Monetary Policy & Asset Prices

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The problem

Model: forward-looking difference equation

\[ y_t = \lambda E_t y_{t+1} + x_{1t} + x_{2t} \]

Questions

- How do we identify \( \lambda \)?
- Do we need to observe the shocks \((x_1, x_2)\)?
- Are forecasts and asset prices helpful? Cross-equation restrictions?
- Connection to methods that identify shocks?

Answers

- We need “enough” observables and structure
- We need to observe something, but one shock can be enough
- Forecasts, asset prices, and economic structure all help
- Identifying \( \lambda \) and \( x \) closely related
Example 1: independent shocks

Independent moving average shocks

Vary what we observe

Apply to model with Taylor rule
Example 1: independent shocks

Model

\[ y_t = \lambda E_t y_{t+1} + x_t \]
\[ x_t = \alpha(L) w_t, \quad w_t \sim \text{NID}(0, 1) \]

[\mid \lambda \mid < 1, \alpha \text{ one-sided and square-summable, } w \text{ fundamental for } x]\n
Solution (agents observe everything)

\[ y_t = \beta(L) w_t, \quad \beta_j(\alpha, \lambda) = \alpha_j + \lambda \alpha_{j+1} + \lambda^2 \alpha_{j+2} \ldots \]

What do we need to estimate \( \lambda \)?
Observable shock

\[(x, y)\]

- Joint process but degenerate (same innovations)
- Cross-equation restrictions on parameters \((\beta \text{ depends on } \lambda, \alpha)\)

Observe variable \(y\) (always)

- Estimate \(\beta\)

Observe shock \(x\)

- Estimate \(\alpha\)
- Use mapping \(\beta(\alpha, \lambda)\) to infer \(\lambda\)
- \(\lambda\) overdetermined if \(x\) is \(\text{MA}(q)\) and \(q > 1\)
Hidden shock

$(x, y)$ still joint process, but we don’t observe $x$

Observe variable $y$

- Estimate $\beta$
- But without $x$, we can’t distinguish roles of $\alpha$ and $\lambda$

Do forecasts help?

$$f_t^k = E_t y_{t+k} = \left[ \frac{\beta(L)}{L^k} \right]_+ w_t$$

[Nope, depends only on $\beta$, can’t tell us about $(\lambda, \alpha)$]
Partly observable shock

Model and solution

\[ y_t = \lambda E_t y_{t+1} + x_{1t} + x_{2t} \]
\[ x_{it} = \alpha_i(L) w_{it}, \quad (w_{1t}, w_{2t}) \sim \text{NID}(0, I) \]
\[ \Rightarrow y_t = \beta_1(L) w_{1t} + \beta_2(L) w_{2t} \]

Observe variable \( y \) and shock \( x_1 \) — but not \( x_2 \)

- Estimate \( \alpha_1 \) and \( \beta_1 \)
- Use mapping \( \beta_1(\alpha_1, \lambda) \) to infer \( \lambda \)

Do forecasts help?

- They help us identify the \( w_i \)'s
- Then we can recover \( x_2 \) from the difference equation
Example 1: independent shocks

Model with Taylor rule

Model \((p = \text{inflation}, \ i = \text{nominal interest rate})\)

\[
\begin{align*}
i_t &= r + E_t p_{t+1} + x_{1t} \quad \text{(Euler equation)} \\
i_t &= r + \tau p_t + x_{2t} \quad \text{(Taylor rule)} \\
x_{it} &= \alpha_i(L)w_{it}, \ (w_{1t}, w_{2t}) \sim \text{NID}(0, I)
\end{align*}
\]

Solution

\[
\begin{align*}
p_t &= \beta_1(L)w_{1t} + \beta_2(L)w_{2t} \\
i_t &= r + \left[\beta_1(L)/L\right]_+ w_{1t} + \left[\beta_2(L)/L\right]_+ w_{2t} + x_{1t}
\end{align*}
\]

What do we need to estimate \(\lambda = \tau^{-1}\)?
Taylor rule: observable shocks

\((x_1, x_2, p, i)\)
- Joint process but degenerate (two-dimensional innovation)
- Cross-equation restrictions on parameters

Observe shocks \((x_1, x_2)\)
- Estimate \(\alpha_i\)’s

Observe variables \((p, i)\)
- Estimate \(\beta_i\)’s
- Use mapping \(\beta_i(\alpha_i, \lambda)\) to infer \(\lambda = \tau^{-1}\)
Taylor rule: hidden shock ("Cochrane version")

Shocks: $x_1 = 0$, $x_2$ hidden

Observe variable $p$

- Estimate $\beta_2$ (possible because shock is one dimensional)
- Can’t distinguish roles of $\alpha_2$ and $\lambda$

Does extra variable $i$ help?

- No, contains no information not in $p$

Do forecasts help?

- No, they add no information beyond $p$
Taylor rule: partly observable shock ("Gertler version")

Observe variables \((p, i)\) and shock \(x_1\)
- Estimate \(\alpha_1\) and \(\beta_1\)
- Use mapping \(\beta_1(\alpha_1, \lambda)\) to infer \(\lambda\)

Key insight
- \(i\) gives us an additional observable when \(x_1 \neq 0\)

How might we observe \(x_1\)?
- Observe \(E_t p_{t+1}\) directly — or derive from VAR in \((p, i)\)
- Then back \(x_1\) out of Euler equation
Example 1: summary

Identification may be possible with partial observability

Observability facilitated by forecasts

Independence needs further investigation
Example 2: correlated shocks

Correlated shocks in state-space model

Vary what we observe

Apply to model with Taylor rule
Example 2: correlated shocks

Model

\[ y_t = \lambda E_t y_{t+1} + e^\top x_t \]
\[ x_{t+1} = A x_t + C w_{t+1}, \quad \{w_t\} \sim \text{NID}(0, I) \]

[\|\lambda\| < 1, A \text{ stable \\& “regular”}] 

Solution

\[ y_t = e^\top (I - \lambda A)^{-1} x_t = \beta^\top x_t \]

What do we need to estimate \( \lambda \)?
Observable state and shock

Observe state $x$ and vector $e$ (hence shock $e^T x$)

- Estimate $A$

Observe variable $y$

- Estimate $\beta$, use $A$ to infer $\lambda$

$$
\lambda \beta^T = (\beta - e)^T A^{-1}
$$

[\lambda \text{ typically overdetermined}]

Forecasts redundant

$$
f_t^k = E_t y_{t+k} = \beta^T A^k x_t
$$
Digression on hidden shock & state

What’s hidden, state $x$ or shock $e^\top x$?

One line of thought (Hansen-Sargent)
- Only part of state observed
- → may not be able to back out innovations [??]

We suggest: observe the state, not the shock
- Lots of econometric approaches
- Or: build it from forecasts of $y$

$$f_t = [y_t, f_t^1, \ldots, f_t^{n-1}]^\top$$

$f_t$ spans state $x$ if $A$ is regular and $n = \text{dim}(x)$
Hidden shock

Shock $e^\top x$ not observed ($e$ not known)

Observe variable $y$, state $x$
  - From $y$: estimate $\beta$
  - From $x$: estimate $A$
  - But since we don’t know $e$, can’t infer $\lambda$

Do forecasts help?
  - They span the state, not otherwise helpful
Partly observable shock

Model

\[ y_t = \lambda E_t y_{t+1} + e_1^T x_t + e_2^T x_t \]

\[ \Rightarrow y_t = (e_1 + e_2)^T (I - \lambda A)^{-1} x_t = \beta^T x_t \]

Observe variable \( y \), state \( x \)

- From \( y \): estimate \( \beta \)
- From \( x \): estimate \( A \)

Is it enough to observe shock \( e_1^T x \)?

- Can’t easily disentangle effects of \( e_1 \) and \( e_2 \)
Partly observable shock (continued)

Reminder: solution is

$$\beta^T = (e_1 + e_2)^T (I - \lambda A)^{-1}$$

What we know: $\beta$ ($n$ knowns)

What we don’t know: $e_1, \lambda$ ($n + 1$ unknowns)

Needed: one or more restrictions

- Uncorrelated shocks
- Other information that restricts the shocks
- Tight economic structure
Model with Taylor rule

Model ($p =$ inflation, $i =$ nominal interest rate)

\[
\begin{align*}
i_t &= r + E_t p_{t+1} + e_1^T x_t & \quad \text{(Euler equation)} \\
i_t &= r + \tau p_t + e_2^T x_t & \quad \text{(Taylor rule)} \\
x_{t+1} &= Ax_t + Cw_{t+1}, \quad \{w_t\} \sim \text{NID}(0, I)
\end{align*}
\]

Solution: $p_t = \beta^T x_t$

\[
\beta^T = (e_1 - e_2)^T (\tau I - A)^{-1}
\]

What do we need to estimate $\lambda = \tau^{-1}$? One restriction on $e_2$
Example 2: correlated shocks

Example 2: summary

Forecasts helpful in observing state

Identification then possible with

- Partial observability of shocks
- Restriction(s) on hidden (monetary policy) shock
Example 3: bond pricing models

Representative agent model

Model

\[ i_t = -\log E_t \left[ m_{t+1} \exp(-p_{t+1}) \right] \]  \hspace{1cm} \text{(Euler equation)}

\[ i_t = r + \tau p_t + e_2^\top x_t \]  \hspace{1cm} \text{(Taylor rule)}

\[ \log m_{t+1} = -\delta - \alpha \log g_{t+1} \]  \hspace{1cm} \text{(pricing kernel)}

\[ \log g_{t+1} = g + e_1^\top x_t \]

\[ x_{t+1} = Ax_t + Cw_{t+1}, \quad \{w_t\} \sim \text{NID}(0, I) \]

Solution: \( p_t = \beta^\top x_t \)

\[ \beta^\top = (\alpha e_1 - e_2)^\top (\tau I - A)^{-1} \]

\[ r = \delta + \alpha g - (\alpha e_1 + \beta)^\top CC^\top (\alpha e_1 + \beta)/2 \]
Representative agent model: identification

What do we need to estimate $\lambda = \tau^{-1}$?

Reminder: solution is

$$
\beta^\top = (\alpha e_1 - e_2)^\top (\tau I - A)^{-1}
$$

$$
r = \delta + \alpha g - (\alpha e_1 + \beta)^\top CC^\top (\alpha e_1 + \beta)/2
$$

Identification strategies

- Observe $\log g_t, e_1$
- Long bond yields span state
- Risk aversion $\alpha$: match equity premium
- One restriction on $e_2$
- Additional information from mean bond yields
Summary and conclusion

Identification never a free lunch

We suggest

- Forecasts and bond yields aid observability of state
- Additional restrictions can identify shock and “Taylor rule”
- Representative agent bond pricing models can also help

The biggest challenge is not identification, but fit
Related work (some of it)

Identifying monetary policy shocks
- Christiano-Eichenbaum-Evans, Cochrane, Hansen-Sargent, Leeper-Sims-Zha

Factor models
- Sargent-Sims, Stock-Watson

Macro bond pricing models
- Gallmeyer-Hollifield-Zin, Piazzesi-Schneider, Rudebusch-Swanson, Smith-Taylor ...