Optimal Fiscal and Monetary Policy With Occasionally Binding Zero Bound Constraints*

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Preliminary

Previous studies on fiscal policy at the zero bound have focused on models with perfect foresight. However, the recent economic environment motivating such research is characterized by a high degree of uncertainty. This paper studies optimal government spending and monetary policy when the nominal interest rate is subject to the zero lower bound constraint in a stochastic environment. In the presence of uncertainty, the government chooses to increase its spending when at the zero lower bound by a larger amount. The welfare effect of fiscal policy is nuanced in the stochastic environment if the government cannot commit. While the access to government spending policy increases welfare in the face of a large deflationary shock, it can decrease welfare during normal times as the government reduces the nominal interest rate less aggressively before reaching the zero lower bound. The implications of optimal fiscal policy for the average inflation rate are also discussed.

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1 Introduction

Previous studies on fiscal policy at the zero bound have focused on models with perfect foresight.\(^1\) For example, several authors recently have analyzed fiscal multipliers at the zero lower bound in the environment in which an exogenous variable that has pushed the nominal interest rate to zero reverts back to its steady-state level in a deterministic manner.\(^2\) However, the current economic environment motivating such analysis is characterized by a high degree of uncertainty. Policymakers during any economic downturn are far from certain as to how severe the recession would become or how long the recession would last, and this might have been particularly true in the recent episode. Thus, it would be useful to know how the presence of uncertainty affects the assessment of fiscal policy at the zero bound.

Accordingly, this paper studies optimal fiscal and monetary policy when the nominal interest rate is subject to the zero lower bound constraint in a stochastic environment. An exogenous variation in the household’s discount rate occasionally forces the government to lower nominal interest rate to zero. The fiscal instrument available to the government is government spending financed by lump-sum taxations. Eggertsson (2001) and Nakata (2011) have shown that, in a deterministic environment, a transitory increase in government spending increases welfare at the zero bound. Analysis is conducted under two timing protocols on the government’s decision. In the first timing protocol, the government makes decisions sequentially, taking as given the policy functions of future government, household, and firms. I refer to this economy as the model without commitment. In the second timing protocol, the government decides a sequence of policy variables for all states for all time periods at the beginning of time one. I refer to this economy as the model with commitment.

In addition to understanding how the presence of uncertainty alters the optimal fiscal policy and allocations at the zero bound, this paper’s another goal is to understand the global implications of fiscal policy on the conduct of monetary policy. Adam and Billi (2006), Adam and Billi (2007), and Nakov (2008) characterized optimal monetary policy with zero lower bound constraints in stochastic environments. In their model, the nominal interest rate is the only policy instrument and the government cannot do anything once the nominal interest rate is at the zero bound. In reality, a myriad of policy instruments are available to the government, and they are actively used. It would be useful to understand how the presence of an additional policy instrument affects the conduct of monetary policy and the allocations both away from and at the zero lower bound.

In the model without commitment, the government increases its spending at the zero bound by a substantially larger amount in the stochastic environment than in the perfect foresight environment. The access to government spending policy directly affects the allocation at the zero bound, but it also affects the allocations away from the zero bound indirectly through its effect on the nominal interest rate policy. The government reduces the nominal interest rate less aggressively when fiscal policy is available, and this can decrease welfare for a wide range of the discount factor shocks.

This somewhat counterintuitive result—the access to fiscal policy can reduce welfare—is driven by the fact that the constraint on government spending policy has two aspects in the model without commitment. While it constrains the government’s choice on its spending today, this constraint represents a commitment to not rely on countercyclical fiscal policy in the future. The government reduces the nominal interest rate more aggressively when it does not have access to government spending policy. A more aggressive reduction in the nominal interest rate improves allocations near the point where the zero bound constraints become

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\(^1\)Eggertsson and Woodford (2003) considered a two-state Markov process for the natural rate of interest with an absorbing state, and many subsequent authors have adopted that process. This setup is useful because it allows one to analytically characterize some properties of the model and to use a variation of Newton-algorithm to solve the model numerically. However, it is not suited for answering the questions this paper is interested in for several reasons that will become clear later.

\(^2\)See, for example, Christiano, Eichenbaum, and Rebelo (2011) and Erceg and Linde (2010).
binding. As a result, the welfare is larger without fiscal policy unless the economy is faced with a large discount factor shock.

In the model with commitment, the presence of uncertainty also leads the government to increase its spending at the zero bound by a larger amount and keep the nominal interest rate at zero for a longer period than in the perfect foresight environment. However, the additional increase in government spending due to uncertainty is very small. Both in the perfect foresight and stochastic environments, the use of fiscal policy does not have large effects on the allocation and welfare. The commitment to keep low nominal interest rates for an extended period can go a long way in improving the allocations, and the marginal effects of fiscal policy are very small even in the stochastic environment.

This paper is the first to examine the consequences of an additional policy instrument in the model with occasionally binding zero bound constraints. It would be interesting to consider the consequences of alternative policy instruments in a similar setting. For example, considering the effect of financial policy in a richer model would be useful. I would expect the analyses conducted in this paper are useful when researchers analyze those models in the future.

Section 2 describes the model and Section 3 formulates the government’s problem. Section 4 discusses calibration and the solution method. Section 5 illustrates the importance of uncertainty at the zero bound using a simpler model with a truncated Taylor rule. Section 6 and 7 respectively discuss the results for the models without and with commitment. Section 8 discusses the implications of optimal fiscal policy on the average inflation rate. Section 9 concludes. Tables and figures follow.

2 Model

This section describes the private sector of the model and defines the equilibrium. The private sector of the economy is given by the standard New Keynesian model. The model is formulated in discrete time with infinite horizon. The economy starts at time one.

2.1 Household

The representative household chooses consumption, labor supply, and bond holdings to maximize the expected discounted sum of the future period utilities. The household likes consumption and government spending, and dislikes labor. The period utility is assumed to be separable. The household problem is given by

$$\max_{C,N,B} \sum_{t=1}^{\infty} \beta t E_{t} \left[ \prod_{s=0}^{t-1} \delta_{s} \right] \left[ C_t^{1-\chi_c} - \frac{N_t^{1+\chi_n,1}}{1+\chi_n,1} + \chi_g,0 G_t^{1-\chi_g,1} \right]$$

subject to

$$P_t C_t + R_t^{-1} B_t \leq W_t N_t + B_{t-1} - P_t T_t + P_t \Phi_t$$

and $\delta_t$ is given. $C_t$ is consumption, $N_t$ is labor supply, and $G_t$ is government spending. $P_t$ is the price of consumption good, $W_t$ is nominal wage, $T_t$ is lump-sum taxation, and $\Phi_t$ is the profit from the intermediate goods producers. $B_t$ is one-period risk free bond that pay one unit of money at $t+1$, and $R_t^{-1}$ is the price of the bond.

The discount rate at time $t$ is given by $\beta \delta_t$. $\delta_t$ is the discount factor shock that alters the weight of the future utility at time $t+1$ relative to the period utility at time $t$. $\delta_t$ follows an AR(1) process:
\[(\delta_t - 1) = \rho(\delta_{t-1} - 1) + \epsilon_t \quad \forall \quad t \geq 2\]

where \(\epsilon_t\) is a shock to the discount factor shock and is distributed as normal with mean 0 and standard deviation \(\sigma_{\epsilon}\. \delta_t\) is the only state variable of this economy. The previous literature considered the case in which \(\sigma_{\epsilon} = 0\.\) This paper analyzes the case with \(\sigma_{\epsilon} = 0\.\) An increase in \(\delta_t\) means that the household increases the valuation of the future utility flows. In the absence of any changes in the nominal interest rate, the household accordingly decreases the consumption today.

2.2 Producers

There is a representative final good producer and a continuum of intermediate goods producers indexed by \(i \in [0,1]\). The representative final good producer purchases the intermediate goods, combines them into the final good using CES technology, and sells it to the household and the government.

\[
\max_{Y_{i,t}, i \in [0,1]} \quad P_t Y_t - \int_0^1 P_{i,t} Y_{i,t} di \\
\text{subject to the CES production function, } Y_t = \left[ \int_0^1 Y_{i,t}^{\theta - 1} di \right]^{\frac{\theta}{\theta - 1}}.
\]

Intermediate-good producers use labor to produce imperfectly substitutable intermediate goods according to linear production function. Each firm sets the price of its own good in order to maximize the expected discounted sum of future profits. Price changes are subject to quadratic adjustment costs.

\[
\max_{P_{i,t}} \quad \mathbb{E} \sum_{t=0}^{\infty} \beta^{t-1} \left[ P_{i,t} Y_{i,t} - W_{i,t} N_{i,t} - P_t \frac{e}{2} \left( \frac{P_{i,t}}{P_{i,t-1}} - 1 \right)^2 Y_t \right] \\
\text{subject to } Y_{i,t} = \left[ \frac{P_{i,t}}{P_t} \right]^{-\theta} Y_t, \text{ and } Y_{i,t} = N_{i,t}. \text{ There is no heterogeneity in the time zero prices across firms. That is, } P_{i,0} = P_0 \text{ for some given constant } P_0 > 0.
\]

2.3 Government’s Policy Instruments

The government’s problem will be introduced in the next section. Here, I describe a set of restrictions on government’s policy instruments.

Throughout the paper, I assume that a lump-sum taxation is used to finance government spending and that the government bond is in zero net supply. Thus, the government budget constraint is given by \(G_t = T_t\.\) While the paper focuses on the lump-sum taxation, it is straightforward to analyze the model with distortionary taxations under the balanced budget assumption. The balanced budget assumption is made mainly in order to simplify the analysis. The more challenging model in which government debt appears as an additional state variable is the subject of ongoing research. However, for economies with large deficits, fiscal policy might be constrained by pay-as-you-go rules that limit the extent of future borrowing, and this assumption may not be too restrictive.

Finally, and most importantly, I impose that the nominal interest rate cannot fall below 1.

\[R_t \geq 1\]
2.4 Market Clearing Conditions

\[ Y_t = C_t + G_t + \int \frac{\varphi}{2} \left[ \frac{P_{i,t}}{P_{t-1}} - 1 \right]^2 Y_t di \]

\[ N_t = \int_0^1 N_{i,t} di \]

\[ B_t = 0 \]

2.5 An Implementable Symmetric Equilibrium

Given \( P_0 \) and \( \{ \delta_t \}_{t=1}^\infty \), an implementable symmetric equilibrium of this economy consists of allocations, \( \{ C_t, N_t, N_{i,t}, Y_t, Y_{i,t} \}_{t=1}^\infty \), prices \( \{ W_t, P_t, P_{i,t} \}_{t=1}^\infty \), and policies \( \{ R_t, G_t, T_t \}_{t=1}^\infty \) such that

- Allocations solve the problem of the household given prices and policies.
- \( P_{i,t} \) solves the problem of firm \( i \).
- \( P_{i,t} = P_{j,t} \) for all \( i \neq j \).
- All markets clear.

It is straightforward to show that a set of implementable symmetric equilibria can be characterized by \( \{ C_t, N_t, Y_t, w_t, \Pi_t, R_t, G_t \}_{t=1}^\infty \equiv \{ d_t \}_{t=1}^\infty \) satisfying

\[ C_t^{\chi_c} = \beta \delta_t R_t E_t C_{t+1}^{\chi_c} \Pi_{t+1}^{-1} \]

\[ w_t = \chi_n \theta N_{t+1}^{\chi_c} C_t^{\chi_c} \]

\[ N_t \langle C_t^{\chi_c} \rangle \left[ \varphi (\Pi_t - 1) \Pi_t - (1 - \theta) - \theta w_t \right] = \beta \delta_t E_t \frac{N_{t+1}^{\chi_c}}{C_{t+1}^{\chi_c}} \varphi (\Pi_{t+1} - 1) \Pi_{t+1} \]

\[ Y_t = C_t + G_t + \frac{\varphi}{2} \left[ \Pi_t - 1 \right]^2 Y_t \]

\[ Y_t = N_t \]

\[ R_t \geq 1 \]

3 Government’s Problem

This section formulates the government’s problem. Previous studies have documented that the government’s ability to commit makes stark differences in the allocations when the nominal interest rate is constrained by the zero lower bound. Thus, I consider two alternative timing protocols. In the first protocol, the government sequentially makes decisions. Each period, the government optimizes taking as given the policy functions of future government, household, and firms. I refer to this model as the model without commitment. In the second protocol, the government decides a sequence of policy variables for all states for all time periods at the beginning of time one, announces it to agents in the privates sector, and adheres to the announced policy in the future. I refer to this model as the model with commitment.
3.1 Without Commitment

For every period t, the government solves the following problem.

\[ V_t(\delta_t) = \max_{\{d_t\}} \left[ C_t^{1-\chi_c} \frac{1}{1-\chi_c} - \frac{N_t^{1+\chi_n,1}}{1+\chi_n,1} + \chi \gamma,0 \frac{G_t^{1-\chi_g,1}}{1-\chi_g,1} \right] + \beta \delta_t E_t V_{t+1}(\delta_{t+1}) \]

subject to the private sector equilibrium conditions stated above and taking as given the next period value and policy functions \{V_{t+1}(\cdot), C_{t+1}(\cdot), N_{t+1}(\cdot), Y_{t+1}(\cdot), w_{t+1}(\cdot), \Pi_{t+1}(\cdot), R_{t+1}(\cdot), G_{t+1}(\cdot)\}.

A Markov-Perfect Equilibrium consists of a set of time-invariant value and policy functions subject to the private sector equilibrium conditions stated above and taking as given the next period value and policy functions \{V(\delta_t), C(\delta_t), N(\delta_t), Y(\delta_t), w(\delta_t), \Pi(\delta_t), G(\delta_t)\} solving the Bellman equation above. In this paper, I focus on policy functions to depend only on the current state, \(\delta_t\). This excludes other time-consistent equilibria where policy and value functions depend on the history of states. My ongoing research analyzes those cases.

3.2 With Commitment

The government with the ability to commit chooses a sequence of policy variables for all states for all times at the beginning of time one.

\[ W(\delta_1) = \max_{\{d_t\}} E_1 \sum_{t=1}^{\infty} \beta_t^{-1} \prod_{s=0}^{t-1} \delta_s \left[ C_t^{1-\chi_c} \frac{1}{1-\chi_c} - \frac{N_t^{1+\chi_n,1}}{1+\chi_n,1} + \chi \gamma,0 \frac{G_t^{1-\chi_g,1}}{1-\chi_g,1} \right] \]

subject to the private sector equilibrium conditions. A Ramsey Equilibrium consists of \{\(C(\delta'), N(\delta'), \Pi(\delta'), G(\delta'), R(\delta')\)\} for all \(\delta'\) satisfying the FONCs of the Ramsey planner’s problem, where \(\delta' \equiv [\delta_1, ..., \delta_t]\) denotes the history of shocks.

Recursive Characterization of the Ramsey Equilibrium

Following Marcet and Marimon (2011), I characterize the Ramsey equilibrium recursively. The government’s problem can be written as:

\[ W(\delta_t, \phi_{1,t-1}, \phi_{2,t-1}) = \max_{\{d_t\}} \min_{\phi_t} \left[ C_t^{1-\chi_c} \frac{1}{1-\chi_c} - \frac{N_t^{1+\chi_n,1}}{1+\chi_n,1} + \chi \gamma,0 \frac{G_t^{1-\chi_g,1}}{1-\chi_g,1} \right] + \beta \delta_t E_t W(\delta_{t+1}, \phi_{1,t}, \phi_{2,t}) \]

subject to the equations characterizing the symmetric equilibrium. \(\phi_t = [\phi_{1,t}, \phi_{2,t}, \phi_{3,t}, \phi_{4,t}]\) is a vector of the Lagrangian multipliers for the four constraints characterizing the symmetric equilibrium. A Ramsey Equilibrium can be characterized by a set of time-invariant policy and value functions, \{\(C(s_t), N(s_t), \Pi(s_t), G(s_t), R(s_t), W(s_t)\)\}, where \(s_t \equiv [\delta_t, \phi_{1,t-1}, \phi_{2,t-1}]\).

4 Calibration and Solution Method

4.1 Calibration

Table 2 lists the parameter values selected. The values chosen for the household’s preference parameters, the elasticity of substitution among intermediate goods, and the price adjustment cost are within the range

\[ 3 \text{The within-period timing assumption that leads to this optimization problem is that government and the agents in the private sector move simultaneously. See the discussion in Eggertsson and Swanson (2008). For alternative within-period timing assumptions, see King and Wolman (2004) and van Zandwegrhe and Wolman (2010). For the detailed discussion on the importance of the within-period timing assumption in Markov Perfect Equilibrium, see Ortigueira (2006).} \]
considered in the literature. The discount rate \( \beta \) is set to \( \frac{1}{1+0.0075} \), which implies the steady state real interest rate of 3\%.

Two parameters describing the evolution of time-preference (\( \rho \) and \( \sigma_\epsilon \)) have important influences on the equilibrium. For the persistence parameter, \( \rho \), I use 0.8, which is the value considered in Adam and Billi (2006), Adam and Billi (2007), and Nakov (2008).

For the variance of shock, \( \sigma_\epsilon \), I choose a value of \( \frac{0.42}{100} \) as a benchmark.\(^4\) This value makes the frequency of hitting the zero lower bound around 8\%.

While this may sound too large, the Federal Reserve has indicated that it is likely to keep its policy rate at exceptionally low levels until mid-2013. The frequency of being at the zero bound over post WWII periods will be about 8\% by that time. The implied unconditional standard deviation of \( \delta_t \), denoted by \( \sigma_\delta \), is 0.007.

4.2 Solution Method

The model is solved by a time iteration method by Coleman (1991). The time-iteration method starts from a guess of policy functions. Assuming that the guessed policy functions are in use for the next period, the FONCs of the government problem is solved to find the policy functions in the current period. This process is repeated until the policy function today becomes arbitrarily close to the policy function tomorrow.

For the model without commitment, I use 501 grids on \([1-5\sigma_\delta, 1+5\sigma_\delta]\) for \( \delta_t \). For the model with commitment, I use 101 grids on \([1-5\sigma_\delta, 1+5\sigma_\delta]\) for \( \delta_t \), 11 grids on \([0,0.05]\) for \( \phi_{1,t} \), 11 grids on \([\delta_2 - 0.02, \delta_2 - 0.02]\) for \( \phi_{2,t} \) where \( \delta_2 \) is the Ramsey steady state of \( \phi_{2,t} \).

5 Simple Illustration With A Truncated Taylor Rule

Before presenting the main results, it is useful to understand why the presence of uncertainty can matter at the zero lower bound in a simpler setting. Thus, this section compares the allocations in the perfect foresight and stochastic economies when the nominal interest rates are determined according to a truncated Taylor rule and government spending is constant. Specifically, throughout this section, I assume that the nominal interest rate and government spending are given by

\[
R_t = \max[1, \frac{1}{\beta} \Pi^2_t]
\]

\[
G_t = 0.21
\]

Also, \( \sigma_\epsilon \) is set to 0.18/100 as the equilibrium does not exist with the benchmark parameter values when the policy variable are chosen suboptimally.

**Figure 1** shows the policy functions in both perfect foresight and stochastic economies. Solid black lines are for the stochastic model, and dashed red lines are for the perfect foresight model. According to the top-left panel, the zero lower bound starts binding as \( \delta \) exceeds above 1.006 in the perfect foresight economy, while the zero bound binds for \( \delta \) larger than 1.0052 in the stochastic economy. For both economies, an increase in the discount factor shock reduces inflation, consumption, and output regardless of the level of the nominal interest rate. However, the reductions in these variables are larger when the nominal interest rates are constrained at the zero lower bound. At the zero lower bound, the increase in the discount factor shock are not offset by the reduction in the nominal interest rate. Thus, the demand for consumption goods decreases more, which in turn leads to sharper reductions in inflation and output.

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\(^4\)The larger the \( \sigma_\epsilon \) is, the more frequently the discount rate \( \beta \delta_t \) exceeds one. The Markov equilibrium does not exist if the discount rate exceeds one sufficiently frequently.
The presence of uncertainty worsens allocations at the zero bound in a quantitatively important way. The declines in inflation, consumption, and output are more dramatic at the zero lower bound in the stochastic economy, while the effect of uncertainty are negligible away from the zero lower bound. The perfect foresight model understates the severity of the output collapse and deflation at the zero lower bound by a factor of 2.

To understand why the effect of uncertainty can be large, consider the following thought experiment in the perfect foresight economy. Suppose that \( \delta_1 = 1.0075 \). Since the agents in the economy knows that there are no shocks in the future, they know that \( \delta_2 = 1.006 \). According to the policy function for consumption, consumption is 0.843 when \( \delta_2 = 1.006 \). Thus, the agents in this perfect foresight economy know their consumption next period is 0.843.

Now, consider asking the following hypothetical question to the agents in the economy. Suppose that there is uncertainty about \( \delta_2 \). That is, \( \epsilon_2 \) may not be zero. There will be no uncertainty beyond \( t=2 \) so that \( \epsilon_t = 0 \) for \( \forall t \geq 3 \). What is the probability distribution of consumption tomorrow in the presence of this hypothetical one-time uncertainty?

Figure 2 shows the agents’ answer to this hypothetical question. The distribution is asymmetric. A negative realization of \( \epsilon_2 \) reduces \( \delta_2 \) and increases consumption tomorrow. The nominal interest rate rises in response to the decline in \( \delta_2 \) and the increase in consumption is not large. Similarly, a positive realization of \( \epsilon_2 \) raises \( \delta_2 \) and reduces consumption tomorrow. However, the increase in \( \delta_2 \) will not be countered by the reduction in nominal interest rates due to the zero bound constraint. Thus, the reduction in consumption tomorrow is larger than the increase in consumption that would result with a negative realization of \( \epsilon_2 \) of the same magnitude. Thus, a mean-preserving spread in the distribution on \( \epsilon_2 \) will lead to declines in the expected consumption tomorrow. Expecting lower consumption tomorrow, forward-looking agents lower their consumption today. The increase in uncertainty similarly reduces the expected inflation and output tomorrow. Expecting lower real wage tomorrow, the intermediate goods producers set lower prices today. For the one-time uncertainty case, the effect may be quantitatively small. But, this effect is amplified when there is uncertainty about \( \delta_2 \) for all time periods. As a result, the allocations in the stochastic economy is lower than those in the perfect foresight economy by a factor of 2 to 3.

6 Results without Commitment

This section characterizes the equilibrium in the model without commitment. I first study how the presence of uncertainty affects the allocations by comparing the stochastic economy with the perfect foresight economy. I then move on to study the effects of fiscal policy on the allocations and welfare. To do so, I solve the model in which government is constrained to keep its spending constant at its deterministic steady-state value for all time periods, and compare the allocations and welfare in this constrained economy with those in the benchmark unconstrained economy.

6.1 Optimal Policy With and Without Uncertainty

Figure 3 shows the policy functions in the stochastic and perfect foresight economies. Solid black lines are for the stochastic economy (\( \sigma_\epsilon = \frac{0.42}{100} \)) and dashed red lines are for the perfect foresight economy (\( \sigma_\epsilon = 0 \)).

In both economies, the government responds to the increase in the discount factor shock by reducing the nominal interest rate. An increase in \( \delta_2 \) makes the household value consumption in the future more and reduces the incentive to spend today. The government tries to offset this effect by offering lower nominal interest rates on the government bonds and thus inducing the household to spend more today. In the perfect foresight model, the government reduces the nominal interest rate linearly as the discount factor shock
increases. However, in the stochastic environment, the government reduces the nominal interest rate more aggressively as the time preference shock becomes large. As a result, for certain values of $\delta_t$, the stochastic economy is in the liquidity trap while the perfect foresight economy is not.

The presence of uncertainty also has an important effect on the conduct of fiscal policy. Since the time preference shock can be completely neutralized by the changes in the nominal interest rate, there is no role for government spending policy if the economy is not at the zero lower bound. However, when monetary policy is constrained at the zero bound, a transitory increase in government spending improves the allocation by increasing the demand for goods, which leads to the increase in real wage and thus inflation through the increased demand for labor. While this is true in both deterministic and stochastic economies, the increases in government spending are quantitatively very different. In the stochastic environment, the government chooses to increase its spending by a substantially larger amount than in the perfect foresight environment.

Despite a more accommodative monetary policy and more aggressive fiscal policy responses, consumption, output, and inflation are substantially lower in the stochastic environment. Agents in the stochastic economy assign positive probability to an increase in the discount factor shock in the future, which is associated with low consumption and inflation if the nominal interest rate is zero. The expectation of low consumption and inflation leads the household and firms today to lower their consumption and set lower prices. In the perfect foresight economy, the discount factor shock gradually reverts back to its steady-state level in a deterministic way. Thus, the agents do not assign any probability to further reductions in consumption and inflation tomorrow, and choose their consumption and prices today accordingly.

To help us better understand these policy functions, Figure 4 shows how differently perfect-foresight and stochastic economies respond to a one-time large increase in the time-preference. Solid black lines are the impulse response functions in the economy with $\sigma_\epsilon = 0.42$ and dashed red lines are the impulse response functions in the economy with $\sigma_\epsilon = 0$. The experiment behind the figures is as following. A large shock hits the economies at time one so that $\delta_1 = 1.021$, which is three standard deviations away from the steady-state. There are no shocks after time one. In the perfect foresight economy, the agents know that there will be no shocks in the future. On the other hand, the agents residing in the stochastic economy think that there will be additional shocks in the future. The presence of uncertainty substantially alters the government’s fiscal and monetary policy responses to this shock. While the government in the perfect foresight economy keeps the nominal interest rate at zero for 4 quarters, the government in the stochastic economy keeps the nominal interest rate at zero for 5 quarters. While the government in the perfect foresight economy raises its spending by about 5 percent, government in the stochastic economy does so by about 10 percent. The declines in consumption, output, and inflation are much larger in the presence of uncertainty.

6.2 Optimal Policy With and Without Fiscal Policy

This subsection and next compare the equilibria with and without government spending policy in order to understand how the access to fiscal policy alters the allocation and welfare. Specifically, I solve the government problem with an additional constraint that government spending has to be constant at the deterministic steady-state level for all states and all time periods. This constrained economy corresponds to the model carefully studied in Adam and Billi (2007) and Nakov (2008) where the nominal interest rate is the only policy instrument.

Figure 5 shows the policy functions with and without government spending policy. Solid black lines are the policy functions in the benchmark economy with government spending policy and dashed red lines are the policy functions in the constrained economy.

Not surprisingly, the decline in output is larger at the zero bound when the government is not allowed
to increase its spending. As a result of the reduced demand for goods, the firms reduce the demand for labor, which leads to the decline in real wage. Such reduction in the real wage is then translated into a lower price due to nominal price rigidities. Thus, the reduction in inflation is larger in the absence of government spending policy. Even though less resources are devoted for government spending, consumption is lower in the absence of government spending policy as the labor supply declines by more than government spending declines.

The availability of fiscal policy affects the nominal interest rate policy in an important way. The government without fiscal policy reduces the nominal interest rate more aggressively in response to an increase in the discount factor. As discussed above, if the government is constrained to keep its spending constant, the allocation deteriorates rapidly as the discount factor shock becomes large. As the discount factor increases, the household and firms assign more probability to visiting those states with a large deflation and output decline in the near future, which in turn decreases inflation and consumption today. Thus, the government without access to fiscal policy lowers the nominal interest rate more aggressively.

One way to look at this difference in the nominal interest rate policy is to compare the frequencies of the economy being at the zero lower bound with and without fiscal policy. The first row of Table 3 shows that the frequencies of hitting the zero lower bound in the model without commitment. The government without access to government spending policy lowers the nominal interest rate to zero about 9.1 percent of the time whereas the government with access to government spending policy reduces the policy rate to zero for 7.8 percent of the time.

Figure 6 shows the model’s response to a one-time increase in the discount factor shock with and without government spending policy. The experiment behind the impulse response functions is the same as in Figure 2, and is described in the previous subsection. The government keeps the nominal interest rate at zero for 4 quarters with and without fiscal policy, but it is slightly slow in raising the rate back to the normal level when fiscal policy is not available. At the beginning of the recession, consumption, output and inflation are substantially lower without government spending policy.

6.3 Welfare Implications of Fiscal Policy

The difference in the nominal interest rate policy described above has important implications on welfare. Top-left panel of Figure 7 shows the difference in the value functions between the unconstrained and constrained economies. It is shown for the range of $\delta_t$ covering four standard deviations away. For a wide range of $\delta_t$, this difference is negative, meaning that the welfare in the constrained economy without government spending policy is larger than in the unconstrained economy. Only when the discount factor shocks are very large—more than three standard deviations above the steady-state—, the unconstrained economy with both fiscal and monetary policy instruments generates higher values.

As discussed in the previous subsection, the nominal interest rate is more aggressively reduced in the absence of government spending policy. This improves allocations for a range of discount factor shocks around the point where the zero bound starts binding. Top-right panel of Figure 7 shows the current period utility associated with different discount factor shocks. Solid black lines and dashed red lines are the today’s utility for the model with and without fiscal policy, respectively. For a range of discount factor shocks around 1.01 at which the zero bound constraint becomes binding, the current period utility is larger without government spending policy than without it. A more aggressive reduction in the nominal interest rate leads to an increase in consumption and labor supply as well as a small reduction in inflation, which combines to raise the current period utility. The expectation of such better allocation also leads to better allocations away from the zero bound as the agents expect to visit to these states in the future. Thus, the
current period utility without fiscal policy dominates the one without it for a wide range of discount factor shocks, except for very large ones. Unless $\delta_t$ exceeds 1.016, the current period utilities are larger without fiscal policy.

Since the discount factor shock gradually reverts to its steady-state level, the economy does not stay too long in the region of very large shocks where the access to fiscal policy leads to higher current period utilities. As the current period utilities is a small portion of the welfare, the value of the problem—the expected discounted sum of future utilities—is larger without government spending policy for an even wider range of discount factor shocks. Top-left panel of Figure 7 says that welfare is larger without fiscal policy unless the discount factor shock is more than three standard deviations above the steady-state level.

This somewhat surprising result on the welfare consequences of fiscal policy is driven by the fact that the constraint on government spending policy has two aspects. On the one hand, it constrains the government’s choice on its spending today. On the other hand, this constraint represents a commitment to not rely on countercyclical fiscal policy in the future. If the future government uses fiscal policy to mitigate the impact of deflationary shocks in the future, the agents in the economy do not expect a large reduction in inflation and consumption. Thus, today’s government has less incentive to reduce nominal interest rates aggressively. This effect can dominate the improved allocations in the states of large discount factor shocks if the economy does not visit those states sufficiently often.

In order to quantify the welfare gain from government spending policy that excludes the negative effect arising from the lack of commitment described above, I formulate a hypothetical government’s problem in which the constrained government is given an opportunity to choose its spending in the current period, but not in the future. I then calculate the amount of one-time transfer of consumption goods—measured as a percentage of the steady state consumption—required to make the constrained government as well-off as the hypothetical government with one-time control on its spending.

The bottom-left panel of Figure 7 shows the welfare gain numbers. Solid black lines are the welfare gains in the stochastic environment and dashed red lines are the welfare gains in the perfect foresight environment. By construction, the welfare gains of one-time deviation are positive. One salient feature is that the welfare gains are substantially larger in the stochastic model than in the perfect foresight model. At $\delta_t = 1.021$, the welfare gain is close to 0.08 percent of the steady-state consumption. This number is not trivial for a one-time deviation experiment.

7 Results with Commitment

This section characterizes the allocations and welfare when government can commit to a sequence of policy variables at time one. As in the previous section, I first study the implications of uncertainty on the conduct of fiscal and monetary policy by comparing the stochastic and perfect foresight economies. I then study the effect of having fiscal policy as an additional policy instrument by comparing the allocations and welfare in the constrained and unconstrained economies.

The policy and value functions in the recursively formulated Ramsey equilibrium are functions of three states: the discount factor shock and two Lagrangian multipliers. Instead of directly analyzing this high-dimensional object, I use the response of the economy to a one-time deflationary shock to discuss the key features of the Ramsey equilibrium. Three-dimensional graphs directly characterizing the policy and values functions are available from the author upon request.
7.1 Optimal Policy With and Without Uncertainty

Figure 8 shows the Ramsey equilibria in response to the same experiment conducted in the Figures 2 and 4: there is a one-time shock to the discount factor that pushes $\delta_1 = 1.021$. There are no further shocks after time one, and the discount factor gradually reverts back to its steady-state. Solid black lines are the impulse response functions in the stochastic economy ($\sigma_\epsilon = 0.42$) and dashed red lines are the impulse response functions in the perfect foresight economy ($\sigma_\epsilon = 0$). The dashed red lines replicate the impulse response functions studied in Nakata (2011). In the perfect foresight economy, the government promises to keep the nominal interest rate for an extended period of time and increase government spending at the initial phase of the zero bound period. An extended period of low nominal interest rates create inflation during the zero bound period, which help to mitigate the deflationary spiral that would occur at the beginning of the recession.

In the presence of uncertainty, government promises an even longer period of low nominal interest rates. In this experiment, the nominal interest rate is held at zero for 9 quarters in the stochastic setting, as opposed to 8 quarters in the perfect foresight setting. As in the model without commitment, government increases its spending by a larger amount in a stochastic environment. However, the additional increase in government spending due to the uncertainty is very small. The consequences of these policy responses are that consumption, inflation, and labor supply responses are little affected by the presence of uncertainty despite the presence of uncertainty.

The Ramsey allocations in the model with commitment are quite different from the allocations in the model without commitment analyzed in Section 5.1. In the absence of commitment, the presence of uncertainty causes large changes in both fiscal and monetary policy responses. Consumption, output and inflation drop substantially despite such large policy responses. Here, despite a negligible change in fiscal policy, the presence of uncertainty does not cause large drop in consumption, output, and inflation because of more accommodative monetary policy response. This strengthens the finding in Nakata (2011) that the marginal value of fiscal policy is very small in the model with commitment because the commitment to an extended period of low nominal interest rates can go a long way in mitigating the deflationary shocks. The analysis in this section demonstrates that that finding holds true even in the stochastic environment.

7.2 Optimal Policy With and Without Fiscal Policy

This subsection compares the Ramsey equilibrium with and without government spending policy. In Figure 9, solid black lines are the impulse response functions in the unconstrained economy and dashed red lines are the impulse response functions in the constrained economy in which the government spending is held constant.

The access to government spending policy does not alter the nominal interest rate policy. In both constrained and unconstrained economies, the government keeps the nominal interest rate at zero for 9 quarters before gradually raising it back to the steady-state level. Consumption is essentially unchanged, and the increase in government spending leads to a one-to-one increase in output. A smoother output path leads to a slightly smoother inflation path. With government spending policy, the initial deflation is slightly contained and the inflation peaks at a slightly lower level. Overall, the limited consequences of government spending policy documented in Nakata (2011) applies to the stochastic environment. In the model with commitment, monetary policy does most of the work and the marginal impact of fiscal policy is small, regardless of the presence of uncertainty.

The limited impact of government spending policy in the model with commitment can be also seen in the frequency of hitting the zero bound. The second row of the Table 3 shows the frequencies of hitting
the zero bound with and without fiscal policy. They are the same up to 2 decimal points.

7.3 Welfare Implications of Fiscal Policy

Unlike in the model without commitment, welfare is always larger in the economy with fiscal policy than without it. Top panel of Figure 10 shows the difference in the value functions between the unconstrained and constrained economies for $\phi_{2,t-1} = \phi_2$. The difference is always positive, meaning that welfare is larger with fiscal policy than without in any states of the economy with $\phi_{2,t-1} = \phi_2$. Although not shown, this is true for other values of $\phi_{2,t-1}$. The constraint on government spending policy can increase welfare in the model without commitment because the constraint acts as a commitment device on the future policy. Such beneficial effect does not arise from imposing constraints on any policy instrument in the model with commitment, and therefore constraints on any policy instruments always lead to lower welfare.

Finally, to quantify the welfare effects of fiscal policy in a way that allows comparison with the model without commitment, I compute welfare gains from a one-time use of government spending policy in the constrained economy, conditional on the Lagrangian multipliers being at their steady-state values in the previous period. In the bottom panel of Figure 10, solid black lines and dashed red lines are welfare gains from the one-time use of fiscal policy in the stochastic and perfect foresight economies, respectively. By construction, the welfare gains are always positive.

Comparing the dashed red lines in the bottom-right panel of Figure 6 and the bottom panel of Figure 10 demonstrates that welfare gains are slightly larger in the model with commitment than without commitment in the perfect foresight model. While this may seem to contradict with the earlier discussion that fiscal policy plays a limited role in the model with commitment, it does not. The larger welfare gains in the model with commitment comes from the fact that the steady state nominal interest rate in this model is lower than in the model without commitment. Thus, the nominal interest rate is reduced to the zero lower bound more often in the model without commitment, and thus the welfare gain from fiscal policy is larger at any discount factor shock.

As the previous discussion suggests, in the model with commitment, the presence of uncertainty makes welfare gains from fiscal policy larger, but by a small amount. Solid black lines are very close to dashed red lines. This is because the government responds to the presence of uncertainty mainly by more accommodative monetary policy, not by more aggressive fiscal policy. As a result, the welfare gain of fiscal policy are little affected by the presence of uncertainty. This is in a sharp contrast to the model without commitment where the welfare gains from fiscal policy increases sharply in the presence of uncertainty (see the bottom-left panel of Figure 6).

8 The Average Inflation Rates Revisited

Although the short term nominal interest rates are still at the zero lower bound in the U.S., many economists and policymakers have already started asking the implications of the zero lower bound on the conduct of monetary policy during normal times. One issue that has been most debated is the implication of the zero bound on the inflation and nominal interest rate targets. Some have argued that the nominal interest rate (and thus the inflation rate) should be set high during normal times so that the central bank has more room to reduce it in the face of a large deflationary shock.\(^5\) Several authors have examined this issue rigorously using dynamic stochastic general equilibrium models with occasionally binding zero bound constraints.\(^6\) However, these studies have focused on models that assign a minor or no role to fiscal policy,

\(^5\) See Blanchard, Dell’Ariceia, and Mauro (2010) for an example.

\(^6\) See Coibion, Gorodnichenko, and Wieland (2011) and Billi (2011) for examples.
and left for the future research the investigation of implications of fiscal or other policy instruments on this debate.

To shed some light on this question, this section documents how the presence of government spending policy alters the average inflation rate in the model studied in this paper. Table 4 tabulates the average inflation rates with and without fiscal policy. The upper and lower sections of Table 4 are respectively for the models with and without commitment. For each entry, the number on the top is the unconditional average inflation rate and the second number in brackets is the conditional average inflation rate when the nominal interest rate is away from the zero bound.

In the model without commitment, the deterministic steady-state level of inflation, or equivalently the stochastic average rate of inflation that would prevail in the absence of the zero lower bound, is positive due to inflation bias. With occasionally binding zero bound constraints, the average inflation rate decreases. This is simply because the economy experiences declines in inflation whenever the economy is at the zero lower bound. Since the expectation of visiting the zero lower bound also reduces inflation outside the zero lower bound, the conditional average inflation rate away from the zero bound is also lower than the deterministic level. As the decline in inflation is smaller if the government has the access to fiscal policy, the average inflation rate is higher with fiscal policy than without it.

In the model with commitment, the deterministic steady-state level of inflation is zero. The Ramsey planner chooses zero inflation rate in order to minimize the resource cost of non-zero inflation. As shown in Adam and Billi (2006), Billi (2011), and Nakov (2008), the presence of the zero lower bound makes the average inflation rate positive. This is because the government induces inflation during the zero bound period to improve allocations by promising to keep low nominal interest rates in the future. Since inflation is positive mostly during the zero bound period, the conditional average inflation rate away from the zero bound is essentially zero. When the government has fiscal policy as an additional instrument, the increase in inflation during the zero bound period is slightly contained (see the Figure 6). Therefore, the average inflation rate is slightly lower with fiscal policy. However, the conditional average inflation rate is again essentially zero.

Overall, the analysis of this section shows that the access to fiscal policy partially unwinds the effect of the zero lower bound constraint on the average inflation rate. The presence of the zero lower bound reduces the average inflation rate in the economy in the model without commitment, but the reduction is smaller when government spending policy is available. The zero lower bound increases the average inflation rate in the economy with commitment, but the increase is smaller when government spending policy is available. Even without fiscal policy, these effects of the zero bound constraints on the average inflation rate tend to be very small. The active use of fiscal policy makes them even smaller.

9 Conclusion

This paper characterized optimal government spending and monetary policy when the nominal interest rate is subject to the zero lower bound in a stochastic environment. In the model without commitment, the government increases its spending when at the zero bound by a larger amount in the stochastic environment than in the perfect foresight environment. The access to government spending policy directly affects the allocation at the zero bound, but it also affects the allocations away from the zero bound indirectly through its effect on the nominal interest rate policy. The government reduces the nominal interest rate less aggressively when fiscal policy is available, and this can decrease welfare for a wide range of the discount factor shocks. In the model with commitment, fiscal policy has very small effects on the allocation and welfare even in the stochastic environment. I also showed that the access to government spending policy neutralizes the effects
of the occasionally binding zero lower bound on the average inflation rate.

This paper focused on government spending policy, but Nakata (2011) have shown that other fiscal instruments, namely labor income and consumption taxations, are more effective in improving the allocation at the zero bound. It is straightforward to extend the analysis in this paper to consider other fiscal instruments under the balanced budget assumption. Preliminary analysis shows that many of the results in this paper are not only robust to these alternative fiscal instruments, but also more pronounced.

My ongoing research extends the model without commitment to include one period risk free nominal debt. The introduction of debt is an important step towards answering many interesting and policy relevant questions. How does debt-ceiling affect allocations at the zero bound? Should the debt level be kept low during normal times so that fiscal stimulus can be provided at the zero bound without accumulating a large amount of debt? My next paper will shed light on these questions.
References


Table 1: Relation to other works on optimal $R_t$ and $G_t$

<table>
<thead>
<tr>
<th></th>
<th>with $R_t$ and $G_t$</th>
<th>with $R_t$</th>
</tr>
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<tr>
<td>Stochastic</td>
<td>This paper</td>
<td>Adam and Billi (2006)</td>
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Table 2: Calibration

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Calibrated Value</th>
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<tr>
<td>$\beta$</td>
<td>Discount rate</td>
<td>$1 + 0.0075 \approx 0.9925$</td>
</tr>
<tr>
<td>$\chi_c$</td>
<td>Inverse intertemporal elasticity of substitution for $C_t$</td>
<td>1.0</td>
</tr>
<tr>
<td>$\chi_n$</td>
<td>Inverse labor supply elasticity</td>
<td>1.0</td>
</tr>
<tr>
<td>$\chi_{g,0}$</td>
<td>Utility weight on $G_t$</td>
<td>0.25</td>
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<td>$\chi_{g,1}$</td>
<td>Intertemporal elasticity of substitution for $G_t$</td>
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<td>$\theta$</td>
<td>Elasticity of substitution among intermediate goods</td>
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</tr>
<tr>
<td>$\varphi$</td>
<td>Price adjustment cost</td>
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<tr>
<td>$\rho$</td>
<td>AR(1) coefficient for the discount factor</td>
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</tr>
<tr>
<td>$\sigma_\epsilon$</td>
<td>The standard deviation of shocks to the discount factor</td>
<td>$[0, 0.42]$</td>
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<tr>
<td>$\sigma_\delta$</td>
<td>The implied unconditional standard deviation of $\delta$</td>
<td>0.007</td>
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Table 3: Frequency of Hitting the Zero Bound With and Without Fiscal Policy

<table>
<thead>
<tr>
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<th>with $R_t$ only</th>
<th>with $R_t$ and $G_t$</th>
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</thead>
<tbody>
<tr>
<td>Without Commitment</td>
<td>9.1 %</td>
<td>7.8 %</td>
</tr>
<tr>
<td>With Commitment</td>
<td>14.2 %</td>
<td>14.3 %</td>
</tr>
</tbody>
</table>

*The frequencies are computed based on 100,000 simulations.

Table 4: Average Inflation Rates With and Without Fiscal Policy

<table>
<thead>
<tr>
<th></th>
<th>Without Zero Bound</th>
<th>With Zero Bound</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>with $R_t$ only</td>
<td>with $R_t$ and $G_t$</td>
</tr>
<tr>
<td>Without Commitment</td>
<td>2.031</td>
<td>1.950</td>
</tr>
<tr>
<td></td>
<td>N/A</td>
<td>(2.013)</td>
</tr>
<tr>
<td>With Commitment</td>
<td>0.0</td>
<td>0.022</td>
</tr>
<tr>
<td></td>
<td>N/A</td>
<td>(0.005)</td>
</tr>
</tbody>
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*The inflation rate is expressed as an annualized percentage. Unbracketed numbers are the unconditional averages from 100,000 simulations. The second numbers in the bracket are the averages conditional on the nominal interest rate strictly larger than zero.
Figure 1: Policy/Allocations with a Truncated Taylor Rule: Perfect-Foresight vs. Stochastic Equilibria

---

**Nominal Interest Rate**
(Annualized Percentage)

<table>
<thead>
<tr>
<th>1.002</th>
<th>1.004</th>
<th>1.006</th>
<th>1.008</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.5</td>
<td>2</td>
<td>1.5</td>
<td>1</td>
</tr>
</tbody>
</table>

| 0     | 0.5   | 1     | 1.5   |

**Inflation**
(Annualized Percentage)

<table>
<thead>
<tr>
<th>1.002</th>
<th>1.004</th>
<th>1.006</th>
<th>1.008</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-1</td>
<td>-2</td>
<td>-3</td>
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**Consumption**

<table>
<thead>
<tr>
<th>0.83</th>
<th>0.835</th>
<th>0.84</th>
<th>0.845</th>
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<tbody>
<tr>
<td>0.83</td>
<td>0.835</td>
<td>0.84</td>
<td>0.845</td>
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</table>

**Labor Supply/Output**

<table>
<thead>
<tr>
<th>1.045</th>
<th>1.05</th>
<th>1.055</th>
<th>1.06</th>
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</thead>
<tbody>
<tr>
<td>1.045</td>
<td>1.05</td>
<td>1.055</td>
<td>1.06</td>
</tr>
</tbody>
</table>

**Solid black line:** Stochastic Model \( \sigma = 0.18 \)

**Dotted red line:** Perfect Foresight Model \( \sigma = 0 \)

*Policy functions are shown for the range of \( \delta \) that covers its steady-state level \( \delta = 1 \) to the level that is 4 standard deviations away from the steady-state \( \delta = 1.012 \).*

---

Figure 2

**Distribution of Consumption Tomorrow**
(Hypothetical One-Time Uncertainty Case)

\[ E[C_{t+1}] = 0.842 \]
Figure 3: Optimal Policy/Allocations Without Commitment: Perfect-Foresight vs. Stochastic Equilibria

Solid black line: Stochastic Model ($\sigma = \frac{0.42}{100}$)
Dotted red line: Perfect Foresight Model ($\sigma = 0$)

*Policy functions are shown for the range of $\delta$ that covers its steady-state level ($\delta = 1$) to the level that is 3 standard deviations away from the steady-state ($\delta = 1.021$).
Figure 4: Recovery from a Recession Without Commitment: Perfect-Foresight vs. Stochastic Equilibria

Solid black line: Stochastic Model ($\sigma = 0.42 \text{NN}$)
Dotted red line: Perfect Foresight Model ($\sigma = 0$)

*The impulse response functions are based on the following experiment: There is a shock at time one that pushes $\delta_1$ to 1.021, which is three standard deviations away from the steady-state value of 1. There are no further shocks after time one."
Figure 5: Optimal Policy/Allocations Without Commitment: With and Without Fiscal Policy

*Policy functions are shown for the range of $\delta$ that covers its steady-state level ($\delta = 1$) to the level that is 3 standard deviations away from the steady-state ($\delta = 1.021$).
Figure 6: Recovery from a Recession Without Commitment: With and Without Fiscal Policy

Solid black line: Both $R_t$ and $G_t$ Optimally Chosen
Dotted red line: $G_t$ Held Constant

*The impulse response functions are based on the following experiment: There is a shock at time one that pushes $\delta_1$ to 1.021, which is three standard deviations away from the steady-state value of 1. There are no further shocks after time one.*
Figure 7: Welfare Consequences Of Fiscal Policy Without Commitment

The value of unconstrained problem minus the value of constrained problem

Welfare Gains from a One−Time Use of Government Spending Policy
(as a % of steady−state consumption)

For the top-right panel
Solid black line: Both $R_t$ and $G_t$ Optimally Chosen, Dotted red line: $G_t$ Held Constant

For the bottom-left panel
Solid black line: Stochastic Model ($\sigma = 0.42100$), Dotted red line: Perfect Foresight Model ($\sigma = 0$)

*For the top-left panel, the range of $\delta$ covers -4 to 4 standard deviations away from the steady-state ($\delta = [-0.972, 1.028]$). For the top-right and bottom-left panels, the range of $\delta$ covers its steady-state ($\delta = 1$) to the level that is 3 standard deviations away from the steady-state ($\delta = 1.021$).
Figure 8: Recovery from a Recession With Commitment: Perfect-Foresight vs. Stochastic Equilibria

*The impulse response functions are based on the following experiment: There is a shock at time one that pushes $\delta_1$ to 1.021, which is three standard deviations away from the steady-state value of 1. There are no further shocks after time one.
Figure 9: Recovery from a Recession With Commitment: With and Without Fiscal Policy

- **Nominal Interest Rate (Annualized Percentage)**

- **Government Spending**

- **Inflation (Annualized Percentage)**

- **Consumption**

Solid black line: Both $R_t$ and $G_t$ Optimally Chosen

Dotted red line: $G_t$ Held Constant

*The impulse response functions are based on the following experiment: There is a shock at time one that pushes $\delta_1$ to 1.021, which is three standard deviations away from the steady-state value of 1. There are no further shocks after time one.*
For the bottom panel: Solid black line: Stochastic Model \((\sigma = 0.42)\)

Dotted red line: Perfect Foresight Model \((\sigma = 0)\)

*For the bottom panel, the range of \(\delta\) covers its steady-state \((\delta = 1)\) to the level that is 3 standard deviations away from the steady-state \((\delta = 1.021)\). \(\phi_{1,t-1}\) and \(\phi_{2,t-1}\) are set to their steady-state values.