Adjusting to Trade Liberalization: 
Reallocation and Labor Market Policies*

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Abstract

Labor market responses to trade liberalization typically exhibit three features: slow net absorption of labor by export-oriented sectors, large reallocation costs for displaced workers, and a disproportionate adjustment burden for older workers. To explain these features and to analyze alternative policy responses, I develop a two-sector small open economy model with overlapping generations, labor market search and matching, and sector-specific human capital accumulated through learning-by-doing. The model is calibrated to Brazilian data in order to study the dynamics of an economy in transition after trade liberalization. The calibrated model shows that search frictions alone cannot explain the sluggishness of adjustment. The interaction of search frictions with sector-specific human capital, on the other hand, can rationalize the observed slow adjustment to reforms. The model also helps to compare the distributional and efficiency effects of alternative worker-assistance programs in general equilibrium. A targeted employment subsidy that rewards mobility not only improves the distribution of income but also enhances efficiency gains from trade by facilitating faster formation of necessary skills during the adjustment period. The market failure corrected by the policy is the disincentives of experienced workers to invest in new skills which is in turn caused by the interaction of rent-sharing and intra-sectoral transferability of human capital to future employers. The paper contributes to a better understanding of trade-induced transitional dynamics and the labor market policies aimed at compensating the losers from trade.

Keywords: Trade Liberalization; Labor Reallocation; Sector-specific Human Capital.

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1 Introduction

Trade liberalization generates efficiency gains by moving resources toward an economy’s comparative advantage.\footnote{Production gains in classical theories of trade are due to exploiting comparative advantages. More recent theories emphasize increasing returns to scale (Krugman (1979)), selection (Melitz (2003)), pro-competitive effects (Melitz and Ottaviano (2008)), and complementarities between trade and multinational production (Ramondo and Rodriguez-Clare (2008)).} As these adjustments occur, however, older workers with experience in import-competing sectors suffer earnings losses, unemployment spells, or both. Despite the centrality of these outcomes to the policy debate, economists have devoted little attention to formally modeling the short to medium-term dynamics that derive from trade liberalization.\footnote{Exceptions are Mayer (1974), Leamer (1980), Mussa (1984), and Davidson and Matusz (2004).} Attempts to rigorously quantify these processes in the context of dynamic structural models are rarer still.\footnote{Artuç et al. (2010) and Kambourov (2009) are two recent contributions.} This has led to a disconnect between economists who stress long-run benefits of openness and policy makers who are concerned with short-run effects on employment and income distribution.

To inform this debate and analyze policy alternatives, I develop and calibrate a dynamic two sector small open economy model that captures both the aggregate effects of trade liberalization and the adjustment experiences of heterogeneous workers. Key features of the model include overlapping generations, labor search and matching, and on-the-job human capital accumulation. Calibrated to aggregate and micro data from Brazil’s pre-liberalization period, the model provides a basis for counterfactual experiments. In particular, it allows me to analyze the distributional and efficiency effects of income support programs that have been used in Brazil and elsewhere to facilitate labor market transitions after trade liberalization. These experiments suggest that targeted compensation programs rewarding work and mobility can bring distributional as well as aggregate welfare gains, while unemployment insurance exacerbates the short-run adverse effects by hampering labor reallocation and skill formation.

The motivation for the model comes from three common patterns of post-liberalization labor market adjustments. First, the transition period is marked by simultaneous creation and destruction of jobs within industries, and a slow net reallocation towards industries with comparative advantage.\footnote{Evidence shows that the dominant channel of labor reallocation in the short run is reshuffling of jobs within sectors rather than between sectors. According to Wacziarg and Wallack (2004), a liberalizing country experiences an increase of yearly inter-sectoral job reallocation from 1.1% to 1.5% within five years after reforms. Annual intra-sectoral excess job reallocation dominates this figure: Haltiwanger et al. (2004) report an 11% for a panel of Latin American countries, ranging from 8.9% in Argentina to 16.4% in Brazil. For Chile, Levinsohn (1999) documents that only about 10% of excess job reallocation is between industries in the seven years subsequent to liberalization. Recent literature also explores how}
Using industry-level panel data, Wacziarg and Wallack (2004) find that trade liberalization leads to little or no inter-industry worker reallocation, depending upon the level of aggregation. The case of Brazil’s liberalization is examined using a linked employer-employee dataset by Menezes-Filho and Muendler (2011), who find that workers were displaced from previously protected industries, but comparative advantage industries failed to absorb them for years.

Second, insofar as inter-industry reallocation takes place, it implies costs for workers who move. These costs take the form of initial unemployment and earning losses upon reemployment. That U.S. workers who change sectors have longer unemployment spells than those who return to the same industry is documented by Murphy and Topel (1987). That they also incur large wage losses when they find employment in a different industry is documented by Neal (1995). Evidence suggests that openness amplifies this link. Using US data, Krishna and Senses (2009) find that higher import penetration in the original industry is associated with larger income shocks to workers who switch industries. This result confirms an earlier finding by Kletzer (2001) that the sector of reemployment is very important in accounting for the variation in earnings losses of trade-related displacements.\(^5\)

Third, reallocation patterns display a life-cycle effect. Older workers face a higher risk of not finding reemployment after being displaced from import competing industries. According to Kletzer (2001), displaced U.S. workers below 45 years old are 11 percentage points more likely to be reemployed within two years than workers 45 years or older at the time of displacement. In the case of Brazil, Gonzaga et al. (2003) document that the propensity to transit from unemployment to self-employment increased dramatically for workers of age 40 and older in Brazil after 1990 (from 20% in 1988 to 40% in 1996) whereas it was flat for workers younger than 24. As a result, sectors that expanded more rapidly in terms of employment did so by hiring young workers at the entry margin. The decomposition of changes in youth employment by Pagés et al. (2009) shows that sectoral reallocation increased demand for young workers in Brazil between 1990 and 2003 (See page 137).\(^6\) Finally, in a survey of transition countries undergoing structural change from a formerly planned economy to a market-

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5 Although there is evidence for displacement-related earnings losses in developing countries, there is no study of whether displacement costs get amplified in the aftermath of trade liberalization. In the case of Brazil, Hoek (2007) and Menezes-Filho (2004) document earning losses associated with displacements over a of period time. For Mexico, Krebs et al. (2008) show that liberalization led to a short-run increase in income risk but they do not explore the channels.

6 Similarly, Kim and Topel (1995) find that the dominant channel of manufacturing sector expansion during the industrialization of South Korea was the hiring of new cohorts in the labor force.
oriented one, Boeri and Terrell (2002) summarize the cross-country evidence that older workers lose ground to younger ones since the value of the experience gained in the sectors favored by the communist regimes was much lower in a free market.

In order to capture these features of post-liberalization adjustment, I build three key features into my model. First, to make worker mobility and adjustment costs age-dependent, agents are finite-lived. Second, to allow for endogenous unemployment spells and job-specific rents, labor markets are subject to search frictions. Young and old workers search for jobs and are randomly matched with firms. Depending on their match-specific productivity draws, they continue or separate. Rents arising from the bilateral monopoly are split by Nash bargaining. Third, to allow for earnings losses when workers switch sectors, employed workers accumulate human capital through learning-by-doing. Skills formed on the job are only transferable to subsequent jobs in the same sector.

Importantly, search frictions and the sector-specificity of skills interact to generate an externality between workers and future employers. The effect is similar to Acemoglu (1997): part of the productivity improvement due to the skill formation is captured by future employers, so workers do not fully internalize the returns to their investment in accepting a job and giving up the opportunity to search for more productive ones. In this sense, switching to an industry in which one has no experience is an investment for which the social return is higher than the private one. This market failure could help explain why post-liberalization labor reallocation takes so long, and it raises the possibility that policies encouraging labor mobility may be efficiency-enhancing. Indeed, my policy experiments suggest that this is the case.

To perform these experiments, I calibrate my model to Brazil’s pre-liberalization age-earning profiles and labor market flows (as well as various macro variables). I then consider a decline in the tariff rate that matches the observed increase in trade intensity during Brazil’s trade liberalization and I solve for the equilibrium transition path to the new steady state. This is a complicated task since the distribution of heterogeneous workers over the state space evolves endogenously during the transition. I use a numerical algorithm similar to Costantini and Melitz (2009) to compute the transition path.

The calibrated model enables me to address two issues. First, I compare the transition period of the complete model with that of a nested model without human capital. The insight from this exercise is that search frictions alone cannot account for the sluggishness of the transition. On the other hand, the interaction of search frictions with sector-specific human capital generates a big impediment to
mobility. Second, I investigate the distributional and aggregate effects of labor market policies observed in Brazil and elsewhere. I first consider an unemployment insurance program that approximates the policy Brazil instituted in 1988, just before the liberalization of trade. As a counterfactual experiment, I design a targeted employment subsidy paid to the initial old employed in the previously protected industry conditional on working in the export-oriented industry. My model is especially suitable for comparing these two policies in general equilibrium since it captures both the endogenous formation of heterogeneously productive matches and on-the-job accumulation of human capital.

The unemployment insurance (actual policy) experiment does a good job in matching post-liberalization unemployment patterns. In particular, it is capable of explaining the initial overshooting in unemployment. On aggregate, it leads to an output loss during the transition by hampering what the economy needs most: reallocation and skill formation in the expanding sector. On the other hand, the employment subsidy (counterfactual policy) experiment suggests that it is possible to not only redistribute income toward workers harmed by the liberalization, but also to increase net output during the transition. The subsidy mitigates the market failure due to the learning externality: the underinvestment in skill formation is especially problematic during the transition which is a time for human capital build-up in the export-oriented industry. A policy that rewards work and mobility for workers adversely affected by trade not only compensates them, but it also speeds up the transition and helps the economy reap the gains from trade earlier on.

**Relation to the Literature** The paper builds on several existing literatures. First, it is related to earlier models that analyze the interactions between imperfect labor markets and international trade. Davidson et al. (1988) and Hosios (1990b) apply a two-sector model with search frictions to a small open economy in order to study the validity of conventional trade theorems. Using a two-country two-sector model of trade, Helpman and Itskhoki (2010) show that the flexibility of labor markets can be a source of comparative advantage. None of these papers deal with transitional dynamics.

Second, it is related to a theoretical literature that characterizes the sectoral reallocation of labor in an overlapping generations framework with human capital specificity. Matsuyama (1992) assumes

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7This policy is inspired by the wage insurance program under the 2002 U.S. Alternative Trade Adjustment Assistance (ATAA) which compensates workers age 50 or older who have lost their jobs as a consequence of increased imports. Recipients receive a wage supplement worth half the difference between their previous and new jobs up to $10,000 over two years. This program is extended in 2009 under the name “Reemployment Trade Adjustment Act”.

8This counterfactual is comparable to the actual policy in that both are financed by the same tax rate on firms’ revenues.
away mobility by allowing occupational choice only when agents enter the labor force. Rogerson (2005) is a two-period model with mobility but old workers prefer non-employment to switching sectors when their sector is adversely affected by a relative price shock. In both models, sectoral adjustment occurs through demographic change rather than the reallocation of existing factors. The stylized nature of these papers, however, restricts their quantitative applicability.

Third, this paper is also related to a literature which analyzes policies aimed at displaced workers. In a macro context, Ljungqvist and Sargent (1998) and Rogerson and Schindler (2002) show that unemployment insurance is a highly distortionary method of assisting displaced workers since it reduces the opportunity cost of unemployment. In a trade context, Davidson and Matusz (2006) analyze the least distortionary policies to compensate workers of different ability levels. They find that, depending on the type being compensated, targeted employment and wage subsidies are generally less costly then unemployment insurance. I emphasize the role of experience rather than ability. Moreover, I find that policies that induce mobility may actually achieve distributional goals without trading off efficiency. This last point resonates with Feenstra and Lewis (1994) who show that when all factors of production are imperfectly immobile, Dixit-Norman type commodity taxation needs to be augmented by policies that give factors an incentive to move in order to restore the Pareto efficiency of free trade.

Finally, my paper is most closely related to several papers that study the transition under barriers to mobility. Artuç et al. (2010) develop a dynamic structural model with costly inter-industry labor mobility. Given the little amount of inter-sectoral reallocation, estimated costs are large in the U.S. data. Artuç (2009) extends their framework to study intergenerational distributional effects of trade shocks. Dix-Carneiro (2010) estimates a structural equilibrium labor model with heterogenous workers, sector-specific experience and costly switching of sectors using Brazilian data. Falvey et al. (2010) analyze distributional and policy-related issues in a Heckscher-Ohlin model with an education sector and skill upgrading. Using a calibrated island model of labor market search, Kambourov (2009) shows that firing distortions can substantially reduce gains from trade by hampering the needed reallocation of resources. Ritter (2011) uses a similar model to study the distributional impact of offshoring in the U.S. The main difference between my model and these island-search models is the interaction of rent-sharing and sector-specificity of human capital. This makes it possible that the transition is suboptimally slow even when there are no institutional barriers. As a result, there is a potential efficiency role for policies that encourage workers with different levels of sectoral experience to switch
The rest of the paper is organized as follows: Section 2 describes the model. Section 3 provides some brief background information on Brazilian trade and labor market reforms. Section 4 calibrates the model and conducts counterfactual experiments to assess the effects of labor market policies. Section 5 concludes.

## 2 The Model

### 2.1 The Environment

The economy is populated by workers who have finite lives with two stages, young and old. Let \( g \in \mathcal{G} = \{y, o\} \) denote these generations. Each worker is born young and faces a constant probability \( \delta_a \in (0, 1) \) of becoming old. Once she becomes old, each worker faces a constant probability \( \delta_m \in (0, 1) \) of death. There is no population growth, and the size of the total population is normalized to one. Preferences are defined by a momentary utility function linear in consumption. Agents discount the future at rate \( \beta \in (0, 1) \) and time is discrete.

**Production** A non-tradable final good is produced competitively using two tradable intermediate goods. By the small open economy assumption, world prices for intermediate goods, \( (p_1, p_2) \), are taken as given. The country has a comparative advantage in the production of good 1 and protects sector 2 with an ad-valorem import tariff \( \tau \geq 0 \). In the absence of trade costs, the domestic price of good 2 is \( p_{2d} = p_2(1 + \tau) \) if it is imported in equilibrium, and that of good 1 is equal to the world price, \( p_{1d} = p_1 \).

Final good production is Cobb-Douglas in the two intermediate inputs,

\[
Y = Q_1^{\gamma}Q_2^{1-\gamma}, \tag{1}
\]

and perfect competition in the final good market results in unit-cost pricing:

\[
p_Y = \frac{p_{1d}^{\gamma}p_{2d}^{1-\gamma}}{\gamma \gamma(1 - \gamma)^{1-\gamma}}. \tag{2}
\]

The production of intermediate goods takes place in bilateral matches between workers and firms.
who randomly meet in a labor market subject to search frictions. At each point in time, a worker is characterized by her labor market status and a vector \( h = (h_1, h_2) \in \mathcal{H} \) of sector-specific human capital stocks in sector 1 and 2 respectively. A match between a sector-\( i \) firm and a worker with human capital \( h \) produces output according to

\[
q_i(z, h) = A_i z h_i,
\]  

where \( A_i \) is sectoral aggregate productivity, and \( z \) is a productivity level idiosyncratic to the match. Relative productivity across sectors, \( A_1 / A_2 > 1 \), is the only source of comparative advantage in the model. I assume that \( A_1 \) is sufficiently large that the country is a net exporter of good 1 and a net importer of good 2 in equilibrium.

**Labor Markets** Unemployed workers search for jobs in an undirected fashion taking the match probability \( \phi_w \) as given. On the other side of the market, there is a measure one of value-maximizing firms owned by workers.\(^9\) Some of these firms are already matched with a worker. Idle firms draw a pair of vacancy creation costs \( (c_1, c_2) \) in terms of the final good independently from a distribution \( F_c(c) \) with support \([0, \infty)\). They then decide whether to create vacancies, and if so, which sector to enter. The economy-wide measure of new matches is

\[
m(U, V) = \frac{UV}{(U^\lambda + V^\lambda)^{1/\lambda}},
\]

where \( U \) is the measure of unemployed workers, and \( V \) is the measure of total vacancies.\(^10\) Matching probabilities for workers and firms are thus defined as \( \phi_w = m(U, V) / U \) and \( \phi_f = m(U, V) / V \) respectively. Defining market tightness as \( \theta = V / U \), these probabilities are given by:

\[
\phi_w = (\theta^{-\lambda} + 1)^{-1/\lambda},
\]

\[
\phi_f = (\theta^\lambda + 1)^{-1/\lambda}.
\]

Conditional on locating a vacancy, the probability of the match being in sector \( i \) is given by \( \mu_i \), an

\(^9\)Since firms constitute a scarce factor, they earn profits redistributed to workers as dividend income \( d \).

\(^10\)This constant-returns-to-scale functional form, proposed by Den Haan et al. (2000), has the desirable feature that it generates matching probabilities bounded between 0 and 1.
endogenous object to be characterized later. The probability that an unemployed worker will match with a sector-$i$ vacancy is

$$\phi_{wi} = \phi_w \mu_i.$$  

Not all matches in this economy are transformed into jobs. A newly formed worker-firm pair draws a match-specific productivity level $z$ from the distribution $F_z(z)$ with support $[0, \infty]$ and density $f_z(z)$. The pair decides whether it is optimal to produce output, taking into consideration their outside options. Since some matches do not result in production, job filling and job finding probabilities differ from matching probabilities. If a pairing generates positive rents, the parties produce output and split the associated surplus through Nash bargaining, with the worker’s share being $\sigma \in (0, 1)$. Match specific productivity is fixed thereafter. Firm-worker pairings are exogenously destroyed with probability $(\delta^y_{JD}, \delta^o_{JD})$ for young and old respectively, or endogenously terminated when the surplus falls below zero because of on-the-job learning. More details on the latter source of separation will be given below.

![Figure 1 – Timing of Events for Idle Firms](image)

Figure 1 summarizes the sequence of events for idle firms. All such firms are ex-ante homogeneous and cost draws are independent across time. In other words, firms do not carry these costs as a state variable and the outside option has the same value for all matched firms. If an idle firm finds its cost draws for both sectors prohibitively expensive, it remains inactive and redraws next period. If it creates a vacancy, matching uncertainty is resolved at the beginning of next period. Vacancy creation costs are sunk before the matching uncertainty is resolved.\(^\text{11}\)

\(^{11}\)In order to pay the sunk cost $c_i$, an idle firm needs to have access to credit markets. The entry process can be decentralized with the following ownership structure. Suppose that there is a mutual fund whose shares are equally owned by workers. It can borrow funds from the market at a rate $1 + r = 1/\beta$ which makes young agents indifferent between lending or not. Borrowed funds are used to finance vacancy creation costs. The mutual funds holds a diversified portfolio and owns productive matches until the debt on them is paid back. Since firms constitute a fixed factor, the fund earns positive profits which is distributed to its owners as dividend.
Human Capital Accumulation

Human capital is sector-specific and accumulated through on-the-job learning. Each newborn worker starts her life with an initial endowment \( h = (h_1, h_2) \) normalized to \( h_1 = h_2 = 1 \). The law of motion for \( h \) depends on the labor market state of the worker:

\[
h_{it+1} = \begin{cases} 
  h_{it}^\alpha H^{1-\alpha} & \text{if employed in sector } i, \\
  \max\{1, (1 - \delta_h)h_{it}\} & \text{otherwise},
\end{cases}
\]

with \( \alpha \in (0, 1) \). This expression implies that human capital is an element in the closed and bounded set \( H = [1, H] \times [1, H] \). Over time, a worker’s sector-\( i \) human capital continues to accumulate as long as she is employed in that sector, approaching \( H \) asymptotically. Also, if a worker is not employed in sector \( i \), her human capital for that sector depreciates at the rate \( \delta_h \in [0, 1) \) per period but never falls below the initial level.

The age-earnings profile implied by this functional form is consistent with the micro-estimates of life-cycle earnings growth. Murphy and Welch (1990) document the concave earning profiles with rapid initial earnings growth and a leveling off after mid-career in the US data. Menezes-Filho et al. (2008) provide a similar picture for Brazil. Note that unlike Mincer (1974) and Ben-Porath (1967) where the worker has to divide her time between production and learning, skill formation here is simply a by-product of market work.

State Space

At any point in time, a worker is either employed in a sector with match specific productivity \( z \), or unemployed. Denote these states by

\[
\ell \in \mathcal{L} = \{\ell_1(z), \ell_2(z), \ell_u\}.
\]

The state space for a worker is a collection of terms indicating her labor market state, human capital stock and generation:

\[
s_w \in \mathcal{S}_w = \mathcal{L} \times \mathcal{H} \times \mathcal{G}.
\]

A firm is either idle, or it is producing with a worker \((h, g)\) in sector \( i \) and has productivity \( z \). Next, I describe the job acceptance and vacancy creation problems.
2.2 Job Acceptance Problem

A firm-worker pair jointly decides to continue or terminate the match, depending on the value of the job and their outside options. Using time subscripts, let $\Pi_{it}(z, h_t, g)$ denote the value at time $t$ of a sector-$i$ job with productivity $z$ and a worker $(h_t, g)$. For an old worker $(g = o)$,

$$\Pi_{it}(z, h_t, o) = p_{idt} q_i(z, h_t) + \beta(1 - \delta_{JD}^o)(1 - \delta_m)I_{at+1}^o(z, h_{t+1}, o)\Pi_{it+1}(z, h_{t+1}, o),$$

(8)

where human capital level $h_{t+1}$ evolves according to the law of motion (7). The term $I_{at+1}^o(\cdot)$ is the job acceptance policy which will be defined below, and equals one if the worker-firm pair decides to continue the match next period. For a young worker $(g = y)$, the same sector-$i$ match has value:

$$\Pi_{it}(z, h_t, y) = p_{idt} q_i(z, h_t) + \beta(1 - \delta_{JD}^y)\left[\delta_a I_{at+1}^a(z, h_{t+1}, o)\Pi_{it+1}(z, h_{t+1}, o)
+ (1 - \delta_a)I_{at+1}^a(z, h_{t+1}, y)\Pi_{it+1}(z, h_{t+1}, y)\right].$$

(9)

Although match-specific productivity is fixed after the initial draw, endogenous separations are still possible. Because of the complementarity between the productivity term $z$ and human capital $h_i$ in the production function (3), a worker may accept a match, accumulate human capital and endogenously separate in order to search for a more productive job. Having $I_{at+1}^a(\cdot)$ in the job value functions captures this possibility.

The continuation values in equations (8) and (9) reflect the different life-cycle shocks faced by young and old agents. An old worker survives the period with probability $(1 - \delta_m)$. A young worker has a probability of $\delta_a$ of becoming old. Otherwise, she remains young. As a result, old agents have a higher effective discounting rate which leads to generational differences in unemployment and inter-sectoral mobility over and above of the level of human capital.

The worker’s outside option is to go back to the unemployment pool, and that of the firm is to become idle and redraw a new pair of costs within the same period. Let the values of their outside options be $W_t(\ell_u, h_t, g)$ and $J_{ut}$ respectively (see Appendix B for the derivation of these expressions).
An accepted job yields a surplus over the sum of worker’s and firm’s outside opportunities:

\[ \Delta_{it}(z, h, g) = \Pi_{it}(z, h, g) - [W_t(\ell_u, h, g) + J_{ut}]. \] (10)

If the job is accepted, the parties split the surplus by Nash bargaining with a worker share \( \sigma \in [0, 1) \). Both parties would accept the match is their share of the surplus is non-negative. Since the value of a job is monotonically increasing in \( z \), the acceptance decision has a cutoff property. For each \((h, g)\), there exists a reservation level \( \tilde{z}_{it}(h, g) \) in sector \( i \), defined by \( \Delta_{it}(\tilde{z}, h, g) = 0 \), such that worker-firm pairings with \( z \geq \tilde{z}_{it}(h, g) \) will produce output. The sectoral job acceptance policy \( T_{it}^0(z, h, g) \) is then defined by the following indicator function:

\[ T_{it}^0(z, h, g) = \begin{cases} 
1 & \text{if } \Delta_{it}(z, h, g) \geq 0, \\
0 & \text{otherwise.} 
\end{cases} \] (11)

The cutoff productivity for a sector is increasing in the human capital stock of the worker in the other sector. This is a result of the increasing value of the outside opportunity in human capital. The higher the experience of a worker in sector 1, the more productive a job in sector 2 has to be for her to give up the opportunity of searching for a job in sector 1. This behavior decreases inter-sectoral mobility as workers gain experience and specialize in a sector.\(^{12}\)

### 2.3 Vacancy Creation Problem

I will now characterize the problem of an idle firm with cost draws \((c_{1t}, c_{2t})\). Besides the matching probability, the firm takes into account the expected value conditional on matching. In order to take this expectation, the firm needs to know the distribution of human capital and generations among the unemployed. Let \( \Psi_t(h|\ell_u, g) \) denote the distribution of human capital among the unemployed of generation \( g \). The fraction of unemployed workers who are young is given by \( \nu_t(y|\ell_u) \) such that \( \nu_t(y|\ell_u) + \nu_t(o|\ell_u) = 1 \). Using the value of a match to the firm \( J_{it}(z, h, g) \) derived in Appendix B, the

\(^{12}\)Since search is undirected, specialization here means a high probability of rejecting matches in the sector in which a worker has little or no experience.
expected value of the firm conditional on being matched is

\[ EJ_{it} = \sum_{g \in \{y, o\}} \nu_t(g|\ell_u) \int_0^z \int_{H} J_{it}(z, h, g) dF(z) d\Psi_t(h|\ell_u, g). \] (12)

Taking the cost draws \((c_{1t}, c_{2t})\) and expected values of matching \((EJ_{1t}, EJ_{2t})\), an entrant creates a vacancy in sector \(i \in \{1, 2\}\) if the discounted expected net gain is greater than the value of starting next period idle:

\[ \phi_{ft} \beta [EJ_{it+1} - J_{ut+1}] \geq p_Y c_{it}, \] (13)

and it dominates entry to the other sector:

\[ \phi_{ft} \beta EJ_{it+1} - p_Y c_{it} \geq \phi_{ft} \beta EJ_{jt+1} - p_Y c_{jt}. \] (14)

Note that (13) is obtained by rearranging the condition that the expected value of posting a vacancy, \(\phi_{ft} \beta EJ_{it+1} + (1 - \phi_{ft}) \beta J_{ut+1} - p_Y c_{it}\), is greater than the value of spending the period inactive and entering next period as an idle firm. These conditions define the vacancy creation policy for a sector:

\[ I_{vit}(c_{1t}, c_{2t}) = \begin{cases} 
1 & \text{if (13) and (14) hold,} \\
0 & \text{otherwise.} 
\end{cases} \] (15)

Figure 2 shows the partition of \((c_1, c_2)\) space into the regions of entry and no entry, as implied by the policy function and the cutoff costs \(\tilde{c}_{it} = \phi_{ft} \beta (EJ_{it+1} - J_{ut+1})/p_Y\), defined by (13). The size of these regions determines the fractions \(\tilde{\mu}_{1t}, \tilde{\mu}_{2t}\) of idle firms who create vacancies in sectors 1 and 2 respectively:

\[ \tilde{\mu}_{it} = \int_{R^+} \int_{R^+} I_{it}(c_i, c_j) dF_c(c_i) dF_c(c_j). \] (16)

The remaining \(1 - (\tilde{\mu}_{1t} + \tilde{\mu}_{2t})\) fraction finds it too costly to enter and remains idle. Conditional on matching, the probability of the match being with a sector-\(i\) vacancy is thus

\[ \mu_{it} = \frac{\tilde{\mu}_{it}}{\tilde{\mu}_{it} + \tilde{\mu}_{jt}}. \] (17)
2.4 Equilibrium

Agents in this economy are heterogeneous in several dimensions. In order to define an equilibrium, I need to describe how the distribution of individual state variables evolves. Note that we only need to keep track of workers because idle firms are ex-ante homogeneous before the cost draws, and those already matched are attached to a worker with a particular state $s_w$.

To proceed, define a probability measure $\Psi_t$ on $(S_w, \mathcal{S}_w)$ where $S_w$ the state space for workers introduced above, and $\mathcal{S}_w$ is the Borel $\sigma$-algebra. For $S_w \in S_w$, $\Psi_t(S_w)$ is the mass of agents whose states lie in $S_w$ at time $t$. A transition function $\Gamma_t : S_w \times S_w \rightarrow [0, 1]$ is needed to characterize the evolution of $\Psi_t(S_w)$. The probability that a worker with individual state vector $s_w$ at $t$ will be in $S_w$ next period is $\Gamma_t(s_w, S_w)$. In Appendix C, I describe how such a transition function can be constructed from individual decision rules and stochastic processes of the model. In the following definition of the equilibrium, I use the notation $\{x_t\}$ to denote a sequence $\{x_t\}_{t=0}^{\infty}$.

**Definition 1:** An *equilibrium* for given paths of world prices $\{p_{1t}, p_{2t}\}$ and a trade policy $\{\tau_t\}$ is a sequence of value functions $\{W_t(\cdot), J_t(\cdot)\}$, decision rules $\{T_{1t}^a(\cdot), T_{2t}^r(\cdot)\}$, matching probabilities $\{\phi_{wt}, \phi_{ft}\}$, sectoral composition of vacancies $\{\mu_{1t}, \mu_{2t}\}$, unemployment rates $\{U_t\}$, domestic prices $\{p_{1dt}, p_{2dt}\}$, final good prices $\{p_{Yt}\}$, net output $\{Y_t\}$, dividend payments $\{d_t\}$, aggregate income $\{I_t\}$ and tariff revenues $\{R_t\}$, intermediate good supplies $\{Q_{1t}, Q_{2t}\}$ and demands $\{Q_{1t}^d, Q_{2t}^d\}$, and the

![Figure 2 - Sectoral Entry Decision of an Idle Firm](image-url)
distribution of workers over the state space \{\Psi_t\} such that:

a) value functions \{W_t(\cdot), J_t(\cdot)\} and associated optimal decision rules \{I_t^a(\cdot), I_t^v(\cdot)\} are the solutions to the job acceptance and vacancy creation problems described in Sections 2.2 and 2.3 respectively. When making these decisions, workers and firms take as given domestic prices, dividends, matching probabilities, the distribution of human capital among the unemployed of generation \(g\),

\[
\Psi_t(h|\ell_u, g) = \frac{\Psi_t(\ell_u, h, g)}{\int_H d\Psi_t(\ell_u, h, g)},
\]

and the fraction of unemployed workers in generation \(g\)

\[
\nu_t(g|\ell_u) = \frac{\int_H d\Psi_t(\ell_u, h, g)}{U_t}.
\]

b) Vacancy posting decisions define sectoral composition of entry \{\tilde{\mu}_1t, \tilde{\mu}_2t\} and that of vacancies \{\mu_1t, \mu_2t\} as in (16) and (17).

c) Matching probabilities are defined by (5) and (6) such that

\[
U_t = \int_{S_w} I(\ell = \ell_u)d\Psi_t(s_w),
\]

\[
V_t = (\tilde{\mu}_1t + \tilde{\mu}_2t)U_t,
\]

where \(I(\ell)\) is an indicator function that assumes the value one if its argument holds. The second line follows from the fact that the measure of idle firms is equal to the measure of unemployed workers, and only a fraction of them posts vacancies as described in Section 2.3. This defines market tightness as \(\theta_t = V_t/U_t = \tilde{\mu}_1t + \tilde{\mu}_2t\).

d) Aggregate supply of intermediate good \(i\) is obtained by aggregating the individual supply function over the distribution of workers:

\[
Q_{it}^s = \int_{S_w} q_{it}(s_w) \, d\Psi_t(s_w),
\]

where \(q_{it}(s_w) = A_i z h_i\) if \(s_w = (l_i(z), h, g)\) for \(g \in \{g, o\}\), and zero otherwise.
e) Tariff revenue on good 2 imports, \( R_t = \max\{\tau_t p_{2t} (Q^d_{2t} - Q^s_{2t}), 0\} \), is rebated in a lump-sum fashion, and aggregate income is

\[
I_t = p_{1dt} Q^s_{1t} + p_{2dt} Q^s_{2t} + R_t. \tag{18}
\]

All income is spent on purchasing the final good in the market, which generates the demand for intermediate goods:

\[
Q^d_{1t} = \frac{\gamma I_t}{p_{1dt}}, \quad Q^d_{2t} = \frac{(1 - \gamma) I_t}{p_{2dt}}. \tag{19}
\]

f) Final goods market clears with the price determined competitively by (2),

\[ p_{Yt} Y_t = I_t. \]

g) \( \Psi_t \) is a probability measure that evolves according the transition function \( \Gamma_t \):

\[
\Psi_{t+1}(S) = \int_{S_w} \Gamma_t(s_w, S) d\Psi_t(s_w). 
\]

In words, the distribution evolves consistently with the decision rules, exogenous job destruction shocks, labor market flows, productivity draws for new matches, the law of motion for human capital accumulation (7), and demographic shocks.

h) By Walras Law, trade balance condition hold. Defining net exports of good i by \( NX_{it} = Q^s_{it} - Q^d_{it} \), and using equations (18) and (19), one can derive

\[ p_{1t} NX_{1t} + p_{2t} NX_{2t} = 0. \]

A *steady state equilibrium* is a special case in which all aggregate variables are constant, policies are time-invariant and there is a stationary distribution \( \Psi \) that replicates itself every period. In Section 4.1, the steady state equilibrium concept will help us to calibrate the model to the pre-reform data from Brazil. In Section 4.3, I will characterize the equilibrium transition path after an unexpected and permanent change in trade policy parameter \( \tau \).
2.5 Discussion

The undirected job search by workers and the entry process for firms are important components of the model that deserve further discussion. First, I assume that all workers enter a common pool when searching, as in Alvarez and Veracierto (1999) and Acemoglu (2001). The alternative approach, directed search, assumes that workers can locate the sector of their choice. In that case, either labor markets within sectors are competitive and all unemployment is due to workers in transit (Lucas and Prescott (1974)), or the matching processes function separately (Hosios (1990b) and Helpman and Itskhoki (2010)). Under either interpretation, directed search implies an extreme selectivity where agents receive no information about jobs in the other industry. Between these two polar cases of directed and undirected search, Moscarini (2001) offers a model in which heterogeneous workers with sector-specific skills decide to search selectively or randomly depending on their comparative advantage. The matching process in my model is similar to the case of random search there. Workers receive offers from both sectors which they can accept or reject.

Second, the vacancy creation process helps to render the model economy diversified by introducing a curvature into firms’ entry decision. Some entrants will draw a low vacancy creation cost for the comparative disadvantage sector. A subset of such vacancies will match with unemployed workers because search is undirected. In order to ensure diversification in equilibrium, firms should expect a positive mass of these matches to be accepted. This requires a positive measure of unemployed workers to have a reservation productivity below $\tilde{z}$ (the upper bound on productivity draws) in the comparative disadvantage sector. A sufficient condition is that newborns have a reservation productivity $\tilde{z}_2(b, y)$ lower than $\tilde{z}$ in the import competing sector. If, for a given set of parameters, the relative productivity of sector 1, $A_1/A_2$, is below a certain level, this condition will hold. I assume that this restriction is satisfied to ensure diversification at the initial prices.

Is this a plausible model of human capital? Kambourov and Manovskii (2009) argue that human capital is more occupational than sectoral. Entertaining such a view of human capital does not require a drastic change in the model: one could rewrite the production functions of the two sectors as using two occupational inputs with different intensities (For example, electronics industry is more intensive in engineers and food products are more intensive in bakers and butchers). Trade liberalization would still lead to shifts in relative returns to occupational human capital. If sectoral intensities are
different enough, returns to experience in an occupation would be very low in the sector which is not intensive in that occupation, making the model essentially identical to one presented here. This would also generalize the model to account for some transferability of experience between sectors. The quantitative results presented in the next section should thus be interpreted as an upper bound for the role of sector specific human capital.

3 Background Information on Brazilian Reforms

This section intends to give a brief summary of Brazilian trade and labor market reforms between 1988 and 1991. The timing of these policy changes, together with the availability of aggregate and micro data moments, makes Brazil a suitable environment for a quantitative application of the model.

Trade Reforms   After years of pursuing an import-substitution policy, Brazil underwent a big trade liberalization between 1988 and 1991. As Figure 3 reveals, the reforms substantially lowered average tariffs. What is not evident in the figure is the removal of all non-tariff barriers in 1991 under the Collor Plan. As a result, import penetration has steadily increased. Furthermore, reforms changed the structure of tariffs across industries. Figure 4 plots tariff rates on fifty-three mining and manufacturing industries before and after the liberalization. The high variation in the pre-reform period and the subsequent harmonization indicate a big change in relative domestic prices across industries. Moreover, Pavcnik et al. (2004) cite evidence that the initial tariff structure granted higher protection to industries where Brazil had low comparative advantage. One would thus expect a substantial reallocation of resources between industries as a response to liberalization. Figures 5 and 6 show that this was not the case. The industry composition of the manufacturing labor force in 1995 was quite similar to the pre-reform period. One of the objectives of the paper is to explain this inertia.

Labor Market Reforms   Trade liberalization was not the only big policy change in Brazil at that time. The country also instituted an unemployment insurance in 1986, and increased its coverage as part of a broader labor market reform in 1988. Other programs initiated by that reform included

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13 According to Menezes-Filho and Muendler (2011), although tariffs decreased gradually starting with late 1980s, the removal of binding non-tariff barriers happened in the first day of the Collor administration. In that sense, Brazilian trade liberalization can be considered as a sudden and unexpected policy change.

14 For further details on Brazilian trade policy, see Pavcnik et al. (2004).
an employment subsidy and active labor market policies such as training, and job search assistance.

The unemployment insurance is paid to claimants who worked in the formal sector within the last six months. The duration of benefits varies between three to five months depending on seniority, and the replacement rate is around 50% of the average wage prior to unemployment. According to Cunningham (2000), the program coverage increased significantly in 1990 when eligibility criteria were relaxed. As of 1990, 43% of workers who had been laid off from formal sector jobs were covered. The employment subsidy program, Abono Salarial (salary bonus), is similar to the US Earned Income Tax Credit in that the government makes a transfer to workers with earnings below a certain threshold. According to Paes de Barros et al. (2006), 5% of the workforce was receiving this wage supplement in 1997.

These programs are financed by a special 0.65% tax levied on firms’ revenues (FAT: Fundo de Amparo ao Trabalhador; Workers Protection Fund). According to Berg et al. (2006), total cost of these policies amounted to around 1% of GDP in 1995. Expenditure on unemployment insurance and employment subsidies constituted roughly 70% and 15% of total governmental labor market expenditures respectively, with the rest going to training programs. Because of the dominant expenditure share of unemployment insurance, I will consider the actual policy change to be a simultaneous lowering of tariffs and introduction of unemployment insurance. Of particular interest will be the comparison of the transition after the actual policy change with the outcomes of counterfactual labor market policies accompanying trade liberalization. In order to run these simulations, I first quantify the model in the following section.

4 Quantitative Analysis

4.1 Calibration

This section calibrates the model to Brazilian data in the pre-reform period. Since labor market reform was only legislated in 1988 and became effective in early 1990s, the only distortion in the calibrated model is the import tariff. After a discussion of steady state outcomes, the next section will then analyze the transition to the new steady state when trade is liberalized.

15 These criteria include employment in the formal sector prior to dismissal and payment of insurance premium for a minimum period.

16 Berg et al. (2006) provides further details about Brazilian labor market policies
I calibrate the model in two stages. In the first stage, I set the parameters that can be identified without solving the model. In the second stage, I pin down the remaining parameters by matching five model-generated moments to data. The algorithm used to solve for the steady state is described in Appendix D.

**Parameters Chosen Without Solving the Model**

Table 1 summarizes the parameters that I set either by modeling choice, normalization, or by matching direct empirical counterparts. The model period is a quarter. *Bosch and Maloney (2007)* report an average unemployment duration of 5.76 months (their Table 3b and Figure 2), so a quarterly frequency is enough to capture the transitions in the Brazilian labor market.

Newborn workers have an expected life span of 40 years. They expect to be young for the first half of that time. This implies $\delta_a = \delta_m = 1/80$. The time discount parameter is set as $\beta = 0.97$ to match an average quarterly real interest rate of 3.1% between 1995-2009, a financially stable period compared to the hyperinflationary episodes before 1995. The real interest rate is a quarterly aggregate of the monthly government primary rate (SELIC) minus the quarterly rate of change in the consumer price index (INPC) obtained from IPEADATA (www.ipeadata.gov.br).

I normalize international prices and the productivity of sector 2 by setting $p_1 = p_2 = A_2 = 1$. The tariff rate $\tau$ is equal to the pre-reform average of 0.63 reported by *Pavcnik et al. (2004)*. The two intermediate goods are assumed to be used with equal intensity in the production of the final good, hence $\gamma = 0.5$. Match-specific productivity draws are uniformly distributed between 0 and 1. Another normalization is the initial level of human capital, set as $(h_1, h_2) = (1, 1)$. Following the survey evidence by *Browning et al. (1999)*, depreciation of human capital is set as $\delta_h = 0$. Workers and firms are assumed to split the rents equally which implies $\sigma = 0.5$.

Separation rates from employment are directly observed from labor market flows in Brazil. The average quarterly separation rate as a fraction of employment in the formal manufacturing sector between 1982 and 1988 is reported as 1.5% by *Hoek (2007)*. Across age groups, workers between ages 24–40 have a separation intensity from formal manufacturing towards unemployment that is 50% higher than those between ages 40–60 according to *Bosch and Maloney (2007)*. I thus set $(\delta_y^y, \delta_y^o) = (0.018, 0.012)$.

**Parameters Obtained by Solving the Model**

To proceed, a functional form has to be cho-
sen for $F_c(c)$, the distribution of vacancy creation costs. As discussed in Section (2.3), this distribution determines the measure of vacancies and hence labor market tightness. Market tightness in turn affects the job finding rate. Since this moment is the only source of discipline for $F_c(c)$, its mean and variance are not separately identified. I thus assume that vacancy creation costs are log-normally distributed with mean zero and standard deviation $C_{sd}$. The first two parameters are the curvature and the upper bound for the skill formation process (7) respectively. $A_1$ is the aggregate productivity of the comparative advantage sector. $\lambda$ is the elasticity parameter in the matching function (4).

The data moments used to calibrate these parameters are summarized in Table 2. The two parameters governing the accumulation of human capital are calibrated to match two moments of the age-earning profile in 1990 reported by Menezes-Filho et al. (2008). Male workers with 5 years of labor market experience earn 41% more than their starting wages on average. This moment pins down the curvature parameter of the skill formation function as $\alpha = 0.974$. Experience of 40 years implies an average gain of 2.43 times the starting wage. This moment is informative for calibrating $H = 2.6$.

The productivity of sector 1 is calibrated as $A_1 = 1.71$ to match the average export/(value added) ratio in formal manufacturing between 1987-1990. The data moment is calculated using the time series of manufacturing export/output ratio reported by Pavcnik et al. (2004) and manufacturing (value added)/output ratio obtained from the Brazilian Input-Output tables published by OECD (2006). Unfortunately, there is no time series on the value added/output ratio for Brazil for the entire period. The IO tables are only available for 1995, 1996 and 2000. However, they all yield similar values. I assume that the average of these values, 33%, applies to the pre-reform period as well. An average exports/output ratio of 9.2% divided by the value added/output ratio yields the data moment as 26.3%.

The matching function elasticity $\lambda$ and the standard deviation $C_{sd}$ of vacancy cost distribution are calibrated using two moments. First, the elasticity of new matches to unemployment in Brazil is estimated as 0.25 by Hoek (2007). In the model, this implies the following relationship:

$$\frac{\partial m(U,V)/m}{\partial U/U} = 1 - (\theta^\lambda + 1)^{-1/\lambda} = 0.25$$

If we had an estimate of market tightness for Brazil, this equation would determine $\lambda$. Although
there is no such estimate that I am aware of, market tightness in the model is equal to the fraction of idle firms who create a vacancy in each quarter. This moment, in turn, is driven by $C_{sd}$. The second moment I target is the job finding rate in the formal manufacturing sector. According to Bosch and Maloney (2007), the quarterly transition probability from unemployment to formal employment is 0.375 between 1987 and 1991—see Figure 7, top right quadrant in their paper. In the model, this is equal to $a \phi_w$ where $a$ is the job acceptance ratio. The two parameters $(\lambda, C_{sd})$ are calibrated to match the elasticity reported above and the job finding probability. This gives us a value for $\lambda = 2.16$ and $C_{sd} = 1.49$.

### 4.2 Steady State Results

Table 3 compares the steady states outcomes of the model to data. The model generates declining sectoral mobility over the life-cycle. Kambourov and Manovskii (2008) document that industry mobility declines with age in the US data. In 1997, the probability of moving between two-digit industries is 30% for non-college educated workers between ages 23-28. For the age group 47-61, the probability falls to 4.8%. In my model, 40% of young workers separating from their jobs switch sectors if they are re-employed within a period, compared to a 5% for old workers. Although there is no empirical study of age-related mobility in Brazil, I expect it to be qualitatively similar to the US evidence. With that qualification in mind, one can say that the model is able to generate the steep decline of mobility over the life-cycle.

A related outcome is the difference in the unemployment rates within generations. In the model, 70% of those who are unemployed are young. This implies a higher than average youth unemployment rate since population shares of the two generations are equal. Note that this is partly due to the higher exogenous separation rate $\delta^y_{JD} > \delta^o_{JD}$. However, the two generations also differ in the job acceptance cutoffs. There are two opposing incentives for the young. They have a lower discount rate, which makes them more willing to tolerate unemployment and search for productive jobs. On the other hand, they forgo learning when unemployed. The net effect is thus ambiguous. With the calibrated values, the cutoff productivity is higher for the young at all levels of $h$. In other words, not only are flows out of employment larger but flows into employment are lower as well. As a result, the ratio of youth unemployment rate to old unemployment rate is 2.4 in the model. In the data, unemployment rate is 3.64 (2.61) times higher among males aged 15-24 compared to males aged 50-64 (25-49) in
Turning to post-separation wage changes, old workers who switch industries experience an average wage loss of 11.6%. The wage drop for old workers who find employment in the same sector is 2.25% only. The average percentage wage change between two subsequent jobs for young workers is 0.51%. Micro level evidence on wage dynamics related to sectoral switching in Brazil is scant. Comparing the wages of Brazilian workers changing jobs involuntarily by going through unemployment, Hoek (2006) finds that switches are associated with an average earnings loss of 23% (Panel A, Column 8 in Table 1). This figure, however, is not controlling for selection based on worker characteristics which could affect the probability of involuntarily separating from one’s job, and thus should be considered as an upper bound on the isolated effect of the human capital.

The model does a good job of matching statistics related job turnover. Because there is no net employment change across sectors in steady state, all reallocation is excess job reallocation. In the model, yearly job destruction rate is 17% which implies a turnover rate of 34%. According to IDB (2004), average job turnover in Brazil between 1987 and 1992 is 32%.

The next section investigates the transitional dynamics of a trade reform using the calibrated model.

4.3 Policy Experiments

This section first replicates the actual policy package in Brazil: simultaneous reduction in tariffs and introduction of unemployment insurance. Two counterfactual policy experiments follow. First, I liberalize trade but do not implement any income support program (Counterfactual I). The comparison with the actual policy experiment indicates that the introduction of unemployment insurance jointly with trade reforms might have affected transitional dynamics adversely in Brazil by hampering the reallocation of labor. Counterfactual I is also useful for comparing the transitional dynamics of the model with and without human capital. The results suggest that the dominant barrier to reallocation is sector-specificity of human capital. I then ask whether a different policy could have compensated the losers and facilitated faster reallocation at the same time. Counterfactual II proposes a targeted employment subsidy geared towards encouraging workers to move to the expanding sector. This policy yields a positive outcome in terms of compensation and aggregate output. All transition paths are

\[17\] Data source is Table 11 in the data appendix CD-Rom of IDB (2004).
solved with a numerical algorithm similar to Costantini and Melitz (2009) described in Appendix D.

4.3.1 The Actual Policy: Trade Liberalization with Unemployment Insurance

Let the economy initially be in the steady state which it is calibrated in Section 4.1 with tariff rate $\tau_h$. In period 0, an unexpected and permanent liberalization lowers the tariff to $\tau_l < \tau_h$, and the government announces that it will tax firms’ revenues in the intermediate good sectors by 1% to finance a unemployment insurance program. Tax revenues are equal to labor market expenditures every period. The equilibrium definition now includes a sequence of unemployment benefits $\{b_t\}_{t=0}^{\infty}$ taken as given by agents, tax revenues equal to

$$G_t = 0.01 \times \int_{s_w} p_{idt} q_{it}(s_w) d\Psi_t(s_w),$$

and balance budget condition

$$G_t = b_t U_t \quad \text{for all} \quad t \in \{0, 1, 2, \ldots\}.$$

Associating the date of reform with the year 1991, I choose $\tau_l$ such that the export/value added ratio matches the data in 1997. Terms of trade effects and aggregate shocks after 1998 move the data in ways that the model cannot capture. I thus confine the data comparison to the 1991-1997 time period. The short-run transitional dynamics of openness and unemployment are depicted in Figures 7 and 8. With the announced policy changes, the economy starts to move towards a new steady state. The initial response of export/output ratio is matched well in the model, but the data shows some business cycle fluctuations which I abstract from. The model does a good job of capturing the unemployment dynamics in the short run. Unemployment overshoots right after the reform. The drop in the value of human capital in the previously protected industry, together with the increase in the outside option for workers (due to unemployment benefits), leads to the termination of some matches accepted before liberalization. There is a mismatch between the skill demand and supply until the economy gradually adjusts to the change. I leave the discussion of the long-run to next subsections in order to compare

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Note that this tax rate is equal to the cost of social safety net policies as a fraction of the GDP in Brazil. It does not matter whether the tax is levied on the final good producer or intermediate good producers. The representative final good producer will pass the incidence of the tax to the consumer and the real value of a match will be the same as when the intermediate good sector is taxed.
it to counterfactual outcomes.

4.3.2 Counterfactual I: Trade Liberalization with No Labor Market Reform

How would the economy respond if Brazil liberalized its trade regime but left labor market policies unchanged? To address this question, I repeat the experiment without unemployment insurance. Figure 10 compares the net output during the transition in response to this counterfactual policy with its path under the actual policy solved above. Several results emerge. In the actual policy path, there is an initial dip in net output in line with the overshooting in unemployment which we do not see in the counterfactual scenario with no unemployment insurance. In the latter case, the flows in and out of employment are enough to accommodate the necessary reallocation in the short run. The message is similar to Ljungqvist and Sargent (1998) who analyze the impact of the welfare state under structural change: unemployment compensation hinders the adjustment of an economy to large shocks. Second, unemployment insurance eats away the gains from trade liberalization. The initial drop in output is followed by convergence to a lower steady state level. Finally, transition is very lengthy under both scenarios. The skill stock of the economy adjusts as more members of newborn cohorts accumulate human capital in the export-oriented sector. In the counterfactual experiment, it takes around 80 years, or two generations, for the economy to get close to its new steady state. The model is thus able to explain the low impact of trade reforms on liberalizing countries in the short run. This inertia is caused by two barriers to instantaneous adjustment: search frictions and sector-specificity of human capital.

In order to further explore the quantitatively dominant mechanism behind the sluggishness of adjustment, I solve the model without human capital by setting $\alpha = 1$ and keeping other parameters the same. The two economies have different steady states. To facilitate comparison, Figure 9 plots a normalized measure of reallocation completed at each point in time during the transition. Half of the overall reallocation towards the new steady state is completed in 6 years in the absence of human capital. Search frictions alone predict a fast reallocation. With search frictions and human capital, the half-life is 34 years, almost a generation’s life-time in the model. As new cohorts enter the workforce and accumulate human capital in the comparative advantage sector, and as the initial cohort phases out, aggregate human capital stocks adjust. This in turn affects the composition of vacancies since firms’ entry decisions take into account the aggregate stocks of human capital in each sector. Sector-
specific skills resemble a form of capital which depreciates very slowly, and takes a long time to build. The combination of finite life-times and experience is thus a powerful mechanism that slows down the adjustment of the economy.

This result is similar to the finding by Alvarez and Shimer (2011) that search frictions alone cannot explain unemployment arising from inter-industry shifts in U.S. data without assuming an unreasonably large cost of moving. Sector-specific human capital acts a barrier to mobility since it increases the opportunity cost of switching sectors for workers.

4.3.3 Counterfactual II: Trade Liberalization with a Targeted Employment Subsidy

I now turn to the analysis of an alternative policy that compensates losers of liberalization while inducing them to work in the expanding sector. The motivation for this policy stems from the increasing interest in compensation policies that reward work. Examples that are not targeted include the Earned Income Tax Credit in the United States, the Working Tax Credit in the United Kingdom, “Prime Pour l’Emploi” in France and the “Abono Salarial” in Brazil. The US Alternative Trade Adjustment Assistance (ATAA) targets trade-displaced workers and provides a wage subsidy that pays 50% of the difference between worker’s old and new wages up to $10,000 for two years.

In this scenario, the government announces an employment subsidy \( \{\eta_t\}_{t=0}^\infty \) simultaneously with trade liberalization. The policy has three features:

i. It is targeted at the initial old employed in sector 2 (previously protected industry) at the time of liberalization.

ii. It is conditional on mobility: it is paid for sector 1 jobs (export-oriented industry) only.

iii. It has limited duration: it is implemented for 20 years only (80 model periods are equivalent to the expected life of the initial old). In other words, \( \eta_t = 0 \) for \( t > 80 \).

The policy is implemented in the following way. Since it has limited duration, the tax rate is set such that it declines linearly from 1% to zero in 80 periods.\(^{19}\) Again, the government runs a balanced budget every period. The policy redistributes income from the beneficiaries of trade (those with experience in sector 1) to losers of trade (workers experienced in sector 2). It turns out that it also brings about

\(^{19}\)Otherwise, firms postpone entry as the economy get closer to \( t = 80 \), and the economy contracts before the phasing out of the policy. A gradual decline in the tax rate avoids this kind of behavior.
gains in aggregate welfare. Figure 11 plots net output during the transition in comparison with the outcome of Counterfactual I (no income support program). After six quarters, net output with the employment subsidy overtakes that under the Counterfactual I and converges to the new steady state at a faster rate. Note that after the phasing out of the employment subsidy, both policies converge to the same long-run value. Remarkably, employment subsidies make the transition path more concave with faster initial adjustment. The discounted value of net output stream under Counterfactual II with employment subsidies is 4.17% higher than its value under the actual policy with unemployment insurance, and 0.61% higher than under Counterfactual I with no income support. By encouraging inter-sectoral mobility, this policy strengthens the feedback loop between skill formation by workers and entry by firms, and thus speeds up reallocation.

The source of inefficiency that calls for a public policy is a learning externality between workers and future employers similar to Acemoglu (1997). The interaction of rent-sharing due to search frictions, intra-sectoral transferability of human capital and the impossibility of contracting with future employers gives rise to a market failure in which workers underinvest in learning. When a firm and a worker form a productive match, they generate a positive externality for potential future employers of the worker: on-the-job learning adds to the stock of sectoral human capital which increases the value of entry to that sector. Workers cannot contract with potential future employers who will benefit from their recent learning through rent-sharing. Neither of the parties in an ongoing match fully internalizes the returns to the skill formation. The resulting inefficiency is likely to be particularly costly when the economy is adjusting to a change in relative prices across sectors because labor reallocation requires an investment in learning by workers.\textsuperscript{20} During transition, workers displaced from the import-competing sector underinvest in skill formation by rejecting some matches in the expanding sector with low starting wages but learning prospects. The employment subsidy is essentially a subsidy for investment in human capital. The returns to such an investment are especially high during the transition period because the skill mix of the economy is very different than its long run value.

Could unemployment insurance ever be welfare improving in this economy? The risk-neutrality of agents assumes away potential gains. Models of optimal unemployment insurance emphasize another

\textsuperscript{20}Note standard search externalities are also present: entry and exit of workers and vacancies into the matching process affect matching probabilities of the other participants. The Hosios condition which ensures efficiency in search models (Hosios (1990a)) does not hold here because the elasticity of the matching function (4) is not constant. As a result, there might be deviations from optimality resulting from search externalities.
source of welfare gains under risk aversion (e.g., Acemoglu and Shimer (1999)). When matches have heterogeneous productivity, risk-averse agents are more likely to accept low productivity jobs when they are liquidity constrained in order not to hit the zero consumption bound. By keeping agents' consumption away from zero, unemployment benefits enable them to search for more productive jobs. Although there is match-specific productivity in my model, there is no risk-aversion. However, one cannot rule out the possibility that agents would be underinvesting in searching for more productive jobs under some parameter values because of rent-sharing. On the other hand, the moral hazard effect is present. Here, it not only reduces employment but the stock of skills as well. The overall effect is thus ambiguous. Also note that I do not claim that the above employment subsidy scheme is the optimal policy. Finding the optimal policy for this environment is beyond the focus of this paper, but it is an interesting open question. The main point of my exercise is to demonstrate the potentially beneficial role for employment subsidies in an economy suffering from skill mismatches while adjusting to a major reallocative shock.

5 Conclusion

I develop and solve a two-sector small open economy model of equilibrium search with overlapping generations and sector-specific human capital in order to analyze inter-sectoral reallocation of labor after trade reforms. Modeling choices are motivated by the evidence that reallocation is very sluggish, it is costly for displaced workers, and these costs are increasing with age. I calibrate the model using aggregate and micro moments of the Brazilian economy before the trade liberalization of 1991. This helps me to perform policy experiments on trade and labor market reforms undertaken simultaneously in Brazil.

The first contribution of the paper is to discern the role of search frictions from sector-specificity of human capital in accounting for the slow pace labor reallocation. A comparison of the transition paths of the model with and without human capital suggests human capital is a much bigger barrier to mobility then search frictions. This result indicates the limitations to the often cited policy prescription that flexible labor markets are key to rapid restructuring after reforms, and hence active labor market policies such as job search assistance could be helpful. If the dominant source of slow adjustment is the disincentives of mid-to-old age workers to accept jobs in new sectors, policies aimed
at matching them with employers will have low returns. The second contribution of the paper is the analysis of a particular policy that addresses these disincentives. Performing a counterfactual policy experiment, I find that a targeted employment subsidy paid to workers experienced in the shrinking sector conditional on employment in the expanding sector not only compensates their welfare loss, but also increases aggregate net output. The market failure behind this result is underinvestment in human capital of the comparative advantage sector. Because of search frictions, workers split their human capital rents with their employers. Not being a full claimant of their investment, they accept sub-optimally low number of jobs. A policy that rewards work and mobility mitigates this market failure. Compensation policies such as the wage insurance under the U.S. Alternative Trade Adjustment Assistance (ATAA) and Reemployment Trade Adjustment Assistance (RTAA) constitute a viable alternative to unemployment insurance or retraining when dealing with the effects of globalization. In contrast to the policy experiment in this paper, ATAA/RTAA are not conditional on the sector of reemployment. Plausibly, there are informational limitations to a government’s ability in choosing the sectors in which employment should be subsidized. Whether public policy can be improved in that direction is an open question.

Finally, the model can be applied to study other instances of sectoral price shifts, such as technological change. The decline of manufacturing and the rise of the service sector is one example. Such structural change, however, is more secular in nature and agents have a longer time horizon to adjust. One can thus expect the room for policy to be smaller.
References


Appendices

A Figures and Tables

Figure 3 – Trade Policy and Openness
Source: All series are simple annual averages for Brazilian mining and manufacturing sectors by Classification Nivel 80 compiled by Marc Muendler, data files tariffs-outp.csv and imp-penetrat.csv are available in http://www.econ.ucsd.edu/muendler/html/brazil.html.

Figure 4 – Industry Level Tariff Rates Before and After Trade Reforms
Figure 5 – Manufacturing Workforce Composition across 3-digit Industries in Brazil
The sub-figures plot the employment share of industries at the 3-digit level of ISIC (Rev.2) at different years. Source: UNIDO INDSTAT3 2005

Figure 6 – Manufacturing Workforce Composition across 3-digit Industries in Brazil
Figure 7 – Openness during the Transition
Source: Openness in the data is measured as \( \frac{\text{Exports}}{\text{Output}} \cdot \frac{\text{Output}}{\text{Value Added}} \) for manufacturing. Data on export share of manufacturing output is from Pavcnik et al. (2004). The value added share of manufacturing output is from OECD (2006). The model unemployment rate is the yearly average over quarters.

Figure 8 – Unemployment During the Transition
Source: ILO-KILM 2009, male unemployment rate. The values for 1991 and 1994 are missing in the original data. The initial level of unemployment in 1990 is set as the average for 1981-1990 which is equal to 3.6%. For 1994, I report the average of 1993-1995 values. The model unemployment rate is the yearly average over quarters.
The lines plot the amount of reallocation completed at each period as a percentage of total share reallocation between initial and terminal steady states. Specifically, let $E_t^1$ be the share of employment in sector 1 at time $t$, $E_0^1$ and $E_T^1$ the initial and terminal shares respectively. Transition paths plot $100 \cdot \frac{E_t^1 - E_0^1}{E_T^1 - E_0^1}$. 

**Figure 9 – Employment Share Reallocation During the Transition**

The lines plot the amount of reallocation completed at each period as a percentage of total share reallocation between initial and terminal steady states. Specifically, let $E_t^1$ be the share of employment in sector 1 at time $t$, $E_0^1$ and $E_T^1$ the initial and terminal shares respectively. Transition paths plot $100 \cdot \frac{E_t^1 - E_0^1}{E_T^1 - E_0^1}$. 

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Figure 10 – Net Output under Unemployment Insurance

Figure 11 – Net Output under Targeted Employment Subsidy
### Parameter Definition Value Source/Target

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Definition</th>
<th>Value</th>
<th>Source/Target</th>
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</thead>
<tbody>
<tr>
<td>$\delta_m$</td>
<td>death probability</td>
<td>$1/80$</td>
<td>20 years of youth</td>
</tr>
<tr>
<td>$\delta_a$</td>
<td>aging probability</td>
<td>$1/80$</td>
<td>20 years of old-age</td>
</tr>
<tr>
<td>$f_z(z)$</td>
<td>productivity</td>
<td>uniform $[0,1]$</td>
<td>-</td>
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<tr>
<td>$\gamma$</td>
<td>Cobb-Douglas share of good</td>
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<td>-</td>
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<tr>
<td>$\sigma$</td>
<td>worker’s bargaining share</td>
<td>0.50</td>
<td>-</td>
</tr>
<tr>
<td>$h_i$</td>
<td>initial HC level</td>
<td>1</td>
<td>normalization</td>
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<tr>
<td>$A_2$</td>
<td>sector 2 productivity</td>
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<td>normalization</td>
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<td>$\tau$</td>
<td>import tariff</td>
<td>0.63</td>
<td>Pavenik et al. (2004)</td>
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<tr>
<td>$\beta$</td>
<td>discounting rate</td>
<td>0.97</td>
<td>real interest rate, IPEA</td>
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<tr>
<td>$\delta_{JD}^y$</td>
<td>job destruction for young</td>
<td>0.018</td>
<td>Bosch and Maloney (2007)</td>
</tr>
<tr>
<td>$\delta_{JD}^o$</td>
<td>job destruction for old</td>
<td>0.012</td>
<td>Bosch and Maloney (2007)</td>
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<tr>
<td>$\delta_h$</td>
<td>depreciation of HC</td>
<td>0</td>
<td>Browning et al. (1999)</td>
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**Table 1 – Parameters Set Without Solving the Model**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Target</th>
<th>Source</th>
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<tr>
<td>$\alpha$</td>
<td>0.98</td>
<td>ave. earnings at 5 years of experience</td>
<td>Menezes-Filho et al. (2008)</td>
</tr>
<tr>
<td>$H$</td>
<td>2.60</td>
<td>ave. beginning of the career earnings</td>
<td>Menezes-Filho et al. (2008)</td>
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<td>$A_1$</td>
<td>1.71</td>
<td>export / (value added)</td>
<td>Pavcnik et al. (2004), OECD (2006)</td>
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<tr>
<td>$\lambda$</td>
<td>2.16</td>
<td>elasticity of hiring to unemployment</td>
<td>Hoek (2007)</td>
</tr>
<tr>
<td>$C_{sd}$</td>
<td>1.49</td>
<td>transition probability from U to E</td>
<td>Domeland and Foss (2006)</td>
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</table>

**Table 2 – Parameters Obtained by Solving the Model**

<table>
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<th>Moment</th>
<th>Model</th>
<th>Data</th>
<th>Source</th>
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</thead>
<tbody>
<tr>
<td>share of youth unemployment</td>
<td>0.71</td>
<td>0.71</td>
<td>IDB (2004), ILO KILM Database</td>
</tr>
<tr>
<td>annual excess job reallocation</td>
<td>0.34</td>
<td>0.32</td>
<td>IDB (2004)</td>
</tr>
<tr>
<td>earning losses of old switchers</td>
<td>0.12</td>
<td>0.22</td>
<td>Hoek (2006)</td>
</tr>
<tr>
<td>hiring costs / wage bill</td>
<td>0.09</td>
<td>0.09</td>
<td>Abowd and Kramarz (2003) – blue-collar workers in France</td>
</tr>
</tbody>
</table>

**Table 3 – Steady State Outcomes**
B Value Functions

The value of a match to the worker is
\[
W_t[\ell_i(z), h_t, g] = \max_{\text{accept, reject}} \left\{ \sigma \Delta_{it}(z, h_t, g) + W_t[\ell_u, h_t, g], W_t[\ell_u, h_t, g] \right\},
\]
(20)

Similarly, the value of a match in sector \( i \) to the firm is given by
\[
J_{it}(z, h, g) = \max_{\text{accept, reject}} \left\{ (1 - \sigma) \Delta_{it}(z, h_t, g) + J_{ut}, J_{ut} \right\},
\]
(21)

where the match is with a worker in state \((h, g)\) and the productivity draw is \( z \). The solutions to these two problems agree: only matches with a positive surplus are accepted, giving rise to the job acceptance policy function (11). I now specify \( W_t[\ell_u, h, g] \), the value of unemployment to the worker, and \( J_{ut} \), the value of being idle to the firm.

B.1 Value of Unemployment to the Worker

All workers own balanced portfolios of firms and receive a dividend payment of \( d_t \) units of the final good at any period. The value of unemployment \((\ell = \ell_u)\) for an old worker is:
\[
W_t(\ell_u, h_t, o) = p_{Yt}d_t + \beta(1 - \delta_m) \left[ \sum_{i=1}^{2} \phi_{w_i t} \int_{0}^{\infty} W_{t+1}(\ell_i(z), h_{t+1}, o)f_z(z)dz \right.
\]
\[
+ (1 - \phi_{w1 t} - \phi_{w2 t})W_{t+1}(\ell_u, h_{t+1}, o) \Big]\right].
\]
(22)

For a young worker:
\[
W_t(\ell_u, h_t, y) = p_{Yt}d_t + \beta(1 - \delta_a) \left[ \sum_{i=1}^{2} \phi_{w_i t} \int_{0}^{\infty} W_{t+1}(\ell_i(z), h_{t+1}, y)f_z(z)dz \right.
\]
\[
+ (1 - \phi_{w1 t} - \phi_{w2 t})W_{t+1}(\ell_u, h_{t+1}, y) \Big]
\]
\[
+ \beta \delta_a \left[ \sum_{i=1}^{2} \phi_{w_i t} \int_{0}^{\infty} W_{t+1}(\ell_i(z), h_{t+1}, o)f_z(z)dz \right.
\]
\[
+ (1 - \phi_{w1 t} - \phi_{w2 t})W_{t+1}(\ell_u, h_{t+1}, o) \Big]\right].
\]
(23)

B.2 Value of Being Idle to the Firm

The value function of an idle firm before drawing the vacancy posting costs for period \( t \) depends, among other things, on the average expected costs conditional on successful entry, \((\hat{c}_{1t}, \hat{c}_{2t})\). These are defined as:
\[
\hat{c}_{it} = \int_{R_+} \int_{R_+} c_i I_{it}(c_i, c_j)dF(c_i)dF(c_j).
\]
(24)
Given the expected value of jobs conditional on matching \( (EJ_{t+1}', EJ_{2t+1}) \), and entry probabilities \( (\bar{\mu}_1, \bar{\mu}_2) \),

\[
J_{ut} = \sum_{i=1}^{2} \phi_{it} \beta EJ_{it+1} + (1 - \phi_{it}) \beta J_{ut+1} - p \gamma \tilde{e}_{it} + (1 - \bar{\mu}_1 - \bar{\mu}_2) \beta J_{ut+1}.
\] (25)

Note that the values of all potential outcomes are discounted since it takes one period for new matches to be effective. The large parenthesis represents the expected value of vacancy posting. The last term represents the case in which the firm does not enter at all.

\section*{C Transition Function for the Distribution}

Define a probability function \( \Gamma_t : S_w \times S_w \rightarrow [0, 1] \) such that \( \Gamma_t(s_w, s'_w) \) is the probability of a worker in state \( s_w \) to be in state \( s'_w \) next period. Note that the state variable \( s_w \) is a vector \( (\ell, h, g) \) which summarizes the labor market state, human capital stock and the generation of a worker. If the worker is matched in sector \( i \) with productivity \( z \), it has the form \( (\ell_i(z), h, g) \). A generic element for an unemployed worker is \( (\ell, h, g) \).

Some transitions are infeasible in this environment. For example, an old worker with \( (\ell, h) \) can not become a young worker with \( h' \geq h \). On the other hand, old workers are replaced by young workers when they die, so a transition from \( (\ell, h, o) \) to \( (\ell_o, h, y) \) is feasible. In order to characterize feasible transitions, let \( h'(h) \) denote human capital stock attained from \( h \) according to the law of motion (7). Noting that \( F_z(\cdot) \) is the distribution function for match specific productivity draws with the density \( f_z(z) \), \( \Gamma_t \) is defined as follows:

\[
\Gamma_t(s_w, s'_w) = \begin{cases} 
(1 - \delta_m) \{(1 - \phi_{w_it}) + \sum_i \phi_{w_it} \tilde{F}_z [\tilde{z}_{it}(h'(h), o)]\} & \text{if } s_w = (\ell_u, h, o) \text{ and } s'_w = (\ell_t, h'(h), o), \\
(1 - \delta_m) \phi_{w_it} f_z(z) \tilde{T}_{it+1}(z, h'(h), o) & \text{if } s_w = (\ell_u, h, o) \text{ and } s'_w = (\ell_t(z), h'(h), o), \\
\delta_m & \text{if } s_w = (\ell, h, o) \text{ and } s'_w = (\ell_u, h, y), \\
(1 - \delta_m) (1 - \delta_{J_D}^y) \tilde{T}_{it+1}(z, h'(h), o) & \text{if } s_w = (\ell_t(z), h, o) \text{ and } s'_w = (\ell_t(z), h'(h), o), \\
(1 - \delta_m) \delta_{J_D}^y & \text{if } s_w = (\ell_t(z), h, y) \text{ and } s'_w = (\ell_t(z), h'(h), y), \\
\delta_a (1 - \delta_{J_D}^y) \tilde{T}_{it+1}^a(z, h'(h), o) & \text{if } s_w = (\ell_t(z), h, y) \text{ and } s'_w = (\ell_t(z), h'(h), o), \\
\delta_a \delta_{J_D}^y & \text{if } s_w = (\ell_t(z), h, y) \text{ and } s'_w = (\ell_t(z), y), \\
(1 - \delta_a) \{(1 - \phi_{w_it}) + \sum_i \phi_{w_it} \tilde{F}_z [\tilde{z}_{it}(h'(h), o)]\} & \text{if } s_w = (\ell_u, h, y) \text{ and } s'_w = (\ell_t, h'(h), y), \\
\delta_a \{(1 - \phi_{w_it}) + \sum_i \phi_{w_it} \tilde{F}_z [\tilde{z}_{it}(h'(h), o)]\} & \text{if } s_w = (\ell_u, h, y) \text{ and } s'_w = (\ell_t, h'(h), o), \\
(1 - \delta_a) \phi_{w_it} f_z(z) \tilde{T}_{it+1}^a(z, h'(h), y) & \text{if } s_w = (\ell_u, h, y) \text{ and } s'_w = (\ell_t(z), h'(h), y), \\
\delta_a \phi_{w_it} f_z(z) \tilde{T}_{it+1}^a(z, h'(h), y) & \text{if } s_w = (\ell_u, h, y) \text{ and } s'_w = (\ell_t(z), h'(h), o), \\
0 & \text{otherwise.}
\end{cases}
\]
D Numerical Implementation and Solution Algorithm

This section describes the numerical solution to the model, and the algorithms used to compute the steady state equilibrium and the transition path.

D.1 The State Space

I use a discrete state space for match-specific productivity $z_i$ and human capital level $h_i$. For $z$, I use 40 equally distanced grid points between $[0, 1]$. I use 200 grid points for $h_i$. For the case with positive depreciation of skills, I use equally distanced grid points between $[1, H]$ and evaluate out-of-grid points through interpolation. For the benchmark case with no depreciation, I construct the grid points in line with the increments implied by the learning function (7).

D.2 Steady State Algorithm

Step 1. Start iteration $j$ with a pair of values for entrants’ expected values of matching $(EJ_1^j, EJ_2^j)$ in the two sectors.

Step 2. Calculate $(J_u, \phi_f, \phi_w, \tilde{\mu}_1, \tilde{\mu}_2)$ by simulating a large number of cost draws for firms from the distribution $F_c(c)$, and using expressions (5), (6), (13), (14), (25) and the fact that market tightness $\theta$ is equal to $\tilde{\mu}_1 + \tilde{\mu}_2$.

Step 3. Solve for the job acceptance cutoffs $\tilde{z}_t(h, g)$, and the value functions (20) and (21) using the following subroutine:

i. Start with old workers. Assume initial set of values for unemployment $W(\ell_u, h, o)$ and matches $\Pi_i(z, h, o)$ for both sectors. Use (10) to find the job acceptance cutoffs, and update $\Pi_i(z, h, o)$ using equation (8).

ii. To update $W(\ell_u, h, o)$, use the job acceptance cutoffs in (22). Iterate until convergence.

iii. Repeat the same steps for young workers, using equation (9).

Step 4. Simulate the economy with a large number of workers drawing demographic shocks, labor market shocks (matching and separating), and match-specific productivity terms. Aggregate the cross-sections of workers to find the distribution of workers $\Psi$.

Step 5. Use the distributions to update $(EJ_1^{j+1}, EJ_2^{j+1})$ using equation (12), iterate until the distances $|EJ_1^{j+1} - EJ_1^j|$, and $|EJ_2^{j+1} - EJ_2^j|$ are sufficiently small.

D.3 Transition Algorithm

In order to solve for the transition between two steady states, I use an algorithm similar to Costantini and Melitz (2009). The basic idea is to start with an initial path of aggregate variables, to solve the decision functions backward and to simulate agents’ behavior forward according to these decision rules and random shocks. The simulation allows us to update the aggregate variables which are iterated upon until convergence. Importantly, I fix the length of the transition at $N = 400$ periods (equivalent to 100 years) such that at period $t = N + 1$, the terminal steady state is attained. To make sure that this is not too restrictive, I check that the distribution of agents over the state space is sufficiently close to the distribution in the terminal steady state. The following description provides the details.
Step 1. Start iteration \( j \) with a path of values for entrants’ expected values of matching \( \{EJ_{1t}^j, EJ_{2t}^j\}_{t=1}^{T=N} \) in the two sectors.

Step 2. Calculate \( \{J_{ut}, \phi_{ft}, \phi_{wt}, \tilde{\mu}_{1t}, \tilde{\mu}_{2t}\}_{t=1}^{T=N} \) by using the same cost draws from \( F_c(c) \) as in the steady state solution. Again, we use expressions (5), (6), (13), (14), (25) and the fact that market tightness \( \theta \) is equal to \( \tilde{\mu}_1 + \tilde{\mu}_2 \).

Step 3. Starting with \( t = N \), solve the job acceptance cutoffs backward, using the value functions specified in Section 2.2 and Appendix B. Store the value functions of firms \( \{J_{it}(z, h, g)\}_{t=1}^{T=N} \).

Step 4. Starting with \( t = 1 \), simulate the economy forward for a large number of workers using the cutoffs obtained in Step 3, random draws for separations, aging, mortality, matching and match-specific productivity. This is the most computationally intense part of the algorithm which can easily be parallelized.

Step 5. Using the simulated economy, compute \( \{\Psi_t(S_w)\}_{t=1}^{T=N} \), the distribution of workers during the transition.

Step 6. Use the distributions \( \{\Psi_t(S_w)\}_{t=1}^{T=N} \), and the stored firm values \( \{J_{it}(z, h, g)\}_{t=1}^{T=N} \) in equation (12) to update \( \{EJ_{1t}^{i+1}, EJ_{2t}^{i+1}\}_{t=1}^{T=N} \). Iterate until the maximum of the Euclidean distances \( ||\{EJ_{it}^{i+1} - EJ_{it}^i\}|| \) for \( i = \{1, 2\} \), and \( ||\Psi_t(S_w) - \Psi_{ss}(S_w)|| \) is sufficiently small. \( \Psi_{ss}(S_w) \) is the distribution in the terminal steady state.

For the counterfactual labor market policy experiments, I also iterate over the paths of unemployment insurance benefits \( \{b_t\} \) and employment subsidy payments \( \{\eta_t\} \). These policy paths are updated by dividing the total revenue obtained by 1% tax on match revenues at each period to the measure of agents eligible for income support.