Credit Scores and Relationship Lending*

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Abstract

This paper analyzes the impact of credit scores on relationship lending and its implications on loan pricing and credit availability. I consider a competitive market with two types of financiers. Credit-score lenders have access to a credit-score technology that enables them to make estimates of borrowers’ default likelihoods at a negligible per-loan cost. Relationship lenders observe less accurate public credit scores about firms, but can invest costly resources in acquiring soft information about the projects for which funds are being sought, which yields better estimates of borrower’s creditworthiness than credit scores. It is shown that competition from credit-score lenders creates a lemons problem, which reduces the value of relationship lending. This distortion leads to an inefficient underprovision of relationship loans, an increase of interest rates, and a shortage of funding for creditworthy projects. The model predicts that high quality firms get funded through credit-score loans, that relationship lenders charge lower interest rates than credit-score lenders to firms of similar quality, that relationship loans exhibit higher price dispersion, and that credit-score lending leads to a higher rate of nonperforming loans. These predictions are consistent with the empirical evidence.

Keywords: Credit scores, relationship lending, credit-score lending
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1 Introduction

Small and medium enterprises (SMEs) play an essential role in the U.S. economy, accounting for around half of its employment and more than half of its output, and bank lending constitutes the most important source of their financing (Bitler, Robb and Wolken, 2001; Mach and Wolken, 2006). Small business lending has historically been a local activity, with community banks gathering "soft" information to assess firms’ creditworthiness (Petersen and Rajan, 1994; Frame and White, 2004). However, over the past two decades, a growing number of banks have incorporated credit-score models in their small business lending operations (Akhavein, Frame and White, 2005; Berger, Cowan and Frame, 2011). The evolution of credit markets towards an increased use of credit scores has fostered a wave of empirical research that has raised a number of concerns about the soundness of lending practices, and the availability and price of credit to small businesses.

This paper provides a theoretical framework to address these concerns. I consider a competitive market with two types of financiers. Credit-score lenders have access to a credit-score technology that enables them to make estimates of borrowers’ default likelihoods at a negligible per-loan cost. Relationship lenders observe less accurate public credit scores about firms, but can invest costly resources in acquiring soft information about the projects for which funds are being sought, which yields better estimates of borrower’s creditworthiness than credit scores.

1 Soft information comprises aspects that are hard to reduce to a number, such as the ability of a manager, or the way she reacts under pressure (Petersen, 1994). It also includes the bank officer’s own assessments of "prospects garnered from past communications with [firm’s] suppliers, customers, or neighboring businesses." (Berger and Udell, 2006).

2 Credit scoring is a method of assigning a numerical value to the credit risk of a potential borrower, based on objective data about the firm and its owner (Feldman, 1997).


4 One of the main benefits of credit scoring is that it dramatically reduces the processing times and the labor inputs required (Frame, Srinivasan and Woosley, 2001). For instance, the average small business loan processing time at Barnett Bank decreased from three or four weeks to a few hours after credit scoring was implemented (Mester, 1997). Berger, Cowan, and Frame, (2011) write: "The primary motive for these banks is likely reduced underwriting costs. This method may exacerbate informational opacity problems, yield less accurate credit terms, and result in greater future credit losses, but may nevertheless be profitable because of the lower costs".

5 Berger, Cowan and Frame (2011) show that, in 2005, half of the small community banks (under $1 billion in assets) were using some form of credit scoring in their lending to small businesses, and that they had been using it for a number of years. They also find that 86% of those banks only use Consumer Credit Scores (CCS) for the principal owner of the firm, which they acquire from external vendors. The study also shows that community banks do not typically use credit scores for automatic loan approvals, but to complement the information gathered through the interaction with the borrower.

6 As an illustration of the interaction between a potential borrower and a loan officer at a bank branch, Hauswald and Agarwal (2010) write: "The application process typically takes four hours to a day to complete from the initial contact between firm and bank. During the branch visit, the manager–owner or firm
I begin by studying the constrained efficient allocation, as the problem of a Planner that has access to the same information as the lenders in the economy. The Planner is confronted with a simple trade-off. Soft information creates value through a reduction of the expected losses induced by potential nonperforming loans. But acquiring soft information entails a cost. Credit scores complement relationship lending, improving the efficiency of lenders’ decisions in confronting this trade-off. For firms of high quality, as evidenced by their score, the ex-ante likelihood of a nonperforming loan is small. Therefore, the cost exceeds the expected gain of acquiring soft information. Hence, credit-score loans are more efficient than relationship loans for firms with high scores. On the contrary, relationship loans increase the value of lending to firms with intermediate scores. For these firms, the expected losses arising from potentially failing loans exceed the cost of inquiring about the firms’ projects.

In a decentralized market, firms with the highest quality—those for which credit-score loans are more efficient—obtain credit from credit-score lenders. However, among the firms for which relationship loans are more efficient, credit-score lenders exploit their ex-ante informational advantage to poach those with the highest quality from any pool of potential borrowers which are observationally equivalent for the relationship lender. This selection creates a lemons problem, whereby the expected quality of potential relationship borrowers is lowered. Hence, the presence of credit-score lenders reduces the value of acquiring soft information. The distortion induced by credit-score lenders leads to an underprovision of relationship loans, as compared to the social optimum. Moreover, the reduction in the value of relationship lending has two direct implications on pricing and availability of credit. First, the loan rates that relationship lenders charge are higher than those that would be offered to observationally equivalent borrowers in the absence of credit-score lenders. Second, the amount of creditworthy projects that are deprived from credit enlarges in the presence of credit-score lenders.

This paper explains some observed regularities on the pricing and relative performance of each type of loan. First, the model predicts that relationship lenders charge lower interest rates than credit-score lenders to firms of similar quality, a fact that has pointed out in Petersen and Rajan (1994), Berger and Udell (1995), Degryse and van Cayseele (2000) and Berger, Frame and Miller (2005). In this paper, although credit-score lenders can exploit their ex-ante informational advantage to capture the highest quality firms, relationship lenders can profit from their ex-post superior information to offer lower loan rates to the firms that do actually get credit from them.

The model also predicts that, conditional on the quality of firms, relationship loans rates exhibit higher dispersion than those of credit-score loans. This pricing pattern is explained by the fact that credit-score lenders’ credit decisions are exclusively based on firms’ scores, which are highly correlated with their quality. Cerqueiro, Degryse and Ongena (2011) provide empirical support for this finding. They attribute it to the use of "discretion" in the loan rate setting process by loan officers, as opposed to the "rules" associated to an automated pricing model.

representative supplies all the relevant data, submits financial and tax information, provides a list of assets, etc., which the local loan officer transcribes [...] Concurrently, the loan officer conducts an in-depth interview with the applicant and gathers soft information in the sense that it would be hard to verify by a third party. In about 8% of the cases, the branch officer will invite the applicant back to follow up on open questions, review discrepancies in submitted information with credit reports, discuss the prospects of the firm, etc...".
Notwithstanding the fact that relationship lenders charge lower rates to firms with similar quality, the (unconditional) average rate of relationship loans may be higher than that of credit-score loans. In this model, high quality firms obtain credit from credit-score lenders, a fact that has been put forward by Petersen and Rajan (2002). Moreover, they obtain loans at lower rates than do firms of worse quality. Hence, if the proportion of high quality firms is high enough, the average price of credit-score loans will be lower. In line with this argument, Martínez-Peria and Mody’s (2004) study of foreign banks in Latin America suggests that foreign banks issue hands-off loans at lower rates than domestic relationship-oriented loans because they serve a segment of firms of higher quality. This finding was put forward in an earlier study of lending in the US by Calomiris and Carey (1994), who find that the average firm that borrows from foreign banks has a higher rating and obtains loans at a lower rate than the average borrower of a domestic bank.

Competition from credit-score lenders with access to better ex-ante information forces relationship lenders to base all its lending decisions on soft information. Otherwise, they would suffer from a winner’s curse. As a consequence, the rate of nonperforming credit-score loans is higher, because soft information leads to more accurate assessments of borrowers’ creditworthiness. Frame and Miller (2005) and DeYoung, Glennon and Nigro (2008) provide an empirical counterpart for this result.

Although the model is general enough to accommodate different lending modes, it is particularly suited to analyzing loans to small business. One of the main differences between large corporations and small firms is that the latter are generally more informationally opaque. For instance, small business do not typically have professionally audited financial statements or public ratings, which facilitate the access to different forms of borrowing. Also, small businesses differ from consumers in their heterogeneity. Berger and Udell (1996) argue that the technology of small businesses lending is essentially different from consumer lending in that consumers demand relatively generic financial services.

The remainder of the paper is organized as follows. The following section describes the institutional framework and discusses the use of credit scoring in the industry. Section 3 lays out the main assumptions of the model. In section 4, I characterize the constrained efficient allocation of loans as the solution to the problem of a Planner. This allocation serves as a benchmark in the welfare analysis. In section 5, I characterize the decentralized equilibrium of an economy with relationship and credit-score lenders. In the following section, I compare the decentralized and the constrained efficient allocations to show the inefficiency of the market equilibrium. In section 7, I outline several testable predictions of the model and discuss them on the light of the empirical evidence. Section 8 concludes. All the proofs are contained in the appendix.

2 Institutional framework

Although there is a long-standing tradition in the use of credit scores in the assessment of consumers’ creditworthiness, the adoption of Small Business Credit Scoring (SBCS) did not

\footnote{This finding is evidenced by the fact that relationship lenders do not use scores to automatically accept potential borrowers, but rely on soft information in their credit decisions (Berger, Cowan, and Frame, 2011; Uchida, Udell and Yamori, forthcoming).}
take place until the early nineties (Mester, 1997). One of the main reasons for the relative delay in its implementation is the lack of reliable data on loan origination, coupled with concerns about small firms heterogeneity.

The first small business credit scores were implemented by large national banks. The size of these institutions and the large volume of proprietary data from their own transactions allowed them to develop and feed-back consistent models, which were particularly suited for their operations. Only in the mid nineties, Fair Isaac made its Small Business Scoring Service (SBSS) available to mid-sized and small community banks, followed by Experian and Dun&Bradstreet in the last years of the decade. However, several concerns have been raised among practitioners and researchers about the reliability of credit scores from vendors. These models draw their inferences from a potentially very large and heterogeneous pool of firms, which may be too dissimilar to that faced in certain markets. Consequently, they may not be as good predictors of delinquencies as customized models (Berger, Cowan, and Frame, 2011).

Despite their potential lack of accuracy, the main benefit of SBCS is the reduction of the cost of processing loan evaluations. Mester (1997), citing a study by the Banking Business Board, reports that loan approval times averaged more than 12 hours (taking around 2 weeks previously), while loan evaluations using credit scores would take less than an hour, including the loan officers’ time when the decision is not exclusively based on the outcome of the credit score alone.

Early research by Frame, Srinivasan and Woosley (2001) shows that the use of credit scores by large banks has been prevalent since the mid-nineties. However, there is ample heterogeneity in the way SBCS are incorporated in the underwriting process of large banks. A handful of large banks usually conduct solicitation campaigns, trying to attract small business, to which they lend based only on their credit score. For instance, Wells Fargo’s portfolio of small business loans rose by about one-third in 1996 as a result of their national solicitation campaign (Strahan and Weston, 1998). A stylized fact across banks is that the use of SBCS lending is limited to loans below a certain amount.

Roughly half of the small community banks use some form of credit scoring in their lending to small businesses (Berger, Cowan and Frame, 2011). A large fraction of those use Consumer Credit Scores (CCS) for the principal owner of the firm, purchased from external vendors. Community banks use credit scores to complement the information obtain through the interaction with the borrower, but do not typically grant loans solely based on the information provided by the scores.

It has been argued that small banks may have a comparative advantage in the provision of relationship loans. Large institutions may benefit from economies of scale in the processing of hard information, but be relatively bad at managing soft information, because it is difficult to reduce to numbers and communicate in large organizations (Stein, 2002). In addition, soft information may often be proprietary to the loan officer and may not be easily observed, verified, or transmitted to others within the financial institution (Berger and Udell, 2006).

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8Consumer credit score lending has been in place since the fifties, and it is nowadays widely used in credit card lending, auto loans and home equity loans (Berger, Cowan, and Frame, 2011).
9Berger and Frame (2007) report that large banks use SBCS for the issuance and pricing of loans below $250,000. Mester (1997) show that there is some heterogeneity across banks. While some may use SBCS for loans up to $1,000,000, others establish an upper threshold of $50,000.
3 The model

3.1 Firms and projects

Consider an economy populated by a continuum of firms. Each firm has an investment project that requires an outlay of $1. Firms are wealth constrained and therefore need to borrow in order to perform the project. Lenders have access to funds at the universal risk-free interest rate \( r_0 \). There are two types of projects in the economy. A good project pays \( A > 1 + r \) with certainty, whereas bad projects pay \( 0 \). Firms can borrow funds from lenders at a certain gross rate \( i \in [1 + r, A] \). Project outcomes are observable and contracts based on outcomes can be enforced.

Firms differ in their ability to select and implement projects. In particular, each firm is characterized by an intrinsic quality index \( \theta \in [0, 1] \), which can be interpreted as representing its ex-ante likelihood of carrying out a good project. Firms’ quality is regarded by lenders as a random variable \( \sim g \) with density \( g \) and c.d.f. \( G \). In order to have a well-behaved problem, I assume:

**Assumption 1:** The density function \( g \) is continuous and positive everywhere in its support \( \Theta \), and common knowledge among all lenders.

3.2 Credit scores (hard information)

A credit score model \( \bar{x} \) is a random variable with conditional density \( f (\cdot | \theta) \) and c.d.f. \( F (\cdot | \theta) \). A credit score \( \bar{x} \) produces a signal \( x \) about a firm, conditional on the true quality parameter \( \theta \). I make the following two assumptions about this conditional distribution.

**Assumption 2.1:** The conditional densities \( f (\cdot | \theta) \) are continuous everywhere in their support \( X (\theta) \subset \mathbb{R} \), all \( \theta \in \Theta \), and common knowledge among all lenders.

**Assumption 2.2:** The family \( f (\cdot | \theta) \) satisfies the Strict Monotone Likelihood Ratio Property (SMLRP), i.e.:

\[
\text{For all } x'' > x', \quad \frac{f (x'' | \theta)}{f (x' | \theta)} \text{ is strictly increasing in } \theta.
\]

The SMRLP signifies that high signal realizations are relatively more likely the higher the quality of the firm. Hence, this assumption states that signals are informative.

For comparison purposes, it will be useful to re-scale signals, so that they represent the expected value of the underlying quality parameter \( \theta \). Formally, a score \( s \) is defined as:

\[
s = \sigma (x) \equiv E \left[ \hat{\theta} | x \right] = \int_{\Theta} \theta \cdot \hat{g} (\theta | x) \, d\theta,
\]

where \( \hat{g} (\cdot | x) \) denote the posterior density of \( \hat{\theta} \), conditional on \( x \).\(^{10}\) The random variable \( \tilde{s} \) induced by the transformation \( \sigma (\cdot) \) is a sufficient statistic for \( \bar{x} \).

\(^{10}\)The posterior density \( \hat{g} \) of \( \hat{\theta} \), conditional on \( x \), is derived by Bayes’ rule as:

\[
\hat{g} (\theta | x) = \frac{f (x | \theta) \cdot g (\theta)}{\int_{\Theta} f (x | t) \cdot g (t) \, dt}.
\]
3.3 Relationship and credit-score lenders

There is a finite number \( N \geq 2 \) of relationship lenders and a finite number \( M \geq 2 \) of credit-score lenders. A relationship lender can be thought of as a bank with local presence and knowledge of the local conditions, which can interact with potential borrowers. Credit-score lenders are financiers that do not have this capacity. For instance, long-distance lenders or on-line lenders would fall under this category.

All lenders can observe a public noisy credit score \( s \) about the firm’s true quality \( \theta \) at no cost.\(^{11}\) In addition, relationship lenders can invest resources in acquiring soft information about potential borrowers. Soft information incorporates the subjective impressions of the bank officer about the owners’ abilities and the prospects of the project for which funds are sought. In particular, I assume that banks can incur the cost \( k \in (0, 1 + r) \) and observe the type of the project (whether good or bad) for which funds are being sought.\(^{12}\)

Credit-score lenders differ from relationship lenders in two aspects. First, they cannot acquire soft information. Second, they have access to a more accurate credit score about the firm’s quality.\(^{13}\) For simplicity, I assume that they can observe a firm’s quality \( \theta \) without noise at zero marginal cost. Recall that \( \theta \) measures the probability that a project is good. Hence, except for the (zero-measure) cases \( \theta \in \{0, 1\} \), a relationship lender can always invest in obtaining strictly better information than the credit-score lender. In the appendix, I show that all qualitative results are robust to assuming that credit-score lenders observe a noisy signal of the firms’ quality—instead of the true quality—, as long as this signal constitutes a sufficient statistic for the less accurate signal \( s \).\(^{14}\)

3.4 Relationship and credit-score loans

There are two types of loans that can be granted in the economy: relationship loans and credit-score loans. The only distinction I make between these categories of loans is the type of information on which the lender relies at loan origination. I refer to a "relationship loan" as one which originates on costly soft information. A loan that relies on hard information alone is denoted "credit-score loan".

4 Constrained efficiency

In this section, I analyze the constrained efficient allocation of loans as the solution to the problem of a Planner that is only limited by the information constraints in the economy.

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\(^{11}\) The credit score \( s \) might be thought of as available from one of the credit bureaus, which relationship lenders typically use in their credit assessments.

\(^{12}\) This assumption is made for simplicity. All the qualitative results would hold if we assumed that relationship lenders observed an (informative) noisy signal of the project’s type.

\(^{13}\) Berger, Cowan and Frame (2011) show that, while most large banks use Small Business Credit Scores (SBCS), community banks mostly rely on Consumer Credit Scores (CCS) acquired from public vendors, which provide less accurate estimates.

\(^{14}\) Mester (1997) reports that, in the typical development of a credit-score model, around 50 to 60 variables are considered as predictors of delinquencies, but that only 8 to 12 end up in the final score. Hence, we can think of better scores as those actually pinning down more relevant variables to predict creditworthiness, which would yield more efficient estimators in the sense of a reduced variance.
Hence, the Planner observes the quality $\theta$ of each firm, as credit-score lenders do. In addition, she can invest an amount $k$ and get informed about the project’s type, just as relationship lenders. This allocation serves as a benchmark for the welfare analysis.

A credit-score loan granted to a firm with quality $\theta$ involves an opportunity cost of $1 + r$ and an expected return of $\theta \cdot A$. Hence, a credit-score loan issued to a firm with quality $\theta$ generates a surplus of:

$$w^C(\theta) = \theta \cdot A - (1 + r).$$

Acquiring soft information entails a cost $k$. A relationship loan is granted if and only if the project is creditworthy. Hence, the expected profit from a relationship loan initiated with a firm with quality $\theta$ yields a surplus of:

$$w^R(\theta) = -k + \theta \cdot [A - (1 + r)].$$

For each $\theta$, the Planner chooses the allocation $\hat{a}(\theta)$ of loans that maximizes the expected surplus, i.e.:

$$\hat{a}(\theta) \in \text{ arg } \max_{a \in \{C, R, ?\}} \{w^C(\theta), w^R(\theta), 0\},$$

The last term captures the possibility of Planner’s inaction.

The following proposition characterizes the constrained efficient allocation of loans.

**Proposition 1** There exist credit score thresholds $\theta^R \equiv \frac{k}{A(1 + r)}$, $\theta^C \equiv \frac{1 + r}{A}$ and $\theta^C \equiv \frac{1 + r - k}{1 + r}$, and a cost threshold $\overline{k} \equiv \theta^C \cdot [A - (1 + r)]$, such that the constrained-efficient allocation $\hat{a}$ of loans is given by:

If $k < \overline{k}$:

$$\hat{a}(\theta) = \begin{cases} 
\text{No loans} & \text{if } \theta < \theta^R \\
\text{Relationship loans} & \text{if } \theta^R \leq \theta < \theta^C \\
\text{Credit-score loans} & \text{if } \theta^C \leq \theta
\end{cases}$$

If $k \geq \overline{k}$:

$$\hat{a}(\theta) = \begin{cases} 
\text{No loans} & \text{if } \theta < \theta^C \\
\text{Credit-score loans} & \text{if } \theta^C \leq \theta
\end{cases}$$

**Proof.** See appendix. ■

Figure 1 illustrates the constrained-efficient allocation of loans. The horizontal axis displays firms’ quality. The surplus created by different forms of lending is represented by solid (relationship) or dashed (credit-score) lines.

The upper relationship line corresponds to a low cost $k_L < \overline{k}$ of acquiring soft information. The thresholds $\theta^R$ and $\theta^C$ are given by its intersection with the horizontal axis and the credit-score loan surplus line, respectively. The surplus originated by relationship loans is greater than that created by credit-score loans if and only if $\theta \leq \theta^C$. Moreover, for $\theta \leq \theta^R$, the surplus created by relationship loans is negative. Hence, efficiency prescribes that relationship loans be issued to firms with quality levels $\theta \in [\theta^R, \theta^C]$. Firms with quality $\theta > \theta^C$ should get credit-score loans, while firms with quality $\theta < \theta^R$ should be deprived from borrowing.
The lower relationship line corresponds to a high cost \( k_H > \bar{k} \) of acquiring soft information. It is simply a downward parallel shift of the low-cost line, reflecting the diminished surplus due to the higher cost. This line is above the credit-score loan surplus line only in some neighborhood of \( \theta = 0 \), at which a relationship loan surplus is negative. When the cost is high, relationship loans are never optimal: either they are dominated by credit-score loans or by inaction (or by both). The threshold \( \theta^0 \) is given by the intersection of the credit-score loan surplus line with the horizontal axis. Credit-score loans are efficient for borrowers \( \theta > \theta^0 \).

\[
\begin{align*}
w^C(\theta) - w^R(\theta) &= k - (1 - \theta) \cdot (1 + r) .
\end{align*}
\]

At a cost \( k \), relationship loans create value through a reduction of the expected loss \((1 - \theta) \cdot (1 + r)\) induced by nonperforming loans. For firms of high quality \( \theta \), the ex-ante likelihood \( 1 - \theta \) of a nonperforming loan is small. Therefore, the cost \( k \) exceeds the expected gain of issuing a relationship loan. Hence, credit-score loans are more efficient for firms with high quality.

Recall that the surplus created by a relationship loan is given by:

\[
\begin{align*}
w^R(\theta) &= -k + \theta \cdot [A - (1 + r)] .
\end{align*}
\]
For firms of low quality $\theta$, the expected gain $\theta \cdot [A - (1 + r)]$ from any loan is smaller than the cost $k$ of acquiring soft information. Hence, it is optimal not to grant loans to firms of low quality.

Credit scores complement relationship lending in that the score improves the efficiency of lenders’ decision in confronting the trade-off they face when deciding whether to invest resources in acquiring soft information. By reducing the amount of nonperforming loans, relationship loans increase the value of lending for firms in the range $[\theta^R, \theta^C]$. Moreover, relationship loans allow creditworthy projects of firms in the quality range $[\theta^R, \theta^C]$ to be funded. Those projects would not obtain funding in a world without relationship lenders.

5 A decentralized economy

5.1 Preliminaries

In this section, I study the equilibrium provision of loans in a decentralized economy in which relationship and credit-score lenders compete. For the rest of the paper, I consider the cases for which the cost of acquiring soft information is $k < \bar{k}$. As seen above, relationship lending is strictly dominated when the cost exceeds $\bar{k}$. Therefore, there is no role for relationship lending in that case.

The timing is as follows. First, each firm is endowed with a quality level $\theta$, which is privately observed by credit-score lenders. Then, conditional on $\theta$, a score $s$ is drawn from $f(\cdot|\theta)$, which is observed by all lenders. Lenders compete in the third stage by simultaneously offering a loan at gross interest rate $i \in [1 + r, A]$ to firms, or may decide not to make any offer. Firms take loans from the lender quoting the lowest rate. Finally, relationship lenders choose whether to issue a relationship or a credit-score loan to firms which have accepted their offers. Firms with funds implement their projects and repay their loans if and only if these are successful.

A credit-score loan granted to a firm with quality $\theta$ at a rate $i$ involves an opportunity cost of $1 + r$ and an expected return of $\theta \cdot i$. Hence, a credit-score loan issued to a firm with quality $\theta$ generates a profit for the lender given by:

$$\pi^C(i, \theta) = \theta \cdot i - (1 + r).$$

Potentially, both credit-score lenders and relationship lenders can issue credit-score loans. The latter do not observe the firm’s quality $\theta$, but a score $s$. The expected profit of issuing a credit-score loan to a firm with score $s$ at rate $i$ is given by:

$$\Pi^C(i, s; \hat{h}) = \int_\theta \pi^C(i, \theta) \cdot \hat{h}(\theta|s) d\theta,$$

where $\hat{h}(\theta|s)$ is some (posterior) density over quality levels $\theta$, conditional on the observation of the score $s$. The function $\hat{h}(\theta|s)$ is an equilibrium object, as specified below.

On the other hand, the expected profit from a relationship loan initiated with a firm with quality $\theta$ and granted at a rate $i$ is given by:

$$\pi^R(i, \theta) = -k + \theta \cdot [i - (1 + r)].$$
The expected profit of a relationship loan issued to a firm with score \( s \) at a rate \( i \) is therefore given by:

\[
\Pi^R(i, s; \hat{h}) = \int_\Theta \pi^R(i, \theta) \cdot \hat{h}(\theta|s) \, d\theta.
\]

For any given interest rate \( i \), firm’s score \( s \) and conditional density \( \hat{h}(\theta|s) \), we can compare each lending mode and determine which one is the most profitable. The expected profit given by the most profitable lending mode is:

\[
\Pi(i, s; \hat{h}) = \max \left\{ \Pi^R(i, s; \hat{h}), \Pi^C(i, s; \hat{h}) \right\}.
\]

### 5.2 Strategies and equilibrium concept

Before describing the strategies and the equilibrium concept, it is useful to note that, for any given firm with quality \( \theta \), and for any given interest rate \( i \), the payoff difference between a credit-score and a relationship loan

\[
\pi^C(i, \theta) - \pi^R(i, \theta) = k - (1 - \theta) \cdot (1 + r)
\]

is independent of the interest rate \( i \).

Consequently, for any given score \( s \), for any fixed density function \( \hat{h}(\theta|s) \), and for any given interest rate \( i \), it follows that the expected difference in profits from a credit-score and a relationship loan

\[
\Pi^C(i, s; \hat{h}) - \Pi^R(i, s; \hat{h}) = \int_\Theta [k - (1 - \theta) \cdot (1 + r)] \cdot \hat{h}(\theta|s) \, d\theta.
\]

is independent of the interest rate \( i \).

The following remark is an immediate consequence of this property and is useful for simplifying the description of strategies and the definition of an equilibrium:

**Remark 1** For any given score \( s \), and any fixed density function \( \hat{h}(\theta|s) \), whether issuing a credit-score or a relationship loan is more profitable for a lender does not depend on the particular interest rate \( i \) charged.

In equilibrium, the interest rate \( i \) charged on a loan to a firm with a score \( s \) does have an influence on the relative profitability of each mode of lending, for it influences the conditional density \( \hat{h}(\theta|s) \). Moreover, different type of loans may potentially (and will) be priced at different interest rates in equilibrium. However, for a given fixed density \( \hat{h}(\theta|s) \), and any given interest rate \( i \), the relative profitability of each lending mode does not (directly) depend on the interest rate \( i \).

Strategies for credit-score lenders are interest rates \( i^C(s, \theta) \) to offer to a firm with score \( s \) and quality \( \theta \). Relationship lenders’ strategies entail interest rates \( i^R(s) \) to offer to a firm with a credit score \( s \), and a lending mode \( a^R(s) \) –credit-score or relationship– for the last stage of the game (although the choice of lending mode for the relationship lender should
potentially depend on the equilibrium interest rates, it follows from Remark 1 that we can suppress the dependence of $a^R(\cdot)$ on the interest rate. I adopt the convention $i = +\infty$ to denote that no loan is offered.

A symmetric Bayesian equilibrium of this game is a profile

$$\left\{ i^C(s, \theta), (i^R(s), a^R(s)); \hat{h}(\theta|s) \right\}$$

of strategies, and beliefs $\hat{h}(\theta|s)$ for relationship lenders, such that:

(i) Given relationship lenders’ strategies $(i^R(s), a^R(s))$ and other credit-score lenders’ strategies $i^C(s, \theta)$, for each potential borrowers $(s, \theta)$, a credit-score lender strategy $i^C(s, \theta)$ satisfies:

$$\text{Either} \quad i^C(s, \theta) \in \{ \text{s.t.} \ i \leq \min \left\{ i^C(s, \theta), i^R(s) \right\} \},$$

$$\pi^C(i, \theta) \geq 0$$

or

$$i^C(s, \theta) = +\infty.$$  

A credit-score lenders’ equilibrium strategy $i^C(s, \theta)$ is a choice of interest rate that maximizes the expected profit $\pi^C(i, \theta)$ from issuing a credit-score loan to a firm with quality $\theta$ and score $s$, provided that the interest rate does not exceed other offers, and yields nonnegative profits. If such interest rate does not exists in the space $[1 + r, A]$ , then no offer is made $(i = +\infty)$.

(ii) Given credit-score lenders’ strategies $i^C(s, \theta)$, other relationship lenders’ strategies $(i^R(s), a^R(s))$, and beliefs $\hat{h}(\theta|s)$, for each potential borrowers $s$, a relationship lender’s strategy $(i^R(s), a^R(s))$ satisfies:

(ii.1) Given any interest rate $i \in [1 + r, A]$ offered in the previous stage of the game, the lending mode $a^R(s)$ solves:

$$a^R(s) \in \arg \max_{a \in \{C, R\}} \left\{ \Pi^C(i, s; \hat{h}), \Pi^R(i, s; \hat{h}) \right\}.15$$

(ii.1) Given its own lending mode $a^R(s) \in \{C, R\}$ in the final stage of the game, the
interest rate \( i_R(s) \) satisfies:

\[
\arg \max_{i \in [1+r, A]} \Pi \left( i, s; \hat{h} \right) \quad \begin{cases} 
E \text{ither} & i_R(s) \in \text{s.t. } i \leq \min \left\{ i^C(s, \theta), i_R(s) \right\} \\
& \Pi(i, s) \geq 0. 
\end{cases}
\]

\[ \text{or} \quad i_R(s) = +\infty. \]

A relationship lenders’ equilibrium pricing strategy \( i_R(s) \) is a choice of interest rate that maximizes the expected profit \( \Pi \left( i, s; \hat{h} \right) \) from issuing the most profitable loan to a firm with score \( s \), provided that the interest rate does not exceed other offers, and yields nonnegative profits. If such interest rate does not exists in the space \( [1 + r, A] \), then no offer is made \( (i = +\infty) \).

(iii) Beliefs \( \hat{h}(\theta|s) \) are derived from prior distributions and equilibrium strategies by Bayes’ rule, whenever possible.

5.3 Decentralized equilibrium without credit-score lenders

The following proposition characterizes the decentralized equilibrium in the absence of credit-score lenders.

**Proposition 2** Let \( \theta^R \equiv \frac{k}{A-\left(1+r\right)} \), \( \theta^0 \equiv \frac{1+r}{A} \) and \( \theta^C \equiv \frac{1+r-k}{1+r} \), as defined in Proposition 1. Let \( i^R(s) = \frac{k}{s} + (1 + r) \) and \( i^C(s) = \frac{1+r}{s} \). Then, the unique equilibrium allocation and pricing of loans in the absence of credit-score lenders is given by:

\[
a(s) = \begin{cases} 
\text{No loan} & \text{if } s < \theta^R \\
\text{Relationship loans at rate } i^R(s) & \text{if } \theta^R \leq s < \theta^C \\
\text{Credit-score loans at rate } i^C(s) & \text{if } \theta^C \leq s 
\end{cases}
\]

**Proof.** See appendix.

Lenders choose the loan type whose associated break-even interest rate is lower, and offer the break-even rate on the chosen loan type. Otherwise, other lenders could undercut their offers. The higher the surplus generated by a loan, the lower the rate that a lender can charge to a firm and break-even. Consequently, break-even rates are lower for, say, relationship loans, if and only if relationship loans are relatively more efficient than credit-score loans.

Figure 2 illustrates the allocation and pricing of loans in the absence of credit-score lenders. The horizontal axis displays firms’ public scores. The solid line represents the
break-even line for relationship loans, while the dashed line represents the break-even line for credit-score loans. We have seen in Proposition 1 that relationship loans yield a negative surplus for $\theta < \theta^R$. Consequently, a relationship lender should charge an interest rate above $A$ in order to make nonnegative profits to firms whose expected quality is below $\theta^R$. For firms with expected quality in the range $[\theta^R, \theta^C]$, relationship loans generate a higher surplus than credit-score loans. Hence, relationship loans can be priced at lower rates. For firms in the range $[\theta^C, 1]$ this pattern is reversed, reflecting the higher efficiency of credit-score loans.

![Figure 2](image)

### 5.4 Decentralized equilibrium

The following lemma is useful in the characterization of the equilibrium. It states that relationship lenders only issue relationship loans in equilibrium, despite the possibility of granting credit-score loans.

**Lemma 1** Relationship lenders do not grant credit-score loans in equilibrium.

**Proof.** See appendix.

Credit-score lenders’ ex-ante information is more accurate than relationship lenders’. Hence, relationship lenders would suffer from the winner’s curse if they were to issue a credit-score loan. The fact that relationship lenders concentrate their lending activities on relationship loans is a direct consequence of the presence of credit-score lenders. In the absence of credit-score lenders with better ex-ante information, relationship lenders would issue credit-score loans to firms with high quality, as shown in Proposition 2.

Uchida, Udell and Yamori (forthcoming), using survey data on Japanese SMEs, show that banks with local presence tend to concentrate their lending activities on soft information acquired at its branches. Berger, Cowan, and Frame (2011) seem to confirm this finding. In
their study of the use of credit scores by American banks, they find that most community lending originates on soft information.

The following proposition characterizes the equilibrium of this game.

**Proposition 3** Let \( \theta^R \equiv \frac{k}{A-(1+r)} \), \( \theta^\theta \equiv \frac{1+r}{A} \) and \( \theta^C \equiv \frac{1+r-k}{1+r} \), as defined in Proposition 1. Let \( \bar{i}^C(\theta) \equiv \frac{1+r}{\theta} \) and \( \bar{i}^R(s) \equiv \frac{k}{s} + (1+r) \). Then, there exists a unique equilibrium, which satisfies the following characteristics:

(i) There exists a threshold \( s^* > \theta^R \) such that:

a) \( i^R(s) = +\infty \), for all \( s < s^* \).

b) \( i^R(s) \in (\bar{i}^R(s), A) \), for all \( s \in [s^*, 1) \).

c) \( i^R(s) \) is continuous and strictly decreasing, with \( \lim_{s \to 1} i^R(s) = \frac{1+r}{\theta} \).

(ii) There exists a threshold \( \bar{\theta}(s) \equiv \frac{1+r}{\bar{i}^R(s)} \), defined in the range \( s \geq s^* \), such that:

\[
\bar{i}^C(s, \theta) = \begin{cases} 
\bar{i}^C(\theta) & \text{if either} \quad s \geq s^* \text{ and } \theta \geq \bar{\theta}(s) \\
\bar{i}^R(s) & \text{or} \quad s < s^* \text{ and } \theta \geq \theta^\theta \\
+\infty & \text{otherwise}
\end{cases}
\]

**Proof.** See appendix. \( \blacksquare \)

Relationship lenders charge a rate that decreases continuously with the public score \( s \) to firms exceeding a certain public score threshold \( s^* \). This threshold is above \( \theta^R \), which is the lower bound for relationship loans in the absence of credit-score loans, as described in Proposition 2. The rate \( i^R(s) \) exceeds \( \bar{i}^R(s) \), the one that would be charged in the absence of credit-score lenders. For firms with a public score above \( s^* \), credit-score lenders issue loans to those for which they can earn nonnegative profits by charging a rate below \( i^R(s) \). Firms with public scores below \( s^* \) and quality above \( \theta^\theta \) obtain credit-score loans.

In order to grasp some intuition for this result, Figure 3 illustrates the selection of firms with a given score \( s \). The horizontal axis displays firms’ quality. The solid line represents the break-even line for relationship loans, while the dashed line represents the break-even line for credit-score loans. For a given interest rate \( i^R(s) \), credit-score lenders can charge \( \bar{i}^C(\theta) < i^R(s) \) to firms with quality \( \theta > \bar{\theta}(s) \) and make nonnegative profits.
Formally, relationship lenders’ expected profits on a borrower with score $s$ in equilibrium is given by:

$$
\Pi^R \left( i, s; \hat{h} \right) = \int_{\Theta} \pi^R (i, \theta) \cdot \hat{h} (\theta | s) \, d\theta,
$$

where $\hat{h} (\theta | s)$ denotes the (posterior) density of $\tilde{\theta}$, conditional on $s$ and $\theta \leq \tilde{\theta} (s)$. Recall that, in the absence of asymmetric information, relationship lenders’ profit function is given by:

$$
\Pi^R (i, s; \hat{g}) = \int_{\Theta} \pi^R (i, \theta) \cdot \hat{g} (\theta | s) \, d\theta,
$$

where $\hat{g} (\theta | s)$ denotes the (posterior) density of $\tilde{\theta}$, conditional on $s$, as derived from prior distributions from Bayes’ rule. Clearly, for any borrower with score $s \in (0, 1)$, for any interest rate $i \in [1 + r, A]$, we have that:

$$
\Pi^R \left( i, s; \hat{h} \right) < \Pi^R (i, s; \hat{g}),
$$

for the latter includes a strictly better pool of borrowers.

The presence of credit-score lenders with access to more accurate credit scores reduces the value of relationship lending, as reflected by the last inequality. Since $\Pi^R (\cdot, \cdot)$ is increasing in the interest rate $i$, relationship lenders must charge a rate $i^R (s)$ in excess of $\tilde{i}^R (s)$. Moreover, the minimum public score $s^*$ required to obtain relationship loans must also exceed $\tilde{\theta}^R$. 

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Figure 3: Fix $s$, $\theta$ on horizontal-axis (privately observed by Credit-Score Lenders)
6 Inefficiency of the decentralized equilibrium

Recall from Proposition 1 that the constrained efficient allocation of loans is given by:

$$\hat{a}(\theta) = \begin{cases} 
\text{No loans} & \text{if } \theta < \theta^R \\
\text{Relationship loans} & \text{if } \theta^R \leq \theta < \theta^C \\
\text{credit-score loans} & \text{if } \theta^C \leq \theta
\end{cases}$$

Figure 4 illustrates the equilibrium allocation of loans and contrasts it with the efficient allocation. The horizontal axis displays firms’ quality. The solid line represents the break-even line for relationship loans, while the dashed line represents the break-even line for credit-score loans. The efficient allocation, depicted in Figure 1, can also be observed in this diagram. For $\theta < \theta^R$, the surplus created by a relationship loan is negative. In this graph, this fact can be mapped to the break-even rate for a relationship loan to firms $\theta < \theta^R$ exceeding the maximum surplus $A$ that a project can generate. For firms in the range $[\theta^R, \theta^C]$, relationship loans are more efficient, which is reflected in the relationship loan break-even rate laying below the credit-score line. For firms with quality $\theta > \theta^C$, the most efficient form of funding is a credit-score loan.

In equilibrium, firms with quality $\theta > \theta^C$ obtain credit-score loans, which is efficient. However, some firms with quality $\theta \in \left[\theta^0, \theta^C\right]$ get credit-score loans, while efficiency prescribes that they get relationship loans. Also, some firms with quality $\theta \in \left[\theta^R, \theta^0\right]$ are deprived from funding, instead of obtaining relationship loans. Finally, some firms with qualities $\theta \in [0, \theta^R]$ inefficiently get relationship loans.
The following immediate result follows:

**Proposition 4** The decentralized equilibrium is constrained inefficient.

We can identify two sources for this inefficiency. First, the lower accuracy of the credit score that relationship lenders have access to leads to an incorrect ex-ante identification of the firms’ quality. The second originates on the distortion that credit-score lenders induce on the equilibrium allocation. The wedge between $i^R(s)$ and $\underline{i}^R(s)$ creates a niche for credit-score lenders, who can poach firms in $\theta \in [\theta^h, \theta^C]$ from relationship lenders. Moreover, the reduction in the value of relationship lending leads to firms $(s, \theta) \in [\theta^R, s^*] \times [\theta^R, \theta^0]$ to be inefficiently deprived from credit.

Not surprisingly, a way to eliminate this inefficiency would be to endow relationship lenders with the same amount of ex-ante information as credit-score lenders. The market equilibrium would then be efficient. In practice, this can be difficult. Developing a reliable credit score model requires a large amount of available data and a big fix investment. While large institutions can exploit their economies of scale to implement credit score lending, small community banks are unlikely to pursue that path. However, different type of lenders could integrate and exploit the complementarity of their lending technologies. The last wave of mergers and acquisitions experienced by the banking sector may have led to an improved efficiency of loan allocations along these lines.

### 7 Empirical implications and supportive evidence

The predictions of the model can help explain several empirical regularities found in the literature. The adoption of the small business credit scoring technology suggests that the extent to which lenders and borrowers meet must have reduced with time, as improvements in the automated models to assess firms’ creditworthiness would facilitate the issuance of credit-score loans based on hard data. Using data from the National Survey of Small Business Finances (NSSBF), Petersen and Rajan (2002) show that in the last two decades there has been a substantial decline in the share of loans in which lender and borrower meet, and attribute it to advances in credit scoring methods.\(^{16}\) Another implication of the adoption of credit-scoring technologies is a diminished role for geographical distance in transactions, as found in Wolken and Rohde (2002). Moreover, Brevoort and Hannan (2007) find that the distance between relationship lenders and borrowers has not reduced, so that the increased distance is due to a raise in the loans originated by hands-off lenders. Hannan’s (2003) analysis of Community Reinvestment Act (CRA) data shows that there has been a considerable increase of the share of distant lenders’ loans, and attributes a large fraction of this to an increase in credit card borrowing by small businesses.

\(^{16}\)They document that this pattern cannot be explained by mergers of distant banks, reduction on lending standards, or changes in firms characteristics.
7.1 Loan interest rates

The first implication of this model for loan pricing is described in the following statement.

**Corollary 1** For any fixed firms’ quality level, relationship loans are priced at a lower rate than credit-score loans. Moreover, relationship loan rates exhibit higher price dispersion than credit-score loan rates.

**Proof.** See appendix. ■

Clearly, for any given quality level $\theta$, firms who obtain relationship loans must necessarily face cheaper rates than those getting credit-score loans. Otherwise, these borrowers could simply get credit-score loans, for their quality is observed by credit-score lenders. Despite credit-score lenders poaching the firms with a higher quality, the ex-post superior information of relationship lenders allows them to retain some borrowers, who get loans at cheaper rates. The higher pricing dispersion of relationship loans is due to the dispersion of public scores across firms with the same quality.

The fact that relationship loans are associated with lower interest rates has been acknowledged in the empirical literature. An early reference is Petersen and Rajan (1994) who, using data from the NSSBF, find a small but positive effect of relationships in reducing the price of credit to firms. This result is found to be stronger by Berger and Udell (1995), who analyzed the same data, but concentrated on a subset of loans that better capture the extent of relationships. Degryse and van Cayseele (2000), using Belgian data, also point out the same link between relationship loans and their rates. Berger, Frame and Miller (2005) provide direct evidence that small business credit scoring is associated with higher average loan rates for loans under $100000.

Cerqueiro, Degryse and Ongena (2011) identify the pattern that loan rates for seemingly identical borrowers exhibit substantial dispersion. They attribute this dispersion to “discretion” (versus "rules") in the loan pricing process at banks: while credit-score loans rates would be the outcome of a precise mapping from predicted firm quality to prices, relationship loan pricing would incorporate bank officers’ assessments, based in soft information acquired through their interaction. An increased use of credit scoring methods should therefore be reflected in a diminished role of discretion over time, particularly for firms whose characteristics posed a bigger challenge for credit-score lenders in the past. Consistent with this fact, Cerqueiro, Degryse and Ongena (2011) identify that the role of discretion has diminished with time, but only for small and less transparent firms. In addition, no significant temporal changes in discretion is identified when attention is reduced to a subgroup of loans that are mainly relationship-driven, which suggests that the reduction in discretion with time might be driven by the implementation of better credit-score lending technologies, as predicted in this model.

Relationship loans are associated with lower rates, **conditional** on the quality of a firm. However, the "**unconditional**" statement might be reversed in markets with sufficiently high concentration of firms with high quality.
Corollary 2  If the proportion of high quality firms in the market is sufficiently high, the average rate of credit-score loans is lower than the average rate of relationship loans.

Proof. See appendix. ■

Proposition 3 shows that firms with higher quality \((\theta \geq \theta^C)\) are better suited to obtain credit-score loans. In markets with a high concentration of high quality firms, credit-score loans should be more likely than relationship loans, which would be limited to firms with lower quality levels. Consequently, average credit-score loan rates would reflect the higher quality of firms in that portfolio and would hence be lower.

Martinez-Peria and Mody (2004) look into data from Argentina, Chile, Colombia, Mexico and Peru and find that foreign bank entrants charge lower rates than national banks, despite facing similar costs for their funds. We can see this pattern as suggesting that foreign credit-score lenders charge lower rates than domestic relationship-driven lenders because the pool of firms that they target has higher quality.\(^{17}\) This view is reinforced by the fact that domestic banks do not respond to the increased competition posed by foreign banks by reducing their loan rates, suggesting that they concentrate on certain niches of firms.

Berger and Udell (1996) analyze data on 900000 loans from the Federal Reserve’s Survey of Terms of Bank Lending to Business and Call Reports. They find that large banks are predicted to charge about 100 basis points less on loans issues to small business than small community banks.\(^{18}\) Moreover, they infer that this effect is driven by the higher share of credit-score borrowers in the portfolio of large and complex lending institutions and the higher average quality of these borrowers. This result should be taken cautiously though, as this effect may well be distorted by a preferential access to funds by large banks.

7.2 Firms quality and default rates

As argued above, an immediate result follows from observation of Proposition 3:

**Corollary 3** The average quality of firms that get credit-score loans is higher than that of firms that obtain relationship loans.

This is consistent with Berger and Udell (1996), who argue that firms who obtain credit from credit-score lenders have better quality than those being funded by Community banks, a fact that was also pointed out by Petersen and Rajan (2002).

Moreover, Petersen and Rajan (2002) find that the access of firms with lower intrinsic quality to credit-score loans has increased with time, and argue that this pattern is associated with technological advances in the credit scoring industry that lead to more efficient

\(^{17}\)Berger, Klapper and Udell (2001) explore date from Argentina and find that foreign-owned institutions experience difficulties extending relationship loans to opaque small firms and, hence, they tend to concentrate their small-firm lending to enterprises with relatively transparent data available in their financial statements.

\(^{18}\)There is ample evidence that large banks focus on credit-score loans, while small banks tend to specialize on the issuance of relationship loans. For instance, Berger, Miller, Petersen, Rajan and Stein (2005) and Cole, Goldberg and White (2004) provide evidence that large banks lend at a greater distance and interact more impersonally with their borrowers.
appraisal of the creditworthiness of a potential borrower. Comparing the results of Proposition 2 and Proposition 3 can help understand this pattern. In a situation in which there are no informational asymmetries, only the best firms, obtain credit-score loans. However, in the presence of informational asymmetries, some firms of lower quality, namely \( \theta \in [\theta^b, \theta^c] \) also obtain loans from credit-score lenders.

One of the concerns following the expansion of credit-score loans is whether these are granted on more solid grounds than relationship loans. We have shown above that relationship loans are issued on the basis of the soft information gathered about borrowers. Since, relationship lenders’ ex-post information is better, it follows that their loans should perform better than credit-score lenders’. This is an immediate result:\(^{19}\)

**Corollary 4**  *The probability of default of credit-score loans is higher.*

This pattern has been evidenced in recent studies assessing the impact of credit-score lending on the performance of loans. Berger, Frame and Miller (2005) study a 1998 Federal Reserve Bank of Atlanta survey on the use of credit scores and find that banks that rely on small business credit-scoring experience higher default ratios. DeYoung, Glennon and Nigro (2008), on 1984-2001 data from the SBA lending program, also finds that credit scoring is associated with a higher rate of nonperforming loans than relationship lending. These studies seem to confirm that credit score lending is associated with riskier loans.

8 Conclusion

This paper provides a framework to study the effect of credit scores on relationship lending. The revolution in the credit-score industry constitutes a source of potential improvement on the provision of loans to firms. However, it threatens the competitiveness, and even the survival, of traditional forms of relationship lending that add value to the provision of loans in certain firm segments. This paper identifies the inefficiency induced by the relative advantage that advancements in the credit score industry confer to credit-score lenders. Competition from credit-score lenders with access to accurate credit scores reduces the value of relationship lending. As a consequence, there is an inefficient underprovision of relationship loans, relationship lenders charge higher prices on their loans and provide less credit to creditworthy projects.

The paper contributes to an emerging literature on credit-score lending to small firms. The availability of new data sets, credit registers and surveys has been followed by a wave of empirical research that attempts to address the impact of credit-scores on the price and availability of credit for Small and Medium Enterprises. This paper provides a theoretical

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19 This result does not depend on the assumption that relationship lenders perfectly observe the quality of the project that they grant funds for. It is robust to assuming that relationship lenders observe a noisy signal of the project’s type, as long as the signal is more accurate than the credit-score lenders’ score and the cost is sufficiently low. To see why, observe that relationship lenders incur a cost \( c > 0 \) in order to obtain this information. In equilibrium, a relationship lender will only spend these resources if its ex-post information is strictly better than that of the credit-score lender. This higher ex-post information about borrowers that get credit explains the lower default rates of relationship loans.
account for some of the regularities found in this literature. In particular, the model predicts that credit-score lenders attract the firms with the highest quality, that relationship lenders charge lower interest rates to firms of similar quality, that relationship loans exhibit higher price variability, and that credit-score lending is associated with a higher rate of nonperforming loans. These predictions are consistent with the received empirical evidence.

A potential policy implication of this analysis is that lenders should tend to integration in order to eliminate the distortion induced by credit-score lenders. Large banks could benefit from the economies of scale involved in the development of accurate credit-score models. But also, by having local presence, they could benefit from the relative advantage that relationship lenders possess over a certain range of firms.

References


A Omitted Proofs

Proof of Proposition 1.

The functions $w^C(\theta)$ and $w^R(\theta)$ are strictly increasing affine functions. Since, by assumption, $k < 1 + r$, it follows that they intersect at $\theta^C \in (0, 1)$. We also have $w^C(0) < w^R(0)$ and $w^C(1) > w^R(1)$. Hence, relationship loans are more efficient than credit-score loans if and only if $\theta < \theta^C$. Moreover, the surplus generated by a relationship loan is positive if and only if $\theta > \theta^R$.

For assessment costs $k < \bar{k}$ it follows that $\theta^R < \theta^C$. Hence, it is efficient to grant relationship loans for $\theta^R < \theta < \theta^C$. Moreover, $w^C(\theta^C) > 0$. Hence, for it is optimal to grant credit-score loans for $\theta > \theta^C$. Finally, for $\theta < \theta^R$, it is optimal not to grant credit.

On the other hand, for assessment costs $k \geq \bar{k}$, we have that either $w^R(s) < w^C(s)$ or $w^R(s) = 0$ (or both). Hence, relationship loans are dominated by either credit-score loans or by inaction. Hence, relationship loans are inefficient. credit-score loans are optimal if and only if $w^C(\theta) > 0$, that is, for $\theta > \theta^\theta$.

Proof of Lemma 1. Suppose that there exists an equilibrium with interest rates $i^C(s, \theta)$ and $i^R(s)$ for credit-score lenders and relationship lenders, respectively. Fix an arbitrary $s'$ and assume, for the sake of contradiction, that a relationship lender issues a credit-score loan at rate $i^R(s')$. Necessarily, we have that profits for the relationship lender are zero, i.e. $\Pi^C(i^R(s'), s; \hat{h}) = 0$. If profits were positive, it could be undercut by another relationship lender. Moreover, by individual rationality profits cannot be negative. By the same token, we have that credit-score lenders’ profits are zero, i.e. $\pi^C(i^C(s', \theta), \theta)$, for all $\theta$. Now, consider the following strategy for a credit-score lender:

$$i^C(s', \theta) = \begin{cases} i^R(s') & \text{if } \theta \geq s' \\ +\infty & \text{otherwise} \end{cases}.$$ 

Hence, we have that $0 = \Pi^C(i^R(s'), s; \hat{h}) < \pi^C(i^C(s', \theta), \theta)$, where the last inequality follows from $\pi^C(i, \cdot)$ being strictly increasing on $\theta$. Hence, there exists a profitable deviation for the credit-score lender.

Proof of Proposition 2. In the absence of credit-score lenders, beliefs are just given by the posterior distribution of quality $\theta$, given $s$, derived by Bayes’ rule as:

$$\hat{g}(\theta|x) = \frac{f(x|\theta) \cdot g(\theta)}{\int_{\Theta} f(x|t) \cdot g(t) dt}.$$ 

Define $\Pi^C(i, s) \equiv \Pi^C(i, s; \hat{g})$ and $\Pi^R(i, s) \equiv \Pi^R(i, s; \hat{g})$. For a given $i \in [1 + r, A]$, since $\pi^C(i, \theta)$ (resp. $\pi^R(i, \theta)$) is an affine function of $\theta$, and $f(x|\theta)$ are continuous for all $\theta \in \Theta$, it follows that $\Pi^C(i, s)$ (resp. $\Pi^R(i, s)$) is an affine function of $s$. Moreover, for a given $i$, $\pi^C(i, \theta)$ (resp. $\pi^R(i, \theta)$) is strictly increasing in $\theta$. Hence, since $f(x|\theta)$ satisfy the SMLRP, it follows that $\Pi^C(i, s)$ (resp. $\Pi^R(i, s)$) is strictly increasing too.

For any given $i \in [1 + r, A]$, we have that $\Pi^R(i, s) > (=) \Pi^C(i, s)$ if an only if $s < (=) \theta^C$. Hence, relationship loans cannot be issued for firms with scores $s > \theta^C$, as by continuity of $\Pi^C(i, s)$ on $s$, any such relationship loan could be profitably undercut by a credit-score loan. Moreover, for $s < \theta^R$, it follows that $\Pi^R(i, s) < 0$ for any $i \in [1 + r, A]$, the last inequality holding with equality for $s = \theta^R$ and $i = A$. Hence, relationship loans granted to firms with such scores yield negative profits. Let $s \in [\theta^R, \theta^C]$ and define the break-even rate $i^R(s)$ associated with a relationship loan to a firm with score $s$ implicitly as $\Pi^R(i^R(s), s) = 0$. By strict monotonicity of $\Pi^R(\cdot, s)$ on $s$, it
follows that $i^R(s)$ is unique. Moreover, by definition of $s$, it follows that $i^R(s) = \frac{R}{s} + (1 + r)$ (which lies in $[1 + r, A]$ for $s \in [\theta^R, \theta^C]$). For any given $s$, any loan with $i(s) > i^R(s)$ could be profitably undercut by some other relationship loan. On the other hand, any loan with $i(s) < i^R(s)$ would lead to losses.

Credit-score loans can only be issued if $s \geq \theta^C$, as by continuity of $\Pi^R(i, s)$ on $s$, any credit-score loan issued for $s < \theta^C$ could be profitably undercut by a relationship loan. Analogously as for relationship loans, define the break-even rate $i^C(s)$ associated with a relationship loan to a firm with score $s$ implicitly as $\Pi^C(i^C(s), s) = 0$, which is given by $i^C(s) = \frac{1 + r}{s}$. Reasoning analogously as above, this is the only possible equilibrium rate.

Let $k \geq \tilde{k}$. For any $i \in [0, 1]$, for all $s$, either $\Pi^R(i, s) < \Pi^C(i, s)$ or $\Pi^R(i, s) < 0$ (or both). Hence, only credit-score loans can be issued in equilibrium. For $s < \theta^0$, it follows that $\Pi^C(i, s) < 0$ for any $i \in [1 + r, A]$, the last inequality holding with equality for $s = \theta^0$ and $i = A$. For $s \geq \theta^0$, any credit-score loan that both leads to nonnegative profits and cannot be undercut must be issued at rate $i^C(s)$.

**Proof of Proposition 3.** Part (i):

Let $\tilde{\theta}(i) \equiv \frac{1 + r}{i}$ and define:

$$
\hat{\Pi}^R(i, s) = \int_{0}^{\tilde{\theta}(i)} \Pi^R(i, \theta) \cdot \hat{h}^R(\theta|s, i) d\theta,
$$

where $\hat{h}^R(\cdot|s, i)$ denote the (posterior) density of $\theta$, conditional on $s$ and $\theta < \tilde{\theta}(i)$.

Assume that the credit-score lender charges $i^C(s, \theta)$, as defined in the statement of the proposition. Then, $i^R(s)$ must be such that relationship lenders’ expected profit from establishing a relationship with a firm with score $s$ and choosing an interest rate $i$ is given by $\hat{\Pi}^R(i, s)$.

First, it is shown that $i^R(s) = +\infty$ for all $s < s^*$.

Since $\pi^R(i, \cdot)$ is continuous in $\theta$ for all $i \in [1 + r, A]$, it follows that $\hat{\Pi}^R(i, \cdot)$ is continuous on $s$ for all $i \in [1 + r, A]$. Moreover, we have that $\pi^R(i, 0) < 0$ for all $i \in [1 + r, A]$, which implies that $\hat{\Pi}^R(i, 0) < 0$ for all $i \in [1 + r, A]$. Hence, by continuity of $\hat{\Pi}^R(i, \cdot)$ on $s$, it follows that $\hat{\Pi}^R(i, s^*) < 0$ for all $i \in [1 + r, A]$, for some $s^* > 0$. Moreover, for any $i \in I$, $h(\cdot|s, i)$ satisfies the SMLRP. Hence, $\Pi^R(i, \cdot)$ is strictly increasing on $s$ for all $i \in I$. Hence, it follows that $\Pi^R(i, s) < 0$ for all $s < s^*$. It remains to show that $s^* > \theta^R$. By definition of $\theta^R$, we have that $\Pi^R(i, \theta^R) = 0$ for all $i \in [1 + r, A]$. But also, we have that $\Pi^R(i, s) < \Pi^R(i, \tilde{\theta}(i))$ for all $s < 1$, all $i \in [1 + r, A]$. Hence, $\Pi^R(i, s) < 0$, for all $i \in [1 + r, A]$.

Now, it is shown that $i^R(s) > \hat{i}^R(s)$.

First, observe that for any $s < 1$, we have that $\hat{\Pi}^R(i, s) < \Pi^R(i, \tilde{\theta}(i))$ for all $i \in [1 + r, A]$. But also, by construction of $\theta^C$, it follows that $\Pi^R(i, \tilde{\theta}(i)) < (=) 0$ for all $i < (=) \frac{1 + r}{\theta^C}$. Hence, relationship lenders must choose interest rates in the interval $I \equiv \left(\frac{1 + r}{\theta^C}, A\right]$.

Fix $s < 1$. We show that if $\hat{\Pi}^R(i, s) > 0$ for some $i$, then there exists $i(s)$, with $\frac{1 + r}{\theta^C} < i(s) < i$, such that $\hat{\Pi}^R(i(s), s) = 0$. As noted above, we have that $\hat{\Pi}^R\left(\frac{1 + r}{\theta^C}, s\right) < 0$. Hence, the result follows immediately by continuity of $\hat{\Pi}^R(\cdot, s)$ with respect to $i$.

We show now that the set $S^+ \equiv \{s : \hat{\Pi}^R(i, s) \geq 0\}$ for some $i \in I$ is non-empty. Observe that $\lim_{s \to 1} \hat{\Pi}^R(i, s) = \Pi^R(i, \tilde{\theta}(i)) > 0$ for any $i \in I$. Hence, by continuity of $\hat{\Pi}^R(i, \cdot)$ with respect to $s$ for all $i \in I$, it follows that there exists $s < 1$ such that $\hat{\Pi}^R(i, s) > 0$ for some $i \in I$.

Define $s^* \equiv \min S^+$. By continuity of $\hat{\Pi}^R(i, \cdot)$ on $s$, it follows that $S^+$ is left-closed. Hence, $s^*$ exists. Moreover, $s^*$ coincides with the upper threshold defined in part (i).
Now, for each \( s < 1 \), define the set \( I ( s ) \equiv \{ i : \Pi^R ( i, s ) = 0 \} \) and let \( \hat{i} ( s ) \equiv \min I ( s ) \). By continuity of \( \Pi^R ( \cdot, s ) \) on \( i \) for all \( s \), it follows that \( I ( s ) \) is a closed set and hence the minimum interest rate \( \hat{i} ( s ) \) for which a relationship lender breaks-even on a loan initiated with a borrower with a credit score \( s \) exists.

Since relationship lenders must break-even, it follows that \( i^R ( s ) = \hat{i} ( s ) \). Also, since \( \Pi^R ( i, s ) > \Pi^R ( i, s ) \) for all \( i \in I \), all \( s < 1 \), it follows that \( i^R ( s ) > i^R ( s ) \), for the expected quality of the pool of applicants is strictly lower.

Finally, it is shown that \( i^R ( \cdot ) \) continuous and strictly decreasing, with \( \lim_{s \to 1} i^R ( s ) = \frac{1 + r}{\theta^C} \).

We have that \( \pi^R ( i, \cdot ) \) is strictly increasing on \( \theta \) for all \( i \in I \). Since \( \Pi^R ( i, \cdot ) \) is strictly increasing on \( s \) for all \( i \in I \), we have that for any \( i \in I ( s ) \), it follows that \( \Pi^R ( i, s^l ) > 0 \) for any \( s^l > s^l \). But then, we have seen above that there exists \( i ( s^l ) \), with \( \frac{1 + r}{\theta^C} < i ( s^l ) < i \), such that \( \Pi^R ( i ( s^l ), s^l ) = 0 \). Hence, \( \hat{i} ( s^l ) \leq i ( s^l ) < i \). Hence, \( i^R \) strictly decreasing.

To show that \( i^R \) is continuous, let \( i^R ( s ) = \max \{ \Pi^R ( i, s ) : i \in I ( s ) \} \), which trivially leads to \( i^R ( s ) = \hat{i} ( s ) \). Then, continuity of \( i^R \) follows from the Berge’s theorem of the maximum, for \( \Pi^R ( \cdot, \cdot ) \) continuous in its two arguments and \( I ( s ) \) compact-valued and continuous.

Finally, observe that \( \lim_{s \to 1} \Pi^R ( i, s ) = \Pi^R ( i, \bar{\theta} ( i ) ) \). Also, \( \Pi^R ( i, \bar{\theta} ( i ) ) \geq 0 \). But also, by construction of \( \theta^C \), it follows that \( \Pi^R ( i, \bar{\theta} ( i ) ) < ( = ) 0 \) for all \( i < ( = ) \frac{1 + r}{\theta^C} \). Hence \( \lim_{s \to 1} i^R ( s ) = \frac{1 + r}{\theta^C} \).

Part (ii):

Since \( \pi^C ( i, \cdot ) \) is continuous in \( \theta \) for all \( i \in [1 + r, A] \) and \( f ( x | \theta ) \) are continuous for all \( \theta \in \Theta \), it follows that \( \Pi^C ( i, \cdot ) \) is continuous in \( \theta \) for all \( i \in [1 + r, A] \). Also, \( \pi^C ( i, \cdot ) \) is strictly increasing in \( \theta \), for all \( i \in [1 + r, A] \). Hence, \( \Pi^C ( i, \cdot ) \) is a strictly increasing function of \( \theta \). Let \( i^R ( s ) \) be the relationship lenders’ strategy and define \( \theta ( s ) \equiv \frac{1 + r}{\Pi^C ( i^R ( s ), \theta ( s ) )} \), defined in the range \( s \geq s^* \), as the score that solves \( \Pi^C ( i^R ( s ), \theta ( s ) ) = 0 \). Let \( s \geq s^* \). Then, for any given \( \theta < \bar{\theta} ( s ) \), charging \( i < i^R ( s ) \) leads to losses. Hence, it is optimal for credit-score lenders to reject any such applicant. From strict monotonicity of \( \Pi^C ( i, \cdot ) \) on \( \theta \), it follows that \( \Pi^C ( i^R ( s ), \theta ) > 0 \) for any \( \theta \geq \bar{\theta} ( s ) \). Hence, for each \( \theta \geq \bar{\theta} ( s ) \), there exists a range of interest rates \( i \in [ \Pi^C ( \theta ), i^R ( s ) ] \) for which credit-score lenders can make nonnegative profits. Competition among credit-score lenders yields the desired result.

Now, let \( s < s^* \). Then, the relationship lender is passive and therefore credit-score lenders charge \( i^C ( \theta ) \) on the firms for which \( \Pi^C ( i, \theta ) \geq 0 \), for some \( i \in [1 + r, A] \). By construction, \( \theta^C \) determined the range of firms for which nonnegative profits can be earned.

**Proof of Corollary 1.** Fix a quality level \( \theta \) and let \( \bar{\theta} ( s ) \) as defined in the above proposition. Credit-score lenders charge \( i^C ( \theta ) \) to all firms with scores \( s < \bar{\theta}^{-1} ( \theta ) \). On the other hand, relationship lenders charge \( i^R ( s ) \) to firms with higher scores \( s > \bar{\theta}^{-1} ( \theta ) \). By construction of \( \bar{\theta} ( s ) \), it follows that \( i^R ( s ) < i^C ( \theta ) \) for all \( s > \bar{\theta}^{-1} ( \theta ) \). Hence, credit-score lenders charge a higher rate to firms of the same quality. Moreover, firms with the same quality obtain different interest rates \( i^R ( s ) \) at relationship lenders, while firms obtaining credit at credit-score lenders get the same interest rate \( i^C ( \theta ) \).

**Proof of Corollary 2.** Let \( i^C \) and \( i^R \) be the unconditional interest rate charged by credit-score and relationship lenders. Let \( \theta^C \equiv \frac{1 + r - k}{1 + r} \) stand for the minimum quality for which credit-score loans outperform relationship loans and let \( \lambda \equiv 1 - G ( \theta ) \) be the measure of firms with quality higher than \( \theta^C \). We can write the unconditional interest rate charged by credit-score lenders as

\[
i^C = \lambda \cdot i^C_{HighQual} + (1 - \lambda) \cdot i^C_{LowQual}
\]

where \( i^C_{HighQual} \) and \( i^C_{LowQual} \) stand for the average interest rates of loans granted to firms with quality levels \( \theta > \theta^C \) and its complement, respectively. From the characterization of the equilibrium, we know that \( i^R ( s ) > \frac{1 + r}{\theta^C} \) for all \( s \), all \( \theta \). Hence, it follows...
that $i^R > \frac{1+r}{\theta^R}$. Also, we have that $i^C(\theta) = \frac{1+r}{\theta^C}$ and hence, $i^C(\theta) < \frac{1+r}{\theta^C}$ for all $\theta > \theta^C$. Hence, $i^C_{HighQual} < \frac{1+r}{\theta^C}$. Therefore, there exist $\lambda \in (0, 1)$ such that $i^C < i^R$. 

B Noisy scores

In this section, I show that the qualitative results of Proposition 3 are robust to assuming that credit-score lenders observe a noisy signal of the firms’ quality instead of the true quality, as long as this signal constitutes a sufficient statistic for the less accurate signal $s$.

Assume that credit-score lenders have access to a credit score $\tilde{x}^C$ that is more accurate than the public credit score $\tilde{x}^R$ that all lenders have access to. For simplicity, we take $\tilde{x}^C$ to be a (Blackwell’s) sufficient statistic for $\tilde{x}^R$. Intuitively, a less informative signal may be obtained from a more informative signal by some random process independent of the true quality parameter. Hence, the correlation between the less informative signal and the true parameter will be lower, as it would include additional noise independent of true underlying quality. Hence, the best possible inference about the true parameter can be made relying on the more informative credit score alone.

Formally, I assume that credit-score lenders have access to a credit score model $\tilde{x}^C$, where the conditional densities $f^C(x^C|\theta)$, all $\theta \in \Theta$, satisfy assumptions (2.1) and (2.2). All lenders, in turn, have access to credit score model $\tilde{x}^R$, which produces a noisy signal $x^R$ of the signal $x^C$ observed by credit-score lenders. I make the following two assumptions about this conditional distribution, which parallel assumptions (2.1) and (2.2):

Assumption 3.1: The conditional densities $f^R(x^R|x^C)$ are continuous in their support $X^R(x^C) \subset R$, all $x^C \in X^C$, and common knowledge among all agents.

Assumption 3.2: The family $\{f^R(x^C|x^C)\}_{x^C \in X^C}$ satisfies the Strict Monotone Likelihood Ratio Property (SMLRP), i.e.:

For all $x^R_1 > x^R_2$, $f^R(x^R_1|x^C)/f^R(x^R_2|x^C)$ is strictly increasing, for all $x^C \in X^C$.

Signals are re-scaled as above, so that they represent the expected value of the underlying quality parameter $\theta$. In particular, abusing notation a bit, $s$ and $z$ stand for the relationship lenders’ and credit-score lenders’ scores, respectively, where:

$$s = \sigma^R(x^R) = E_{\theta^R}[x^R]$$
$$z = \sigma^C(x^C) = E_{\theta^C}[x^C].$$

The following proposition is expressed in similar terms to Proposition 3:

Proposition 5 Let $\theta^R = \frac{k}{A-(1+r)}$, $\theta^0 = \frac{1+r}{A}$ and $\theta^C = \frac{1+r-k}{1+r}$, as defined in Proposition 1. Let $i^C(\theta) = \frac{1+r}{\theta^C}$ and $i^R(s) = \frac{k}{s} + (1+r)$. Then, there exists a unique equilibrium, which satisfies the following characteristics:

(i) There exists a threshold $s^* > \theta^R$ such that:

a) $i^R(s) = +\infty$, for all $s < s^*$.
b) $i^R(s) \in (i^R(s), A]$, for all $s \in [s^*, 1]$.
c) $i^R(s)$ is continuous and strictly decreasing, with $\lim_{s \to 1} i^R(s) = \frac{1+r}{\theta^C}$. 

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(ii) There exists a threshold \( z(s) \equiv \frac{1 + p}{m(s)} \), defined in the range \( s \geq s^* \), such that:

\[
i_C(s, z) = \begin{cases} 
i_C(z) & \text{if either } s \geq s^* \text{ and } \theta \geq z(s) \\
+\infty & \text{or } s < s^* \text{ and } \theta \geq \theta^0 
\end{cases}
\]

Proof. The density of \( x^C \), given \( s \), is given by:

\[
\varphi_C^s(x^C|s) = \int_{\theta} f_C^s(x^C|\theta) \cdot g^R(\theta|s) \, d\theta,
\]

with associated c.d.f. \( \chi^C|s(\cdot|s) \). Hence, the density of \( x^C \), conditional on both \( s \) and \( z \leq z^C \), for some \( z^C \in [0, 1] \), is given by

\[
\varphi_C^s,z^C(x^C|s, z \leq z^C) = \frac{\varphi_C^s(x^C|s)}{\chi^C|s(\sigma^{-1}(z^C)|s)} \cdot 1_{z \leq z^C},
\]

where \( 1_{z \leq z^C} \) is an indicator that assigns takes on values of 1 if and only if \( z \leq z^C \).

The density \( h^R(\cdot|s, \tilde{z}(s)) \) of \( \theta \), conditional on \( s \) and \( z < \tilde{z}(s) \), is given by:

\[
h^R(\theta|s, \tilde{z}(s)) = \tilde{g}^C(\theta|z) \cdot \varphi_C^s,z^C(\sigma^{-1}(\theta)|s, z \leq \tilde{z}(s)) .
\]

Since \( z \) is a sufficient statistic for \( s \), it follows that for any \( s \), beliefs about borrowers’ types \( \theta \) are given by \( \tilde{g}^C(\cdot|z) \), the posterior distribution of \( \theta \) given signal \( z \) alone. Moreover, since \( f_C^s(\cdot|\theta) \) satisfies SMLRP, so do \( \varphi_C^s,z^C(\cdot|s, z \leq z^C) \) and \( \tilde{g}^C(\cdot|z) \). Consequently, \( h^R(\theta|s, \tilde{z}(s)) \) satisfies SMLRP.

The rest of the proof is equivalent to the proof of Proposition 3. ■