Delegating Multiple Decisions

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Abstract
This paper shows how to extend the heuristic of capping an agent against her bias to delegation problems over multiple decisions. These caps may be exactly optimal when the agent has constant biases, in which case a cap corresponds to a ceiling on the weighted average of actions. In more general settings caps give approximately first-best payoffs when there are many independent decisions. The geometric logic of a cap translates into economic intuition on how to let agents trade off increases on one action for decreases on other actions. I consider specific applications to political delegation, capital investments, monopoly price regulation, and tariff policy.

1 Introduction
Consider a principal who delegates decisionmaking authority to a more informed agent. For instance, an executive places an administrator in charge of certain political decisions. The administrator has private information about the appropriate policies for each decision. But her own ideal policy choices may differ from those of the executive. The executive anticipates the administrator’s biases, and exerts control by requiring the administrator to choose policies from a restricted set.

This so-called delegation problem was introduced by Holmström (1977, 1984). It has been used to analyze issues such as the investment levels a manager may choose; the prices that a regulated monopolist may charge; or the set of tariff levels allowed by a trade agreement. When there is a single decision to be made, Melumad and Shibano (1991), Martimort and Semenov (2006), Goltsman et al. (2007), Alonso and Matouschek (2008), Ambrus and Egorov (2009), Kovac and Mylovanov (2009), and Amador and Bagwell (2011) give a variety of

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conditions under which it is optimal for the principal to cap the agent’s choices against the direction of her bias.\textsuperscript{1} When the administrator is more liberal than the executive, the administrator is allowed to choose any policy that is sufficiently conservative. Likewise, a manager who is biased towards investing too much money is given a spending cap; a monopolist is given a price ceiling; a trade agreement sets a maximum tariff level. The more biased is the agent, the tighter is the cap.

The above analysis considers the delegation of a single decision. But the political administrator may make decisions on multiple policies; a manager may invest in more than one project; a country may place tariffs on multiple goods; the monopolist may be pricing a number of separate products. How should we extend the logic of the one-dimensional caps to delegation problems over multiple decisions? Can we do better than independently capping each decision?

Previous work in mechanism design and strategic communication develops a robust intuition that a principal can often improve payoffs by linking together an agent’s incentive constraints over multiple decisions, even when the decisions themselves are independent. See for example Chakraborty and Harbaugh (2007), Jackson and Sonnenschein (2007), or Frankel (Forthcoming). An executive delegating three decisions might require the administrator to make one liberal decision, one centrist one, and one conservative one. The administrator now faces trade-offs across decisions: even if she always prefer liberal actions, she can only take such an action once. So she takes the liberal action for the decision at which a liberal action is most appropriate. Well-designed quotas can often give the principal high (approximately first-best) payoffs when there are many independent decisions.\textsuperscript{2}

The logic for the incentive compatibility of such quotas assumes that the agent has identical biases across decisions. That is, the political administrator must have an identical liberal bias for each of three policies she is choosing. If the administrator is very liberal on health care but is centrist or conservative on defense and education, then offered a quota over these three policies she will always just take the liberal action on health care. Likewise, a monopolist may prefer much higher prices on goods with less elastic demand, regardless of her marginal costs. The literature gives little guidance on how to account for an agent with different preferences across decisions.

\textsuperscript{1}Athey et al. (2005) and Amador et al. (2006) argue for similar caps in the context of decisions taken over time, when the agent has time-inconsistent preferences.

\textsuperscript{2}Quotas are not generally fully optimal. Restricted to a single decision, a quota corresponds to a predeetermined action with no input from the agent. As discussed above, more flexible rules such as caps often do better. Frankel (2010) and Frankel (Forthcoming) give models in which quotas may be optimal when the principal is uncertain about the agent’s preferences over large enough set of possibilities.
In this paper I argue for extending the geometric intuition of \textit{capping an agent’s choices against the direction of her bias} to the multiple decision environment. This rule can properly align an agent’s incentives even when she has different biases across decisions, and in special cases is fully optimal. Moreover, the shape of these caps translates into economic prescriptions on how to let the agent make tradeoffs across decisions.

Caps against the bias are most simply illustrated in a setting where the agent has a constant bias relative to the principal: given any principal ideal actions $a = (a_1, ..., a_N)$, the agent’s ideal point is $a + \lambda = (a_1 + \lambda_1, ..., a_N + \lambda_N)$ for a fixed bias vector $\lambda$. Assuming quadratic loss utility functions, a cap against the agent’s bias corresponds to a half-space delegation set where the boundary is normal to the agent’s bias vector. See Figure 1.

Figure 1: For any principal ideal point $(a_1, a_2)$, the agent’s ideal point is $(a_1 + \lambda_1, a_2 + \lambda_2)$. The set $D$ illustrates a half-space normal to the agent’s bias.

Mathematically, the feasible action vectors given this cap are those which satisfy $\sum_i \lambda_i a_i \leq K$. We cap a weighted sum or average of actions, where the weight is proportional to the agent’s bias on that decision. In economic terms, these weights can be interpreted as prices. If the administrator is strongly liberal on health care, she must expend a lot of her budget to make health care policy more liberal. If the administrator is slightly conservative on national defense, she gains a small amount of flexibility on other decisions when she makes defense policy more liberal.

Section 3.1 establishes a benchmark optimality result for this quadratic loss constant bias setting. Caps against the agent’s bias are exactly optimal if the ex ante distribution of states (principal ideal points) is iid normal. Section 3.2 establishes that this form of delegation rule
gives high payoffs under much more general distributions of states. In particular, they deliver approximately first-best payoffs when there are many independent decisions.

Armstrong et al. (1994) have previously argued that if a multiproduct monopolist has unknown costs, capping a weighted average of prices is likely to improve outcomes relative to predetermined prices or independent caps across products. The above results show that, in some sense, caps on the average are exactly the right solution for problems with quadratic loss constant bias utilities. But utilities will tend to take other functional forms when preferences are derived from a monopolist’s profit maximization problem.

Caps against an agent’s bias can be naturally defined for a broader set of utility functions, though, including those derived from a monopolist’s problem. Let the principal’s payoff from action $a_i$ on decision $i$ be an arbitrary function of the underlying state of the world. Let the agent share the principal’s payoffs, plus an additional payoff term $G_i(a_i)$ that depends only on the action taken. Relative to the principal, the agent is biased towards actions with higher values of $G_i$. In this setting a cap against the agent’s bias corresponds to a restriction of the form $\sum_i G_i(a_i) \leq K$. For these more general utility functions, Section 4 shows that caps against the agent’s bias extend the robustness results of the constant bias setting. Again, payoffs are approximately first best when there are many independent decisions. In this sense, caps provide the right incentives for agents to make tradeoffs across decisions.

These utilities and caps cover the constant bias preferences above, modeling a political administrator or a manager who always wants to overinvest. They also capture monopoly pricing and import tariff regulations. In the monopoly regulation application, the firm setting prices does not internalize the effect of its price increases on consumer surplus. So a cap against the agent’s bias translates into a requirement that the firm chooses prices subject to a minimum level of consumer surplus across all markets. For the tariff cap application, the importing country ignores profits of foreign firms and so a cap sets a minimum foreign profit summed up across goods.

We can also use these preferences to analyze a political delegation problem in which the agent may be “moderate” or “extreme.” In delegation problems over a single decision these preferences have been modeled with quadratic loss linear bias utilities. Melumad and Shibano (1991) and Alonso and Matouschek (2008) show that it may be optimal to force a moderate agent to pick an extreme action outside of a bounded interval, and to force an extreme agent to pick a moderate action inside of an interval. In my setting with multiple

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3 Armstrong and Vickers (2000) solve for optimal regulation mechanisms under the assumption that goods have a binary distribution of underlying costs.
decisions, this intuition extends to capping an extreme agent by forcing her to choose an action inside of an ellipse (or its higher dimensional equivalent). A cap on a moderate agent, on the other hand, requires her to choose an extreme action outside of an ellipse.

Section 5 then considers some straightforward extensions of the model. I show that caps against the bias continue to align incentives when decisions are taken sequentially rather than all at once, and are approximately optimal for a fixed number of potentially correlated decisions when the agent is strongly biased.

The vast majority of the literature solving for optimal delegation sets focuses on single-decision problems in which the agent’s bias is known (see the papers above). Frankel (Forthcoming) considers multiple-decision problems in which the agent’s bias is unknown. In that paper, decisions can be linked because the agent’s unknown bias is taken to be identical across decisions. The current paper bridges a gap, looking at multiple-decision problems in which the agent’s bias is known. This lets me explore how to provide incentives and create tradeoffs for agents whose preferences differ across decisions.

Koessler and Martimort (2012) consider an alternative extension of a delegation model to multiple decisions, or to a single multidimensional decision. The agent has known biases which may differ across two dimensions of actions, but in contrast to the current paper there is a one-dimensional underlying state. This can be thought of as modeling a joint restriction on the price and quality of a single product, rather than a restriction on the prices of two distinct products. Alonso et al. (2011) study a model where multiple decisions are delegated to different agents, under an exogenous budget constraint across decisions.

Multiple decisions have also been studied in contexts where the principal elicits “cheap talk” information from the agent but cannot commit to a decision rule. The ability to commit can only improve the principal’s payoffs; anything that can be achieved in a cheap talk environment can also be achieved under delegation. Chakraborty and Harbaugh (2007) show how the principal can get high payoffs without commitment when states are iid and biases are identical across decisions. Chakraborty and Harbaugh (2010) show how to incentivize the agent to make appropriate tradeoffs across decisions when her biases across decisions differ in a general way, but her preferences are state-independent. Battaglini (2002) looks at a cheap talk environment with both multiple decisions and multiple informed agents (senders). He shows that there may be robust equilibria in which senders with different biases are induced to fully reveal the state.

4 Other recent work on exploring the delegated choice of a single decision includes Dessein (2002), Krishna and Morgan (2008), Armstrong and Vickers (2010), and Nocke and Whinston (2011).
Finally, the corporate finance literature has argued that forms of credit lines—budgets implemented over time—can be components of optimal contracts to constrain agents who want to invest too much of the principal’s money; see for instance DeMarzo and Sannikov (2006), DeMarzo and Fishman (2007), and Malenko (2011). In Malenko (2011) the agent has state-independent preferences and her payoff is linear in the amount of money spent. The author points out that if the agent is known to prefer certain types of projects over others, then the budget should correct for this (as in the current paper) by putting a higher “price” on the preferred investments.

2 The Model

A principal and agent jointly make $N < \infty$ decisions, indexed by $i = 1, \ldots, N$. Each decision has an exogenous underlying state $\theta_i \in \mathbb{R}$, and an action $a_i \in \mathbb{R}$ is to be taken. Principal and agent stage payoffs on decision $i$ depend on the action and the state, and are given by $U_{Pi}(a_i|\theta_i)$ and $U_{Ai}(a_i|\theta_i)$. For each state $\theta_i$, let $a_i^*(\theta_i)$ denote some principal-optimal action in $\arg\max_{a_i} U_{Pi}(a_i|\theta_i)$. Lifetime payoffs $V_P$ and $V_A$ are additively separable across decisions:

$$V_P = \sum_i U_{Pi}(a_i|\theta_i)$$
$$V_A = \sum_i U_{Ai}(a_i|\theta_i).$$

At the start of the game, the principal knows the number of decisions, $N$; the utility functions $U_{Pi}$ and $U_{Ai}$; and he has some prior belief about the joint distribution of the states. Only the agent will observe the state realizations. To try to make use of the agent’s information, the principal “delegates” the decision. He lets the agent choose actions, subject to certain constraints on the actions that she is allowed to take. The game is as follows:

1. The principal chooses a closed delegation set $D \subseteq \mathbb{R}^N$.
2. The agent observes the vector of underlying states $\theta = (\theta_1, \ldots, \theta_N)$.
3. The agent chooses a vector of actions $a = (a_1, \ldots, a_N)$ from the set $D$ to maximize $V_A$.

Because the principal knows the agent’s utility functions $U_{Ai}$, he can predict which actions the agent will take under any delegation set $D$ and any state realizations $\theta$. So he can calculate his expected payoff from any proposed delegation set $D$, taking expectation over $\theta$ with respect to his prior. Denote this payoff as $\tilde{V}_P(D) = \mathbb{E} [\sum_i U_{Pi}(a_i|\theta_i) \mid D]$.

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In this paper I will show two main classes of results: optimal delegation sets for a fixed delegation problem, and approximate first-best payoffs in a sequence of delegation problems.

A delegation set is optimal for a delegation problem if it maximizes the principal’s expected payoff \( \tilde{V}_P(D) \) over all delegation sets \( D \).

Given some sequence of delegation problems, a sequence of corresponding delegation sets gives the principal approximately first-best payoffs if the expected payoff per decision goes to the first-best level. To be concrete, let us define an independent sequence of delegation problems as follows. Fix an infinite sequence of principal stage utility functions \( (U_{P_i})_i \); an infinite sequence of agent utility functions \( (U_{A_i})_i \); and an infinite sequence of distributions \( (F_i)_i \) over \( \mathbb{R} \). The \( N \)th delegation problem in this independent sequence has \( N \) decisions, with payoff functions \( U_{P_i} \) and \( U_{A_i} \) on decision \( i \leq N \), and state \( \theta_i \) distributed independently according to \( F_i \). Given a corresponding sequence of delegation sets \( (D^{(N)})_N \) in \( \mathbb{R}^N \), the expected payoff to the principal on the \( N \)th delegation problem is \( \tilde{V}_P^{(N)}(D^{(N)}) \). We achieve approximately first-best payoffs if

\[
\mathbb{E}_{\theta_i \sim F_i} \left[ \frac{\sum_{i \leq N} U_{P_i}(a^*_i(\theta_i)|\theta_i) - \tilde{V}_P^{(N)}(D^{(N)})}{N} \right] \rightarrow 0.
\]

We can also be precise about the rate at which payoffs per decision go to first-best as the number of decisions grows. Fix some nonnegative function \( q(N) \) which goes to 0 in \( N \). Say that the payoffs per decision go to first-best at a rate of \( q(N) \) if there exist constants \( 0 < \eta < \bar{\eta} \) such that for all \( N \) large enough, the principal’s payoff loss relative to first-best \( \mathbb{E}_{\theta_i \sim F_i} \left[ \sum_{i \leq N} U_{P_i}(a^*_i(\theta_i)|\theta_i) - \tilde{V}_P^{(N)}(D^{(N)}) \right] \) is contained in \( (\eta q(N), \bar{\eta} q(N)) \). The payoffs per decision go to first-best at a rate of at worst \( q(N) \) if we have the limiting upper bound on payoff loss \( \bar{\eta} q(N) \), but not necessarily a lower bound \( \eta q(N) \).

### 3 Quadratic Loss, Constant Bias Utilities

The players have quadratic loss, constant bias utilities if

\[
U_{P_i}(a_i|\theta_i) = -(a_i - \theta_i)^2, \quad \text{and} \quad U_{A_i}(a_i|\theta_i) = -(a_i - \lambda_i - \theta_i)^2.
\]

So the principal wants to choose action \( a_i \) to match the state \( \theta_i \), while the agent prefers \( a_i = \theta_i + \lambda_i \). The players have quadratic losses from taking actions away from their respective
ideal points of $\mathbf{\theta}$ and $\mathbf{\theta} + \mathbf{\lambda}$. I call $\mathbf{\lambda} = (\lambda_1, ..., \lambda_N)$ the bias of the agent. If the principal’s ideal action is taken at each decision, he would receive a first-best payoff normalized to 0.

This is a natural model of the delegation of a number of political decisions, from an executive to an administrator or from a legislative body to a committee. A lower action $a_i$ represents a more liberal policy on decision $i$, and a higher action represents a more conservative policy. The executive (principal) does not know what policies are best; the administrator (agent) is better informed about the executive’s preferred policy $\theta_i$ on decision $i$. But the players’ policy preferences disagree. The administrator may be much more liberal than the executive on health care ($\lambda$ strongly negative) and slightly more conservative on issues of national defense ($\lambda$ mildly positive), say.

These utilities can also model a school delegating grading decisions to a teacher who is a grade-inflator or grade-deflator. Student $i$’s performance is given by $\theta_i$, which is privately observed by the teacher, and the student will be assigned a grade $a_i$. Here we would presumably think of the teacher having a uniform bias across all students, in which case $\lambda_i$ would be identical across all $i$. Likewise, we can model an empire-building manager choosing investments for a firm. The underlying productivity of the project determines the amount $\theta_i$ that the firm would want to invest, but the manager wants to invest $\theta_i + \lambda_i > \theta_i$. Again, the manager’s bias $\lambda_i$ would be identical across decisions if her preferences towards overinvestment didn’t depend on the identity of the project.

3.1 Benchmark Environment: iid Normal States

3.1.1 Optimality of half-space delegation

Suppose that states are iid normally distributed. The spherical symmetry of the distribution lets us focus on delegation sets that counter the agent’s biases, rather than those which are fine-tuned to follow the shape of the distribution. Without further loss of generality, I let the distribution of each $\theta_i$ be normal with mean 0 and variance 1.

I begin with the analysis of $N = 1$, the delegation of a single decision. For an unbiased agent with $\lambda = 0$, complete freedom would be optimal – that is, $D = \mathbb{R}$. The agent would then choose the principal’s first-best action.

To find the optimal single-decision delegation set when the agent is biased, we can apply prior work such as Kovac and Mylovanov (2009). The principal should cap the agent against her bias: use an action ceiling if the bias is positive, or a floor if the bias is negative. More
precisely, if the agent has positive bias $\lambda > 0$, it follows from the increasing hazard rate of the normal distribution that the optimal delegation set would be $D = \{ \theta \mid a \leq \kappa(\lambda) \}$, where the action ceiling $\kappa(\lambda)$ satisfies $\kappa(\lambda) = \mathbb{E}_{\theta \sim \mathcal{N}(0,1)}[\theta | \theta \geq \kappa(\lambda) - \lambda]$. Interpreting this delegation set, the agent chooses her ideal action of $a = \theta + \lambda$ for low states ($\theta \leq \kappa - \lambda$), and sets the action at the ceiling $\kappa$ for higher states. The ceiling is set so that conditional on the agent’s choosing an action at the cap, the average state (expected principal ideal point) is equal to ceiling. Details are given in the proof of Lemma 1.

Symmetrically, with a negative bias $\lambda < 0$, the optimal delegation set is a floor rather than a ceiling: $D = \{ a \mid a \geq -\kappa(-\lambda) \}$. We can sum up the optimal delegation set for all nonzero biases as

$$D = \{ a \mid \lambda a \leq |\lambda| \cdot \bar{k}(|\lambda|) \}.$$ Following Kovac and Mylovanov (2009), these delegation sets are optimal even in an extended environment in which the agent may be given stochastic delegation sets, i.e., delegation sets containing lotteries over actions. See that paper for details of the stochastic mechanism design problem.

Lemma 1 (Application of Kovac and Mylovanov (2009)). Let $N = 1$, and let $\theta \sim \mathcal{N}(0,1)$.

1. If $\lambda = 0$, then $D^* = \mathbb{R}$ is optimal. If $\lambda \neq 0$, then the unique optimal delegation set is $D^* = \{ a \mid \lambda a \leq |\lambda| \cdot \bar{k}(|\lambda|) \}$, where the function $\bar{k} : \mathbb{R}_{++} \rightarrow \mathbb{R}_{++}$ is uniquely defined by

$$\bar{k}(x) = \mathbb{E}_{\theta}[\theta | \theta \geq \kappa(x) - x].$$

2. $D^*$ remains optimal in the extended class of stochastic delegation mechanisms.

Theorem 1 builds on this prior work to give a benchmark optimality result for the delegation of multiple decisions. Under iid normal states, the optimal multidimensional delegation set is a half-space with boundary normal to the agent’s bias vector; see Figure 2. This new result generalizes the concept of a “cap against the bias” to arbitrarily many decisions, and arbitrary biases on each decision.

Theorem 1. Let the players have quadratic loss, constant bias utilities and let the principal’s prior on states be iid normal: $\theta \sim \mathcal{N}(0, I)$. If $\lambda = 0$, the optimal delegation set is $\mathbb{R}^N$. If

5With a single decision, I omit the subscripts on $a_1, \lambda_1,$ and $\theta_1$.

6Other papers such as Alonso and Matouschek (2008) can be applied to derive the optimal single-decision delegation set, but do not have results about stochastic mechanisms. I rely on the results about stochastic mechanisms when deriving the optimal multiple-decision delegation set in the proof of Theorem 1.
\( \lambda \neq 0 \), then
\[
D^* = \left\{ a \mid \sum \lambda_i a_i \leq K(\lambda) \right\}, 
\text{for } K(\lambda) = |\lambda| \cdot f(|\lambda|)
\]
is an optimal delegation set.

The formal proof of the theorem is given in Appendix A; I sketch the proof below. All other proofs, including that of Lemma 1, are in Appendix B.

**Proof Sketch.** First consider an “augmented game” in which the principal can cheat: he gets to choose the delegation set after learning on which line parallel to the vector \( \lambda \) is the realized state. In other words, he learns the line containing both his and the agent’s ideal points in the \( N \)-dimensional space of actions. Whatever payoffs he can achieve in the augmented game are an upper bound on his true payoffs.

After cheating, the principal faces a one-dimensional problem embedded in \( N \) dimensions: he knows that the state is distributed normally on a line, and the agent’s bias is parallel to this line. So he can use the one-dimensional optimum described above to find the optimal augmented delegation set. In particular, he caps the agent against her bias by picking the optimal half-bounded interval contained in this line, as given by Lemma 1.
To show that this interval is in fact an optimal delegation set in the augmented game, we need to know that there is no way that the principal can ever do better by inducing the agent to take an action vector off of the line. In fact, the payoffs to the principal and agent from taking an action off of the line are mathematically equivalent to those from taking a randomized action on the line. So the result of no randomization in Lemma 1 part 2 yields the desired conclusion.

Finally, I show that the delegation set $D^*$ in the original non-augmented game implements the same allocation as the half-intervals that constitute the augmented optimal delegation sets. This last step is where the assumption of iid normality becomes crucial; the identical allocations follow from the independence as well as spherical symmetry of the underlying distribution. Putting this all together, $D^*$ achieves an upper bound on payoffs in the original game. So it must be optimal.

There is a simple geometric interpretation of these half-space delegation sets. We can think of the space of actions in $\mathbb{R}^N$ as generated by an orthogonal basis consisting of $\lambda$ along with $N-1$ perpendicular vectors. Under the new basis, the agent has a bias of 0 in each of the $N-1$ perpendicular dimensions; her preferences coincide with the principal’s. So the agent is given complete freedom along these dimensions of common interest. But on the dimension of disagreement parallel to $\lambda$, the principal caps the agent’s choices to prevent her from taking actions that are too extreme in the direction of the bias.

In this delegation set, we can think of the cap on action $j$ as depending on all of the other actions taken: $\lambda_j a_j \leq K - \sum_{i \neq j} \lambda_i a_i$. A naive principal might cap each decision separately rather than linking constraints in this manner. However, the joint constraint improves on naive caps by further aligning the agent’s incentives. Consider a problem where the bias $\lambda_j$ is positive; the agent prefers action $a_j$ to be higher than what the principal wants. A hard cap on $a_j$ is a coarse way of getting the agent to take lower actions. The principal would like a finer tool. With transfer payments, for instance, the principal could give increasing payments to the agent for lowering her action. We do not have transfer payments in this problem, but the existence other decisions can simulate transfers. Under the optimal delegation set $\sum_i \lambda_i a_i \leq K$, the principal rewards the agent for low $a_j$ by giving her additional freedom on other decisions. The principal punishes her for high $a_j$ by further

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7 As for the converse, independence plus spherical symmetry imply that a distribution must be iid normal by the so-called Herschel-Maxwell derivation (Jaynes, 2003, Chapter 7). So my argument is specific to the iid normal distribution.

8 Following Kovac and Mylovanov (2009), I show in the proof that the delegation set $D^*$ is optimal even in an extended problem in which the agent may choose lotteries over actions rather than just actions.
constraining her on other decisions. See Figure 3.

Figure 3: $D^*$ gives the agent more freedom on $a_2$ if she moves $a_1$ against the direction of her bias. This linking of constraints improves payoffs relative to $D^{ind}$, a delegation set with independent constraints on the $a_1$ and $a_2$.

Here $D^{ind}$ is the best delegation set with independent constraints across decisions. To give context to these payoff values, the principal’s first-best outcome (from taking $a = \theta$) gives $V_P = 0$ while taking action 0 (an uninformed principal’s ex ante choice) gives $E[V_P] = -2$, minus the sum of the variances. With many decisions, $\tilde{V}_P(D^*)$ is bounded below at $-1$ while $\tilde{V}_P(D^{ind})$ may be unboundedly negative.

We now have an economic interpretation of the cap to go along with the geometric one. The principal gives the agent $K$ units of artificial resources (“delegation dollars”), and tells the agent to spend up to this amount on buying a bundle of actions. The price of each unit of action $a_i$ is set to be equal to the agent’s bias along that dimension, $\lambda_i$. So if the administrator has a strong liberal bias on health care, then she gets a lot of credit – in the form of flexibility on other decisions – for making health care policy a little more conservative. To make health care decisions more liberal, she must likewise give up a lot of flexibility on other decisions. If the administrator has a small conservative bias on national defense, then she gets a little bit of credit for making defense more liberal. And if she shares the executive’s
preferences on education, then she is free to choose any education policy without affecting the constraints on her other decisions.

If biases are the same on all decisions – as they may be for an empire-building investment manager, or a grade-inflating teacher – then the policy simply caps the unweighted sum or average of decisions. So the manager is given a budget of dollars to spend across all investments, and the teacher is given a GPA ceiling for her class. Indeed, a GPA ceiling of exactly this form is used at places such as the business school of the University of Chicago.

### 3.1.2 Comparative Statics and Payoffs

For nonzero biases, the theorem states that the optimal delegation set is a half-space. The half-space always contains the origin, the uninformed principal’s *ex ante* preferred action. The bounding hyperplane of this half-space, $\sum_i \lambda_i a_i = K$, is normal to the bias vector $\lambda$ and is a distance of $k = \frac{K}{|\lambda|} > 0$ from the origin. The further is the boundary from the origin, the larger is the set of actions that the agent is allowed to take. We can derive the following comparative statics on the distance $k$ of the cap to the origin.

**Proposition 1.** The distance $k(|\lambda|)$ of the origin to the boundary of the delegation set $D^*$:

1. strictly decreases in $|\lambda|$,
2. approaches 0 as $|\lambda| \to \infty$,
3. approaches $\infty$ as $|\lambda| \to 0^+$.

So the stronger is the agent’s bias, the less freedom she is given. An unbiased agent is given complete freedom, and an agent with a very strong bias is only allowed to take actions on a half-space which just barely contains the origin.

With a single decision, an unbiased agent with complete freedom would give the principal his first-best payoff of 0. As the agent’s bias grew and the cap moved in to the origin, the payoff would move negative, but it would be bounded below. Even for an infinitely large bias, the agent would almost always take actions at the ceiling of $a = 0$, the average state; this would give the principal a payoff of minus the variance of the state, or $-1$.

With multiple decisions, the principal’s payoff is exactly as if the agent were given an optimal cap on a single decision with bias of $|\lambda|$. This comes from the spherical symmetry of the normal distribution; payoffs and distributions are unchanged if we rotate the problem so that the bias is along a single dimension $i$, in which case an optimal delegation set just caps the agent’s choices on $a_i$. So the principal would get the first-best payoff of 0 on $N-1$
unbiased decision, and the payoff of an optimal cap on the one biased decision. As we add decisions with arbitrary biases, then, the principal’s total expected payoff approaches a constant value of at worst $-1$, the minimum payoff on a single decision. His average payoff per decision goes to the first-best level of 0 at a rate of $\frac{1}{N}$.

**Proposition 2.** Fix an independent sequence of delegation problems in which the players have quadratic loss, constant bias utilities; the agent’s sequence of biases $(\lambda_i)_i$ is not identically equal to 0; and states are drawn from the standard normal distribution. Then the corresponding sequence of optimal delegation sets $D^*$ gives approximately first-best payoffs, with payoffs per decision going to first-best at a rate of $\frac{1}{N}$.

If the principal used a naive delegation rule with independent caps for each decision, then payoffs per decision would not approach the first-best level of 0 as we added decisions. With an identical nonzero bias on each decision, for instance, the payoff per decision would be negative and constant in $N$.

### 3.2 Half-space caps under general distributions

The optimality results above required an assumption that states were iid normal. However, the geometric and economic intuition for half-space delegation sets as caps against the agent’s bias came from the preferences, not the details of the distribution. In this section I argue that these delegation sets are still a good rule of thumb for the principal under much broader distributional assumptions, even though the rule may not be fully optimal. In particular, I will show that these delegation sets achieve approximately first-best payoffs when there are many independent decisions.

Let us begin with some notation. Fixing an agent bias $\lambda$, let $D^H(K) \equiv \{a | \sum_i \lambda_i a_i = K\}$ be a hyperplane delegation set normal to $\lambda$. Let $D^{HS}(K) \equiv \{a | \sum_i \lambda_i a_i \leq K\}$ be the half-space capped by this hyperplane. These sets align incentives between the principal and agent in the following manner:

**Lemma 2.** Suppose that players have quadratic loss, constant bias utilities. Conditional on any realized states $\theta$,

1. The agent plays in delegation set $D^H(K)$ as if she shared the principal’s utility function.
2. The principal’s payoff from the agent’s optimal choice in $D^{HS}(K)$ is at least a high as his payoff from the agent’s optimal choice in $D^H(K)$. 

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In the language of Frankel (Forthcoming), part 1 of the lemma states that the hyperspace delegation sets $D^H(K)$ induce *aligned delegation*: given the constraints of the mechanism, the agent plays exactly as the principal would want her to. Part 2 says that this aligned play under the hyperplane $D^H(K)$ gives a lower bound for the principal’s payoff from the half-space $D^{HS}(K)$. Any time a positively biased agent wants to take an action below the cap, the principal must want the lower action even more strongly.

Frankel (Forthcoming) considers an environment where biases are identical across decisions, but the bias itself is unknown. In that case, a hyperplane delegation set corresponds to fixing the sum or average of actions at some predetermined level. The manager is required to spend all of her investment budget, or a teacher is required to hit an exact GPA level. No matter what is the magnitude of the agent’s biases, this strict budget incentivizes the agent to take appropriate actions in the sense of aligned delegation. We see here that we do better by allowing the agent to go under budget. Of course, to know which direction is “under” the budget level we must know the direction of the bias – positive or negative.

In the analysis below, I will focus on hyperplane and half-space delegation sets defined by one particular budget level. Setting $K = \sum_i \lambda_i E[\theta_i]$, the hyperplane (or, for half-spaces, the bounding hyperplane) cuts through the mean state. Call these the mean hyperplane $D^{MH}$ and the mean half-space $D^{MHS}$:

$$D^{MH} \equiv \{a : \sum_i \lambda_i a_i = \sum_i \lambda_i E[\theta_i]\}$$

$$D^{MHS} \equiv \{a : \sum_i \lambda_i a_i \leq \sum_i \lambda_i E[\theta_i]\}.$$

While $D^{MHS}$ is not necessarily the principal’s favorite half-space delegation set (as seen in the iid normal case, where $K > 0$ was optimal), the hyperplane $D^{MH}$ is in fact preferred to any other hyperplane $D^H(K)$. And it is easy to calculate the principal’s payoff from $D^{MH}$, for any joint distribution of states. This gives a lower bound on payoffs from $D^{MHS}$.

**Lemma 3.** Suppose that players have quadratic loss, constant bias utilities. The principal’s payoff from choosing delegation set $D^{MH}$ is $-\text{Var}\left(\sum_i \frac{\lambda_i}{|\lambda|} \theta_i\right)$. In the case of independent states, this corresponds to $-\sum_i \frac{\lambda^2_i}{|\lambda|^2} \text{Var}(\theta_i)$; for iid states, $-\text{Var}(\theta_1)$.

For comparison, if the principal were required to choose actions without input from the

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The alignment of incentives by hyperplane delegation sets is related to Chakraborty and Harbaugh (2010), which shows that $N - 1$ dimensions of private information (e.g., a hyperplane) can be revealed by agents with state-independent preferences, corresponding to a particular form of very large bias.
principal, he would choose \( \mathbf{a} = \mathbb{E}[\mathbf{\theta}] \) and would get a payoff of \(-\sum_i \text{Var}(\theta_i)\).

The formula in Lemma 3 shows that if we were to normalize all biases to be positive, then a positive correlation across states would reduce payoffs in \( D^{MH} \). As we approached perfect correlation, the principal would no longer benefit from linking the decisions with \( D^{MH} \) instead of treating them separately.\(^{10}\)

We can now apply Lemma 3 to find conditions under which the payoff per decision goes to 0 as we add decisions. When states are independent, the principal’s payoff per decision from the mean hyperplane is at worst \(-\frac{\max_i [\text{Var}(\theta_i)]}{N}\). So the payoff per decision goes to 0 as we add decisions if states are independent and have uniformly bounded variance. Indeed, the payoff per decision goes to 0 at a rate of at worst \( \frac{1}{N} \). And the half-space \( D^{MHS} \) weakly improves on these payoffs, by Lemma 2.

**Proposition 3.** Fix an independent sequence of decision problems with quadratic loss, constant bias utilities, and state distributions \((F_i)_i\) with uniformly bounded variance. The respective delegation sets \( D^{MHS} \) give approximately first-best payoffs, with payoffs per decision going to first-best at a rate of at worst \( \frac{1}{N} \).

The proposition implies that the mean half-space guarantees a rate of convergence to first-best of \( \frac{1}{N} \) whenever states are iid from some distribution with finite variance. From Proposition 2, we know that the rate of \( \frac{1}{N} \) is in fact optimal for one particular iid distribution — the normal. So the \( \frac{1}{N} \) convergence rate is tight; in the language of Satterthwaite and Williams (2002), the half-space delegation sets \( D^{MHS} \) are worst-case asymptotic optimal.\(^{11}\)

I conjecture that the rate of \( \frac{1}{N} \) is optimal in a much larger family of distributions, but have no formal results on this. Converging at a faster rate would mean that the absolute payoff (as opposed to payoff per decision) would increase to the first-best level of 0 as we added decisions.

\(^{10}\) Koessler and Martimort (2012) shows how the principal may benefit from linking perfectly correlated states through other mechanisms than these hyperplanes, when the agent has different biases across the states. In the notation of the current paper, they derive optimal delegation mechanisms for the case of \( N = 2 \) where \( \theta_1 = \theta_2 \sim U[0, 1] \).

\(^{11}\) The double auctions studied by Satterthwaite and Williams (2002) are particularly appealing for their application because they are “belief-free” for the designer, requiring no information about underlying distributions. The delegation sets \( D^{MHS} \) which I consider require the principal to input the agent’s bias and also the mean of the distribution of states.
4 General Bias Utilities

Now consider more general utility functions, where the agent may no longer have a constant bias. The players have *general bias utilities* if the principal has arbitrary stage utility functions $U_{P_i}(a_i|\theta_i)$, and the agent’s utilities are of the form $U_{A_i}(a_i|\theta_i) = U_{P_i}(a_i|\theta_i) + G_i(a_i) + H_i(\theta_i)$. This functional form implies that the agent’s preferences are partially aligned with those of the principal. When choosing actions, she maximizes the principal’s payoffs plus an additional bias term $G_i$ that depends only on actions. That is, the agent’s action payoffs vary with the state in the same way as the principal’s do, but she has an additional preference for taking actions with large $G_i$ values. The nuisance payoff term $H_i$ is convenient to include for applications, but does not affect preferences. I assume that the functions $U_{P_i}$ and $G_i$ are continuous.

Principal: $V_P = \sum_i U_{P_i}(a_i|\theta_i)$

Agent: $V_A = \sum_i U_{A_i}(a_i|\theta_i) = \sum_i \left( U_{P_i}(a_i|\theta_i) + G_i(a_i) + H_i(\theta_i) \right)$

The agent pushes for action vectors with higher $\sum_i G_i(a_i)$. So I say that a *cap against the agent’s biases* corresponds to a delegation set $D(K)$ that caps the sum $\sum_i G_i(a_i)$ at a budget level $K$:

$$D(K) = \left\{ a \mid \sum_i G_i(a_i) \leq K \right\}.$$  

**Example 1** (Quadratic Loss, Constant Biases). Let the players have quadratic loss, constant bias utilities. We can write utilities in the following manner:

Principal: $V_P = \sum_i -\left( a_i - \theta_i \right)^2$

Agent: $V_A = \sum_i -\left( a_i - \lambda_i - \theta_i \right)^2$

$$= \sum_i \left( -\left( a_i - \theta_i \right)^2 + 2\lambda_i a_i + \left( -2\lambda_i \theta_i - \lambda_i^2 \right) \right).$$

A cap against the bias is a ceiling on the sum $\sum_i G_i(a_i) = \sum_i 2\lambda_i a_i$. Modulo the factor of 2, these are exactly the half-space delegation sets $D^{HS}(K)$ discussed in Section 3. As we have
seen, half-space caps against the agent’s bias are exactly optimal under iid normal states and give approximately first-best payoffs when there are many independent decisions.

**Example 2** (Asymmetric significances). Fix some general bias stage utility functions: $U_{Pi}$ for the principal, and $U_{Ai} = U_{Pi} + G_i + H_i$ for the agent. But now suppose that the decisions are asymmetrically significant. The principal and agent maximize $V_P = \sum_i \alpha_i U_{Pi}(a_i|\theta_i)$ and $V_A = \sum_i \alpha_i U_{Ai}(a_i|\theta_i)$, with $\alpha_i > 0$. The $\alpha_i$ parameters indicate the significance of a decision: the number of people affected by a policy, the size of a market, etc. A straightforward transformation to $\tilde{U}_{Pi} = \alpha_i U_{Pi}$ and $\tilde{G}_i = \alpha_i G_i$ shows that the new payoffs are also of the general bias form. A cap corresponds to a ceiling on $\sum_i \alpha_i G_i(a_i)$. For the quadratic loss, constant bias utilities, for instance, a cap is now of the form $\sum_i \alpha_i \lambda_i a_i \leq K$: the price of increasing an action is proportional to both the bias and the significance of the decision.

For these general bias utilities, Section 4.1 extends the approximate optimality results of Section 3.2, while Section 4.2 discusses additional applications and examples. In a very general sense, these caps tell us how to put together decisions to incentivize the agent to act in the principal’s interests. Under the interpretation of a cap against the bias as a budget from which the agent buys actions, this analysis tells us how to get prices right. Note that I do not have additional optimality results for this setting. By taking into account the details of the state distribution, one might be able to improve upon these caps.\(^{12}\)

### 4.1 Payoff Results

This section establishes that caps against the bias give approximately first-best payoffs when there are many independent decisions. To avoid technical complications, I will make a number of boundedness assumptions on this environment. These assumptions could be relaxed in various ways, but making the assumptions simplifies the analysis and allows me to keep utility functions in a very general form.

**Assumption A** (Boundedness). In an independent sequence of decision problems with general biases, define $\mu_i$ as the expectation of $G_i(a^*_i(\theta_i))$ and $\sigma_i$ as the standard deviation of $G_i(a^*_i(\theta_i))$. The following conditions hold:

\(^{12}\)For any number of decisions $N$ and any fixed preferences across decisions, caps against the agent’s bias are completely characterized by the budget level $K$. These caps allow for only one dimension of flexibility on the delegation set across all possible joint distributions of the $N$ underlying states. So it is perhaps unsurprising that outside of a benchmark constant bias iid normal setting, caps will not tend to be exactly optimal.
1. For all $i$, $\theta_i$ has compact support $\Theta_i \subseteq \mathbb{R}$.

2. There exist uniform utility bounds $\underline{U} < U$ such that for all $i$ and all $\theta_i, \theta'_i \in \Theta_i$, it holds that $\underline{U} \leq U_{P_i}(a^*_i(\theta'_i)|\theta_i) \leq U$.

3. There exist uniform bias bounds $G < \overline{G}$ such that for all $i$ and for all $\theta_i \in \Theta_i$, we have $G \leq G_i(a^*_i(\theta_i)) \leq \overline{G}$. Define $G_i = \min_{\theta \in \Theta} G_i(a^*_i(\theta))$ and $\overline{G}_i = \max_{\theta \in \Theta} G_i(a^*_i(\theta))$.

4. There exists $\sigma > 0$ such that the number of decisions $i \leq N$ with $\sigma_i < \sigma$ is $O(\sqrt{N})$.\footnote{That is, for $N$ large enough, there exists $\eta > 0$ such that the number of decisions $i \leq N$ with $\sigma_i < \sigma$ is less than $\eta \sqrt{N}$. This assumption is stronger than necessary, but guarantees that we can take trade off actions across sufficiently many decisions.}

Under these assumptions, caps against the bias give convergence to first-best payoffs at a rate of at worst approximately $\frac{1}{\sqrt{N}}$.

**Proposition 4.** Consider an independent sequence of decision problems with general bias utilities, satisfying Assumption A. For decision problem $N$, let $K^{(N)} = \sum_{i=1}^{N} \mu_i$. Fix any $\epsilon > 0$. Then the sequence of delegation sets $D(K^{(N)})$ achieves approximately first-best payoffs, with payoffs per decision going to first-best at a rate of at worst $\frac{1}{N^{1/2-\epsilon}}$.

In fact, in this general class of utilities and distributions, caps against the bias sometimes converge to first-best at exactly the rate of $\frac{1}{\sqrt{N}}$\footnote{I thank Jason Hartline for the following example. Let $\theta_i \in \{0, 1\}$, let $U_{P_i} = -|a_i - \theta_i|$ for $a_i \in [0, 1]$. Let the agent have some bias of $G_i(a_i) = \lambda a_i$ for $\lambda > 1$, so that for any decision $i$ she prefers to take the action as high as possible regardless of $\theta_i$. A cap against the bias corresponds to a requirement that $\sum_i a_i \leq K$ (dividing out the constant factor $\lambda$). The agent’s optimal strategy, ignoring a minor integer constraint, is to treat this as a quota in which exactly $K$ actions are chosen at $a_i = 1$. The agent will positively sort actions and states, choosing $a_i = 1$ for as many $\theta_i = 1$ decisions as possible, and if there are any remaining spots will choose $a_i = 1$ for some decisions with $\theta_i = 0$. The principal’s payoff loss relative to first-best is the number of misallocated decisions, which is proportional to $\sqrt{N}$ in expectation – the expected absolute deviation from the mean of a sum of $N$ Bernoulli random variables. So the loss per decision goes as $\frac{1}{\sqrt{N}}$.} So while we know that it is sometimes possible to do much better – e.g., quadratic loss payoffs with constant biases give a $\frac{1}{N}$ convergence rate – the convergence rate I find is essentially the best possible guaranteed rate.

### 4.2 Applications

#### 4.2.1 Monopoly Price Regulation

Suppose, as in models considered in Armstrong et al. (1994) and Armstrong and Vickers (2000), that there is a multiproduct monopolist. A regulator seeks to regulate prices in such a way as to maximize total surplus, but the monopolist has private information on her
marginal costs for each good. Demand curves for each of the products are commonly known and do not interact.

We can put this into the general bias utility formulation, for arbitrary demand curves. Let the marginal cost of product \( i \) be \( \theta_i \), and the chosen price be \( a_i \). Profits in market \( i \) are \( \Pi_i(a_i|\theta_i) \), while consumer surplus is \( CS_i(a_i) \) — consumer surplus depends on the prices but not the costs. Payoffs for the two players are

\[
\text{Principal: } V_P = \sum_i U_{P_i} \left( \frac{CS_i(a_i) + \Pi_i(a_i|\theta_i)}{u_{P_i}} \right)
\]

\[
\text{Agent: } V_A = \sum_i \Pi_i(a_i|\theta_i) = \sum_i \left( \frac{CS_i(a_i) + \Pi_i(a_i|\theta_i)}{u_{P_i}} - CS_i(a_i) \right) G_i
\]

The delegation set \( D(K) \) fixing \( \sum_i G_i(a_i) \leq K \) corresponds to a regulation that the monopolist choose prices subject to a minimum level of consumer surplus:\(^{15}\)

\[
\sum_i CS_i(a_i) \geq -K.
\]

Indeed, this form of policy is suggested in Armstrong, Cowan, and Vickers (1994) as a simple way to do better than any rule which fixes the prices at any predetermined level. My results show that this cap is an effective way to get high payoffs when there are many decisions. The monopolist will trade off prices across markets in a way that aligns her incentives with those of the consumers.

It is useful to interpret \( CS_i'(a_i) \) as the marginal flexibility cost of increasing price \( a_i \). The higher this is, the more the agent must decrease other prices in order to be allowed to raise the price on good \( i \) by a small amount. It is a standard result in microeconomics that the derivative of consumer surplus with respect to price is just the quantity demanded. So we see that the marginal flexibility cost is proportional to the market size. The agent must give up more flexibility to increase the price on larger markets. Moreover, the higher is the price on good \( i \), the lower is the marginal flexibility cost of raising it further (because quantity falls as price increases). Put another way, the agent gets less credit from reducing a high price than reducing a low price. And for goods with inelastic demand, the marginal flexibility cost is relatively flat; for goods with elastic demand, the flexibility cost goes down quickly as the price rises.

\(^{15}\)The same rule obtains if the regulator puts a different weight on consumer surplus than on profit.
4.2.2 Tariff Policy

In Amador and Bagwell (2011), tariff policy is modeled in a delegation framework. An importing country enters into a trade agreement which seeks to maximize ex ante surplus. Once the agreement is entered, the importing country observes a “political shock” that affects how much it values domestic industry profits. Here the agent is an importing government setting tariff levels that induce domestic producer profits of $a_i$ on industry $i$, and corresponding tariff revenue plus consumer surplus of $b_i(a_i)$. (Producer profits are an increasing function of tariff levels, so there is a one-to-one map between profits $a_i$ and implied tariff levels.) The agent has payoffs of $\theta_i a_i + b_i(a_i)$ on decision $i$, where $\theta_i > 0$ represents the realization of the political shock. The principal is a social planner who values importing country welfare (including the political payoffs of domestic profits) plus foreign profits of $v_i(a_i)$ on industry $i$.\(^{16}\) This gives payoffs of

\[
\text{Principal: } V_P = \sum_i \left( \theta_i a_i + b_i(a_i) + v_i(a_i) \right)
\]

\[
\text{Agent: } V_A = \sum_i \left( \theta_i a_i + b_i(a_i) \right) = \sum_i \left( \frac{\theta_i a_i + b_i(a_i) + v_i(a_i) + -v_i(a_i)}{U_{P_i}} \right)
\]

Amador and Bagwell consider the case of a single good, and develop conditions under which the optimal policy is a cap on domestic profits $a$, or equivalently a cap on the maximum tariff.\(^{17}\) I now consider a variety of imported goods with noninteracting production or demand. In this context, a cap against the bias fixing $\sum_i G_i(a_i) \leq K$ corresponds to a trade agreement requiring the importing government to choose tariffs subject to a minimum level of foreign profits:

\[
\sum_i v_i(a_i) \geq -K.
\]

Rather than have distinct caps on each separate good, I argue for a joint tariff cap across all of the goods. In the event of a particularly strong protectionist shock to one industry, the country has the flexibility to place a very high tariff on that product. To compensate, though, it must lower the tariffs on other goods. The floor on foreign profits tells us exactly how much we must trade off increases on one good’s tariffs for decreases on others.

\(^{16}\)I use $\theta$ for their variable $\gamma$, and I use $a$ for their variable $\pi$.

\(^{17}\)They additionally find that these policies are optimal even when the planner has the power to force “money burning” of the importing country in exchange for higher tariffs; money burning is not used.
4.2.3 Quadratic Loss, Linear Bias Utilities

Papers such as Melumad and Shibano (1991) and Alonso and Matouschek (2008) consider agents with linear biases in a quadratic loss framework over a single decision. This can model political delegation in which the administrator is extreme or moderate relative to the executive, in addition to being more conservative or liberal. The principal has ideal point normalized to $a_i = \theta_i$ on decision $i$. The agent has ideal point $a_i = \zeta_i \theta_i + \lambda_i$ for $\zeta_i \in \mathbb{R}_+$, $\lambda_i \in \mathbb{R}$. The $\zeta_i$ represents the agent’s relative sensitivity to state changes. The agent is “extreme” on dimension $i$ if $\zeta_i > 1$, and “moderate” if $\zeta_i < 1$. We can put this into the functional form of general bias utilities in the following way:\(^{18}\)

Principal: $V_P = \sum_i \frac{U_{P_i}}{-(a_i - \theta_i)^2}$

Agent: $V_A = \sum_i \left( \frac{1}{\zeta_i} (a_i - \zeta_i \theta_i - \lambda_i)^2 \right)

= \sum_i \left( \frac{-(a_i - \theta_i)^2}{U_{P_i}} + \left( \frac{a_i^2}{\zeta_i} \left( 1 - \frac{1}{\zeta_i} \right) + \frac{2\lambda_i a_i}{\zeta_i} \right) + \left( \frac{\theta_i^2 - \zeta_i \theta_i^2 - 2\theta_i \lambda_i - \frac{\lambda_i^2}{\zeta_i}}{G_i} \right) \right)$.\(^{18}\)

This embeds the constant bias stage utility for $\zeta_i = 1$. With $\zeta_i < 1$, the agent’s bias pushes her ideal point inwards to some “agreement state” $\hat{\theta}_i = \frac{\lambda_i}{1 - \zeta_i}$. With $\zeta_i < 1$, the bias points outwards from the agreement state.

We know the optimal single-decision delegation sets from Melumad and Shibano (1991) and Alonso and Matouschek (2008), under appropriate assumptions on the state distribution. For a very moderate agent, one who is inwardly biased, we forbid moderate actions and require the agent to take actions from either a low or a high interval. For an extreme agent, we forbid extreme actions and require the agent to take actions from an interval bounded on both ends.

\(^{18}\)The scaling of the agent’s payoffs by the leading coefficient of $\frac{1}{\zeta_i}$ – an irrelevant transformation in a single-decision problem – allows payoffs which can be put into my general bias functional form. In either case, the principal has the payoffs of an agent with $\lambda_i = 0$ and $\zeta_i = 1$.\(^{22}\)
The corresponding caps that I propose are sets $D(K)$ defined by action vectors with
\[
\sum_i \left( a_i^2 \left( 1 - \frac{1}{\zeta_i} \right) + 2\lambda_i a_i \right) \leq K.
\]
Focusing on decision $i$ while holding all other actions fixed, this corresponds to three possible cases. For $\zeta_i > 1$, the coefficient on $a_i^2$ is positive, so the agent is given both a ceiling and a floor – the extreme biased agent is forbidden from taking extreme actions. For $\zeta_i < 1$, the coefficient on $a_i^2$ is negative, so the agent is allowed to take actions either below some low level or above some high level – the moderate agent is forbidden from taking moderate actions. For $\zeta_i = 1$ we have our old result that an agent with positive bias $\lambda_i > 0$ is given a ceiling, an agent with a negative bias $\lambda_i < 0$ is given a floor, and an agent with no bias $\lambda_i = 0$ is given full freedom. In other words, the sets look like the one-dimensional caps found by Melumad and Shibano (1991) and Alonso and Matouschek (2008).\(^{19}\)

With multiple decisions, a delegation set $D(K)$ is a region bounded by a quadric (an $N$-dimensional analog of a conic section). For instance, suppose there are two decisions; see Figure 4. If the agent is extreme on both dimensions, the delegation set is the interior of an ellipse (Panel (a)). If the agent is moderate on both decisions, the delegation set is the exterior of an ellipse (Panel (b)). When the agent is mixed, being moderate on one and extreme on the other, the delegation set is a region bounded by a hyperbola (Panels (c) and (d)). This last case may correspond to a connected or a disconnected delegation set. With more decisions, an agent who was moderate on all decisions would take actions in the exterior of an $N$-dimensional ellipsoid, and one who was always extreme would take actions on the interior of such an ellipsoid.

5 Extensions

In this section I consider some other ways in which caps against the agent’s bias may work well as delegation sets. I first consider an extension in which I change the timing of the model, so that decisions are taken sequentially rather than simultaneously. In that case the results from Sections 3.2 and 4.1 regarding high payoffs under many decisions continue to hold for caps against the bias. I then discuss a form of payoff alignment for these delegation

\(^{19}\)Under the distributional assumptions of these two papers, the forbidding of moderate actions is shown to be optimal in one-decision examples with $\zeta < \frac{1}{2}$. The delegation sets I propose involve forbidding moderate actions under broader circumstances, i.e., $\zeta < 1$. 
Figure 4: Possible delegation sets with linear biases, $N = 2$.

(a) Extreme on both decisions:
$\zeta_1 = 2, \zeta_2 = 4$

(b) Moderate on both decisions:
$\zeta_1 = \frac{1}{2}, \zeta_2 = \frac{1}{4}$

(c) Moderate on 1, Extreme on 2; Low $K$:
$\zeta_1 = \frac{1}{2}, \zeta_2 = 2$

(d) Moderate on 1, Extreme on 2; High $K$:
$\zeta_1 = \frac{1}{2}, \zeta_2 = 2$

The arrows point from a state (the principal’s ideal point) to the agent’s ideal point. The shaded areas show a delegation set capping the agent against her bias. Panel (a): If the agent is extreme on both decisions, she chooses from inside an ellipse. Panel (b): If she is moderate on both decisions, she chooses from outside an ellipse. Panels (c) and (d): If her biases are mixed, the boundary of the delegation set is a hyperbola. Increasing the cap $K$ makes the delegation sets larger.
sets that holds for a fixed number of decisions, rather than as the number of decisions grows large. Fixing any joint distribution of states, caps against the bias become approximately optimal as the agent’s bias grows stronger.

5.1 Sequential Decisions

Suppose decisions are taken sequentially rather than all at once; the agent is given a delegation set \( D \subseteq \mathbb{R}^N \) from which to choose actions, but she must choose action \( a_i \) before observing state \( \theta_{i+1} \). There is less information available at the time decisions are made. Even so, I show that in this environment, caps against the bias continue to align incentives well.

5.1.1 Quadratic Loss, Constant Bias Utilities

Lemma 4. Consider a sequential problem with independent states and quadratic loss, constant bias utilities. The principal’s payoff from the mean hyperplane \( D^{MH} \) is

\[
-\sum_{i=1}^{N} \frac{\lambda_i^2 \text{Var}(\theta_i)}{\sum_{j=1}^{N} \lambda_j^2}.
\]

The principal’s payoff from the mean half-space \( D^{MHS} \) is weakly higher.

For iid states, the payoff per decision is the variance of a state times \(-\frac{1}{N} \cdot \sum_{i=1}^{N} \frac{\lambda_i^2}{\sum_{j=1}^{N} \lambda_j^2}\). For independent states with bounded variance, the payoff per decision is at worst the maximum variance times the same expression. There do exist sequences of biases for which the expression does not go to 0. For instance, suppose \(|\lambda_i| = 2^{-\frac{i}{2}}\). Then each term \(\frac{\lambda_i^2}{\sum_{j=1}^{N} \lambda_j^2}\) would be greater than or equal to \(\frac{1}{2}\), and so in an iid problem with \(N\) decisions the payoff per decision would be \(-\frac{1}{N} \cdot \sum_{i=1}^{N} \frac{\lambda_i^2}{\sum_{j=1}^{N} \lambda_j^2} < -\frac{1}{2}\), bounded away from 0.

However, the payoff per decision will tend to go to 0 for independent states under less pathological sequences of biases. For intuition, if all biases were of equal magnitude \(L > 0\), then the payoff per decision from \( D^{MH} \) would simplify to \(-\frac{L^2}{N} \sum_{j=1}^{N} \frac{1}{j}\). This would go to 0 at a rate of \(\frac{\log N}{N}\). It is straightforward to show that if (all but finitely many) biases are bounded in magnitude between two positive values, then payoffs per decision continue go to 0 at a rate of \(\frac{\log N}{N}\):

Proposition 5. Consider an independent sequence of sequential decision problems, in which players have quadratic loss, constant bias utilities; all but finitely many biases \(\lambda_i\) are bounded

\[20\text{In the simultaneous setting, delegation sets were without loss of generality as contracting mechanisms, supposing that the principal only had the power to restrict the agent’s choices. In a sequential setting, the principal might be able to do better if he can observe signals of past states and use this information to adjust the agent’s future action sets. For instance, the principal might limit the agent’s freedom on decision } i+1 \text{ if his utility realization on decision } i \text{ was low.}\]
in magnitude between two positive values; and the distribution of states at each decision has bounded variance. Then the respective delegation sets $D_{MHS}$ give approximately first-best payoffs, with payoffs per decision going to 0 at a rate of at worst $\frac{\log N}{N}$.

5.1.2 General Bias Utilities

For general bias utilities, we can maintain the worst-case rate of convergence to first-best payoffs of approximately $\sqrt{N}$ in the sequential problem, the same as for simultaneous decisions.

**Proposition 6.** Consider an independent sequence of sequential decision problems with general bias utilities, satisfying Assumption $A$. For decision problem $N$, let $K^{(N)} = \sum_{i=1}^{N} \mu_i$. Fix any $\epsilon > 0$. Then the sequence of delegation sets $D(K^{(N)})$ achieves approximately first-best payoffs, with payoffs per decision going to first-best at a rate of at worst $\frac{1}{N^{1/2-\epsilon}}$.

5.2 Strong Biases

Another sense in which caps against the agent’s bias align incentives well is that they are approximately optimal when the agent has very strong biases. In a quadratic loss, constant bias setting with iid normal states, Proposition 1 tells us that the optimal delegation set approaches the mean half-space $D_{MHS}$ when the magnitude of the bias is large. In fact, for an arbitrary state distributions, the mean half-space becomes approximately optimal with large biases. This is because the strongly biased agent tries to take an action vector as far as possible in the direction of the bias. Conditional on being any given distance in the direction of the bias, though, the agent’s incentives are completely aligned with the principal. So a half-space normal to the bias, capping how far the agent may push the action vector in the direction of the bias, is approximately optimal. I present one formalization of this approximate optimality result in Appendix C.

The logic follows that in Frankel (Forthcoming), which shows that if an agent has identical biases on each decision, but the bias magnitude is unknown and may be extreme, then hyperplane delegation sets of the form $\sum_i a_i = K$ are max-min optimal. The insight of the current paper is to show that we can use a hyperplane $\sum_i \lambda_i a_i = K$ to correct for asymmetric biases, and that half-spaces are at least as good as hyperplanes (Lemma 2 part 2).

For general bias utilities, likewise, we could fix some principal payoffs and some joint distribution of $N$ states while scaling up each of the $G_i$ bias functions. As these bias functions grew large, the agent would approximately try to maximize $\sum_i G_i(a_i)$. And caps on this
sum give the agent maximal freedom on this dimension of alignment. So, again, caps against the bias would become approximately optimal under appropriate regularity conditions.

6 Conclusion

Developing general methods to solve for optimal multiple-decision delegation sets – under general joint distributions of states, general payoffs (perhaps nonseparable across decisions), and when decisions may be sequential instead of simultaneous – is an open topic for future research. This paper provides some basic intuition on what we might expect the optimal sets may look like: caps against a multidimensional bias, with suitable adjustments to follow the distribution of underlying states.\(^{21}\)

These caps have a simple and intuitive economic interpretation. Rather than treat the decisions independently, the principal gives the agent a single budget out of which she “purchases” all of her actions. To induce the agent to make the proper trade-offs across decisions, the “price” of increasing actions depends on the agent’s biases. For the constant bias setting, for instance, the price is proportional to an agent’s bias on that decision. In some sense, the existence of multiple decisions acts as a surrogate form of transfer payments: we reward and punish the agent by giving her more or less flexibility on other decisions.

References


\(^{21}\)In a discussion of the difficulties in solving for optimal multidimensional delegation sets, Armstrong (1995) writes: In order to gain tractable results it may be that ad hoc families of sets such as rectangles or circles would need to be considered... Moreover, in a multi-dimensional setting it will often be precisely the shape of the choice set that is of interest. For instance, consider the problem of finding the optimal form of price cap regulation for a multiproduct firm... In this case, what is of interest is the shape of the allowed set of prices (e.g. to what extent does it resemble such commonly used mechanisms as ‘average revenue’ price cap regulation) as much as anything else.


A Proof of Theorem 1

If \( \lambda = 0 \), then the agent shares the principal’s utility function and it is optimal for the principal to give the agent no constraints: the principal achieves a first-best payoff by choosing \( D = \mathbb{R}^N \). For the rest of this proof I suppose that \( \lambda \neq 0 \).

I prove the theorem by first considering an augmented game in which the principal can “cheat” and learn some information about the states before choosing \( D \). Conditional on this extra information, the principal does at least as well as in the original problem. So the optimal payoff in the augmented game gives an upper bound on what the principal can achieve in the original game. Then I show that the delegation set \( D^* \) in the original game exactly implements the optimal outcomes of the augmented game. Because this set achieves a theoretical upper bound on payoffs, it must be optimal.

In the augmented game, before the principal chooses a delegation set \( D \), he learns some information about the state. In particular, he observes the projection \( P_{\theta} \) of \( \theta \) onto the hyperplane defined by \( \{ x \in \mathbb{R}^N | \sum_i \lambda_i x_i = 0 \} \):

\[
P_{\theta} = \theta - \frac{\sum_i \lambda_i \theta_i}{\sum_i \lambda_i^2} \lambda.
\]

So in the augmented game, we add a period 0 to the original description:

0. The principal observes \( P_{\theta} \).

1. The principal chooses a delegation set \( D \subseteq \mathbb{R}^N \).

2. The agent observes the underlying states, \( \theta \).

3. The agent chooses a vector of actions \( a \) from \( D \) to maximize her payoff \( V_A \).

Learning the projection \( P_{\theta} \) is equivalent to learning on which line parallel to the agent’s bias \( \lambda \) is the state \( \theta \). Denote the line parallel to \( \lambda \) intersecting \( P_{\theta} \) by \( M_{\theta} \), and parametrize points on the line by the distance \( k \in \mathbb{R} \) from \( P_{\theta} \) (where \( \lambda \) is taken to be pointing in the positive direction):

\[
M_{\theta}(k) = P_{\theta} + k \frac{\lambda}{|\lambda|}.
\]

The proof that the augmented optimum implements the outcome from \( D^* \) will come from putting together three observations, each one represented graphically in Figure 5.

First, after observing \( P_{\theta} \), the principal now believes that the state is normally distributed on the line \( M_{\theta} \):

**Observation 1.** Conditional on observing \( P_{\theta} \), the principal’s posterior belief on \( \theta \) is that \( \theta = M_{\theta}(k) \) for \( k \) distributed normally with mean 0 and variance 1.

This is seen most easily by recalling that by independence, conditional on any known \( \theta_{-i} \), the coordinate \( \theta_i \) has a standard normal distribution. By the spherical symmetry of the
In the augmented game, before the principal chooses a delegation set he observes $P_\theta$, the projection of $\theta$ onto the hyperplane defined by $\sum_i a_i \lambda_i = 0$. That is, he learns the line $M_\theta$ parallel to $\lambda$ on which $\theta$ lies. The principal’s posterior is that the distance $k$ from $P_\theta$ to $\theta$ has a standard normal distribution.

Conditional on the information that $\theta$ lies on $M_\theta$, the thick black line $D^{aug}$ is an optimal delegation set. $D^{aug}$ is the subset of $M_\theta$ for which the distance $k$ from the hyperplane $\sum_i a_i \lambda_i = 0$ is in the range $(-\infty, K]$.

Each $D^{aug}$ set is a translation of any other in a direction normal to $\lambda$. Taking the union of $D^{aug}$ sets over all $M_\theta$ lines gives a halfspace $D^* = \{\sum_i a_i \lambda_i \leq K\}$. For any $\theta$, the agent chooses identically from $D^*$ in the original game as from $D^{aug}$ in the augmented game. So $D^*$ is an optimal delegation set.
distribution of $\theta$, these lines parallel to a coordinate axis are not special. The principal’s posterior belief on $\theta$ conditional on knowing that it is on any line $M_\theta$ must have a standard normal distribution on that line.

So once the principal has observed $P_\theta$, he now knows that the state has a standard normal distribution on a line (embedded in $N$ dimensions) parallel to the agent’s bias vector. If we knew that the optimal delegation set in $\mathbb{R}^N$ would restrict the agent’s actions to this line, then Lemma 1 part 2 would tell us exactly which subset of the line to allow: a ray, capped against the direction of the agent’s bias, extending $\overline{k}(|\lambda|)$ from the mean. But how do we know that the principal cannot do better by getting the agent to take actions which are sometimes off of the line?

This step is the key technical point in the paper. It follows from Lemma 1 part 3, that the optimal one-dimensional delegation set is not improved by randomization. In payoff terms, randomization is exactly equivalent to taking an action off of the line. In a one dimensional problem, a lottery over actions with mean $a$ and variance $s^2$ gives the principal and agent their payoffs from $a$, minus the quadratic loss term $s^2$. In the multi-dimensional problem where both players’ ideal points are on the line $M_\theta$, taking an action which is a distance $s$ from the line and projects to $a$ on $M_\theta$ gives the principal and agent their payoff from action $a$, minus the quadratic loss term $s^2$. So if random allocations cannot improve payoffs in the one dimensional delegation problem, then points off of the line cannot improve payoffs after embedding the one dimensional delegation problem in multiple dimensions.

**Observation 2.** Given the observation of $P_\theta$, the set $D_{\text{aug}}(P_\theta) \equiv \{ M_\theta(k) | k \in (\mathbb{R^N} \setminus \overline{k}(|\lambda|)) \}$ is an optimal delegation set in the augmented game.

The cutoff value of $\overline{k}$ on this delegation set does not depend on the observation of $P_\theta$. This fact follows from the assumption that $\theta$ was iid normally distributed, and in particular that as we translate a line along any perpendicular dimension, the conditional distributions of $\theta$ on this line remain constant. So if we take the union of all of the augmented delegation rays over all possible observations of $P_\theta$, we have a half-space – indeed, the set $D^*$ as defined before. The boundary is a hyperplane normal to the agent’s bias. If the agent were given this half-space as a delegation set from the start, she would choose her ideal point if feasible, or else would choose the projection of her ideal point onto the bounding hyperplane. In either case, her chosen action would be on the line $M_\theta$. So she would choose the same actions as in the augmented game.\(^{22}\)

**Observation 3.** 1. The set $D^*$ defined in Theorem 1 satisfies

$$D^* = \bigcup_{\theta \in \mathbb{R^N}} D_{\text{aug}}(P_\theta).$$

\(^{22}\)Notice that this proof technique would fail under an alternate distribution of states. The conditional distribution along a line would vary as we translated the line, and therefore the optimal delegation ray cutoffs (corresponding to $\overline{k}$) would vary with $P_\theta$. So if we took the union of augmented delegation rays, the set would be curved rather than a hyperplane. That would mean that the agent’s ideal point projected onto the closest point of the boundary would not necessarily be in the same line $M_\theta$ as the state. Therefore that the union delegation set would implement different outcomes than the augmented delegation sets.
2. For any \( \theta \), the agent’s optimal choice of \( a \) from \( D^* \) in the original game is equal to her optimal choice of \( a \) from \( D^{aug}(P_\theta) \) in the augmented game.

This completes the proof of Theorem 1. Giving the agent a delegation set \( D^* \) with \( K \) set to \( k \cdot |\lambda| \) implements the same outcomes as giving the optimal delegation sets in the augmented game. And in expectation the principal must do weakly better in the augmented game, in which she has strictly more information than in the original game. So no delegation set in the original game can give a higher expected payoff than does \( D^* \). From this argument, we see that the delegation set \( D^* \) implements the optimal outcomes not just of any deterministic mechanism, but of any stochastic one as well. \( \blacksquare \)

B Additional Proofs

Proof of Lemma 1. An agent with \( \lambda = 0 \) who is given delegation set \( D^* = \mathbb{R} \) will always take first-best actions, so this delegation set is clearly optimal. So consider an agent with \( \lambda > 0 \); the problem with \( \lambda_1 < 0 \) is symmetric. In this case I seek to show the optimality of the delegation set \( D^* = \{a | a \leq k\} \).

In order to apply the characterization from Kovac and Mylovanov (2009), I first define \( g(\theta) \equiv 1 - \Phi(\theta - \lambda) - \lambda \phi(\theta - \lambda) \) for \( \Phi \) and \( \phi \) the cdf and pdf of a standard normal. This function satisfies the following technical conditions:

Claim 1. 1. The function \( g(\theta) \) is strictly decreasing when \( g(\theta) \in [0, 1] \).

2. \( g(\theta) < 1 \), for all \( \theta \).

3. There exists a unique value \( \beta_1 \) such that \( g(\beta_1) = 0 \), with \( g(\theta) > 0 \) for \( \theta < \beta_1 \) and \( g(\theta) < 0 \) for \( \theta > \beta_1 \). Furthermore, there exists a unique \( \beta_0 < \beta_1 \) such that \( \int_{\beta_0}^{\beta_1} g(\theta) d\theta = -\int_{\beta_1}^{\infty} g(\theta) d\theta \).

Proofs are below. The first item is the regularity condition labeled Assumption 1 in Kovac and Mylovanov (2009).\(^{23}\) It follows from the fact that the normal distribution has an increasing hazard rate. Given this condition, Proposition 1 of Kovac and Mylovanov (2009) implies that the optimal delegation set is a closed interval from a lower action \( \alpha_0 \) (possibly negative infinity, given the unbounded action space) to an upper action \( \beta_0 \) (possibly positive infinity, given that states are drawn from a compact support, but this is not necessary for the result. We could approximate the normal distribution through a sequence of increasing hazard rate distributions with compact support; index these distributions by \( n \). For any fixed delegation set \( D \), the sequence of payoffs \( \bar{V}_P^n(D) \) would approach the payoff under the normal distribution; this is because the normal has finite variance, so the contribution to (quadratic loss) payoffs from states with \( |\theta| > l \) for any action \( a \in \mathbb{R} \) goes to 0 for \( l \to \infty \). Let \( \bar{V}_P^n(D^*) \) indicate the payoff from the proposed delegation set under the normal distribution, and thus the limit of \( \bar{V}_P^n(D) \).

Now suppose for the sake of contradiction that an alternative delegation set \( D' \) gives a higher normal payoff of \( \bar{V}_P^n(D') = \bar{V}_P^n(D^*) + \epsilon \), with \( \epsilon > 0 \). Then for \( n \) large enough, \( D' \) gives a payoff of \( \bar{V}_P^n(D') > \bar{V}_P^n(D^*) + \epsilon/2 \). But Kovac and Mylovanov’s result formally solves for the sequence of optimal delegation sets for these compact distributions, and shows that these optimal delegation sets approach \( D^* \). So for \( n \) large enough, \( \bar{V}_P^n(D) < \bar{V}_P^n(D^*) + \epsilon/2 \) for all \( D \). Contradiction.
infinity). Following Kovac and Mylovanov (2009) the second item implies that \( \alpha_0 = -\infty \), because \( \int_a^b g(\theta) d\theta < b - a \) for any \( a < b \). The third item characterizes the upper limit \( \beta_0 \in \mathbb{R} \). (For \( \lambda < 0 \), we would instead get \( \beta_0 = \infty \) and \( \alpha_0 \in \mathbb{R} \).)

Therefore the delegation set \( D = \{ a | a \leq \beta_0 \} \) is optimal, and optimal among stochastic delegation sets as well. I now need only show that the value \( \beta_0 \) derived above is identical to \( \bar{k}(\lambda) \) defined in Lemma 1.

**Claim 2.** \( \beta_0 \) is the unique solution to \( \beta_0 = \mathbb{E}_{\theta \sim \mathcal{N}(0, 1)}[\theta | \theta \geq \beta_0 - \lambda] \).

This claim completes the proof.

**Proof of Claim 1.**

1. As noted in footnote 10 of Kovac and Mylovanov (2009), it suffices to show that the normal distribution has an increasing hazard rate. This can be confirmed by recalling that the hazard rate of the standard normal distribution at \( x \) is exactly the conditional expectation of a normal distribution truncated below at \( x \): \( \mathbb{E}_{\theta \sim \mathcal{N}(0, 1)}[\theta | \theta \geq x] = \frac{\phi(x)}{1 - \Phi(x)} \). The left-hand side is increasing in \( x \), thus so too is the right-hand side.

2. Given part 1, that \( g(\theta) \) is decreasing when \( g(\theta) \in [0, 1] \), it suffices to confirm that \( \lim_{\theta \to -\infty} g(\theta) = 1 \). This follows from the definition of \( g(\cdot) \).

3. First I show that there exists \( \beta_1 \) such that \( g(\beta_1) = 0 \); if so, part 1 implies that \( g(\theta) > 0 \) for \( \theta < \beta_1 \) and \( g(\theta) < 0 \) for \( \theta > \beta_1 \). To show this, recalling the identity that \( \phi'(\theta) = -\theta \phi(\theta) \),

\[
g'(\theta) = -\phi(\theta - \lambda) - \lambda \phi'(\theta - \lambda) = -\phi(\theta - \lambda) \cdot (1 - \lambda(\theta - \lambda))
\]

So \( g \) is increasing if \( \theta > \frac{1}{\lambda} + \lambda \). But \( \lim_{\theta \to -\infty} g(\theta) = 0 \), and therefore \( g(\theta) < 0 \) for some large \( \theta \in \mathbb{R} \). Because \( g(\theta) > 0 \) for some small \( \theta \in \mathbb{R} \), by continuity there exists an intersection of \( g(\theta) \) with 0.

To show the result about \( \beta_0 \), we observe that the function \( \int_{X}^{\beta_1} g(\theta) d\theta \) becomes arbitrarily large as \( X \) goes to minus infinity (because \( \lim_{\theta \to -\infty} g(\theta) = 1 \)); it decreases in \( X \) for \( X < \beta_1 \); and it goes to 0 as \( X \) goes to \( \beta_1 \). So there is a unique solution \( \beta_0 \) on the left-hand side, with \( \beta_0 < \beta_1 \), so long as the right-hand side is finite. Integrating by parts:

\[
- \int_{\beta_1}^{\infty} g(\theta) d\theta = - \int_{\beta_1}^{\infty} (1 - \Phi(\theta - \lambda) - \lambda \phi(\theta - \lambda)) d\theta
\]

\[
= \lambda (1 - \Phi(\beta_1 - \lambda)) + (1 - \Phi(\beta_1 - \lambda)) \beta_1 - \int_{\beta_1}^{\infty} \theta \phi(\theta - \lambda) d\theta < \infty
\]

□

**Proof of Claim 2.** \( \beta_0 \) is the unique point satisfying \( \int_{\beta_0}^{\beta_1} g(\theta) d\theta = - \int_{\beta_1}^{\infty} g(\theta) d\theta \), and because \( g(\theta) \) is continuous and has a single crossing with 0 it is also the unique point satisfying
\[ \int_{\beta_0}^{\infty} g(\theta)d\theta = 0. \] A point \( x \) satisfies \( \int_{x}^{\infty} g(\theta)d\theta = 0 \) if and only if it also satisfies \( x = \mathbb{E}_{\theta \sim \mathcal{N}(0,1)}[\theta|\theta \geq x - \lambda] \), the expression defining \( \bar{k} \):

\[
\int_{\beta_0}^{\infty} g(\theta)d\theta = 0 \iff \int_{x-\lambda}^{\infty} (1 - \Phi(\theta) - \lambda_1 \phi(\theta))d\theta = 0 \\
\iff \int_{x-\lambda}^{\infty} (1 - \Phi(\theta(x - \lambda)) - (1 - \Phi(x - \lambda))x - \lambda) + \int_{x-\lambda}^{\infty} \theta \phi(\theta)d\theta = 0 \\
\iff \int_{x-\lambda}^{\infty} \theta \phi(\theta)d\theta = (1 - \Phi(x - \lambda))x \\
\iff \frac{\int_{x-\lambda}^{\infty} \theta \phi(\theta)d\theta}{1 - \Phi(x - \lambda)} = x \] \( \square \)

**Proof of Proposition 1.** Recall from the proof of Lemma 1 that \( \bar{k}(|\lambda|) \) could be derived as the value defined as \( \beta_0 \) in Claim 1. So for a bias magnitude \( \lambda > 0 \) it holds that \( \bar{k}(\lambda) \) satisfies

\[ \int_{\bar{k}(\lambda)-\lambda}^{\infty} (1 - \Phi(\theta) - \lambda \phi(\theta))d\theta = 0. \]

Taking derivative of both sides with respect to \( x \),

\[
\left(1 - \frac{d\bar{k}(\lambda)}{d\lambda}\right) (1 - \Phi(\bar{k}(\lambda) - \lambda) - \lambda \phi(\bar{k}(\lambda) - \lambda) - \int_{\bar{k}(\lambda)-\lambda}^{\infty} \phi(\theta)d\theta = 0 \\
\Rightarrow \frac{d\bar{k}(\lambda)}{d\lambda} = \frac{-\lambda \phi(\bar{k}(\lambda) - \lambda)}{1 - \Phi(\bar{k}(\lambda) - \lambda) - \lambda \phi(\bar{k}(\lambda) - \lambda)}
\]

Furthermore, from the definition of \( \beta_0 \) we know that \( 1 - \Phi(\bar{k}(\lambda) - \lambda) - \lambda \phi(\bar{k}(\lambda) - \lambda) > 0 \) (because, in the language of Claim 1, \( g(\beta_0) > 0 \)). Therefore \( \frac{d\bar{k}(\lambda)}{d\lambda} < 0 \) for \( \lambda > 0 \).

Now switch to the alternate characterization of \( \bar{k}(\lambda) \) as the unique solution to \( \bar{k}(\lambda) = \mathbb{E}_{\theta \sim \mathcal{N}(0,1)}[\theta|\theta \geq \bar{k}(\lambda) - \lambda] \). It is convenient to let \( X = \bar{k}(\lambda) - \lambda \). For any \( X \in \mathbb{R} \), I can find a corresponding bias magnitude \( \lambda = \mathbb{E}_{\theta \sim \mathcal{N}(0,1)}[\theta|\theta \geq X] - X \) for which \( \bar{k}(\lambda) = \mathbb{E}_{\theta \sim \mathcal{N}(0,1)}[\theta|\theta \geq X] \). By monotonicity of \( \bar{k}(\lambda) \), to show that \( \lim_{\lambda \to \infty} \bar{k}(\lambda) = 0 \) it suffices to show that I can find some \( \bar{k}(\lambda) \) arbitrarily close to 0 for \( \lambda \) large. Indeed, letting \( X \) go to minus infinity, \( \mathbb{E}_{\theta \sim \mathcal{N}(0,1)}[\theta|\theta \geq X] \) is approximately 0. This gives us a bias of \( \lambda = \mathbb{E}_{\theta \sim \mathcal{N}(0,1)}[\theta|\theta \geq X] - X \) going to infinity for \( \bar{k}(\lambda) = \mathbb{E}_{\theta \sim \mathcal{N}(0,1)}[\theta|\theta \geq X] \) going to 0.

Likewise, to show that \( \lim_{\lambda \to 0^+} \bar{k}(\lambda) = \infty \) it suffices to show that I can find some \( \bar{k}(\lambda) \) arbitrarily large for some \( \lambda \) close to 0. For \( X \) large and positive, \( \mathbb{E}_{\theta \sim \mathcal{N}(0,1)}[\theta|\theta \geq X] - X \) goes to 0. This gives us a bias of \( \lambda = \mathbb{E}_{\theta \sim \mathcal{N}(0,1)}[\theta|\theta \geq X] - X \) going to 0 for \( \bar{k}(\lambda) = \mathbb{E}_{\theta \sim \mathcal{N}(0,1)}[\theta|\theta \geq X] \) going to infinity. \( \square \)
Proof of Proposition 2. Follows from the argument in the text. One can confirm that the principal’s payoffs decrease in the magnitude of the bias by observing that for any fixed cap, the principal’s payoffs decrease in the magnitude of the bias. So it holds for the optimal caps as well.

Proof of Lemma 2. 1. Given $D^H(K)$, we can expand out the agent’s problem as follows:

$$
\max_a - \sum_i (a_i - \theta_i)^2 \text{ s.t. } \sum_i \lambda_i a_i = K \\
\max_a - \sum_i (a_i - \theta_i)^2 + \sum_i [2(a_i - \theta_i)\lambda_i - \sum_i \lambda_i^2] \text{ s.t. } \sum_i \lambda_i a_i = K \\
\max_a - \sum_i (a_i - \theta_i)^2 + \left[2K - 2\sum_i \theta_i \lambda_i - \sum_i \lambda_i^2\right] \text{ s.t. } \sum_i \lambda_i a_i = K
$$

This has identical argmax as the problem

$$
\max_a - \sum_i (a_i - \theta_i)^2 \text{ s.t. } \sum_i \lambda_i a_i = K
$$

2. Let $a^H$ be an agent-optimal choice from $D^H(K)$, and $a^{HS}$ be an agent-optimal choice from $D^{HS}(K)$. I seek to show that the principal weakly prefers $a^{HS}$ to $a^H$.

The agent must weakly prefer $a^{HS}$ to $a^H$ because it is a solution to a more relaxed optimization. Write the agent’s objective function as

$$
\left[-\sum_i (a_i - \theta_i)^2\right] + \left[2\sum_i a_i \lambda_i\right] + \left[-2\sum_i \theta_i \lambda_i - \sum_i \lambda_i^2\right]
$$

From right to left, the terms in the third bracket are independent of $a$. The second bracket is weakly lower for $a^{HS}$ than for $a^H$ (strictly so, if $a^{HS} \not\in D^H$). Therefore, the first bracket is weakly higher for $a^{HS}$. That is, the principal weakly prefers $a^{HS}$ to $a^H$.

Proof of Lemma 3. Given $\theta$, the agent chooses an action $a$ in the $D^{MH}$ closest to $\theta$ (recall that by Lemma 2 part 1 she maximizes the principal’s payoff). Letting $k$ be the distance from $a$ to $\theta$, we can solve for $k$ (using dot-product notation) as

$$
k = \lambda |\lambda| \cdot \theta - \lambda |\lambda| \cdot E[\theta]
$$

The principal’s payoff is minus the expectation of this quantity squared, which evaluates to $-\text{Var} \left(\sum_i \lambda_i \theta_i\right)$.

Proof of Proposition 5. Follows from Lemma 2 part 2 and Lemma 3.
Proof of Propositions 4 and 6. Define the delegation set \( \tilde{D}(K) = \{ a \mid \sum_i G_i(a_i) = K \} \) as the boundary of \( D(K) \). I will construct a strategy which is feasible in \( \tilde{D}(K) \) and gives payoffs per decision going to first-best levels; this will provide a lower bound on the principal’s payoffs from the agent’s optimal strategy in \( D(K) \).

1. The principal’s expected payoffs from the agent-optimal strategy under delegation set \( \tilde{D}(K) \) (in a simultaneous or sequential problem) is a lower bound on the principal’s payoffs from the optimal strategy given \( D(K) \). This is because the strategy for \( \tilde{D}(K) \) is feasible in \( D(K) \) and therefore gives the agent weakly lower payoffs \( V_A = \sum_i (U_{P_i}(a_i|\theta_i) + G_i(a_i) + H_i(\theta_i)) \) than the strategy for \( D(K) \). But the expected sum \( \sum_i G_i(a_i) \) is higher in the strategy for \( \tilde{D}(K) \) – it is exactly \( K \), whereas \( K \) is an upper bound in the strategy for \( D(K) \) – and so the expectation of \( V_P = \sum_i U_{P_i}(a_i|\theta_i) \) is weakly higher in the strategy for \( D(K) \) than for \( \tilde{D}(K) \).

2. Take an arbitrary agent strategy that is feasible in \( \tilde{D}(K) \). The principal’s payoffs from this strategy give a lower bound on the principal’s payoffs from the agent’s optimal strategy in \( \tilde{D}(K) \), because conditional on the restriction that \( \sum_i G_i(a_i) = K \) the principal and agent have fully aligned preferences over actions – the set \( \tilde{D}(K) \) satisfies the property of “aligned delegation.” The optimal agent choice is optimal for the principal as well.

3. I will now construct a strategy which can be played for delegation set \( \tilde{D}(K) \) in a sequential or a simultaneous problem – the action on decision \( i \) depends only on information available from decisions 1 through \( i \). Say an action \( a_i \) at decision \( i \) in problem \( N \) is sequential-feasible if \( G_i(a_i) \in \left[ K - \sum_{j=1}^{i-1} G_j(a_j) - \sum_{j=i+1}^{N} G_j, K - \sum_{j=1}^{i-1} G_j(a_j) - \sum_{j=i+1}^{N} G_j \right] \). The strategy is as follows: At decision \( i \), play \( a_i^*(\theta_i) \) if this action is sequential-feasible, and otherwise play an arbitrary sequential-feasible action.

I seek to show that under delegation sets with budgets \( K^{(N)} \), this strategy yields non-first-best actions for a number of decisions on the order of \( N^\frac{1}{2}+\epsilon \). If this holds, then by the uniform bound on utilities the absolute expected payoff loss is bounded above by an expression on the order of \( N^\frac{1}{2}+\epsilon \cdot (\bar{U} - \underline{U}) \). Per-decision, this goes to 0 in \( N \) at a rate of \( \frac{1}{N^{\frac{1}{2}-\epsilon}} \).

Plugging in for \( K \), the strategy yields first-best decisions on decision \( i \) in the \( N^{th} \) problem if

\[
\sum_{j=1}^{i} (G_j(a_j^*(\theta_j)) - \mu_j) \leq \left[ \sum_{j=i+1}^{N} (\mu_j - \bar{G}_j), \sum_{j=i+1}^{N} (\mu_j - \bar{G}_j) \right].
\]

It suffices to show that the probability that this holds goes to 1 as we increase \( N \) and consider \( i \) at \( i^{(N)} \approx N - N^{\frac{1}{2}+\epsilon} \) for some fixed \( \epsilon > 0 \).

- Show probability of \( \sum_{j=1}^{i^{(N)}} (G_j(a_j^*(\theta_j)) - \mu_j) > \sum_{j=i^{(N)}+1}^{N} (\mu_j - \bar{G}_j) \) goes to 1:

Divide both sides by the standard deviation of the left-hand side, \( \sqrt{\sum_{j=1}^{i^{(N)}} \sigma_j^2} \). We
are looking for the probability that
\[
\frac{\sum_{j=1}^{(N)} (G_j(\theta_j) - \mu_j)}{\sqrt{\sum_{j=1}^{(N)} \sigma_j^2}} > -\frac{\sum_{j=i(N)+1}^{N} (\bar{G}_j - \mu_j)}{\sqrt{\sum_{j=1}^{(N)} \sigma_j^2}}.
\]

The left-hand side has mean 0 and standard deviation 1. The numerator on the right-hand side contains approximately \(N^{\frac{1}{2}+\epsilon}\) nonnegative terms, of which at least \(N^{\frac{1}{2}+\epsilon}\) are bounded away from 0 (from Assumption A part 4 – a lower bound on the variance implies a lower bound on \(\bar{G}_j - \mu_j\)). So the numerator increases at a rate of \(N^{\frac{1}{2}+\epsilon}\). The denominator on the right-hand side is positive and increases at a rate of at most proportional to the square root of the number of terms in the sum, \(\sqrt{N - N^{\frac{1}{2}+\epsilon}}\), because each term in the summation is uniformly bounded above (from Assumption Apart 3 – a bounded range of \(\bar{G}_j - G_j\) implies a bounded variance). So the denominator increases at a rate of at most \(N^{\frac{1}{2}}\). For \(N\) large enough the fraction becomes arbitrarily highly positive, and so the right-hand side becomes arbitrarily negative.

By Chebyshev’s inequality, the probability that the left-hand side is greater than the right-hand side goes to 1.

- Show probability of \(\sum_{j=1}^{(N)} (G_j(\theta_j) - \mu_j) < \sum_{j=i(N)+1}^{N} (\mu_j - \bar{G}_j(\theta_j))\) goes to 1:
  Identical argument as above, where the right-hand side now becomes arbitrarily large. Chebyshev again implies the result.

We see from the proof that the assumptions were unnecessarily strict for the simultaneous case, because we restricted ourselves to a sequential-feasible strategy. Even retaining a “greedy” strategy which picked as many first-best actions as possible, we might be able to get more first-best actions if we reordered the decisions, or picked actions in an order which depended on state realizations.

Proof of Lemma 4. Lemma 2 goes through unchanged in the sequential problem. The agent plays in a hyperplane \(D^H(K) = \{a| \sum_i \lambda_i a_i = K\}\) as if she were unbiased (the delegation set satisfies aligned delegation), and the principal’s payoff from \(D^{HS}(K) = \{a| \sum_i \lambda_i a_i \leq K\}\) is weakly higher than that from \(D^H(K)\).

Given a delegation set \(D^H(K)\) and a list of actions \(a_1, \ldots, a_{i-1}\) through period \(i - 1\), let \(K_i = K - \sum_{j=1}^{i-1} a_j\) be the budget remaining at period \(i\): the agent must choose remaining actions \(a_i, \ldots, a_N\) so that \(\sum_{j=i}^{N} \lambda_j a_j = K_i\). I claim that at period \(i\) with remaining budget \(K_i\):

(i) Given \(\theta_i\), the agent chooses
\[
a_i = \theta_i + \frac{\lambda_i K_i - \theta_i \lambda_i^2 - \lambda_i \sum_{j=i+1}^{N} \lambda_j \mathbb{E}[\theta_j]}{\sum_{j=1}^{N} \lambda_j^2}.
\]

(ii) Prior to the realization of \(\theta_i\), the principal’s expected payoff from actions \(i\) through \(N\)
is

\[- \sum_{j=i}^{N} \lambda_j^2 \text{Var}[\theta_j] - \left( \frac{K_i - \sum_{j=i}^{N} \lambda_j E[\theta_j]}{\sum_{j=i}^{N} \lambda_j^2} \right)^2 \]

By the sequential version of Lemma 2, the agent acts to maximize the principal’s payoff. Given that, I prove (i) and (ii) by backwards induction:

Base case – show (i) and (ii) for \( i = N \):

At period \( N \), the agent must choose \( a_i = K_i \lambda_i \) and this gives the principal a payoff of

\[- \mathbb{E} [(a_i - \theta_i)^2] = -\text{Var}[\theta_i] - \left( \frac{K_i}{\lambda_i} - E[\theta_i] \right)^2.\]

Plugging in, we can see that this confirms the inductive hypotheses (i) and (ii).

Inductive case – show (i) and (ii) for \( i \), supposing they hold for periods \( i + 1 \) and beyond:

By inductive hypothesis (ii), the principal’s payoff (which the agent maximizes) for periods \( i \) through \( N \), given \( K_i \) and \( \theta_i \), is

\[- (a_i - \theta_i)^2 - \sum_{j=i+1}^{N} \lambda_j^2 \text{Var}[\theta_j] - \left( \frac{K_i - \lambda_i a_i - \sum_{j=i+1}^{N} \lambda_j E[\theta_j]}{\sum_{j=i+1}^{N} \lambda_j^2} \right)^2 \]

Taking the first order condition of this payoff with respect to \( a_i \) confirms (i) for period \( i \). Plugging this optimal action back into the payoff and taking expectation over \( \theta_i \) confirms (ii).

Plugging in \( K_1 = \sum_{j=1}^{N} \lambda_j E[\theta_j] \) into (ii) at \( i = 1 \) establishes the payoff result for \( D^{MH} \).

We can also confirm that this is the value of \( K \) which maximizes the principal’s payoff. ■

C Strong Biases

**Proposition 7.** Fix a number of states \( N \), a joint distribution of \( \theta \) over \( \mathbb{R}^N \) with finite first and second moments, and an \( N \)-dimensional unit vector \( \kappa \). Consider a sequence of delegation problems with constant bias utilities in which the agent’s bias in problem \( n \) is \( \lambda^{(n)} = c^{(n)} \kappa \), with \( c^{(n)} \to \infty \). For \( n \) large, the mean half-space \( D^{MHS} \) is approximately optimal.

By approximately optimal I mean that for any \( \epsilon > 0 \), there exists \( \bar{n} \) large enough so that if \( n > \bar{n} \) then \( \hat{V}^{(n)}(D^{MHS}) \geq \max_{D} \hat{V}^{(n)}(D) - \epsilon \). Notice that the mean half-space is constant over the sequence of decision problems: \( D^{MHS} = \{ a | \kappa_i a_i \leq \kappa_i E[\theta_i] \} \).

**Proof of Proposition 7.** By Lemma 2, it suffices to show that there is some strategy under the hyperplane delegation set \( D^{MH} = \{ a | \sum_i \kappa_i a_i = \sum_i \kappa_i E[\theta_i] \} \) which achieves payoffs greater than or approximately equal to those from an arbitrary set \( D \). I show this in two steps.

**Step 1:** Show that for any delegation set \( D \), there exists some \( K^D \) such that the hyperplane delegation set \( \{ a | \sum_i \kappa_i a_i = K^D \} \) approximately matches the payoffs from \( D \) as \( c \) gets large.
The agent’s payoff is

\[ -\sum_i (a_i - \theta_i)^2 + 2c \left( \sum_i a_i \kappa_i \right) + \left( -2c \sum_i \theta_i \kappa_i - c^2 \sum_i \kappa_i^2 \right) \]

Fix some \( \delta > 0 \) and some \( K \in \mathbb{R} \). Suppose it is that case that there exists a point \( a \in D \) such that \( \sum_i \kappa_i a_i \geq K \). Then fix \( a^I \in D \) (an “inside” action) such that \( \sum_i a_i^I \kappa_i \geq K \), and take any \( a^O(\theta) \) (an “outside” action, not necessarily in \( D \)) such that \( \sum_i a_i^O \kappa_i \leq K - \delta \). The payoff to the agent from \( a^I \) minus the payoff from \( a^O(\theta) \) is at worst \(-L^2 + 2\delta c\).\(^{24}\) This is positive for \( 2c\delta > L^2 \). So for \( L = \sqrt{2c\delta} \), it is the case that for every state within a radius \( L \) of \( a^I \), the agent chooses an action such that \( \sum_i \kappa_i a_i > K - \delta \).

Now we can consider two cases. Case 1 is that \( \max_{a \in D} \sum_i \kappa_i a_i \) does not exist – no action vector achieves the maximum. Then taking \( c \to \infty \), it holds that the principal’s payoffs from \( D \) go to minus infinity.\(^{25}\) Therefore the hyperplane delegation set \( \{ a | \sum_i \kappa_i a_i = K^D \} \) for any \( K^D \in \mathbb{R} \) improves on \( D \) for \( c \) large enough (where this hyperplane gives a finite payoff which is independent of \( c \) – see Lemma 2 part 1).

Case 2 is that the maximum is achieved, in which we can take \( K^D = \max_{a \in D} \sum_i \kappa_i a_i \) and fix \( a^I \) to be some action such that \( \sum_i \kappa_i a_i = K^D \). Then take \( \delta \) to 0 and \( c \) to \( \infty \) in such a way that \( L = \sqrt{c\delta} \) goes to infinity, while \( L\delta \) goes to 0. So for \( \delta \) arbitrarily small, for \( c \) large enough, an arbitrarily high proportion of states have actions such that \( \sum_i \kappa_i a_i \geq K^D - \delta \). For each such action, there is an action with \( \sum_i \kappa_i a_i = K^D \) within a distance \( \delta \). The action with this sum gives the principal a payoff of at worst \( 4L\delta + \delta^2 \) less than the payoff from the action in \( D \).\(^{26}\) This goes to 0 as \( \delta \) and \( \delta \) go to 0. So the principal’s worst case payoff loss on an arbitrarily high proportion of states goes to 0, and the payoff contribution from other states goes to 0 (by the assumption of finite variance), hence the payoff from the hyperplane delegation set approximately matches that from \( D \).

**Step 2:** Out of all hyperplane delegation sets \( \{ a | \sum_i \kappa_i a_i = K \} \), a value \( K = \sum_i \kappa_i \mathbb{E}[\theta_i] \) is optimal. (The agent’s play is independent of \( c \) – see Lemma 2 part 1).

Given any states, the agent chooses an action equal to the projection of the state onto the relevant hyperplane. This gives the principal an expected payoff equal to minus the distance squared from the state onto the hyperplane. This expected payoff is maximized by choosing a hyperplane which intersects the expected state, i.e., setting \( K = \sum_i \kappa_i \mathbb{E}[\theta_i] \).

\(^{24}\) The first bracket in the payoff term is at worst \(-L^2 \) for \( a^I \), and at best 0 for \( a^O \); the second bracket is \( 2c \) times the difference in \( \sum_j a_i \kappa_i \), which is at worst \( \delta \); and the third bracket cancels out.

\(^{25}\) If almost all states imply actions outside of some bounded region \( B \subseteq \mathbb{R}^N \), for \( B \) growing to take up the whole space, then the principal’s payoff must go to minus infinity because almost all states imply actions very far from the state. And indeed, taking \( K \to \sup_{a \in D} \sum_i \kappa_i a_i \), \( \delta \to 0 \), then \( c \to \infty \), we see that this is the case. All actions for which \( \sum_i \kappa_i a_i \geq K - \delta \) are eventually outside of a bounded region which grows to take up the space as \( c \) grows, and some such action is taken for a set of states with measure approaching one.

\(^{26}\) The worst case is that the action in \( D^H \) is \( \delta \) greater distance than the action in \( D \) from \( \theta \), in which case the payoff difference is \( (|a - \theta| + \delta)^2 - |a - \theta|^2 = 2|a - \theta|\delta + \delta^2 \leq 4L\delta + \delta^2 \).