Information Revelation and (Anti-)Pandering in Elections

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Abstract

We study a Downsian model where office-motivated candidates have policy-relevant private information. A conventional view is that electoral competition will lead to efficient information aggregation because policy positions reveal private information. A countering perspective is that competition causes candidates to pander to the median voter’s beliefs, thereby preventing information aggregation. We find both intuitions to be incorrect. In our model, candidates have an incentive to overreact to private information, which is precisely the opposite of pandering. Indeed, there is a fully-revealing equilibrium with overreaction. But all equilibria (including fully-revealing equilibria) are dominated in terms of the electorate’s welfare by “unbiased dictatorship,” in which one candidate chooses a policy position that is efficient based on his information alone and is always elected. We further show that pandering can actually be desirable. Compared to the unbiased dictatorship equilibrium, the electorate’s welfare can be improved by both candidates pandering. Instead of interfering with information aggregation and efficiency, pandering turns out to be a complement of information aggregation to establish efficiency.

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1 Introduction

Candidates who run for political office are generally better informed than voters about policy-relevant variables.\(^1\) Among various reasons, one is that candidates and their parties have broad access to policy experts whereas voters have limited incentives to invest in information acquisition. The asymmetric information between candidates and voters implies that a candidate’s electoral platform is a signal to voters about his policy-relevant private information. Consequently, the efficiency of a representative democracy depends on whether elections efficiently aggregate politicians’ private information. This can be decomposed into two (related) questions: to what extent do politicians reveal their policy-relevant information through their platforms, and does any information revelation occur without creating policy distortions?

There are competing intuitions regarding these issues. On the one hand, a conventional view is that electoral competition between office-motivated candidates should lead to full revelation and efficient aggregation of the candidates’ private information. On the other hand, more recent formal models have suggested just the opposite: the conjunction of office motivation and competition leads to inefficient pandering. That is, not only do candidates’ platforms not fully reveal their private information, but in fact the platforms are systematically biased toward the electorate’s ex-ante beliefs about optimal policy (e.g. Heidhues and Lagerlof, 2003).

This paper studies a simple model of Downsian electoral competition in which both these intuitions turn out to be incorrect. There are two purely office-motivated candidates, each of whom receives a noisy private signal about some policy-relevant but unobservable state of the world. The state is drawn from a normal distribution and each candidate’s signal is the true state plus a mean-zero random shock that is also normally distributed. Both candidates simultaneously commit to

\(^1\)This is consistent with the notion that voters are initially uncertain and ambivalent on issues, and learn or refine their preferences during the course of an election. Supporting evidence is found in studies on framing in polls (e.g. Schuman and Presser, 1981), experiments on priming (e.g. Iyengar and Kinder, 1987), experiments on deliberative polling (e.g. Fishkin, 1997), and empirical studies on information effects on opinions (e.g. Zaller, 1992; Althaus, 1998; Gilens, 2001).
policy platforms, followed by which a (median) voter updates her beliefs about the true state and
elects one of the two candidates. Finally, the elected candidate implements his platform. The voter’s
welfare is given by a quadratic loss function in the distance between the implemented policy and the
true state.

In this setting, given that the voter wants policy to match the true state, a benchmark strategy
for each candidate is to choose a platform that is his best estimate of the state given his private signal;
call this an unbiased (or naive, or truthful) strategy. Our first analytical observation is that unbiased
strategies do not constitute an equilibrium. Perhaps surprisingly, this is not because candidates have
incentives to distort their platform toward the prior expected state (i.e., to pander to prior beliefs);
rather, a profitable deviation comes from overreacting to information by choosing a platform that
puts more weight on one’s private signal than prescribed by an unbiased Bayesian estimate. We show
that this is because when both candidates play unbiased strategies, it is optimal for the voter to elect
the candidate with the more extreme platform. The intuition is that given unbiased strategies, the
voter is able to infer two signals from the pair of platforms, and hence her posterior on the state puts
less weight on the prior than either candidate’s individual unbiased estimate.

Building on this logic, we find that there is in fact is a fully-revealing equilibrium in which
candidates overreact to their private information. In other words, there is an equilibrium in which
the voter is able to infer each candidate’s signal from his platform, but the platforms entail just the
opposite of pandering. Hence, the intuition that electoral competition creates an incentive to pander
and thereby precludes information revelation is doubly incorrect in our model.

That information can be fully revealed in equilibrium does not imply that efficiency can be
attained. Because the fully-revealing equilibrium entails overreaction, there is a distortion in platform
choice, and we show that this distortion has severe consequences for efficiency. Specifically, this
equilibrium is dominated in voter welfare by “unbiased dictatorship” equilibria, which are asymmetric
equilibria where one candidate adopts an unbiased strategy and is always elected. Indeed, we find
that unbiased dictatorship equilibria cannot be improved upon (from the point of view of voter welfare) by any equilibrium. In other words, elections can do no better that efficiently aggregate one candidate’s signal in the current model. To the extent that one finds symmetric equilibria more plausible than asymmetric equilibria, this result also shows that competition between candidates harms voter welfare.

Somewhat paradoxically, the reason that any equilibrium is weakly dominated by asymmetric dictatorship equilibria is because there is no pandering in the fully-revealing equilibria. We show that there is a sense in which pandering is beneficial for the electorate’s welfare. Specifically, we show that subject to some technical assumptions, a social planner who maximizes the voter’s welfare would require candidates to use strategies that entail pandering. Put differently, pandering would be an equilibrium phenomenon if the candidates were benevolent instead of being purely office-motivated.

In sum, this paper suggests a substantial rethinking of common intuitions regarding information revelation in elections when office-motivated candidates have policy-relevant private information. First, competition creates incentives to overreact to private information rather than to pander to the electorate’s prior. While full revelation of information is possible, it entails severe policy distortions. Indeed, instead of promoting efficiency, competition between candidates harms voters when attention is restricted to symmetric equilibria. Finally, from a normative point of view, pandering does not instead interfering with efficiency: an appropriate degree of pandering can be the efficient way in which candidates should reveal their private information.

Our work is most closely related to the literature on electoral competition when candidates have policy-relevant private information. Perhaps the most important benchmark is a nice paper by Heidhues and Lagerlof (2003). Their main message is that candidates have an incentive to pander to the electorate’s prior belief, and that pandering is detrimental because it mitigates information revelation. Our findings are quite the opposite, despite candidates in both models being office motivated and imperfectly informed about the policy-relevant state. The key is that we consider a
setting where there are a continuum of possible states, signals, and policies; whereas Heidues and Lagerlof’s model has binary states, binary signals, and binary policies. Naturally, with binary signals, one cannot see the logic of why and how candidates wish to exaggerate their signal, which is central to our results. Loertscher (2010) extends Heidues and Lagerlof (2003) by permitting a continuum of policies but also finds that there cannot be full revelation in his leading case because he maintains the binary signal/state structure.

In a model that is otherwise similar to Heidues and Lagerlof (2003), Laslier and Van de Straeten (2004) show that if voters have their own private information about the policy-relevant state, then there are equilibria in which candidates fully reveal their private information; see also Gratton (2010) and Klumpp (2011). By contrast, voters in our model do not possess any private information.

In Schultz (1996), two candidates are perfectly informed about the policy-relevant state and are ideologically or policy motivated. He finds that when the candidates’ ideological preferences are sufficiently extreme, platforms cannot reveal the true state; however, because of the perfect information assumption (and no uncertainty about the median voter’s preferences), full revelation can be sustained when ideological preferences are not too extreme. Martinelli (2001) shows that even extreme ideologies can permit full revelation if voters have their own private information. Alternatively, Martinelli and Matsui (2002) show that if the ideologically-motivated parties are risk averse over policy, then the assumption of perfect information can be exploited to induce information revelation even if voters do not have their own private information.

There are others models of pandering in politics that are less directly related to the current paper. In an early contribution, Harrington (1993) studies a setting where an incumbent has privately knows his own policy preference rather than a policy-relevant state. He shows that the incumbent may pander toward the electorate’s ex-ante preferred policy in order to increase his reelection probability. Related models where incumbents may have some private information about a policy-relevant state

\[2\text{See also Roemer (1994) for a non-Bayesian model with a similar theme.}\]
but also try to signal either competence or preference through policy choice include Cukierman and Tommasi (1998), Canes-Wrone, Herron and Shotts (2001), Majumdar and Mukand (2004), and Maskin and Tirole (2004). Building on Kartik and McAfee (2007), Honryo (2011) studies Downsian electoral competition where some candidates are perfectly informed about the policy-relevant state while others have no information; this leads to candidates trying to signal their competence through their platforms and generates polarization.

Finally, we note that there is a small literature on electoral competition where the private information of candidates is about the location of the median voter rather than about a policy-relevant variable; see, for example, Ottaviani and Sorensen (2006) and Bernhardt, Duggan and Squintani (2007, 2009).

2 Model

An electorate is represented in reduced-form by a single median voter, whose preferences depend upon the implemented policy, $y \in \mathbb{R}$, and an unknown state of the world, $\theta \in \mathbb{R}$. We assume that the voter’s preferences can be represented by a von-Neumann utility function, $U(y, \theta) = -(y - \theta)^2$. The state $\theta$ is unobservable, but is drawn from a normal distribution with mean 0 and a finite precision $\alpha$ (i.e. variance $1/\alpha$). There are two candidates: $A$ and $B$, each of whom is purely office motivated and hence maximizes the probability of winning the election. Each candidate $i$ privately observes a signal $\theta_i = \theta + \varepsilon_i$, where each $\varepsilon_i$ is drawn independently of any other random variable from a Normal distribution with mean 0 and finite precision $\beta$.

After privately observing their signals, both candidates simultaneously choose platforms, $y_A \in \mathbb{R}$ and $y_B \in \mathbb{R}$ respectively. Upon observing the pair of platforms, the median voter updates her belief about the state and then elects one of the two candidates. The elected candidate implements his

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3This assumption that both candidates receive equally precise signals is for expositional simplicity only; all the results extend to the case where one candidate is known to receive a more precise signal than the other.
platform as final policy, i.e., platforms are policy commitments in the Downsian tradition. All aspects of the model except the candidates’ privately observed signals are common knowledge.

With some abuse of notation, a pure strategy for a candidate $i$ will be denoted $y_i(\theta_i)$, which is the platform chosen when his signal is $\theta_i$. A (mixed) strategy for the voter is denoted by $p(y_A, y_B)$, which is the probability with which she elects candidate $A$ when the platforms are $y_A$ and $y_B$. We study perfect Bayesian equilibria of the electoral game, which implies that the voter elects candidate $i$ if $y_i$ is strictly preferred to $y_{-i}$. As is common, we require that the voter randomizes with equal probability between the two candidates if she is indifferent between $y_A$ and $y_B$.

Furthermore, for technical reasons, we restrict to attention to equilibria in which for any given policy of one candidate, say $y_A$, the voting function $p(y_A, \cdot)$ has only at most a countable number of discontinuities, and analogously for $p(\cdot, y_B)$ for any $y_B$.

2.1 Terminology and Preliminaries

A pure strategy $y_i(\cdot)$ is informative if it is not constant, and it is fully revealing if it is a one-to-one function, i.e., if the candidate’s signal can be inferred from his platform. As is well known (Degroot, 1970), the Normality assumptions imply that the expected value of $\theta$ given a single signal $\theta_i$ is:

$$E[\theta|\theta_i] = \frac{\beta}{\alpha + \beta} \theta_i,$$

whereas conditional on both signals, we have

$$E[\theta|\theta_A, \theta_B] = \frac{\beta}{\alpha + 2\beta} (\theta_A + \theta_B).$$

Because of quadratic utility, the optimal policy for a voter is the conditional expectation of the state given all available information. Since the only information a candidate has when he selects his platform is his own signal, we refer to the strategy $y_i(\theta_i) = E[\theta|\theta_i] = \frac{\beta}{\alpha + \beta} \theta_i$ as the unbiased

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4Indeed, in a full-fledged model with a continuum voters of heterogenous ideologies, this would be necessary whenever $y_A \neq y_B$ and yet the median voter is indifferent between the two platforms.
strategy. Plainly, this strategy is full revealing. We say that a strategy $y_i(\cdot)$ displays pandering to the median voter's beliefs if it is informative and yet $|y_i(\theta_i)| < |E[\theta]|$ for all $\theta_i \neq 0$. In other words, a candidate panders if his platform conveys some information to the voter about his signals, but his platforms are systematically distorted from his unbiased estimate of the best policy toward the ex-ante optimal policy.\(^5\) Similarly, we say that $y_i(\cdot)$ displays overreaction to private information if $|y_i(\theta_i)| > |E[\theta]|$ for $\theta_i \neq 0$.

Whenever we refer to an equilibrium satisfies any of the above properties, we mean that both candidates' strategies satisfy the property. In addition, we say that an equilibrium is symmetric if both candidates use the same strategy, and is linear if both candidates play linear pure strategies.

3 Results

Given that the voter desires policies as close as possible to the true state, and that a candidate's only information when choosing his policy is his private signal, one might conjecture that a candidate can do no better than playing an unbiased strategy, particularly if the opponent is using an unbiased strategy. However:

**Proposition 1.** The profile of unbiased strategies is not an equilibrium.

(The proof of this result, and all others not in the text, is in the Appendix.)

Crucially, the reason that unbiased strategies do not form an equilibrium is that candidates have an incentive to overreact to their private signal rather than pander toward the prior. The incentive to overreact comes from the fact that if both candidates play unbiased strategies, the voter optimally selects the candidate with a more extreme platform. A crude intuition is that since unbiased strategies are fully revealing, the voter can infer two signal from the candidates’ platforms, and accordingly puts less weight on the prior when forming her estimate of the state than does either candidate's

\(^5\)Note that a constant strategy of $y_i(\cdot) = 0$ is not pandering according to our terminology, because it uninformative.
unbiased estimate.

To see the logic more precisely, suppose that both candidates adopt the unbiased strategy, $y_i(\theta_i) = \frac{\beta}{\alpha + \beta} \theta_i$. Since these strategies are fully revealing, it follows from (2) that for any $\theta_A, \theta_B$, the voter’s posterior expectation of the state is $\frac{\beta(\theta_A + \theta_B)}{\alpha + 2\beta}$. On the other hand, the mid-point of the two platforms is $\frac{\beta(\theta_A + \theta_B)}{2\alpha + 2\beta}$. Since these two statistics have the same sign but the former is larger in magnitude, the voter optimally elects the candidate with the more extreme platform (i.e. whose platform has larger magnitude). Hence, each candidate would like to raise his probability of playing the more extreme platform, which can achieved by placing more weight on the private signal than prescribed by the unbiased strategy. In this sense, a profitable deviation involves overreaction rather than pandering.

The above logic suggests that equilibria with informative strategies should entail overreaction. Somewhat surprisingly, we find that an appropriate degree of overreaction can induce full revelation in equilibrium:

**Proposition 2.** There is a symmetric fully revealing equilibrium with overreaction where each candidate $i$ plays

$$y_i(\theta_i) = \frac{2\beta}{\alpha + 2\beta} \theta_i.$$ (3)

In this equilibrium, each candidate is elected with probability $1/2$ regardless of the signal realizations $\theta_A$ and $\theta_B$. This is the unique symmetric linear fully revealing equilibrium.

**Proof.** Because $y_A$ and $y_B$ are one-to-one and onto, studying the choice of platforms $y_i'$ is equivalent to saying that $i$ submits a report $\theta_i'$ which then turns into the platform $y_i(\theta_i')$. To establish existence, we then need to prove that if the opponent $j$ truthfully reports $\theta_j' = \theta_j$, then either player $i = A, B$ has no incentive to deviate from truthfully reporting $\theta_i' = \theta_i$. We now show that the voter is indifferent between the two candidates for any pair of reports that they submit, when forming equilibrium beliefs according to the strategies given by (3). As a consequence of this, each candidate $i$ is elected with

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6If $\theta_A = -\theta_B$, so that $y_A(\theta_A) = -y_B(\theta_B)$, then the voter is indifferent between the two candidates and each is elected with probability $1/2$. 

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probability 1/2 regardless of the pair of reports chosen and hence has not profitable deviation from truthfully reporting $\theta_i' = \theta_i$. Formally, we want to show that:

$$\left( y_A (\theta_i') - \mathbb{E} [\theta|\theta_A', \theta_B'] \right)^2 = \left( y_B (\theta_B') - \mathbb{E} [\theta|\theta_A', \theta_B'] \right)^2. \quad (4)$$

Using (2) and (3), the above equality can be rewritten as

$$\left( \frac{2\beta}{\alpha + 2\beta} \theta_i' - \frac{\beta}{\alpha + 2\beta} (\theta_A' + \theta_B') \right)^2 = \left( \frac{2\beta}{\alpha + 2\beta} \theta_B' - \frac{\beta}{\alpha + 2\beta} (\theta_A' + \theta_B') \right)^2,$$

which is true.

The proof of the uniqueness claim is relegated to the Appendix. \hfill \square

Despite full revealing private information, the equilibrium of Proposition 2 would appear to entail inefficiency from the point of view of voter welfare because of the candidates’ overreaction. But even if it is inefficient, is it the best equilibrium for the voter? To address this question, we first demonstrate a very different kind of equilibrium. Say that an equilibrium has dictatorship if one candidate is always elected (hence is a “dictator”), no matter which on-path platforms are realized. Furthermore, a dictatorship equilibrium has unbiased dictatorship if the dictator uses an unbiased strategy. While Proposition 1 says that unbiased dictatorship cannot be supported in an equilibrium where candidates play symmetrically, the following result shows that it can be supported by asymmetric strategies.

**Proposition 3.** There is a fully revealing equilibrium with unbiased dictatorship: candidate $i$ plays $y_i(\theta_i) = \frac{\beta}{\alpha + \beta} \theta_i$ and the candidate $-i$ plays $y_{-i}(\theta_{-i}) = \theta_{-i}$; candidate $i$ is always elected regardless of the realization of $\theta_A, \theta_B$. Up to permuting the candidates, this is the unique pure strategy fully revealing unbiased dictatorship equilibrium.

Note that because the above equilibrium is fully revealing, all platform pairs are on the equilibrium
path and hence there is no issue of off-path beliefs for the voter. Nevertheless, the fact that the losing candidate full reveals his signal is irrelevant for voter welfare: in any unbiased dictatorship equilibrium, the voter’s welfare is the same as if there were just a single candidate who plays an unbiased strategy. In other words, in any unbiased dictatorship equilibrium, there is the efficient aggregation of one signal. Quite strikingly, this level of information aggregation cannot be improved upon:

**Proposition 4.** There is no equilibrium (including equilibria where candidates where may play mixed strategies) that yields the voter a higher ex-ante expected utility than an unbiased dictatorship equilibrium. Any equilibrium in which both candidates have a strictly positive ex-ante probability of winning yields the voter a strictly lower ex-ante utility than an unbiased dictatorship equilibrium.

The key insight behind this Proposition is that any equilibrium must in fact be an ex-post equilibrium in the sense that the voter's strategy $p(y_A, y_B)$ is constant over almost all on-path platform pairs. (Plainly, this property is satisfied in the equilibria of Proposition 2 and Proposition 3.) The ex-post property is proved by exploiting the fact that for any given strategy of the voter, the induced game between the two candidates is a constant-sum Bayesian game, whose equilibria are correlated equilibria of a complete-information constant-sum game. The ex-post property of all equilibria then implies that the voter’s welfare in any equilibrium can be evaluated as though there is a dictator, holding fixed the dictator’s strategies. But then, the best equilibrium for the voter cannot provide her higher ex-ante utility than when the dictator plays an unbiased strategy, which is attained in an unbiased dictatorship equilibrium. Furthermore, in any equilibrium where both candidates have a strictly positive ex-ante probability of winning, each must win with probability $1/2$ for almost all on-path platforms, which implies that the voter’s utility can be evaluated by treating either candidate as the dictator (holding fixed his strategy), but we already know from Proposition 1 that both candidates cannot be using unbiased strategies; hence, the voter’s ex-ante utility must be strictly

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7It is also possible to sustain unbiased dictatorship in an equilibrium where the losing candidate $-i$ plays an uninformative strategy, e.g. $y_{-i}(\theta_{-i}) = 0$. But such a construction is less convincing because it raises questions about what beliefs are “reasonable” for the voter if she were to observe a deviation by candidate $-i$ to some off-path platform.
lower than under unbiased dictatorship.

**Proposition 4** provides a sense in which competition between candidate harms voter welfare, because any equilibrium in which one candidate is not a dictator — in which case competition plays no role — reduces voter welfare compared to having just one candidate (and selecting the best equilibrium of that game).

Is the anti-pandering or overreaction incentive identified in Proposition 1 to blame for the above negative welfare conclusions? We answer this question by providing the following result, which identifies a sense in which pandering would actually be beneficial for voter welfare. To state it, we consider an alternative game, called the *benevolent politicians* games in which the candidates are not office-motivated, but rather maximize the voter’s welfare. This alternative game is useful because the candidate’s strategies in the Pareto-dominant equilibria of that game would be a solution to a social planner’s problem of choosing candidate strategies to maximize voter welfare.

**Proposition 5.** *Any continuously-differentiable equilibrium of the benevolent politicians’ game has both candidates using strategies with pandering.*

The above results show that rather than harming voters, an optimally selected amount of pandering by politicians may actually be the best way in which to promote voter welfare. However, such pandering is incompatible with information revelation in the equilibrium of the Downsian game, because candidates are opportunistic. The final word of our analysis is that some amount of civic responsibility may be necessary for politicians to correctly reveal their information to voters in the electoral game.

4 Conclusion

This paper has studied a model of Downsian electoral competition where office-motivated electoral candidates have private information that is valuable to the electorate because it concerns some
policy-relevant state variable. A candidate’s platform choices is then a signal about his private information. Contrary to common intuitions about pandering to the electorate’s prior belief, we find that competition tends to cause candidates to overreact to information. We construct a symmetric fully-revealing equilibrium where candidates put more weight on their private signal than an unbiased Bayesian estimate, i.e. they overreact to the private information, which is just the opposite of pandering. Despite revealing information however, this equilibrium is not efficient. In fact, any equilibrium in which both candidates win with positive probability is dominated (in terms of the electorate’s welfare) by “unbiased dictatorship” equilibria, where one candidate always wins the election and chooses a platform that is socially optimal based on his information alone. Pandering can aid efficiency in the sense that the electorate would be even better off if both candidates revealed their private information through an appropriate degree of pandering; while this is not an equilibrium with office-motivated candidates, it is one when candidates are benevolent and maximize the electorate’s welfare. The final word of our analysis is that some amount of civic responsibility may be necessary for politicians to correctly reveal their information to voters in the electoral game, and achieve the social optimum.
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A Omitted Proofs

Proof of Proposition 1. Because $y_A$ and $y_B$ are one-to-one and onto, studying the choice of platforms $y_i'$ is equivalent to saying that $i$ submits a report $\theta_i'$ which then turns into the platform $y_i(\theta_i')$.

First we simplify the median voter’s expected utility when platform $y$ is adopted, conditional on the belief that the signal realizations are $\theta_A$ and $\theta_B$:

$$E[U(y, \theta) | \theta_A, \theta_B] = -E[(y - \theta)^2 | \theta_A, \theta_B]$$

$$= -[y^2 + E(\theta^2 | \theta_A, \theta_B) - 2yE(\theta | \theta_A, \theta_B)]$$

$$= -[y^2 + E(\theta | \theta_A, \theta_B)^2 - 2yE(\theta | \theta_A, \theta_B)] - E(\theta^2 | \theta_A, \theta_B) + E(\theta | \theta_A, \theta_B)^2$$

$$= -[y - E(\theta | \theta_A, \theta_B)]^2 - \text{Var} (\theta | \theta_A, \theta_B).$$

Now, we can proceed checking A’s deviation. The median voter prefers $A$ to $B$ if

$$(y_B (\theta_B) - E[\theta | \theta_B, \theta_A'])^2 > (y_A (\theta_A') - E[\theta | \theta_B, \theta_A'])^2,$$

where, $E[\theta | \theta_B, \theta_A'] = \frac{\beta_B \theta_B + \beta_A \theta_A'}{\alpha + \beta_A + \beta_B}$.

So, candidate $A$ wins if

$$\left(\frac{\beta_B \theta_B - \beta_B \theta_B + \beta_A \theta_A'}{\alpha + \beta_A + \beta_B}\right)^2 > \left(\frac{\beta_A \theta_A' - \beta_B \theta_B + \beta_A \theta_A'}{\alpha + \beta_A + \beta_B}\right)^2,$$

or after simplification, if

$$\left(\frac{\beta_A \theta_A' - \beta_A \theta_B}{\alpha + \beta_A}\right)^2 > \left(\frac{\beta_B \theta_B - \beta_A \theta_A'}{\alpha + \beta_A - \beta_B}\right)^2.$$

Because $\beta_A \geq \beta_B$ and $\beta_A / (\alpha + \beta_A) < 1$, it follows that candidate $A$ is more likely to win if over-reporting $\theta_A'$.

The following Lemma will be used in the proofs of Proposition 2 and Proposition 3.

Lemma A.1. Pick any equilibrium in which candidate $i$ is always elected, range$[y_i(\cdot)] = \mathbb{R}$, and $y_i(\cdot)$ is a continuous function that is fully revealing. Then the following condition must hold:

$$\forall y \in \text{range} [y_{-i}(\cdot)]: E[\theta | y_i = y_{-i} = y] = y.$$  

Proof. Suppose, to contradiction, that $E[\theta | y_i = y_{-i} = y] > y$ for some $y \in \text{range} [y_{-i}(\cdot)]$; the case of reverse inequality is analogous. Since $y_i(\theta_i)$ is continuous, $E[\theta | y_i = y - \varepsilon, y_{-i} = y]$ is continuous in $\varepsilon$. Thus, for small enough $\varepsilon > 0$, $E[\theta | y_i = y - \varepsilon, y_{-i} = y] > y$. It follows that for small enough $\varepsilon > 0$, the voter must elect candidate $-i$ upon seeing $y_i = y - \varepsilon$ and $y_{-i} = y$. This contradicts the
hypothesis that \( i \) is always elected.

**Proof of Proposition 2.** It remains only to show that the unique symmetric linear fully revealing equilibrium has both candidates playing according to (3). Fix any symmetric linear fully revealing equilibrium. The proof of Proposition 4 will show that there is some \( p^* \in \{0, 1/2, 1\} \) such that the voter’s strategy \( p(y_A, y_B) = p^* \) for almost all on-path \( y_A, y_B \). Since no symmetric linear fully revealing equilibrium can satisfy the property that for any \( y \in \mathbb{R} \), \( \mathbb{E}[\theta | y_A = y_B = y] = y \), Lemma A.1 implies that \( p^* = 1/2 \). Denote the candidates’ strategies by \( a + b \theta_i \) for some \( a \) and \( b \neq 0 \).

Since \( p^* = 1/2 \), the indifference condition (4) must hold for the voter for almost all \( \theta_A, \theta_B \), which is equivalent to

\[
\left( a + b \theta_A - \frac{\beta}{\alpha + 2 \beta} (\theta_A + \theta_B) \right)^2 = \left( a + b \theta_B - \frac{\beta}{\alpha + 2 \beta} (\theta_A + \theta_B) \right)^2 \quad \text{for almost all } \theta_A, \theta_B,
\]

which requires that \( a = 0 \) and \( b = \frac{2 \beta}{\alpha + 2 \beta} \).

**Proof of Proposition 3.** First, we verify the constructed equilibrium. That the candidates are each playing optimally given the voter’s strategy is obvious, so it suffices to show that the voter’s behavior is optimal. Fix the candidate strategies \( y_i(\theta_i) = \frac{\beta}{\alpha + \beta} \theta_i \) and \( y_{-i}(\theta_{-i}) = \theta_{-i} \). Since both strategies are fully revealing, it suffices to verify that for any \( \theta_A, \theta_B \), the midpoint of the two platforms is larger than \( \mathbb{E}[\theta | \theta_A, \theta_B] \) if and only if candidate \( i \)’s platform is lower than candidate \( -i \). Using (2) and the stipulated strategies, this condition simplifies to

\[
\frac{2 \beta}{\alpha + 2 \beta} (\theta_i + \theta_{-i}) < \frac{\beta}{\alpha + \beta} \theta_i + \theta_{-i} \iff \frac{\beta}{\alpha + \beta} \theta_i < \theta_{-i}.
\]

To see that the above equivalence is indeed true, observe that

\[
\frac{2 \beta}{\alpha + 2 \beta} (\theta_i + \theta_{-i}) < \frac{\beta}{\alpha + \beta} \theta_i + \theta_{-i} \iff \theta_i \left( \frac{2 \beta}{\alpha + 2 \beta} - \frac{\beta}{\alpha + \beta} \right) < \frac{\alpha}{\alpha + 2 \beta} \theta_{-i}
\]

\[
\iff \theta_i \frac{\beta \alpha}{(\alpha + 2 \beta)(\alpha + \beta)} < \frac{\alpha}{\alpha + 2 \beta} \theta_{-i}
\]

\[
\iff \theta_i \frac{\beta}{\alpha + \beta} < \theta_{-i}.
\]

Now we prove the 2nd statement of the Proposition. Fix any pure strategy full revealing unbiased dictatorship equilibrium in which \( i \) always wins. Then for any \( y \in \text{range } [y_{-i} (\cdot)] \), Lemma A.1 implies that \( \mathbb{E}[\theta | y_i = y_{-i} = y] = \frac{\beta}{\alpha + 2 \beta} \left( \frac{\alpha + \beta}{\beta} y + y_{-i}^{-1} (y) \right) = y \). Rearranging yields \( y_{-i}^{-1} (y) = y \). Since this is true for all \( y \in \text{range } [y_{-i} (\cdot)] \), it follows that \( y_{-i} (\theta_{-i}) = \theta_{-i} \).

The following Lemma is used in the proof of Proposition 4. To state it, we need some notation. Given any equilibrium (which may have mixing by candidates), let \( \Pi_i(y_i; \theta_i) \) denote the expected
utility (i.e. win probability) for candidate \( i \) when his type is \( \theta_i \) and he plays platform \( y_i \), and let \( Y_i \) denote the set of platforms that \( i \) plays with strictly positive ex-ante probability. Given any equilibrium, when we refer to “for almost all on-path platforms”, we mean for all but a set of platforms that have ex-ante probability zero with respect to the prior over types and the equilibrium strategies. Similarly for statements about generic platforms.

Lemma A.2. Given any equilibrium and any \( i \), for almost all on-path platforms, \( y_i, y_i' \), and almost all types \( \theta_i, \theta_i' \),

\[
\Pi_i(y_i ; \theta_i) = \Pi_i(y_i' ; \theta_i').
\]

Proof. Fix any equilibrium. Given the voter’s strategy, \( p(y_A, y_B) \), the induced game between the two candidates is a zero-sum Bayesian game. Any equilibrium of this Bayesian game between the candidates is clearly a correlated equilibrium of a complete-information zero-sum game between the two candidates where each chooses an action \( y_i \in \mathbb{R} \) and for any profile \( (y_A, y_B) \), the payoff to candidate \( A \) is \( p(y_A, y_B) \) while the payoff to candidate \( B \) is \( 1 - p(y_A, y_B) \). The Lemma follows from a general fact about zero-sum games that is stated and proved as Proposition B.1 in Appendix B.

Proof of Proposition 4. It suffices to show that in any equilibrium, \( p(y_A, y_B) \) is constant over almost all on-path platforms, because of the argument given in the paragraph following Proposition 4. Without loss, we will show that for a generic platform of candidate \( A \), \( A \)'s winning probability is almost-everywhere constant over \( B \)'s platforms. So fix an equilibrium and pick an arbitrary finite partition of the range of player \( B \)'s on-path platforms, \( \{ Y_B^1, \ldots, Y_B^m \} \), with each \( Y_B^j \) is a convex set.

For a generic on-path platform of player \( A \), \( \bar{y}_A \), Lemma A.2 implies that there is some \( v_A^* \) such that

\[
v_A^* = \Pi_A(\bar{y}_A ; \theta_A) = \Pi_A(\bar{y}_A; \theta_A') \quad \text{for almost all } \theta_A, \theta_A'.
\]

Let \( q(Y_B^j | \theta_A) \) be the probability that \( B \) plays a platform in the set \( Y_B^j \) given his possibly-mixed strategy \( \sigma_B(\cdot) \) and that his type is distributed according to the conditional distribution given \( \theta_A \), i.e. \( \theta_B | \theta_A \sim \mathcal{N}\left( \frac{\beta_A}{\alpha + \beta_A} \theta_A i + \frac{\beta_A}{\alpha + \beta_A} + \frac{1}{\beta_B} \right) \). Let \( p(\bar{y}_A | Y_B^j ; \theta_A) \) be the probability with which \( A \) of type \( \theta_A \) expects to win when he chooses platform \( y_A \) given that the opponent’s platform falls in the set \( Y_B^j \); notice that because \( p(\cdot, \cdot) \) is locally constant (as our restriction is to equilibria where \( p(\bar{y}_A, \cdot) \) has only at most a countable number of discontinuities), the dependence on \( \theta_A \) can be dropped if each \( Y_B^j \) has been chosen as a sufficiently small interval, because then the distribution of \( B \)'s platforms within \( Y_B^j \) is irrelevant. Therefore, with the understanding that each \( Y_B^j \) is a small enough interval, we write \( p(\bar{y}_A | Y_B^j) \). Note that wlog we can take \( m > 1 \), because if \( m = 1 \) then the result is trivial.
Therefore, for any generic $m$ types of player $A$, $(\theta^1_A, \ldots, \theta^m_A)$, we have

\[
\begin{pmatrix}
q(Y^1_B|\theta^1_A) & \cdots & q(Y^m_B|\theta^1_A) \\
\vdots & \ddots & \vdots \\
q(Y^1_B|\theta^m_A) & \cdots & q(Y^m_B|\theta^m_A) \\
\end{pmatrix}
\begin{pmatrix}
p(\bar{y}_A|Y^1_B) \\
\vdots \\
p(\bar{y}_A|Y^m_B) \\
\end{pmatrix}
= 
\begin{pmatrix}
v^*_A \\
v^*_B \\
\end{pmatrix}.
\]

The unknowns above are $p(\bar{y}_A|Y^j_B)$; clearly one solution is for each $p(\bar{y}_A, Y^j_B) = v^*_A$ (which requires $v^*_A \in \{0, 1/2, 1\}$). If we prove that this is the unique solution for some generic choice of $(\theta^1_A, \ldots, \theta^m_A)$, we are done, because $\bar{y}_A$ was a generic platform for $A$ and the partition $\{Y^1_B, \ldots, Y^m_B\}$ was arbitrary (subject to each $Y^j_B$ being a small enough interval). The Rouché-Capelli Theorem implies that it suffices to show that for some choice of $(\theta^1_A, \ldots, \theta^m_A)$, the coefficient matrix of $q(\cdot|\cdot)$ above has non-zero determinant. Suppose that for some selection of distinct types $(\theta^1_A, \ldots, \theta^m_A)$, the coefficient matrix has zero determinant. Since $q(\cdot|\theta^j_A)$ changes non-linearly in $\theta^j_A$ (because $m > 1$ and $\theta_B|\theta^j_A$ is normally distributed), it follows that the determinant cannot remain zero for all perturbations of $(\theta^1_A, \ldots, \theta^m_A)$.

\[\Box\]

**Proof of Proposition 5.** Consider the strategies $y_A$ and $y_B$ and suppose that they constitute an equilibrium.

Let $i$ be either candidate $A$ or $B$, and $j$ denote the opponent. Let $l_i(\theta_i) = y_i(\theta_i) - 2\beta\theta_i/ (\alpha + 2\beta)$.

For any $\theta_i$, and choice $\theta'_i$ of player $i$, the probability that $i$ wins is:

\[
\frac{1}{2} \Pr \left\{ \left( y_j(\theta_j) - E[\theta|\theta'_i, \theta_j] \right)^2 = (y_i(\theta'_i) - E[\theta|\theta'_i, \theta_j])^2 | \theta_i \right\} \\
+ \Pr \left\{ \left( y_j(\theta_j) - E[\theta|\theta'_i, \theta_j] \right)^2 > (y_i(\theta'_i) - E[\theta|\theta'_i, \theta_j])^2 | \theta_i \right\}.
\]

First, note that

\[
(y_j(\theta_j) - E[\theta|\theta'_i, \theta_j])^2 - (y_i(\theta'_i) - E[\theta|\theta'_i, \theta_j])^2 > 0
\]

simplifies to

\[
[y_j(\theta_j) - E[\theta|\theta'_i, \theta_j] + y_i(\theta'_i) - E[\theta|\theta'_i, \theta_j]] [y_j(\theta_j) - E[\theta|\theta'_i, \theta_j] - y_i(\theta'_i) + E[\theta|\theta'_i, \theta_j]] > 0
\]

i.e.,

\[
[y_j(\theta_j) + y_i(\theta'_i) - 2E[\theta|\theta'_i, \theta_j]] [y_j(\theta_j) - y_i(\theta'_i)] > 0
\]

So $i$ wins the election if either one of the two following possibility realizes:

1. $y_j(\theta_j) > y_i(\theta'_i)$ and $\frac{y_j(\theta_j) + y_i(\theta'_i)}{2} > E[\theta|\theta'_i, \theta_j] = \frac{\beta_j\theta_j + \beta\theta'_i}{\alpha + 2\beta}$,
2. $y_j(\theta_j) < y_i(\theta'_i)$ and $\frac{y_j(\theta_j) + y_i(\theta'_i)}{2} < E[\theta|\theta'_i, \theta_j] = \frac{\beta\theta_j + \beta\theta'_i}{\alpha + 2\beta}$.
Rearranging the conditions $\frac{y_j(\theta_j) + y_i(\theta'_i)}{2} > (<) \frac{\beta \theta_j + \beta \theta_i}{\alpha + 2\beta}$, we obtain:

$$l_j (\theta_j) = y_j (\theta_j) - 2\beta \theta_j / (\alpha + 2\beta) > (<) 2\beta \theta_i / (\alpha + 2\beta) - y_i (\theta_i) = -l_i (\theta'_i).$$

Hence, we obtain that for any $\theta_i$, and $\theta'_i$, player $i$'s the probability of winning is:

$$\frac{1}{2} \Pr (y_j (\theta_j) = y_i (\theta'_i) \text{ or } l_j (\theta_j) = -l_i (\theta'_i) | \theta_i)$$

$$+ \Pr (y_j (\theta_j) > y_i (\theta'_i) \text{ and } l_j (\theta_j) > -l_i (\theta'_i) | \theta_i) + \Pr (y_j (\theta_j) < y_i (\theta'_i) \text{ and } l_j (\theta_j) < -l_i (\theta'_i) | \theta_i).$$

We momentarily focus on the case in which the first term in the expression is equal to zero.

Let $f (\theta_j | \theta_i)$ be the distribution of $\theta_j$ given $\theta_i$. Because the strategies $y_A$ and $y_B$ are continuous, it can be decomposed as follows. There exists collections $\{c_k (\theta'_i), d_k (\theta'_i)\}_{k=1}^{K}$, and $\{a_k (\theta'_i), b_k (\theta'_i)\}_{k=1}^{K'}$, where $K$ and $K'$ are possibly infinite, for every $k$ and $k'$, $c_k (\theta'_i) < d_k (\theta'_i) < y_j^{-1} (y_i (\theta'_i)) < a_k (\theta'_i) < b_k (\theta'_i)$, $l_j (a_k (\theta'_i)) = l_j (b_k (\theta'_i)) = l_j (c_k (\theta'_i)) = -l_i (\theta'_i)$, $l'_j (a_k (\theta'_i)) > 0$, $l'_j (b_k (\theta'_i)) < 0$, $l'_j (c_k (\theta'_i)) < -l_i (\theta'_i)$, and $l'_j (d_k (\theta'_i)) > 0$. The role of these collections is the following.

If $l_j (y_j^{-1} (y_i (\theta'_i))) < -l_i (\theta'_i)$, then the probability that $i$ wins with $\theta'_i$ is:

$$\sum_{k=1}^{K} \int_{c_k (\theta'_i)}^{d_k (\theta'_i)} f (\theta_j | \theta_i) d\theta_j + \int_{y_j^{-1} (y_i (\theta'_i))}^{b_0 (\theta'_i)} f (\theta_j | \theta_i) d\theta_j + \sum_{k'=1}^{K'} \int_{a_{k'} (\theta'_i)}^{b_{k'} (\theta'_i)} f (\theta_j | \theta_i) d\theta_j$$

for some appropriate $b_0 (\theta'_i) > y_j^{-1} (y_i (\theta'_i))$ such that $l_j (b_0 (\theta'_i)) = -l_i (\theta'_i)$ and $l'_j (b_0 (\theta'_i)) < 0$.

Hence, the payoff of player $i$ is:

$$- \sum_{k=1}^{K} \int_{c_k (\theta'_i)}^{d_k (\theta'_i)} \left[y_i (\theta'_i) - \frac{\beta (\theta_j + \theta_i)}{\alpha + 2\beta}\right]^2 f (\theta_j | \theta_i) d\theta_j - \sum_{k=1}^{K} \int_{d_k (\theta'_i)}^{y_j^{-1} (y_i (\theta'_i))} \left[y_j (\theta_j) - \frac{\beta (\theta_j + \theta_i)}{\alpha + 2\beta}\right]^2 f (\theta_j | \theta_i) d\theta_j$$

$$- \int_{y_j^{-1} (y_i (\theta'_i))}^{b_0 (\theta'_i)} \left[y_i (\theta'_i) - \frac{\beta (\theta_j + \theta_i)}{\alpha + 2\beta}\right]^2 f (\theta_j | \theta_i) d\theta_j - \sum_{k'=1}^{K'} \int_{a_{k'} (\theta'_i)}^{b_{k'} (\theta'_i)} \left[y_i (\theta'_i) - \frac{\beta (\theta_j + \theta_i)}{\alpha + 2\beta}\right]^2 f (\theta_j | \theta_i) d\theta_j$$

If $l_j (y_j^{-1} (y_i (\theta'_i))) < -l_i (\theta'_i)$, then the probability that $i$ wins is:

$$\sum_{k=1}^{K} \int_{c_k (\theta'_i)}^{d_k (\theta'_i)} f (\theta_j | \theta_i) d\theta_j + \int_{y_j^{-1} (y_i (\theta'_i))}^{y_j^{-1} (y_i (\theta'_i))} f (\theta_j | \theta_i) d\theta_j + \sum_{k'=1}^{K'} \int_{a_{k'} (\theta'_i)}^{b_{k'} (\theta'_i)} f (\theta_j | \theta_i) d\theta_j$$

for some appropriate $c_0 (\theta'_i) < y_j^{-1} (y_i (\theta'_i))$ such that $l_j (c_0 (\theta'_i)) = -l_i (\theta'_i)$ and $l'_j (c_0 (\theta'_i)) > 0$. 
Hence, the payoff of player $i$ is:

$$ \sum_{k=1}^{K} \int_{c_k'(\theta'_i)}^{d_k'(\theta'_i)} \left( y_i(\theta'_i) - \frac{\beta (\theta_j + \theta_i)}{\alpha + 2\beta} \right)^2 f(\theta_j|\theta_i) \, d\theta_j - \sum_{k=1}^{K} \int_{c_{k+1}'(\theta'_i)}^{d_k'(\theta'_i)} \left( y_j(\theta_j) - \frac{\beta (\theta_j + \theta_i)}{\alpha + 2\beta} \right)^2 f(\theta_j|\theta_i) \, d\theta_j - \sum_{k'=1}^{K'} \int_{b_{k'}'(\theta'_i)}^{a_{k'}'(\theta'_i)} \left( y_j(\theta_j) - \frac{\beta (\theta_j + \theta_i)}{\alpha + 2\beta} \right)^2 f(\theta_j|\theta_i) \, d\theta_j $$

If $l_j(y_j^{-1}(y_i(\theta'_i))) = -l_i(\theta'_i)$, then the probability that $i$ wins is:

$$ \sum_{k=1}^{K} \int_{c_k'(\theta'_i)}^{d_k'(\theta'_i)} f(\theta_j|\theta_i) \, d\theta_j + \sum_{k'=1}^{K'} \int_{b_{k'}'(\theta'_i)}^{a_{k'}'(\theta'_i)} f(\theta_j|\theta_i) \, d\theta_j. $$

Hence, the payoff of player $i$ is:

$$ \sum_{k=1}^{K} \int_{c_k'(\theta'_i)}^{d_k'(\theta'_i)} \left( y_i(\theta'_i) - \frac{\beta (\theta_j + \theta_i)}{\alpha + 2\beta} \right)^2 f(\theta_j|\theta_i) \, d\theta_j - \sum_{k=1}^{K} \int_{c_{k+1}'(\theta'_i)}^{d_k'(\theta'_i)} \left( y_j(\theta_j) - \frac{\beta (\theta_j + \theta_i)}{\alpha + 2\beta} \right)^2 f(\theta_j|\theta_i) \, d\theta_j - \sum_{k'=1}^{K'} \int_{b_{k'}'(\theta'_i)}^{a_{k'}'(\theta'_i)} \left( y_j(\theta_j) - \frac{\beta (\theta_j + \theta_i)}{\alpha + 2\beta} \right)^2 f(\theta_j|\theta_i) \, d\theta_j $$

Differentiating the above payoff expressions with respect to $\theta'_i$ we obtain the following expressions.
1. For $l_j(y_j^{-1}(y_i(\theta'_i))) > -l_i(\theta'_i)$,

$$-2\dot{y}_i(\theta'_i) \sum_{k=1}^{K} \int_{c_k(\theta'_i)}^{d_k(\theta'_i)} \left( y_i(\theta'_i) - \frac{\beta(\theta_j + \theta_i)}{\alpha + 2\beta} \right) f(\theta_j|\theta_i) \, d\theta_j$$

$$- \sum_{k=1}^{K} \left( y_i(\theta'_i) - \frac{\beta (d_k(\theta'_i) + \theta_i)}{\alpha + 2\beta} \right)^2 f(d_k(\theta'_i)|\theta_i) \dot{a}_k(\theta'_i)$$

$$+ \sum_{k=1}^{K} \left( y_i(\theta'_i) - \frac{\beta (c_k(\theta'_i) + \theta_i)}{\alpha + 2\beta} \right)^2 f(c_k(\theta'_i)|\theta_i) \dot{c}_k(\theta'_i)$$

$$- \sum_{k=1}^{K} \left( y_j(d_k(\theta'_i)) - \frac{\beta (d_k(\theta'_i) + \theta_i)}{\alpha + 2\beta} \right)^2 f(d_k(\theta'_i)|\theta_i) \dot{d}_k(\theta'_i)$$

$$- \sum_{k=1}^{K} \left( y_j(c_k(\theta'_i)) - \frac{\beta (c_k(\theta'_i) + \theta_i)}{\alpha + 2\beta} \right)^2 f(c_k(\theta'_i)|\theta_i) \dot{c}_k(\theta'_i)$$

$$- \left( y_i(\theta'_i) - \frac{\beta (y_j^{-1}(y_i(\theta'_i)) + \theta_i)}{\alpha + 2\beta} \right)^2 f(y_j^{-1}(y_i(\theta'_i))|\theta_i) \frac{d}{d\theta'_i} y_j^{-1}(y_i(\theta'_i))$$

$$+ \left( y_j(\theta_j) - \frac{\beta (y_j^{-1}(y_i(\theta'_i)) + \theta_i)}{\alpha + 2\beta} \right)^2 f(y_j^{-1}(y_i(\theta'_i))|\theta_i) \frac{d}{d\theta'_i} y_j^{-1}(y_i(\theta'_i))$$

$$-2\dot{y}_i(\theta'_i) \int_{y_j^{-1}(y_i(\theta'_i))}^{b_i(\theta'_i)} \left( y_i(\theta'_i) - \frac{\beta(\theta_j + \theta_i)}{\alpha + 2\beta} \right) f(\theta_j|\theta_i) \, d\theta_j$$

$$-2\dot{y}_i(\theta'_i) \sum_{k=1}^{K} \int_{a_k(\theta'_i)}^{b_k(\theta'_i)} \left( y_i(\theta'_i) - \frac{\beta(\theta_j + \theta_i)}{\alpha + 2\beta} \right) f(\theta_j|\theta_i) \, d\theta_j$$

$$- \sum_{k=1}^{K} \left( y_i(\theta'_i) - \frac{\beta (a_k(\theta'_i) + \theta_i)}{\alpha + 2\beta} \right)^2 f(a_k(\theta'_i)|\theta_i) \dot{a}_k(\theta'_i)$$

$$+ \sum_{k=1}^{K} \left( y_i(\theta'_i) - \frac{\beta (b_k(\theta'_i) + \theta_i)}{\alpha + 2\beta} \right)^2 f(b_k(\theta'_i)|\theta_i) \dot{b}_k(\theta'_i)$$

$$+ \sum_{k=1}^{K} \left( y_j(b_k(\theta'_i)) - \frac{\beta (b_k(\theta'_i) + \theta_i)}{\alpha + 2\beta} \right)^2 f(b_k(\theta'_i)|\theta_i) \dot{b}_k(\theta'_i)$$

$$- \sum_{k=1}^{K} \left( y_j(a_k(\theta'_i)) - \frac{\beta (a_k(\theta'_i) + \theta_i)}{\alpha + 2\beta} \right)^2 f(a_k(\theta'_i)|\theta_i) \dot{a}_k(\theta'_i)$$
Consider
\[
- \sum_{k=1}^{K} \left( y_i (\theta_i') - \frac{\beta (d_k(\theta_i') + \theta_i)}{\alpha + 2\beta} \right)^2 f (d_k(\theta_i') \mid \theta_i) \, d_k(\theta_i') \\
+ \sum_{k=1}^{K} \left( y_j (d_k(\theta_i')) - \frac{\beta (d_k(\theta_i') + \theta_i)}{\alpha + 2\beta} \right)^2 f (d_k(\theta_i') \mid \theta_i) \, d_k(\theta_i')
\]

Using the equilibrium condition, we have:
\[
\sum_{k=1}^{K} \left[ - \left( y_i (\theta_i) - \frac{\beta (d_k(\theta_i) + \theta_i)}{\alpha + 2\beta} \right)^2 + \left( y_j (d_k(\theta_i)) - \frac{\beta (d_k(\theta_i) + \theta_i)}{\alpha + 2\beta} \right)^2 \right] f (d_k(\theta_i) \mid \theta_i) \, d_k(\theta_i)
\]

Decomposing term by term, we obtain:
\[
\left[ - \left( y_i (\theta_i) - \frac{\beta (d_k(\theta_i) + \theta_i)}{\alpha + 2\beta} \right)^2 + \left( y_j (d_k(\theta_i)) - \frac{\beta (d_k(\theta_i) + \theta_i)}{\alpha + 2\beta} \right)^2 \right] = (y_j (d_k(\theta_i) - y_i (\theta_i)) \left( y_j (d_k(\theta_i) + y_i (\theta_i) - \frac{2\beta (d_k(\theta_i) + \theta_i)}{\alpha + 2\beta} \right) \\
= (y_j (d_k(\theta_i) - y_i (\theta_i)) (l_j (d_k(\theta_i) + l_i (\theta_i))
\]

Because, by definition of \( d_k(\theta_i) \), \( l_j (d_k(\theta_i) = -l_i (\theta_i) \), the terms equal zero.

Hence, the whole derivative reduces to:
\[
-2\hat{y}_i (\theta_i) \sum_{k=1}^{K} \int_{c_k(\theta_i)}^{d_k(\theta_i)} \left( y_i (\theta_i) - \frac{\beta (\theta_j + \theta_i)}{\alpha + 2\beta} \right) f (\theta_j \mid \theta_i) \, d\theta_j \\
-2\hat{y}_i (\theta_i) \int_{y_j^{-1}(y_i(\theta_i))}^{\theta_i} \left( y_i (\theta_i') - \frac{\beta (\theta_j + \theta_i)}{\alpha + 2\beta} \right) f (\theta_j \mid \theta_i) \, d\theta_j \\
-2\hat{y}_i (\theta_i') \sum_{k=1}^{K} \int_{\alpha_k(\theta_i)}^{b_k(\theta_i)} \left( y_i (\theta_i) - \frac{\beta (\theta_j + \theta_i)}{\alpha + 2\beta} \right) f (\theta_j \mid \theta_i) \, d\theta_j
\]

Setting this equal to zero, we obtain:
\[
\sum_{k=1}^{K} \int_{c_k(\theta_i)}^{d_k(\theta_i)} \left( y_i - \frac{\beta (\theta_j + \theta_i)}{\alpha + 2\beta} \right) f (\theta_j \mid \theta_i) \, d\theta_j + \int_{y_j^{-1}(y_i(\theta_i))}^{\theta_i} \left( y_i - \frac{\beta (\theta_j + \theta_i)}{\alpha + 2\beta} \right) f (\theta_j \mid \theta_i) \, d\theta_j \\
+ \sum_{k=1}^{K} \int_{\alpha_k(\theta_i)}^{b_k(\theta_i)} \left( y_i - \frac{\beta (\theta_j + \theta_i)}{\alpha + 2\beta} \right) f (\theta_j \mid \theta_i) \, d\theta_j = 0
\]

Next, we use the fact that \( \theta_j \mid \theta_i \sim \mathcal{N} \left( \frac{\beta}{\alpha + \beta} \theta_i, \frac{\beta}{\alpha + \beta} + \frac{1}{\beta} \right) \), and the formula of the expectation of
truncated normals, if \( x \sim N(\mu, \sigma^2) \), then \( E[x|a < x < b] = \mu - \frac{\phi\left(\frac{b-\mu}{\sigma}\right) - \phi\left(\frac{a-\mu}{\sigma}\right)}{\Phi\left(\frac{b-\mu}{\sigma}\right) - \Phi\left(\frac{a-\mu}{\sigma}\right)} \sigma. \)

We obtain the equation:

\[
y_i = \beta \left( \frac{\beta}{\alpha + \beta} \theta_i + \theta_i \right) \alpha + 2 \beta + C(\alpha, \beta, \sigma, \theta_i) = 0, \tag{A.1}
\]

where the “correction term” \( C(\alpha, \beta, \sigma, \theta_i) \) comes from the second terms in the expressions for \( E[\theta_j|a < \theta_j < b] \) and \( E[\theta_j|c < \theta_j < d] \). Because \( \frac{\beta}{\alpha + 2 \beta} \frac{\theta_i}{\alpha + \beta} = \frac{\beta}{\alpha + \beta} \theta_i. \)

The cases 2. and 3. are treated with analogous arguments.

To complete the proof, we now allow for the possibility that \( \Pr(l_j(\theta_j) = -l_i(\theta'_i)|\theta_i) > 0 \) for some \( \theta_i \). Note that the only modification in the payoff \( \Pr(i \text{ wins with } \theta'_i|\theta_i) \) is that there is a collection of intervals \( \{(w_{k''}(\theta'_i), z_{k''}(\theta'_i))\}_{k'' \in K''} \) where for every \( k'' \), \( w_{k''}(\theta'_i) < z_{k''}(\theta'_i) \) and \( l_j(\theta_j) = -l_i(\theta'_i) \) for all \( \theta_j \in (w_{k''}(\theta'_i), z_{k''}(\theta'_i)) \), that induce the extra terms

\[
\frac{1}{2} \sum_{k'' \in K''} \int_{w_{k''}(\theta'_i)}^{z_{k''}(\theta'_i)} f(\theta_j|\theta_i) \, d\theta_i.
\]

But the addition of these terms in the payoff makes it discontinous in \( \theta'_i \), and hence the continuously differentiable strategies \( y_i, y_j \) cannot be equilibria.

Because, in any case the equation governing equilibrium is:

\[
y_i = \beta \left( \frac{\beta}{\alpha + \beta} \theta_i + \theta_i \right) \alpha + 2 \beta + C(\alpha, \beta, \sigma, \theta_i)
\]

which displays pandering, this concludes the proof of the Proposition. \( \Box \)

24
B  A Useful Fact about Correlated Equilibria of Two-Player Zero-Sum Games

In this section, we state and prove a key auxiliary result, Proposition B.1 below, that is used in the proof of Proposition 4. As we are not aware of this result being proved elsewhere, we state it in some generality.

A two-player zero-sum game is given by \((S, u_1, u_2)\) where \(S := S_1 \times S_2\) with each \(S_i\) a measurable action (i.e. pure strategy) space for player \(i\), and each \(u_i : S \to \mathbb{R}\) is a bounded utility function for player \(i\) such that for \(s \in S\), \(u_1(s) = -u_2(s)\). Payoffs are extended to mixed strategies in the usual way.\(^8\) We write \(\Delta(S_i)\) and \(\Delta(S)\) as the spaces of mixed strategies and mixed strategy profiles respectively. For any \(\mu \in \Delta(S)\), we write \(\mu(\cdot|s_i) \in \Delta(S_{-i})\) as the conditional distribution of \(\mu\) over \(S_{-i}\) given \(s_i\).

\(\mu \in \Delta(S)\) is a correlated equilibrium of this game if for any \(i\) and \(s_i \in \text{supp}[\mu]\),

\[
u_i := u_i(s_i, \mu(\cdot|s_i)) = \sup_{s'_i \in S_i} u_i(s'_i, \mu(\cdot|s_i)).
\]

The game has a value if there exists

\[
u_1^* := \max_{\sigma_1} \min_{\sigma_2} u_1(\sigma_1, \sigma_2) = \min_{\sigma_2} \max_{\sigma_1} u_1(\sigma_1, \sigma_2).
\]

We say that \(\nu_1^*\) is player 1’s value and that any solution to the above maxmin problem is an optimal strategy for player 1. Analogously, \(\nu_2^* = -\nu_1^*\) is player 2’s value (and solves the analogous version of the above minmax problem).

Throughout what follows, we fix an arbitrary two-player zero-sum game.

Lemma B.1. If \(\mu\) is a correlated equilibrium, then the game has a value, and each player’s payoff from \(\mu\) is his value. Moreover, the marginal induced by \(\mu\) for each player is an optimal strategy for that player.

Proof. Let \(v_i = u_i(\mu)\) be player \(i\)’s payoff in the correlated equilibrium; clearly \(v_1 = -v_2\). Let \(\sigma_i \in \Delta(S_i)\) be the marginal distribution over player \(i\)’s actions induced by \(\mu\). Since \(\mu\) is a correlated equilibrium, it must be that for any \(s_1 \in S_1\), \(u_1(s_1, \sigma_2) \leq v_1\) (otherwise, for some recommendation, \(s_1\) would be a profitable deviation), and hence \(u_2(s_1, \sigma_2) \geq -v_1 = v_2\). So player 2 has a strategy that guarantees him at least \(v_2\). By a symmetric argument, player 1 has a strategy that guarantees him at least \(v_1 = -v_2\). It follows that \((v_1, v_2)\) is the value of the game; furthermore, each \(\sigma_i\) is an optimal strategy. \(\Box\)

Lemma B.2. If \(\mu\) is a correlated equilibrium, then for \(\mu\)-a.e. \(s_i\), \(u_i(s_i, \mu(\cdot|s_i)) = v_i^*\).

\(^8\)Measurability and boundedness ensure that expected utility is well defined.
Proof. By Lemma B.1, the game has a value and each player has an optimal strategy. Since any \( s_i \in \text{supp}[s_i] \) must be a best response against \( \mu(\cdot|s_i) \), it follows that

\[
\text{for any } s_i \in \text{supp}[\mu], u_i(s_i, \mu(\cdot|s_i)) \geq v_i^*.
\] (B.1)

But this implies that under \( \mu \), neither player can have a positive-probability set of actions that all yield him an expected payoff strictly larger than \( v_i^* \), because then by (B.1) his expected payoff from \( \mu \) would be strictly larger than \( v_i^* \), implying that the opponent’s payoff from \( \mu \) is strictly lower than \( v_{-i}^* \), a contradiction.

Lemma B.3. If \( \mu \) is a correlated equilibrium, then for \( \mu \text{-a.e. } s_i, \mu(\cdot|s_i) \) is an optimal strategy for player \(-i\).

Proof. Wlog, assume \( i = 1 \). By Lemma B.2, \( u_1(s_1, \mu(\cdot|s_1)) = v_1^* \) for \( \mu \text{-a.e. } s_1 \). Hence, by best responses in a correlated equilibrium, it follows that for \( \mu \text{-a.e. } s_1 \),

\[
v_1^* = \max_{s_1'} u_1(s_1', \mu(\cdot|s_1)) = -\min_{s_1'} (-u_1(s_1', \mu(\cdot|s_1))) = -\min_{s_1'} u_2(s_1', \mu(\cdot|s_1)).
\]

Since \( v_1^* = -v_2^* \), we conclude that for \( \mu \text{-a.e. } s_1, v_2^* = \min_{s_1'} u_2(s_1', \mu(\cdot|s_1)) \), i.e. \( \mu(\cdot|s_1) \) guarantees player 2 a payoff of \( v_2^* \), hence \( \mu(\cdot|s_1) \) is an optimal strategy for player 2.

Proposition B.1. If \( \mu \) is a correlated equilibrium, then for \( \mu \text{-a.e. } s_i \) and \( s_i', u_i(s_i', \mu(\cdot|s_i)) = v_i^* \) (and hence \( s_i' \) is a best response to \( \mu(\cdot|s_i) \)).

Proof. Wlog, let \( i = 1 \). Fix any \( s_1 \) that is generic w.r.t. the measure \( \mu \). From Lemma B.3, we have that for any \( s_1' \), \( u_1(s_1', \mu(\cdot|s_1)) \leq v_1^* \). So suppose that there is a set, \( S_1' \), such that \( \mu(S_1') > 0 \) and for each \( s_1' \in S_1' \), \( u_1(s_1', \mu(\cdot|s_1)) < v_1^* \), or equivalently that \( u_2(s_1', \mu(\cdot|s_1)) > v_2^* \). Then, there must be some set \( S_2' \) such that \( \mu(S_2') > 0 \) and \( \mu(S_1'|S_2') > 0 \). By playing \( \mu(\cdot|s_1) \) whenever \( \mu \) recommends any \( s_2 \in S_2' \), player 2 has an expected payoff strictly larger than \( v_2^* \) for a positive-probability set of recommendations, a contradiction with Lemma B.2.