - F(y^k_2 - x^j_2 + s(y^k_0 - x^j_0) - \alpha) = \frac{x^j_2 - y^k_2 - s(y^k_0 - x^j_0) + \alpha}{\theta} + \frac{1}{2}.$$  

(1)

3.1 No Primaries

First suppose there is only a general election. Then each party’s candidates maximize the probability of winning that election. It follows from (1) that the uniquely optimal strategy for each candidate is to offer the maximal amount of transfers (i.e., the entire dollar) to group 2. The first remark summarizes the resulting allocation and voting strategies.

Remark 1 Transfers and Voting Under No Primaries. Without primaries, all candidates offer the transfer vector

$$\vec{x}^a = \vec{x}^b = \vec{y}^a = \vec{y}^b = \begin{cases} (0, 0, \frac{1}{n_2}, 0) & \text{if } \frac{1}{n_2} > s \\ (1, 0, 0, 0) & \text{otherwise.} \end{cases}$$

(2)

Group 1 and 3 members vote for the party $X$ and $Y$ candidates, respectively. Group 2 members vote for party $X$’s candidate if $\gamma_2 > -\alpha$ and for party $Y$’s candidate if $\gamma_2 < -\alpha$.

These strategies imply that party $X$’s probability of victory is $1/2 + \alpha/\theta$. The equilibrium expected utilities of each group’s members are then:

$${E}_1^G = \begin{cases} \left(\frac{1}{2} + \frac{\alpha}{\theta}\right) \gamma_1 & \text{if } \frac{1}{n_2} > s \\ \left(\frac{1}{2} + \frac{\alpha}{\theta}\right) \gamma_1 + s & \text{otherwise.} \end{cases}$$

$${E}_2^G = \begin{cases} \frac{1}{n_2} + \frac{\theta}{8} + \frac{\alpha}{2} + \frac{\alpha^2}{2\theta} & \text{if } \frac{1}{n_2} > s \\ \frac{\theta}{8} + \frac{\alpha}{2} + \frac{\alpha^2}{2\theta} + s & \text{otherwise.} \end{cases}$$

$${E}_3^G = \begin{cases} \left(\frac{1}{2} + \frac{\alpha}{\theta}\right) \gamma_3 & \text{if } \frac{1}{n_2} > s \\ \left(\frac{1}{2} + \frac{\alpha}{\theta}\right) + s & \text{otherwise.} \end{cases}$$

$^7$The last three terms in $E_2^G$ are the expected utility from the fixed issue and valence, or $\int_{-\alpha}^{\gamma_2} (\gamma_2 + \alpha)/\theta \ d\gamma_2$. 

7
3.2 Primaries

Now suppose that there are primary elections in each party, with group 1 and half of group 2 voting in party X’s primary, and group 3 and half of group 2 voting in party Y’s primary.

Primary voters are forward-looking when voting in the primary, taking into account the expected outcome in the general election. Assume that \( n_1 > n_2/2 \) and \( n_3 > n_2/2 \), so group 1 is a majority in party X’s primary and group 3 is a majority in party Y’s primary. Then candidates running in party X’s primary both offer to maximize expected utility of a group-1 voter. This means trading off optimally (from a group-1 voter’s point of view) between winning the general election and giving transfers to group 1. Similarly, candidates running in party Y’s primary both offer to maximize the expected utility of a group-3 voter.

We derive a pure strategy equilibrium by finding the optimal platform strategy within each party, given an expected winning platform from the opposing party. Let \( \pi \) and \( \gamma \) denote arbitrary platforms from parties X and Y. The expected utilities of group-1 and group-3 voters are then:

\[
E_1(\pi, \gamma) = \left[ \frac{x_2 - y_2 - s(y_0 - x_0)}{\theta} + \frac{1}{2} \right] (x_1 - y_1 + s(x_0 - y_0) + \gamma_1) + y_1 + sy_0 \tag{3}
\]

\[
E_3(\pi, \gamma) = \left[ \frac{x_2 - y_2 - s(y_0 - x_0)}{\theta} + \frac{1}{2} \right] (x_3 - y_3 + s(x_0 - y_0) + \gamma_3) + y_3 + sy_0 \tag{4}
\]

The first important property of these objective functions is that the efficiency of the public good \( s \) plays an important role in determining the kinds of goods that are offered. As intuition would suggest, for \( s \) sufficiently low, any best response must include only private goods, and for \( s \) sufficiently high, any best response must have only the public good. The problem becomes more interesting for intermediate values of \( s \). When \( s \) is between \( 1/(n_1+n_2) \) and \( \max\{1/n_1, 1/n_2\} \), then it offers private goods to at most one group. A symmetric result holds for party Y and groups 2 and 3.

**Lemma 1** Efficiency of public goods and best responses. Party X candidates’ best responses satisfy the following:

(i) \( x_3^* = 0 \).

(ii) If \( s \leq 1/(n_1 + n_2) \), then \( x_0^* = 0 \).

(iii) If \( s > 1/n_i \) (\( i \in \{1, 2\} \)), then \( x_i^* = 0 \); if \( s > \max\{1/n_1, 1/n_2\} \), then \( x_0^* = 1 \).

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8 Respondents in the presidential primary exit poll surveys claim to value electability when deciding how to vote. For example in the 2004 Democratic primaries, more exit poll respondents cited the ability to “defeat George W. Bush” than any other response to the question “Which ONE candidate quality mattered most in deciding how you voted today?”
(iv) If \( s \in (1/(n_1 + n_2), \max\{1/n_1, 1/n_2\}] \), then either \( x_1^* = 0 \) or \( x_2^* = 0 \).

Part (iv) of this result implies that the group receiving zero private benefits will be the one that receives smaller per capita benefits in equilibrium. Intuitively, group 1 members would accept reduced benefits if \( \gamma_1 \) is high, so that they care greatly about the ideological benefits of winning and therefore want candidates to increase payments to group 2. The opposite would happen when group 1 is moderate.

The next important property of (3) and (4) affects the approach we take toward characterizing the equilibrium. These objective functions are never locally concave when \( s \) is high enough for public goods to be offered. This also implies that (as Lemma 1 establishes) at any best response, solutions are at a corner: \( x_0, x_1, \) or \( x_2 \) will equal zero.

**Lemma 2** Nonconcavity of party objectives. *Party X and Y’s objective functions are never locally concave for \( s > 1/(n_1 + n_2) \) and \( s > 1/(n_3 + n_2) \), respectively.*

### 3.2.1 Private Goods Equilibrium

We first consider equilibria in which candidates offer only private goods to voters. Such equilibria clearly exist; for example, if \( s = 1 \), then there is no incentive for candidates to offer public goods because any public good allocation could be replicated simply by offering an allocation of \( s \) in private goods to each group.

The main feature of the private goods equilibrium is that in contrast to the no-primaries private goods equilibrium (see Remark 1), candidates will usually offer positive allocations to their core groups. This is because citizens in groups 1 and 3 would prefer a small allocation and a slightly reduced probability of winning to a zero allocation that maximized their party’s probability of winning. In fact, allocations to core voters are strategic complements: a higher payment to core voters in one party raise the expected return to core allocations in the other. Importantly, more extreme core voters demand less private goods, as they are relatively more interested in winning in order to achieve ideological goals.

Proposition 1 characterizes the private goods equilibrium and establishes a simple condition on \( s \) for its existence and uniqueness. The characterization is simplified by the fact that each candidate’s objective functions is strictly concave, and \(|\gamma_1| \) and \(|\gamma_3| \) are large enough to prevent candidates from offering anything to the opposing party’s core group. This generates a unique platform that maximizes the utility of core voters, and as a result both candidates will adopt it in equilibrium.
Proposition 1 Private Goods Equilibrium. At an interior solution of an equilibrium with only private goods:

\[(x_1^P, y_3^P) = \left( \frac{3n_2\theta + 2n_2\alpha + 2n_3\gamma_3}{6n_1} - \frac{2\gamma_1}{3}, \frac{3n_2\theta - 2n_2\alpha - 2n_1\gamma_1 + 2\gamma_3}{3} \right). \]

When \((x_1^P, y_3^P)\) is not interior, the following corner solutions arise:

\[
(x_1^P, y_3^P) = \begin{cases}
(0, 0) & \text{if } n_2(\theta/2 + \alpha) \leq n_1\gamma_1 \text{ and } n_2(\theta/2 - \alpha) \leq -n_3\gamma_3 \\
\left(0, \frac{n_3\gamma_3 + n_2(\theta/2 - \alpha)}{2n_3}\right) & \text{if } n_2(3\theta/2 + \alpha) \leq 2n_1\gamma_1 - n_3\gamma_3 \text{ and } n_2(\theta/2 - \alpha) \in (-n_3\gamma_3, 2 - n_3\gamma_3) \\
(0, \frac{1}{n_3}) & \text{if } n_2(\theta/2 + \alpha) \leq n_1\gamma_1 - 1 \text{ and } n_2(\theta/2 - \alpha) \geq 2 - n_3\gamma_3 \\
\left(-\frac{n_1\gamma_1 + n_2(\theta/2 + \alpha)}{2n_1}, 0\right) & \text{if } n_2(\theta/2 + \alpha) \in (n_1\gamma_1, 2 + n_1\gamma_1) \text{ and } n_2(3\theta/2 - \alpha) \leq n_1\gamma_1 - 2n_3\gamma_3 \\
\left(\frac{1}{n_1}, 0\right) & \text{if } n_2(\theta/2 + \alpha) \geq 2 + n_1\gamma_1 \text{ and } n_2(\theta/2 - \alpha) \leq -1 - n_3\gamma_3 \\
\left(\frac{1}{n_1}, \frac{1+n_3\gamma_3+n_2(\theta/2-\alpha)}{2n_3}\right) & \text{if } n_2(3\theta/2 + \alpha) \geq 3 + 2n_1\gamma_1 - n_3\gamma_3 \text{ and } n_2(\theta/2 - \alpha) \in (-1 - n_3\gamma_3, 1 - n_3\gamma_3) \\
\left(\frac{1-n_1\gamma_1+n_2(\theta/2+\alpha)}{2n_1}, \frac{1}{n_3}\right) & \text{if } n_2(\theta/2 + \alpha) \in (n_1\gamma_1 - 1, 1 + n_1\gamma_1) \text{ and } n_2(3\theta/2 - \alpha) \geq 3 + n_1\gamma_1 - 2n_3\gamma_3 \\
\left(\frac{1}{n_1}, \frac{1}{n_3}\right) & \text{if } n_2(\theta/2 + \alpha) \geq 1 + n_1\gamma_1 \text{ and } n_2(\theta/2 - \alpha) \geq 1 - n_3\gamma_3.
\end{cases}
\]

This is the unique equilibrium for \(s < \min\{1/(n_1+n_2), 1/(n_2+n_3)\}. \]

With one exception, which occurs when both core groups are large and extreme, some core voters receive positive allocations in equilibrium. Primaries also tend to benefit both parties’ core groups because their party’s probability of victory increases when the opposition party reduces its allocation to group 2. By contrast, when only private goods are offered, primaries typically hurt group 2 citizens relative to a world in which there are no primaries.

Elsewhere (Hirano, Snyder, and Ting 2009), we show that each party gives more (per capita) to its core group when it shrinks in size relative to the swing group, when it becomes more moderate, when electoral uncertainty \((\theta)\) increases, and when its relative valence advantage increases. We also show that each party’s probability of victory is increasing in the size and ideological extremism of its core group, as well as in the size of its relative valence advantage.
### 3.2.2 Public Goods Equilibrium

We now turn to the case of higher values of \( s \), so that public goods are efficient enough to be a part of equilibrium platforms. By Lemma 1, one case is straightforward: when \( s \) is sufficiently high, then the unique optimal strategy is to provide public goods.

When \( s \) is intermediate, then equilibrium platforms may blend public and private goods. To characterize the possible best responses, recall that at most one of \( x_1 \) and \( x_2 \) can be strictly positive at an optimal platform. For party \( X \) candidates this implies that \( x_0 + n_i x_i = 1 \) for some \( i \in \{1, 2\} \). We rewrite the party \( X \) objective (3) in terms of \( x_0 \) by substituting this constraint for each group \( i \). In the first case, where \( x_1 > 0 \) and \( x_2 = 0 \) we have:

\[
E_1(x, y) = \left[ \frac{-y_2 - s(y_0 - x_0) + \alpha}{\theta} + \frac{1}{2} \right] \left( 1 - \frac{x_0}{n_1} - y_1 + s(x_0 - y_0) + \gamma_1 \right) + y_1 + s y_0 \quad (5)
\]

This is concave in \( x_1 \) when \( n_1 s < 1 \). By Lemma 1, this condition is necessary for an interior solution for \( x_1 \), as the public good would clearly be preferable otherwise. The interior solution is:

\[
x_1 = \frac{n_1(\gamma_1 - y_1) + (2n_1 s - 1)(1 - y_0)}{2n_1(n_1 s - 1)} + \frac{\alpha + \theta/2 - y_2}{2n_1 s} \quad (6)
\]

\[
x_{01} = \frac{(2n_1 s - 1)y_0 - n_1(\gamma_1 - y_1) - 1}{2(n_1 s - 1)} - \frac{\alpha + \theta/2 - y_2}{2s} \quad (7)
\]

In the second case, \( x_1 = 0 \) and \( x_2 > 0 \), and the objective is concave if \( n_2 s < 1 \). Again, this condition is necessary for an interior value of \( x_2 \) to be chosen, for otherwise party \( X \) candidates would prefer the public good. Straightforward maximization yields the following interior solutions:

\[
x_2 = \frac{(n_2 s - 1)(\gamma_1 - y_1) - s(2n_2 s - 1)(y_0 - 1)}{2n_2 s(n_2 s - 1)} + \frac{\alpha + \theta/2 - y_2}{2(n_2 s - 1)} \quad (8)
\]

\[
x_{02} = \frac{-(n_2 s - 1)(\gamma_1 - y_1) + s(2n_2 s y_0 - y_0 - 1)}{2s(n_2 s - 1)} - \frac{n_2(\alpha + \theta/2 - y_2)}{2(n_2 s - 1)} \quad (9)
\]

Likewise, for party \( Y \) candidates the necessity of corner solution implies \( y_0 + n_i y_i = 1 \) for some \( i \in \{2, 3\} \). There are again two cases; when \( y_3 > 0, y_2 = 0 \) and \( n_3 s < 1 \), we have the following interior solutions:

\[
\tilde{y}_3 = \frac{(1 - 2n_3 s)(x_0 - 1) - n_3(\gamma_3 + x_3)}{2n_3(n_3 s - 1)} - \frac{\alpha - \theta/2 + x_2}{2n_3 s} \quad (10)
\]

\[
\tilde{y}_{01} = \frac{(2n_3 s - 1)x_0 + n_3(\gamma_3 + x_3) - 1}{2(n_3 s - 1)} + \frac{\alpha - \theta/2 + x_2}{2s} \quad (11)
\]
And finally when \( y_3 = 0, y_2 > 0 \) and \( n_2s < 1 \), we have:

\[
\hat{\gamma}_2 = \frac{(1 - n_2s)(\gamma_3 + x_3) - s(2n_2s - 1)(x_0 - 1)}{2n_2s(n_2s - 1)} - \frac{\alpha - \theta/2 + x_2}{2(n_2s - 1)} \tag{12}
\]

\[
\hat{\gamma}_0 = \frac{(n_2s - 1)(\gamma_3 + x_3) + s(2n_2sx_0 - x_0 - 1)}{2s(n_2s - 1)} + \frac{n_2(2\alpha - \theta + 2x_2)}{4(n_2s - 1)} \tag{13}
\]

Note that by Lemma 1, \( x_3 = 0 \) and \( y_1 = 0 \) in optimal party platforms.

The following result establishes the best responses for each party’s candidates; i.e., the conditions under which candidates promise “swing” or “core” groups private goods as a best response.

**Lemma 3** Best responses and recipient of private goods when \( s \) is intermediate. 

(i) For \( s > \frac{1}{n_1 + n_2} \), \( x_1^* = \min\{1/n_1, \bar{x}_1\} \) and \( x_2^* = 0 \) if:

\[
(2n_1s^2 - s)(1 - y_0) + (1 - n_1s)(y_2 - \alpha - \theta/2) + n_1s(\gamma_1 - y_1) < 0, \tag{14}
\]

and \( x_1^* = 0 \) and \( x_2^* = \min\{1/n_2, \bar{x}_2\} \) if:

\[
(2n_2s^2 - s)(1 - y_0) - n_2s(y_2 - \alpha - \theta/2) - (1 - n_2s)(\gamma_1 - y_1) < 0. \tag{15}
\]

Otherwise, \( x_0^* = 1 \).

(ii) For \( s > \frac{1}{n_2 + n_3} \), \( y_3^* = \min\{1/n_3, \bar{y}_3\} \) and \( y_2^* = 0 \) if:

\[
(2n_3s^2 - s)(1 - x_0) + (1 - n_3s)(x_2 + \alpha - \theta/2) - n_3s(\gamma_3 + x_3) < 0, \tag{16}
\]

and \( y_3^* = 0 \) and \( y_2^* = \min\{1/n_2, \bar{y}_2\} \) if:

\[
(2n_2s^2 - s)(1 - x_0) - n_2s(x_2 + \alpha - \theta/2) + (1 - n_2s)(\gamma_3 + x_3) < 0. \tag{17}
\]

Otherwise, \( y_0^* = 1 \). 

We may use this result and other features of the candidates’ best responses to establish generally the existence of a pure strategy equilibrium. As we will show, there is a unique equilibrium under a broad range of parameter values, but uniqueness is not guaranteed in general.

**Proposition 2** Equilibrium existence. There exists a pure strategy equilibrium.
We begin to derive equilibrium strategies by focusing on cases where $s$ is relatively high. In these cases, at least one party is guaranteed to offer only public goods. The next proposition follows immediately from Lemma 1 and manipulation of (10) and is stated without proof.

**Proposition 3** Equilibrium with high $s$. (i) If $s \geq \max\{1/n_1, 1/n_2, 1/n_3\}$, then $x_0^* = y_0^* = 1$.

(ii) If $s \geq \max\{1/n_1, 1/n_2\}$ and $s < 1/n_3$, then $x_0^* = 1$ and $y_3^* = \max\{0, \min\{\bar{y}_3, 1/n_3\}\}$, where:

$$\bar{y}_3 = \frac{\gamma_3}{2(1-n_3s)} + \frac{\theta/2 - \alpha}{2n_3s}.$$

(iii) If $s \geq \max\{1/n_2, 1/n_3\}$ and $s < 1/n_1$, then $y_0^* = 1$ and $x_3^* = \max\{0, \min\{\bar{x}_1, 1/n_1\}\}$, where:

$$\bar{x}_1 = -\frac{\gamma_1}{2(1-n_1s)} + \frac{\theta/2 + \alpha}{2n_1s}.$$

Parts (ii) and (iii) of Proposition 3 illustrates the general point that smaller groups will tend to receive private goods. This is due to the higher per capita value of a given amount of transfers for small groups. Notably, the result implies that there are conditions (e.g., $|\gamma_i|$ small, $\theta$ large) under which public goods are not promised by all candidates even though $s > 1/n_2$. Remark 1 established that the efficiency threshold for providing public goods when there are no primaries is $s = 1/n_2$, and thus a small core group can result in a party providing a lower level of public goods under a primary system.

Our next result addresses the central question of the paper: compared to a world with no primaries, when do primaries encourage the provision of public goods? In particular, can public goods be provided when $s < 1/n_2$? The threshold at $s = 1/n_2$ generates an inefficiency because public goods provide benefit society whenever $s > 1$. Lemma 1 implies that some inefficiencies must persist in the presence of primaries: party $X$ and $Y$ candidate candidates never offer public goods when $s$ is below $1/(n_1 + n_2)$ and $1/(n_3 + n_2)$, respectively. Part (i) of Proposition 4 shows that when the swing group is the smallest group, then primaries are strictly better for producing public goods in the sense of reducing the threshold value $s$ needed to induce all candidates to offer only public goods. Part (ii) provides a simple lower bound on $s$ for when party $X$ candidates choose platforms with strictly positive public good levels. A symmetric result holds for party $Y$.

**Proposition 4** Public goods under primaries. (i) If $n_2 < \min\{n_1, n_3\}$, then $x_0^* = y_0^* = 1$ for all $s > \tilde{s}$, where $\tilde{s} < 1/n_2$. 

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(ii) If $s > \frac{1}{n_1 + n_2}$, then $x_0^* > 0$ if:

$$s > \max\left\{ \frac{\alpha + \theta/2}{n_1(\gamma_1 + \alpha + \theta/2)}, \frac{\gamma_1}{n_2(\gamma_1 + \alpha + \theta/2 - 1)} \right\}.$$  

In contrast with Proposition 3, Proposition 4(i) requires that group 2 be the smallest in order to reduce the per capita value private goods for core groups. As the proof of this result makes clear, a similar sufficient condition for candidates in just one party to offer only public goods is more easily met: each party’s candidates will do so if the swing group is smaller than their core group.

The bound on $s$ in Proposition 4(ii) is for many parameter values quite low. In particular, for sufficiently large values of $\alpha$, $\theta$, and $\gamma_1$, the bound shows that the threshold for partial adoption of public goods under primaries can be much lower than the $1/n_2$ threshold for adoption without primaries. As an example, suppose $n_1 = 0.4$, $n_2 = 0.3$, $\theta = 10$, $\alpha = 2$, and $\gamma_1 = 4$. By Lemma 1, party $X$ offers no public goods if $s < 1/(n_1 + n_2) \approx 1.43$, and $X$ offers only public goods if $s > 3.33$. The threshold from the result is about 1.59, and so public goods are partially adopted by parties even when they are relatively close to being dominated by private goods.

We now consider the more difficult analysis of lower values of $s$. The next lemma builds on Lemma 3 to establish some simple conditions under which a party’s candidates will never choose either its core or swing group. When the core group is sufficiently moderate, platforms never include private goods for the swing group, and when the core group is sufficiently extreme, platforms never include the core group. The result simplifies the characterization equilibrium because the presence of a “dominant” group reduces each party’s maximization problem to a single-dimensional choice between it and the level of public good to provide.

The bounds used in the result are defined as follows:

$$\gamma_1 \equiv \frac{\max\{-2n_1s^2 + s, 0\} + (1 - n_1s)(\alpha + \theta/2)}{n_1s}$$

(18)

$$\gamma_1 \equiv \frac{n_2s(\alpha + \theta/2)}{1 - n_2s} - 2s$$

(19)

$$\gamma_3 \equiv \min\{2n_3s^2 - s, 0\} + (1 - n_3s)(\alpha - \theta/2)$$

(20)

$$\gamma_3 \equiv \frac{n_2s(\alpha - \theta/2)}{1 - n_2s} + 2s.$$  

(21)

Importantly, these bounds typically exclude only a “small” set of values of $\gamma_1$ and $\gamma_3$, ...
and in many cases exclude none at all.\footnote{For example, when $n_1 = n_2 = n_3 = 1/3$, Lemma 4 implies that there are no restrictions on $\gamma_1$ if $\alpha + \theta/2 > 2.5$ and $s > 2.033$. There are no restrictions on $\gamma_3$ if $\alpha - \theta/2 < -0.5$ and $s > 2.753$.}

**Lemma 4** Independence of private goods recipient in best response. (i) For $s > \frac{1}{n_1+n_2}$, $x_1^* = 0$ if $\gamma_1 \geq \tilde{\gamma}_1$, and $x_2^* = 0$ if $\gamma_1 \leq \tilde{\gamma}_1$.

(ii) For $s > \frac{1}{n_2+n_3}$, $y_3^* = 0$ if $\gamma_3 \leq \tilde{\gamma}_3$, and $y_2^* = 0$ if $\gamma_3 \geq \tilde{\gamma}_3$.

This result allows us to characterize a unique equilibrium for many of the salient parameter ranges of the game. The result identifies the unique interior equilibrium (where possible) under the mild parameter restrictions derived in Lemma 4. Under these restrictions, targeting one group is “dominant” in the sense that that group will receive private goods by candidates of a party regardless of the other party’s platform. For each case, the equilibrium platform can then be derived simply by solving for the appropriate system from equations chosen from expressions (7) through (13).

**Proposition 5** Equilibrium characterization, general case. Let $s \geq \max\{1/(n_1+n_2), 1/(n_3+n_2)\}$.

(i) If $\gamma_1 \leq \tilde{\gamma}_1$ and $\gamma_3 \geq \tilde{\gamma}_3$, the non-zero private good allocations at an interior equilibrium are:

$$x_1^* = \frac{(1-n_3s)(2n_1s(\theta+\gamma_1) - \alpha - 3\theta/2) - n_3s\gamma_3(1-2n_1s)}{n_1s(2n_1s + 2n_3s - 3)}$$

$$y_3^* = \frac{(1-n_1s)(2n_3s(\theta-\gamma_3) + \alpha - 3\theta/2) + n_1s\gamma_1(1-2n_3s)}{n_3s(2n_1s + 2n_3s - 3)}$$

(ii) If $\gamma_1 \geq \tilde{\gamma}_1$ and $\gamma_3 \geq \tilde{\gamma}_3$, the non-zero private good allocations at an interior equilibrium are:

$$x_2^* = \frac{(1-n_3s)(2n_2s(\theta+\gamma_1) - 2\gamma_1 + \alpha - \theta/2) - n_3s\gamma_3(1-2n_2s)}{2n_2s^2 - n_2s + n_3s - 1}$$

$$y_3^* = \frac{(1-n_3s-n_2s)(n_2s(\alpha-3\theta/2) + (1-n_2s)\gamma_1) + 2n_2n_3s^2((1-n_2s)(\gamma_1-\gamma_3) - n_2s\theta)}{n_3s(2n_2s^2 - n_2s + n_3s - 1)}$$
(iii) If $\gamma_1 \leq \gamma_3$ and $\gamma_3 \leq \gamma_1$, the non-zero private good allocations at an interior equilibrium are:

\[
x_1^* = \frac{(1 - n_1 s - n_2 s)(n_2 s(-\alpha - 3\theta/2) - (1 - n_2 s)\gamma_3) + 2n_2 n_1 s^2((1 - n_2 s)(\gamma_1 - \gamma_3) - n_2 s\theta)}{n_1 s(2n_2 s^2 - n_2 s + n_1 s - 1)}
\]

\[
y_2^* = \frac{(1 - n_1 s)(2n_2 s(\theta - \gamma_3) + 2\gamma_3 - \alpha - \theta/2) + n_1 s\gamma_1(1 - 2n_2 s)}{2n_2 s^2 - n_2 s + n_1 s - 1}
\]

(iv) If $\gamma_1 \geq \gamma_3$ and $\gamma_3 \leq \gamma_1$, there is no interior equilibrium. $x_2^* = 1/n_2$ and $y_2^* = 1/n_2$ for $\gamma_1$ and $|\gamma_3|$ sufficiently large.

One implication of this result is that, similar to the private good equilibrium, more extreme core groups receive no private goods. For such voters, the ideological payoff from victory generates an incentive to maximize the probability of victory, thereby shifting all private good provision to the swing group. In the case where all core voters are extreme (i.e., case (iv)), all candidates allocate their entire budget to private good for group 2. In most cases, public goods soften the choice between core and swing groups, and allow extreme core groups to benefit from non-ideological payoffs. By contrast, moderate core voters are more willing to trade off between private goods and the probability of victory, and therefore receive private goods in equilibrium.

3.2.3 Ideological Symmetry

While Proposition 5 is suggestive of the role of ideology in party strategies, it does not provide comparative statics over the full range of ideological polarization. We can obtain sharper predictions by considering a special case of the game with symmetric ideological parameters. In particular, suppose core groups have the same ideological motivation (i.e., $\gamma \equiv \gamma_1 = -\gamma_3$), and that the swing group is not biased toward either party (i.e., $\alpha = 0$). There are two subcases; in the first, the core groups are also of the same size, while in the second, one core group is larger than the other groups.

The first subcase allows us to isolate the pure effect of ideological polarization. Proposition 6 confirms the general intuition about the effects of extremism in Proposition 5. As the ideological extremism of both core groups increases, candidates shift benefits from core groups to the swing group. Public goods then help to “smooth” the transition between private goods for the two groups.
Proposition 6 Equilibrium with symmetric core groups. Suppose \( s \in (\max\{\frac{1}{2n_1}, \frac{1}{2n_2}\}, \max\{\frac{1}{n_1}, \frac{1}{n_2}\}) \), \( n_1 = n_3 \), \( \alpha = 0 \), and \( \gamma_1 = -\gamma_3 = \gamma \). In the unique equilibrium:

\[
\begin{align*}
x_1^* = y_3^* &= \min \left\{ \frac{1}{n_1}, \frac{1 - n_1s}{2n_1s} - \gamma \right\}, \\
x_0^* = y_0^* &= 1 \quad \text{if } \gamma \leq \frac{(1-n_1s)\theta}{2n_1s} \text{ and } s \leq \frac{1}{n_1}, \\
x_0^* &= 1 - n_1x_1^* \quad \text{if } \gamma \in \left( \frac{(1-n_1s)\theta}{2n_1s}, \frac{2n_2s\theta}{n_2s\theta} \right), \\
x_0^* &= y_0^* = 1 \quad \text{or } \gamma \leq \frac{(1-n_1s)\theta}{2n_1s} \text{ and } s > \frac{1}{n_1}, \\
x_2^* = y_2^* &= \frac{1}{n_1} \quad \text{or } \gamma \geq \frac{2n_2s\theta}{(1-n_2s)} \text{ and } s > \frac{1}{n_2}, \\
x_2^* &= y_2^* = \frac{1}{n_1} \quad \text{if } \gamma \geq \frac{2n_2s\theta}{(1-n_2s)} \quad \text{and } s \leq \frac{1}{n_2}. \quad \square
\end{align*}
\]

The symmetry of the parameters core groups here implies that equilibrium platforms are also symmetric. When \( s \) is small enough so that private goods are undominated (i.e., \( s < \min\{1/n_1, 1/n_2\} \)), the allocation strategies as a function of \( \gamma \) are as follows. For the lowest values of \( \gamma \), candidates promise a mix of private goods for the core group and public goods. These private goods are linearly decreasing in \( \gamma \) and increasing in \( \theta \). For higher values of \( \gamma \) the allocation hits a corner entirely public goods. Finally, for extreme values of \( \gamma \) candidates promise only private goods to the swing group. Interestingly, as suggested by Proposition 5(iv), the transition from public goods to private goods for the swing group is discontinuous at \( \gamma = n_2s\theta/(2 - 2n_2s) \): there is no “interior” solution for private goods to the swing group. Overall, then, the provision of public goods is non-monotonic in \( \gamma \). Figure 1 depicts the allocations to each group as a function of \( \gamma \) for the special case where all group sizes are equal. The equilibrium is similar when \( s \) is large enough to dominate the provision of a private good; in these cases the public good is simply substituted for the private good.

Several comparative statics on the range of \( \gamma \) for which public goods are offered by all candidates follow immediately from Proposition 6. The size of this range is increasing in \( \theta \), which measures electoral uncertainty. It also shifts “upwards” as \( \theta \) increases, which implies that as elections become more uncertain, the value of contributions to the swing group decreases while the value of contributions to the core group increases. Next, as \( n_1 \) increases (implying that \( n_2 \) decreases), this range shifts “downwards.” This reflects the dilution of the value of core contributions as \( n_1 \) increases along with the concentration of swing contributions as \( n_2 \) decreases. Finally, an increase in the efficiency \( s \) of the public good strictly increases the appeal of offering public goods.

The second subcase adds an asymmetry in group sizes. Suppose that \( n_1 = n_2 \), and \( n_3 > n_1 \), so that party \( Y \) faces a larger constituency. In addition, suppose that \( s < 1/(n_1 + n_2) \) and \( s > 1/(n_2 + n_3) \) (which also implies that \( s < 1/n_i \) for all \( i \)). Thus party \( X \) offers only private goods, while party \( Y \) may offer some public goods.
Figure 1: Equilibrium Allocations in a Symmetric Equilibrium. Here $n_1 = n_2 = n_3 = 1/3$, $\gamma = \gamma_1 = -\gamma_3$, $\alpha = 0$, $s = 2$, and $\theta = 20$. This figure plots the per capita allocations for each platform component for candidates of both parties, as a function of $\gamma$.

Proposition 7 shows that the basic platform strategy of offering increasingly valuable allocations to swing voters as extremism increases remains when the opposite party uses only private goods. Figure 2 plots the party $Y$ candidates’ allocation strategies as a function of $\gamma$. The main notable difference between this figure and Figure 1 is the smoother transition from public goods to private goods for the swing group. By contrast, party $X$ candidates simply trade core allocations for swing allocations as $\gamma$ increases.

**Proposition 7** Equilibrium with asymmetric core groups. Suppose $s < 1/(n_1 + n_2)$, $s > 1/(n_2 + n_3)$, $n_1 = n_2$, $\alpha = 0$, and $\gamma_1 = -\gamma_3 = \gamma$. In equilibrium there exist $\gamma'$ and $\gamma''$ such that:

(i) If $\gamma < \gamma'$, then $y_2^* = 0$ and $y_3^*$ is piecewise linear and weakly decreasing in $\gamma$.

(ii) If $\gamma > \gamma''$, then $y_3^* = 0$.

(iii) If $\gamma \in [\gamma', \gamma'']$, then $y_0^* = 1$.

At an interior solution, $\gamma' = \frac{(1-n_3s)(2s-1/n_2+3\theta/2)-2n_3s^2}{1+n_3s}$ and $\gamma'' = \frac{s(3-4n_2s-3n_3\theta/2)}{n_2s-2}$.

The cutpoints $\gamma'$ and $\gamma''$ correspond to the endpoints of the interval where only public goods are offered. (This interval also appears in Proposition 6.) It can be shown that this
interval is nonempty whenever $\gamma' > 1/n_2$, which holds for $\theta$ sufficiently large. Since $\gamma > 1/n_2$ by assumption, the condition $\gamma' > 1/n_2$ is sufficient for the existence of a region where only party $Y$ provides public goods.

4 Extension: Pivotal Swing Voters

An important assumption in the previous results was that the pivotal voter in each party’s primary election belonged to a core group. However, a broad-based party might have more swing than core voters. If instead the pivotal voter belonged to the swing group, then that party’s candidates could focus exclusively on the general election. In this section, we examine the case where party $Y$’s pivotal primary voter belongs to group 2.

It is clear that party $Y$ candidates will choose to maximize the expected payoffs of group 2 voters. Thus their platform strategies will be identical to those in the no-primaries world:

$$\mathbf{y}^Y = \mathbf{y}^H = \begin{cases} (0, 0, \frac{1}{n_2}, 0) & \text{if } 1/n_2 > s \\ (1, 0, 0, 0) & \text{otherwise.} \end{cases}$$

The party $Y$ strategies generate two cases. When public goods are highly valuable (i.e.,
s > 1/n_2), party Y candidates offer only public goods, and when public goods are less valuable, they focus exclusively on private goods for group 2. The following result characterizes the equilibrium platforms for party X in both cases. We restrict attention here to values of s that are high enough to ensure that public goods are undominated (i.e., s < 1/(n_1 + n_2)).

**Remark 2** Pivotal swing voters in party Y primary. Suppose that group 2 voters are a majority of the party Y primary electorate.

(i) If s > 1/n_2, then x_0^* = 1 - n_1 x_1^* and

\[
x_1^* = \begin{cases} 0 & \text{if } n_1 \geq \frac{\alpha + \theta/2}{s(\gamma_1 + \alpha + \theta/2)} \\
\max \left\{ \frac{1 - \frac{1}{n_1}}{2(n_1 s - 1)} + \frac{s(\gamma_1 + \alpha + \theta/2)}{2n_1} \right\} & \text{otherwise.}
\end{cases}
\]

(ii) If s \in (1/(n_1 + n_2), 1/n_2), then x_0^* = 1 - n_1 x_1^* - n_2 x_2^* and

\[
x_1^* = \begin{cases} 0 & \text{if } n_1 \geq \frac{s + \frac{\alpha + \theta/2 - 1/n_2}{2s + \gamma_1 + \alpha + \theta/2 - 1/n_2}} \\
\max \left\{ \frac{1 - \frac{1}{n_1}}{2(n_1 s - 1)} + \frac{s(\gamma_1 + \alpha + \theta/2)}{2n_1} \right\} & \text{otherwise,}
\end{cases}
\]

\[
x_2^* = \begin{cases} 0 & \text{if } n_1 \geq \frac{s + \frac{\alpha + \gamma_1}{2s + \gamma_1 + \alpha + \theta/2 - 1/n_2}} \\
\max \left\{ \frac{1 - \frac{1}{n_2}}{2n_2 s} + \frac{s(2 + \gamma_1 + \alpha + \theta/2 - 2)}{2n_2(n_2 s - 1)} \right\} & \text{otherwise.}
\end{cases}
\]

Party X’s equilibrium platforms are derived simply from Lemma 1 and equations (6) and (8). The basic properties of the original game continue to hold here. For example, while it is possible for either group 1 or group 2 to benefit from private goods in part (ii), Lemma 3 continues to hold, and thus at most one of x_1 and x_2 can be positive. Additionally, consistent with the logic of Proposition 4, part (i) implies that x_1^* > 0 only if n_1 < n_2. Again, small group sizes are conducive to offering private goods, even when the opposition can be considerably more appealing to swing voters.

One clear implication of this environment is that it helps party Y to win. A more interesting question is how party X candidates respond. In a world without public goods, pivotal swing voters in party Y generally induce party X candidates to allocate more to the swing group in order to compensate for their reduced probability of victory (Hirano, Snyder, and Ting 2009). Public goods can muddle this result. For example, compare what happens when party X offers some private goods to group 1 regardless of which group controls the party Y primary (hence, s < 1/n_1). When s > 1/n_2, swing voter control has no effect on y_2 but weakly increases y_0 to 1. If s > 1/(2n_1), then this shift by party Y candidates increases party X’s allocation to its core group, while if s < 1/(2n_1), the relationship is reversed. The
increase in core allocations is due to the high value of the public good: with increased payoffs from losing, group 1 members are willing to accept a lower probability of victory and higher payoffs conditional upon victory. By contrast, when \( s < 1/n_2 \), party Y no longer provides public goods and so under the same parametric assumptions party X candidates respond by allocating more to public goods.

5 Conclusions

This paper investigates the effect of primary elections on the distribution of public spending. The main intuition is that primary elections provide an incentive for candidates to increase the provision of public as opposed to particularistic goods. The incentive is generated by the simple observation that public goods simultaneously benefit both core and swing voters. Thus they can present candidates with a more efficient way of maximizing the utility of both core voters in a primary election as well as swing voters in the general election.

The model produces a number of non-obvious predictions. First, despite the preceding intuition, primaries do not necessarily increase the provision of public goods. Second, large core groups encourage the provision of public goods. A sufficient condition for primaries to make the provision of public goods “easier” is for the swing group to be the smallest group, thus diluting the value of private goods for core groups. Finally, public goods will be particularly appealing when core voters are moderately extreme. In this case, a public goods equilibrium might arise even when the public goods are not efficient enough to be offered in the absence of primary elections.

The model also shows that primaries produce mixed distributional consequences. In the cases of “intermediate” \( s \), any increase in the use of public goods also increases aggregate social utility. And as Proposition 1 demonstrates, in a world with only private goods, the introduction of primaries draws allocations away from swing voters and toward core voters. With both intermediate-valued public goods and primaries, the model also generally predicts higher payoffs for core voters at the expense of swing voters. This is true even when candidates offer zero private goods to core voters, because the public good is still less valuable than the private good to swing voters. Primaries have no effect on distributions when both core groups are sufficiently extreme, candidates simply maximize their offer of private goods to the swing group in both games.

Our model is simple and may be extended in several ways. For example, what if one candidate had an incumbency advantage, modeled as a candidate-specific valence term? How
would the model be affected by endogenous decisions to adopt primaries? The robustness of our results is also worth exploration. The corner solutions in the model are driven in part by the linearity of voter utility, and interior solutions might follow from non-linear utility over public goods. While our results are robust to multiple candidates in each party, the effects of multiple parties, more groups, or alternative nomination systems are not clear.

Finally, the results suggest some avenues for empirical research. The main challenge lies in classifying government spending as public versus particularistic. Previous research has grappled with this problem, but there is no clear consensus regarding classification schemes. U.S. states and localities spend on a variety of goods and services – education, health, transportation, police, fire departments, courts, sewerage and trash pickup, etc. – and these are all partially public and partially excludable and targetable goods.

An alternative measure, which we plan to explore in future work, is based on “project size.” Within a relatively narrow category of spending, projects that larger in scale are “more public” than smaller projects. Compare for example, a hospital with a 1000 beds centrally located in a county to a 10 hospitals scattered throughout the county each with 100 beds. The former is closer to the theoretical ideal of a public good than the latter. One way to measure project size is from data on local or state government bond issues.
APPENDIX

Proof of Lemma 1. Throughout, suppose that a party \( X \) candidate allocates some \( \pi \), where (by weak dominance) \( \pi \) satisfies the budget constraint \( x_0 + n_1x_1 + n_2x_2 + n_3x_3 = 1 \).

(i) By inspection of (3), any \( x_3 > 0 \) is dominated by \( x_3 = 0 \) and allocating the budget toward any other group or the public good.

(ii) We derive the condition on \( s \) for the party \( X \) candidate to reallocate all of \( x_0 \) to private goods of equal value for groups 1 and 2 at lower cost. A platform giving \( sx_0 + x_1 \) to group 1 voters and \( sx_0 + x_2 \) to group 2 voters is feasible if \( n_1(sx_0 + x_1) + n_2(sx_0 + x_2) \leq 1 \). Rearranging and applying the budget constraint, we obtain:

\[
\frac{n_1sx_0 + n_2sx_0}{s} \leq x_0 \leq \frac{1}{n_1 + n_2}.
\]

(iii) To show the result for \( s > 1/n_i \), observe that a party \( X \) candidate could replace \( x_i \) with \( n_i x_i \) units of \( x_0 \). This revised allocation strictly benefits all voters.

To show the result for \( s > \max\{1/n_1, 1/n_2\} \), we derive the condition on \( s \) for the party \( X \) candidate to reallocate all of \( x_1 \) and \( x_2 \) to the public goods and benefit groups 1 and 2 at lower cost. The platform \( x'_0 = 1 \) is feasible and gives \( s \) to all voters. It provides greater utility to voters in group \( i \) if \( s > sx_0 + x_i \). Since the right-hand side is maximized either at \( x_i = 1/n_i \) (implying \( x_0 = 0 \)), or 0 (implying \( x_0 = 1 \)), the condition holds for any \( s > 1/n_i \).

(iv) Suppose that \( x_i > x_j > 0 \) for \( i, j \in \{1, 2\} \) and \( j \neq i \). We provide conditions under which a party \( X \) candidate would do strictly better by offering a different platform \( \pi' \) where \( x'_{j} = 0 \), \( x'_i = x_i - x_j \), and \( x'_0 = x_0 + x_j/s \). This platform clearly provides all voters in groups 1 and 2 identical utility as \( \pi \). Thus we need only verify the feasibility of \( \pi' \), which, given part (i), is assured if:

\[
\frac{n_i(x_i - x_j) + x_0 + x_j/s}{s} < \frac{1}{n_1 + n_2}.
\]

Therefore for any such \( s \), an optimal platform must have \( x_1 = 0 \) or \( x_2 = 0 \). By part (ii), for \( s > \max\{1/n_1, 1/n_2\} \), this is strengthened to \( x_1 = 0 \) and \( x_2 = 0 \). \( \square \)

Proof of Lemma 2. The budget constraints and weak domination imply that \( x_2 = (1 - n_1 x_1 - n_3 x_3 - x_0)/n_2 \) and \( y_2 = (1 - n_1 y_1 - n_3 y_3 - y_0)/n_2 \). Further, Lemma (i) implies \( x'_3 = y'_1 = 0 \). Substituting these into (3) and (4) yields:

\[
E_1(\pi, \bar{y}) = \left[ \frac{n_3y_3 - n_1x_1 - (n_2s-1)(y_0-x_0)}{\theta n_2} + \frac{\alpha}{\theta} + \frac{1}{2} \right] (x_1 + s(x_0-y_0) + \gamma_1) + sy_0 \quad (22)
\]

\[
E_3(\pi, \bar{y}) = \left[ \frac{n_3y_3 - n_1x_1 - (n_2s-1)(y_0-x_0)}{\theta n_2} + \frac{\alpha}{\theta} + \frac{1}{2} \right] (-y_3 + s(x_0-y_0) + \gamma_3)
+ y_3 + sy_0. \quad (23)
\]
There are two cases. First, if \( n_2s > 1 \), then it is clear that (22) and (23) are strictly convex in \( x_0 \) and \( y_0 \), respectively. Second, if \( n_2s < 1 \), then we may write the first order conditions as follows.

\[
\begin{align*}
\frac{\partial E_1(x, y)}{\partial x_1} &= -\frac{2n_1}{\theta n_2} x_1 + \frac{n_3y_3 - n_1\gamma_1 - (n_2s - n_1s - 1)(y_0 - x_0)}{\theta n_2} + \frac{\alpha}{\theta} + \frac{1}{2} \\
\frac{\partial E_1(x, y)}{\partial x_0} &= \frac{2(n_2s - 1)s}{\theta n_2} x_0 + \frac{(n_2s - 1)(x_1 - sy_0 + \gamma_1) + s(n_3y_3 - n_1x_1 - (n_2s - 1)y_0)}{\theta n_2} + \left(\frac{\alpha}{\theta} + \frac{1}{2}\right)s \\
\frac{\partial E_3(x, y)}{\partial y_3} &= -\frac{2n_3}{\theta n_2} y_3 + \frac{n_1x_1 + n_3\gamma_3 + (n_2s - n_3s - 1)(y_0 - x_0)}{\theta n_2} - \frac{\alpha}{\theta} + \frac{1}{2} \\
\frac{\partial E_3(x, y)}{\partial y_0} &= \frac{2(n_2s - 1)s}{\theta n_2} y_0 - \frac{(n_2s - 1)(-y_3 + sx_0 + \gamma_3) + s(n_3y_3 - n_1x_1 + (n_2s - 1)x_0)}{\theta n_2} - \left(\frac{\alpha}{\theta} - \frac{1}{2}\right)s.
\end{align*}
\]

Now consider whether the conditions for local concavity are possible. For party \( X \), the Hessian is:

\[
\begin{vmatrix}
-\frac{2n_1}{\theta n_2} & \frac{n_3 - n_1s - 1}{\theta n_2} \\
\frac{n_3 - n_1s - 1}{\theta n_2} & \frac{2(n_2s - 1)s}{\theta n_2}
\end{vmatrix}
\]

The diagonal elements are clearly negative, and the determinant is non-negative if:

\[
\begin{align*}
-4n_1(n_2s - 1)s - (n_2s - n_1s - 1)^2 &\geq 0 \\
1 - 2n_1n_2s^2 - (n_1s - 1)^2 - (n_2s - 1)^2 &\geq 0
\end{align*}
\]

It is straightforward to show that this expression is never positive, and can be satisfied with equality if and only if \( n_1s + n_2s = 1 \). But when \( s = 1/(n_1 + n_2) \), party \( X \) does just as well by giving private goods \( x_1 = x_2 = 1/(n_1 + n_2) \). Thus the objective has no local maxima whenever \( s \) is such that public goods might be optimal. The analysis for party \( Y \) is symmetrical and therefore omitted.

**Proof of Proposition 1.** By Lemma 1(ii), the assumptions on \( s \) allow us to restrict attention to strategies where \( x_0 = y_0 = 0 \).

We now characterize the unique equilibrium platforms. The budget constraints and weak domination imply that \( x_2 = (1 - n_1x_1 - n_3x_3)/n_2 \) and \( y_2 = (1 - n_1y_1 - n_3y_3)/n_2 \). Substituting these into (3) and (4) yields:

\[
\begin{align*}
E_1(x, y) &= \left[\frac{\alpha + (n_1y_1 + n_3y_3 - n_1x_1 - n_3x_3)/n_2}{\theta} + \frac{1}{2}\right] (x_1 - y_1 + \gamma_1) + y_1 \\
E_3(x, y) &= \left[\frac{\alpha + (n_1y_1 + n_3y_3 - n_1x_1 - n_3x_3)/n_2}{\theta} + \frac{1}{2}\right] (x_3 - y_3 + \gamma_3) + y_3.
\end{align*}
\]

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Clearly, $\frac{\partial E_1}{\partial x_3}(\vec{x}, \vec{y}) < 0$ and $\frac{\partial E_3}{\partial x_3}(\vec{x}, \vec{y}) < 0$ for all $(\vec{x}, \vec{y})$, so $x_3^p = y_1^p = 0$. The expected utilities of group-1 and group-3 voters can then be written:

$$E_1(\vec{x}, \vec{y}) = \left[ \frac{n_3 y_3 - n_1 x_1}{\theta n_2} + \frac{\alpha}{\theta} + \frac{1}{2} \right] (x_1 + \gamma_1)$$

$$E_3(\vec{x}, \vec{y}) = \left[ \frac{n_3 y_3 - n_1 x_1}{\theta n_2} + \frac{\alpha}{\theta} + \frac{1}{2} \right] (-y_3 + \gamma_3) + y_3$$

These are now concave one-variable choice problems. Thus for any $\vec{x}$ (respectively, $\vec{y}$), there is a unique platform for party $Y$ (respectively, $X$) that maximizes the utility of the pivotal voter in group 3 (respectively, 1). Each party’s candidates must therefore choose the same platform in equilibrium. The two party $X$ candidates simply choose $x_1 \in [0, 1/n_1]$ and the two party $Y$ candidates simply choose $y_3 \in [0, 1/n_3]$.

While candidates within each party choose the same allocation vectors, each party’s platform might be different. Denoting the equilibrium transfer vectors $\vec{x}^p$ and $\vec{y}^p$, the first-order conditions can now be written:

$$x_1^p = \frac{n_3 y_3^p - n_1 \gamma_1 + n_2 (\theta/2 + \alpha)}{2n_1}$$

$$y_3^p = \frac{n_1 x_1^p + n_3 \gamma_3 + n_2 (\theta/2 - \alpha)}{2n_3}$$

Solving these yields the stated unique equilibrium allocations.

**Proof of Lemma 3.** (i) Let $E_i(x_i, \vec{y})$ denote group 1 utility $E_1(\vec{x}, \vec{y})$ when $x_i = (1-x_0)/n_i$ and all other private good allocations are zero.

We first show that $\frac{dE_{i1}}{dx_0}(1, \vec{y}) \geq 0$ for either $i = 1$ or 2. Substituting into (3) and differentiating gives us the following expressions:

$$\frac{dE_{11}}{dx_0} = \frac{2n_1 s^2 (x_0 - y_0) + (1 - n_1 s)(y_2 - \alpha - \theta/2) + s(1 - 2x_0 + y_0) + n_1 s(\gamma_1 - y_1)}{n_1 \theta}$$

$$\frac{dE_{12}}{dx_0} = \frac{2n_2 s^2 (x_0 - y_0) - n_2 s(y_2 - \alpha - \theta/2) + s(1 - 2x_0 + y_0) - (1 - n_2 s)(\gamma_1 - y_1)}{n_2 \theta}$$

Suppose that $\frac{dE_{11}}{dx_0}(1, \vec{y}) < 0$ and $\frac{dE_{12}}{dx_0}(1, \vec{y}) < 0$. Clearly, for each $i$, $\frac{dE_{ii}}{dx_0}(1, \vec{y}) < 0$ iff the corresponding numerator in the above expressions is negative. This implies that the sum of the numerators evaluated at $x_0 = 1$ must also be negative. Substituting and simplifying, we obtain:

$$2n_1 s^2 (1 - y_0) + (1 - n_1 s)(y_2 - \alpha - \theta/2) + s(y_0 - 1 + n_1 (\gamma_1 - y_1)) +$$

$$2n_2 s^2 (1 - y_0) - n_2 s(y_2 - \alpha - \theta/2) + s(y_0 - 1 + n_2 (\gamma_1 - y_1) - \gamma_1 + y_1$$

$$= (1 - (n_1 + n_2)s)[2s(y_0 - 1) + y_2 - \alpha - \theta/2 - \gamma_1 + y_1].$$

By assumption, $s > 1/(n_1 + n_2)$, and thus $1 - (n_1 + n_2)s < 0$. The above expression is then positive if $y_1 + y_2 < \gamma_1$, which always holds since $y_1 + y_2 \leq \max\{1/n_1, 1/n_2\} < \gamma_1$: contradiction. Thus either $\frac{dE_{11}}{dx_0}(1, \vec{y}) > 0$ or $\frac{dE_{12}}{dx_0}(1, \vec{y}) > 0$, or both.
By the concavity of $E_{1i}(x_0, \overline{y})$, if $\frac{dE_{1i}}{dx_0}(1, \overline{y}) > 0$ then the optimal platform that excludes group $j \neq i$ is $x_0 = 1$ and $x_i = 0$. And if $\frac{dE_{1i}}{dx_0}(1, \overline{y}) < 0$, then the optimal platform that excludes group $j \neq i$ is either a corner at $x_0 = 0$ and $x_i = 1/n_i$, or the interior solution given by $\tilde{x}_i$ and $\tilde{x}_0i$ from the appropriate expression in (6)-(9). Denoting the public good level of this solution by $\tilde{x}_0$, it is clear that $E_{1i}(\tilde{x}_0, \overline{y}) > E_{1i}(1, \overline{y}) = E_{1j}(1, \overline{y})$. Thus if $\frac{dE_{1i}}{dx_0}(1, \overline{y}) < 0$ the unique optimal platform is the solution $(\tilde{x}_0, \tilde{x}_i)$ promising private goods to group $i$, and if $\frac{dE_{1j}}{dx_0}(1, \overline{y}) > 0$ and $\frac{dE_{12}}{dx_0}(1, \overline{y}) > 0$, the optimal platform is $x_0^* = 1$.

(iii) Let $E_{3i}(\overline{x}, y_0)$ denote group 3 utility $E_3(\overline{x}, \overline{y})$ when $y_i = (1 - y_0)/n_i$ and all other private good allocations are zero.

We show that $\frac{dE_{3i}}{dy_0} > 0$ for either $i = 2$ or 3. Straightforward differentiation gives us the following expressions:

$$
\frac{dE_{33}}{dy_0} = \frac{2n_3s^2(y_0 - x_0) + (1 - n_3s)(x_2 + \alpha - \theta/2) + s(1 - 2y_0 + x_0) - n_3s(\gamma_3 + x_3)}{\theta n_3}
$$

$$
\frac{dE_{32}}{dy_0} = \frac{2n_2s^2(y_0 - x_0) - n_2s(x_2 + \alpha - \theta/2) + s(1 - 2y_0 + x_0) + (1 - n_2s)(\gamma_3 + x_3)}{\theta n_2}
$$

Suppose that $\frac{dE_{3i}}{dy_0}(\overline{x}, 1) < 0$ and $\frac{dE_{3i}}{dy_0}(\overline{x}, 1) < 0$. Clearly, for each $i$, $\frac{dE_{3i}}{dy_0}(1, \overline{y}) < 0$ iff the corresponding numerator in the above expressions is negative. This implies that the sum of the numerators evaluated at $y_0 = 1$ must also be negative. Substituting and simplifying, we obtain:

$$
2n_3s^2(1 - x_0) + (1 - n_3s)(x_2 + \alpha - \theta/2) + s(x_0 - 1 - n_3(\gamma_3 + x_3)) +
$$

$$
2n_2s^2(1 - x_0) - n_2s(x_2 + \alpha - \theta/2) + s(x_0 - 1 - n_2(\gamma_3 + x_3)) + \gamma_3 + x_3
$$

$$
= (1 - (n_2 + n_3)s)(2s(x_0 - 1) + x_2 - \alpha - \theta/2 + \gamma_3 + x_3).
$$

By assumption, $s > 1/(n_2 + n_3)$, and thus $1 - (n_2 + n_3)s < 0$. The above expression is then positive if $x_2 + x_3 < |\gamma_3|$, which always holds since $x_2 + x_3 \leq \max\{1/n_2, 1/n_3\} < |\gamma_3|$: contradiction. Thus either $\frac{dE_{3i}}{dy_0}(\overline{x}, 1) > 0$ or $\frac{dE_{3i}}{dy_0}(\overline{x}, 1) > 0$, or both.

By the concavity of $E_{3i}(\overline{x}, y_0)$, if $\frac{dE_{3i}}{dy_0}(\overline{x}, 1) > 0$ then the optimal platform that excludes group $j \neq i$ is $y_0 = 1$ and $y_i = 0$. And if $\frac{dE_{3i}}{dy_0}(\overline{x}, 1) < 0$, then the optimal platform that excludes group $j \neq i$ is either a corner at $y_0 = 0$ and $y_i = 1/n_i$, or the interior solution given by $\tilde{y}_i$ and $\tilde{y}_0i$ from the appropriate expression in (10)-(13). Denoting the public good level of this solution by $\tilde{y}_0$, it is clear that $E_{3i}(\overline{x}, \tilde{y}_0) > E_{3i}(\overline{x}, 1) = E_{3j}(\overline{x}, 1)$. Thus if $\frac{dE_{3i}}{dx_0}(\overline{x}, 1) < 0$ the unique optimal platform is the solution $(\tilde{y}_0i, \tilde{y}_i)$ promising private goods to group $i$, and if $\frac{dE_{3i}}{dy_0}(\overline{x}, 1) > 0$ and $\frac{dE_{3i}}{dy_0}(\overline{x}, 1) > 0$, the optimal platform is $y_0^* = 1$.

**Proof of Lemma 4.** (i) Reversing the inequality in (14), we obtain the following condition under which $x_i^* = 0$ must always obtain: $\gamma_i \geq \frac{-2(n_1s^2s)(1 - y_0) - (1 - n_1s)(y_2 - \alpha - \theta/2)}{n_1s}$. To establish the upper bound on the right-hand side of this expression, letting $y_2 = 0$ and $y_0 = 1$ (0) if $2n_1s > (<) 1$ yields the result.

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Next, reversing the inequality in (15), we obtain the following condition under which \( x_2^* = 0 \) must always obtain: \( \gamma_1 \leq \frac{(2n_3s^2-s)(1-y_0)-n_2s(y_2-\alpha-\theta/2)}{1-n_2s} \). To establish the lower bound on the right-hand side of this expression, let \( y_2 = 1/n_2 \) and \( y_0 = 0 \). This yields \( \frac{n_2s(\alpha-\theta/2)}{1-n_2s} - 2s \).

(ii) Reversing the inequality in (16), we obtain the following condition under which \( y_3^* = 0 \) must always obtain: \( \gamma_3 \leq \frac{(2n_1s^2-s)(1-x_0)+(1-n_3s)(x_2+\alpha-\theta/2)}{n_3s} \). To establish the lower bound on the right-hand side of this expression, letting \( x_2 = 0 \) and \( x_0 = 1 \) (0) if \( 2n_3s > (<) 1 \), yields the result.

Next, reversing the inequality in (17), we obtain the following condition under which \( y_2^* = 0 \) must always obtain: \( \gamma_3 \geq \frac{-(2n_2s^2-s)(1-x_0)+n_2s(x_2+\alpha-\theta/2)}{1-n_2s} \). To establish the lower bound on the right-hand side of this expression, let \( x_2 = 1/n_2 \) and \( x_0 = 0 \). This yields \( \frac{n_2s(\alpha-\theta/2)}{1-n_2s} + 2s \).

\[ \blacksquare \]

**Proof of Proposition 2.** There are three cases. First, for \( s < \min\{1/(n_1+n_2), 1/(n_2+n_3)\} \), existence is demonstrated by Proposition 1.

Second, suppose \( s \geq \max\{1/(n_1+n_2), 1/(n_2+n_3)\} \). Since candidates from both parties adopt the platform that maximizes the expected utility of their core voters, it suffices to establish a fixed point in the best response functions for each party’s common platform. We show that the pure strategy best responses satisfy the conditions of Brouwer’s fixed point theorem. Observe that each party’s set of feasible pure strategies is a simplex, and thus the set of strategy profiles is obviously compact, convex, and non-empty.

Next, we show that party X’s best response is single-valued. Let \( E_{1i}(x_0, \overline{y}) \) denote group 1 utility \( E_1(\overline{x}, \overline{y}) \) when \( x_i = (1 - x_0)/n_i \) and all other private good allocations are zero. Note that \( E_{1i}(x_0, \overline{y}) \) is concave in \( x_0 \) for \( s < 1/n_i \) and that \( E_{1i}(\cdot) \) is maximized at \( x_0 = 1 \) for \( s \geq 1/n_i \). Thus, when restricted to a choice between group \( i \) and the public good, there unique solutions \( \tilde{x}_{01} \) (7) and \( \tilde{x}_{02} \) (9). Party X’s best response is \( \tilde{x}_{0i} \) if \( E_{1i}(\tilde{x}_{0i}, \overline{y}) > E_{1j}(\tilde{x}_{0i}, \overline{y}) \) \((j \neq i)\). By the argument in the proof of Lemma 3, \( E_{11}(\tilde{x}_{01}, \overline{y}) = E_{12}(\tilde{x}_{02}, \overline{y}) \) if and only if \( \tilde{x}_{01} = \tilde{x}_{02} = 1 \). Thus, party X’s best response is single-valued. An identical argument establishes that party Y’s best response is also single-valued. The best response to each strategy profile is therefore single-valued.

Finally, we show that the best response function is continuous. By the concavity of \( E_{11}(x_0, \overline{y}), E_{12}(x_0, \overline{y}) \), the solutions \( \tilde{x}_{01} \) and \( \tilde{x}_{02} \) are continuous in \( \overline{y} \). By the argument in the proof of Lemma 3, either \( \tilde{x}_{01} = 1 \) or \( \tilde{x}_{02} = 1 \), and the platform implied by \( \tilde{x}_{0i} \) (i.e., \( x_0 = x_{0i}, x_i = (1 - x_0)/n_i \)) is optimal if \( \tilde{x}_{0i} < 1 \) \((j \neq i)\). Thus for any \( \overline{y} \) the best response is either \( \tilde{x}_{0i} < 1 \) for some \( i \in \{1, 2\} \), or \( x_0 = 1 \), which occurs when \( \tilde{x}_{01} = \tilde{x}_{02} = 1 \). The resulting best response for party X is then:

\[
\begin{align*}
\tilde{x}_{01} \quad & \text{if } \tilde{x}_{01} < 1, \tilde{x}_{02} = 1 \\
1 \quad & \text{if } \tilde{x}_{01} = \tilde{x}_{02} = 1 \\
\tilde{x}_{02} \quad & \text{if } \tilde{x}_{02} < 1, \tilde{x}_{01} = 1.
\end{align*}
\]

This function inherits continuity in \( \overline{y} \) from the continuity of \( \tilde{x}_{01} \) and \( \tilde{x}_{02} \). An identical argument holds for party Y’s best response.
Third, suppose \( s \in [\min\{1/(n_1+n_2), 1/(n_2+n_3)\}, \max\{1/(n_1+n_2), 1/(n_2+n_3)\}] \). Without loss of generality, assume that \( 1/(n_1+n_2) < 1/(n_2+n_3) \). We again show that the pure strategy best responses satisfy the conditions of Brouwer’s fixed point theorem. For party \( X \)’s best responses the analysis is identical to the second case. For party \( Y \), we show that the best response is single-valued and continuous.

Since \( s \leq 1/(n_2+n_3) \), it is clear that \( y_0 = 0 \) in any best response. Noting that \( y_1 = 0 \) and \( y_2 = (1-n_3y_3)/n_2 \) in any best response, we rewrite (4) as the party \( Y \) objective as follows:

\[
E_3(x, y) = \left[ x - (1-n_3y_3)/n_2 + sx_0 + \frac{\alpha}{\theta} + \frac{1}{2} \right] (x - y_3 + sx_0 + \gamma_3) + y_3 + sy_0
\]

This expression is obviously concave in \( y_3 \). Straightforward maximization yields the solution \( y_3^* = \frac{1-n_3x_1+(n_3-s-n_2)x_0+n_3\gamma_3+n_2(\theta/2-\alpha-x_2)}{2n_3} \), which is obviously single-valued and continuous. 

**Proof of Proposition 4.** (i) Note that by Lemma 1, the best responses by candidates in both parties under the stated condition are to offer only public goods when \( s > 1/n_2 \).

Now consider the platform choices of party \( X \) candidates when \( s \leq 1/n_2 \). Observe that the left-hand side of expressions (14)-(15) in Lemma 3 are the numerators of the derivatives of the party \( X \) objectives \( E_{11}(x_0, \bar{y}) \) and \( E_{12}(x_0, \bar{y}) \) with respect to \( x_0 \), where the objectives are restricted to \( x_2 = 0 \) and \( x_1 = 0 \), respectively. Since the denominators of the derivatives are strictly positive, the signs of these expressions is sufficient for signing the derivative.

Evaluating (15) at \( s = 1/n_2 \), the first-order condition of \( E_{12}(\cdot) \) is positive if:

\[
\frac{1-y_0}{n_2} > y_2 - \alpha - \theta/2.
\]

Since \( \theta > \alpha \geq 0 \) and the budget constraint implies \( y_2 \leq (1-y_0)/n_2 \), this expression always holds. Thus \( \frac{dE_{12}}{dx_0}(1, \bar{y}) > 0 \), and by Lemma 3 \( x_0 = 1 \) is the optimal strategy for party \( X \) candidates when \( x_1 = 0 \). By the continuity of \( E_{12}(x_0, \bar{y}) \), there exists a nonempty set \( S = [\bar{s}, 1/n_2] \) such that for all \( s \in S \), \( \frac{dE_{12}}{dx_0}(1, \bar{y}) > 0 \).

Now consider \( E_{11}(x_0, \bar{y}) \). Clearly, for all \( s > 1/n_1 \), the optimal strategy for party \( X \) candidates when \( x_2 = 0 \) is \( x_0 = 1 \). Since \( n_1 > n_2 \), the region \([1/n_1, 1/n_2] \cap S \) is non-empty. Party \( X \) candidates will then choose \( x_0 = 1 \) regardless of whether their best response is to maximize \( E_{11}(x_0, \bar{y}) \) or \( E_{12}(x_0, \bar{y}) \) when \( s \geq \min\{[1/n_1, 1/n_2] \cap S \} \). Thus, for \( n_1 > n_2 \), party \( X \) candidates will offer only public goods for some \( s \) strictly less than \( 1/n_2 \).

The analysis for party \( Y \) candidates is symmetric and therefore omitted. Combining the statements for party \( X \) and \( Y \) yields a threshold \( \bar{s} \).

(ii) By Lemma 1, we require \( s > \frac{1}{n_1+n_2} \) for \( x_0 > 0 \) in equilibrium. Since only \( x_1 \) or \( x_2 \) can be strictly positive at an optimal platform, it is sufficient to derive conditions under which the possible private good allocations for \( x_1 \) and \( x_2 \), \( \bar{x}_1 \) and \( \bar{x}_2 \), are not maximized. Observe that for \( s > 1/n_i \), \( x_i = 0 \), and so we restrict attention to \( s \leq 1/n_i \) for \( i = 1, 2 \). By Lemma 3(i), \( \bar{x}_1 < 1/n_1 \) if:

\[
\frac{n_1(\gamma_1 - y_1) + (2n_1s-1)(1-y_0)}{2n_1(n_1s-1)} + \frac{\alpha + \theta/2 - y_2}{2n_1s} < \frac{1}{n_1}
\]

\[
sn_1\gamma_1 + s - s(2n_1s-1)y_0 + (\alpha + \theta/2)(n_1s-1) > 0
\]

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The preceding simplification used the facts that $y_1 = 0$ in any optimal party $Y$ platform, and $y_2 \geq 0$. Noting that $(2n_1s - 1)y_0$ is bounded from above by 1, this expression simplifies to:

$$s > \frac{\alpha + \theta/2}{n_1(\gamma_1 + \alpha + \theta/2)}.$$ 

Likewise, $\tilde{x}_2 < 1/n_2$ if:

$$\frac{(n_2s - 1)(\gamma_1 - y_1) - s(2n_2s - 1)(y_0 - 1)}{2n_2s(n_2s - 1)} + \frac{\alpha + \theta/2 - y_2}{2(n_2s - 1)} < \frac{1}{n_2}$$

$$(n_2s - 1)\gamma_1 - s(2n_2s - 1)y_0 + s + n_2s(\alpha + \theta/2 - 1) > 0$$

The preceding simplification used the facts that $y_1 = 0$ in any optimal party $Y$ platform, and $y_2 \geq 0$. Noting that $(2n_2s - 1)y_0$ is bounded from above by 1, this expression simplifies to:

$$s > \frac{\gamma_1}{n_2(\gamma_1 + \alpha + \theta/2 - 1)}.$$ 

Combining the two expressions for $s$ yields the result. 

**Proof of Proposition 5.** Combining Lemmas 1 and 4, we have the following choices for party $X$ candidates. For $s \leq \min\{1/n_1, 1/n_2\}$, $x_2^* = \min\{\max\{0, \tilde{x}_2\}, 1/n_2\}$ if (18) holds, and $x_1^* = \min\{\max\{0, \tilde{x}_1\}, 1/n_1\}$ if (19) holds. For $s \in (1/n_1, 1/n_2)$, $x_1^* = 1/n_1$ if (19) holds, and for $s \in (1/n_2, 1/n_1)$, $x_2^* = 1/n_2$ if (18) holds. Finally if $s \geq \max\{1/n_1, 1/n_2\}$, $x_0^* = 1$ for all $\theta$.

Likewise, we have the following choices for party $Y$ candidates. For $s \leq \min\{1/n_3, 1/n_2\}$, $y_2^* = \min\{\max\{0, \tilde{y}_2\}, 1/n_2\}$ if (20) holds, and $y_3^* = \min\{\max\{0, \tilde{y}_3\}, 1/n_3\}$ if (21) holds. For $s \in (1/n_3, 1/n_2)$, $x_3^* = 1/n_3$ if (21) holds, and for $s \in (1/n_2, 1/n_3)$, $x_2^* = 1/n_2$ if (20) holds. Finally if $s \geq \max\{1/n_3, 1/n_2\}$, $y_0^* = 1$ for all $\theta$.

For parts (i)-(iii), the interior platforms are derived from straightforward solutions of the linear systems implied by these equilibrium best response platforms. For part (iv), the system yields:

$$x_2 = y_2 + \frac{\gamma_1}{2n_2s} - \frac{\theta + 2\alpha}{4(1 - n_2s)}$$
$$y_2 = x_2 - \frac{\gamma_3}{2n_2s} - \frac{\theta - 2\alpha}{4(1 - n_2s)}$$

There is clearly no generic solution for this system. For $\gamma_1$ and $|\gamma_3|$ sufficiently large, the unique solution is the corner at $x_2 = 1/n_2$ and $y_2 = 1/n_2$. 

**Proof of Proposition 6.** We first establish party $X$ candidates’ optimal platforms for $\gamma > \frac{(1-n_1s)\theta}{2n_1s}$. By Lemma 3, the solution for party $X$ candidates is $x_1 = \max\{1/n_1, \tilde{x}_1\}$ if (14) holds:

$$(2n_1s^2 - s)(1 - y_0) + (1 - n_1s)(y_2 - \theta/2) + n_1s\gamma < 0.$$
The left-hand side of this expression is increasing in $\gamma$. Substituting in $\gamma = \frac{(1-n_1 s)\theta}{2n_1 s}$, this expression reduces to:

$$(2n_1 s^2 - s)(1 - y_0) + (1 - n_1 s)y_2 < 0.$$ 

This expression cannot hold for any $s > \frac{1}{2n_1}$, and thus there is no optimal platform where $x_1 > 0$ when $\gamma \geq \frac{(1-n_1 s)\theta}{2n_1 s}$. By Lemmas 1(ii) and 2, this implies that at any party $X$ best response, the entire budget is used on $x_0$ and $x_2$, or equivalently, $n_2x_2 + x_0 = 1$.

To derive the value of $x_2$, observe that by Lemma 1(iii), $s > 1/n_2$ implies $x_2 = 0$. For $s \leq 1/n_2$, substituting $n_2x_2 + x_0 = 1$ and $n_2y_2 + y_0 = 1$ into the expressions for $\tilde{x}_2$ and $\tilde{y}_2$ ((8) and (12)) yields the system of unconstrained solutions given by (28) and (29). Substituting in the assumed parameter restrictions, this system simplifies to:

$$x_2 = y_2 + \frac{\gamma}{2n_2 s} - \frac{\theta}{4(1-n_2 s)}$$ \hspace{1cm} (30)

$$y_2 = x_2 + \frac{\gamma}{2n_2 s} - \frac{\theta}{4(1-n_2 s)}.$$ \hspace{1cm} (31)

There is clearly no generic interior solution for this system. Since $\frac{\gamma}{2n_2 s} - \frac{\theta}{4(1-n_2 s)} > (\gamma) 0$ for $\gamma > (\gamma)\frac{n_2 s \theta}{2(1-n_2 s)}$, the unique solution is $x_2 = y_2 = 1/n_2$ ($= 0$) for $\gamma > (\gamma)\frac{n_2 s \theta}{2(1-n_2 s)}$.

Now consider the party $X$ candidates’ optimal platforms for $\gamma \leq \frac{(1-n_1 s)\theta}{2n_1 s}$. Observe that $\frac{(1-n_1 s)\theta}{2n_1 s} < \frac{n_2 s \theta}{2(1-n_2 s)}$ for all $s > \frac{1}{n_1+n_2}$, as assumed. We first establish that $x_2 = 0$ for any best response. To see this, note first that the system (30)-(31) imply that for $\gamma \leq \frac{(1-n_1 s)\theta}{2n_1 s}$, there can be no solution where $x_2 > 0$ and $y_2 > 0$; thus, $x_2 > 0$ requires $y_2 = 0$. By Lemma 3, party $X$ chooses $x_2 > 0$ if and only if (15) holds:

$$(2n_2 s^2 - s)(1 - y_0) - n_2 s(y_2 - \theta/2) - (1 - n_2 s)\gamma < 0.$$ 

The left-hand side of this expression is decreasing in $\gamma$. Substituting in $\gamma = \frac{n_2 s \theta}{2(1-n_2 s)}$ and $y_2 = 0$, this expression can be satisfied only if:

$$(2n_2 s^2 - s)(1 - y_0) < 0.$$ 

This expression cannot hold for any $s > \frac{1}{2n_2}$, and thus all best responses must satisfy $x_0 + n_1 x_1 = 1$. A symmetric analysis holds for party $Y$ candidates.

To derive the value of $x_1$, observe that by Lemma 1(iii), $s > 1/n_1$ implies $x_1 = y_3 = 0$. For $s \leq 1/n_1$, an interior solution must be given by $\tilde{x}_1$ and $\tilde{y}_1$, as defined by (6) and (10). The unique solution of this system is $x_1^* = y_3^* = \min\left\{\frac{1}{n_1}, \frac{(1-n_1 s)\theta}{2n_1 s} - \gamma\right\}$; these are clearly non-negative for all $\gamma \leq \frac{(1-n_1 s)\theta}{2n_1 s}$. ■

**Proof of Proposition 7.** By Lemma 1, since $s < 1/(n_1 + n_2)$, $x_0^* = 0$. Substituting into (3) and maximizing then gives the the interior solutions for $x_1$ and $x_2$ in terms of $y_2$:

$$x_1^* = \frac{1 - n_1 \gamma - n_2 (y_2 - \theta/2)}{2n_1}$$ \hspace{1cm} (32)

$$x_2^* = \frac{1 + n_1 \gamma + n_2 (y_2 - \theta/2)}{2n_2}$$ \hspace{1cm} (33)
By the concavity of (3), the obvious corner solutions are \( x_1^* = 0 \) and \( x_2^* = 1/n_2 \), and \( x_1^* = 1/n_1 \) and \( x_2^* = 0 \). We derive features of the party \( Y \) platforms by applying Lemma 3(ii).

(i) Simplifying from (16), \( y_3^* = \min \{ 1/n_3, \tilde{y}_3 \} \) and \( y_2^* = 0 \) if:

\[
2n_3s^2 - s + (1 - n_3s)(x_2 - \theta/2) + n_3s\gamma < 0
\]

\[
\gamma < \frac{s - 2n_3s^2 - (1 - n_3s)(x_2 - \theta/2)}{n_3s}
\]  

(34)

Since \( y_2^* = 0 \) in this region, we have \( x_2^* = \max \{ 0, \min \{ \frac{1}{n_2}, \frac{1}{2n_2} + \frac{\gamma}{s} - \frac{\theta}{4} \} \} \). The upper bound implied by (34) is linear in \( x_2 \), and \( x_2^* \) is continuous, bounded, and piecewise linear in \( \gamma \). Thus a solution \( \gamma' \) for (34) in terms of \( \gamma \) exists and is weakly decreasing and piecewise linear. It is straightforward to calculate that at an interior solution this bound is:

\[
\gamma' = \frac{(1 - n_3s)(2s - 1/n_2 + 3\theta/2) - 2n_3s^2}{1 + n_3s}
\]

Finally, substituting appropriately into (10), we have the following expression for \( \tilde{y}_3 \):

\[
\tilde{y}_3 = \frac{2n_3s - 1}{2n_3(n_3s - 1)} - \frac{x_2 - \theta/2}{2n_3s}
\]

This expression is decreasing in \( \gamma \) for \( s < 1/n_3 \), and \( y_3^* = 0 \) for \( s \geq 1/n_3 \) when (16) holds. Thus, \( y_3^* \) is weakly decreasing in this region.

(ii) Simplifying from (17), \( y_3^* = 0 \) and \( y_2^* = \min \{ 1/n_2, \tilde{y}_2 \} \) if:

\[
2n_2s^2 - s - n_2s(x_2 - \theta/2) - (1 - n_2s)\gamma < 0
\]

\[
\gamma > \frac{2n_2s^2 - s - n_2s(x_2 - \theta/2)}{1 - n_2s}
\]

(36)

Simplifying (13) yields the following expression for \( \tilde{y}_2 \):

\[
\tilde{y}_2 = \frac{s(2n_2s - 1) - (1 - n_2s)\gamma}{2n_2s(n_2s - 1)} - \frac{x_2 - \theta/2}{2(n_2s - 1)}
\]

The expressions for \( y_2^* \) and \( x_2^* \) are continuous and piecewise linear in \( y_2 \) and \( x_2 \), respectively, and bounded. Thus there exists a solution to the system. At an interior solution we have:

\[
x_2^* = \frac{1}{n_2} + \frac{(2n_2s^2 - n_2s - 1)\gamma}{n_2s(4n_2s - 3)} + \frac{(3/2 - n_2s)\theta}{4n_2s - 3}
\]

\[
y_2^* = \frac{1}{n_2} + \frac{(2 - n_2s)\gamma}{3n_2s - 4n_2s^2} - \frac{3\theta}{6 - 8n_2s}
\]

To characterize \( \gamma'' \), note that since \( x_2^* \geq 0 \), the lower bound on \( \gamma'' \) can be derived by substuting \( x_2 = 1/n_2 \) into (36), yielding \( \gamma'' \geq \frac{s(2n_2s - 2 + n_2\theta/2)}{1 - n_2s} \). Substituting the interior value of \( x_2^* \) into (36), at an interior solution the minimum value of \( \gamma \) for this solution to obtain is:

\[
\gamma'' = \frac{s(3 - 4n_2s - 3n_2\theta/2)}{n_2s - 2}
\]
It is straightforward to verify that $\gamma'' > \gamma'$ whenever $\gamma' > 1/n_2$.

(iii) By Lemma 3(ii), for all $\gamma$ not satisfying the conditions of parts (i) and (ii), $y_0^* = 1$.

**Proof of Remark 2.** (i) Since $s > 1/n_2$, Lemma 1 implies that $x_2^* = 0$. We use Lemma 3(i) to establish the condition under which party $X$ candidates can offer private goods to group 1. Substituting into equation (14) yields:

$$n_1 < \frac{\alpha + \theta/2}{s(\gamma_1 + \alpha + \theta/2)}.$$  \hfill (37)

When this condition is not satisfied, $x_0^* = 1$. When (37) is satisfied, $x_1^* = \max\{1/n_1, \tilde{x}_1\}$, as given by substituting $y_0 = 1$ and $y_1 = y_2 = 0$ into (6):

$$x_1^* = \frac{\gamma_1}{2(n_1s - 1)} + \frac{\alpha + \theta/2}{2n_1s}.$$

(ii) Now suppose that $s < 1/n_2$, which implies $y_2^* = 1/n_2$. We again apply Lemma 3(i). Substituting $y_0 = y_1 = 0$ and $y_2 = 1/n_2$ into equation (14) yields the following condition:

$$2n_1s^2 - 1 - n_1s\left(1 - \frac{\alpha - \theta}{2}\right) + n_1s\gamma_1 < 0$$

$$n_1 < \frac{s + \alpha + \theta/2 - 1/n_2}{s(2s + \gamma_1 + \alpha + \theta/2 - 1/n_2)}. \hfill (38)$$

When (38) is satisfied, $x_1^* = \max\{1/n_1, \tilde{x}_1\}$, as given by substituting appropriately into (6):

$$x_1^* = \frac{\gamma_1 + s}{2(n_1s - 1)} + \frac{1}{2n_1}\left(1 - \frac{1}{n_2s}\right) + \frac{\alpha + \theta/2}{2n_1s}.$$

Similarly, party $X$ candidates may offer private goods to group 2. Substituting into equation (15) yields the following condition:

$$2n_2s^2 - s - n_2s\left(1 - \frac{\alpha - \theta}{2}\right) + (1 - n_2s)\gamma_1 < 0$$

$$n_2 < \frac{s + \gamma_1}{s(2s + \gamma_1 + \alpha + \theta/2 - 1/n_2)}. \hfill (39)$$

When (39) is satisfied, $x_2^* = \max\{1/n_2, \tilde{x}_2\}$, as given by substituting appropriately into (8):

$$x_2^* = \frac{\gamma_1}{2n_2s} + \frac{n_2(2s + \alpha + \theta/2 - 1)}{2n_2(n_2s - 1)}.$$  

When neither (38) nor (39) are satisfied, party $X$ provides only public goods. \hfill ■
REFERENCES


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